Bit Allocation and Statistical Precoding for Correlated MIMO Channels With Limited Feedback

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***Abstract*—In this paper, we jointly consider statistical precoding and feedback of bit allocation (BA) for multiple-input–multiple-ouput (MIMO) systems over correlated channels. The proposed system will be termed a BA system. We assume that the statistics of the channel is known to the transmitter and a reverse link of very low rate is available so that the receiver can send back the quantized BA. Based on the statistics of the channel, we derive the statistical precoder so that bounds of the error rate averaged over the random correlated channel are minimized. Due to statistical precoding, the distribution of the BA is highly nonuniform. Treating BA as a vector signal, we quantize it using vector quantization (VQ), which is known to be particularly useful for quantizing signals with nonuniform distributions. As the distribution of BA is taken into consideration in the codebook design, a good tradeoff between performance and feedback rate can be achieved. Simulations show that the combination of statistical precoding and VQ-based quantization for BA leads to good performance with a small number of feedback bits.**

***Index Terms*—Bit allocation, correlated channel, MIMO, precoding, VQ.**

I. Introduction

MULTIPLE-input–multiple-output (MIMO) systems with limited feedback have recently received great interest. System performance in terms of transmission rate or error rate can be significantly improved with a limited amount of feedback from the receiver through a reverse channel [1]. It is generally assumed that the transmitter has no knowledge of the forward link channel and that only the receiver has knowledge of the channel state information. The feedback of the complete channel information to the transmitter will require an infinite number of bits. In practice, the reverse channel can support only a limited transmission rate, and it is desirable to have a feedback rate as low as possible.

Recently precoded spatial multiplexing systems with finite-rate feedback have been extensively investigated. The receiver chooses the optimal precoder from a codebook and sends the index back to the transmitter. Optimal codebook designs of unitary precoders using Grassmannian subspace packing for different criteria are developed in [2]. The optimal unitary precoder for minimizing bit error rate (BER) using the infinite feedback rate is given in [3], and the generalized Lloyd algorithm is used for constructing codebooks. Capacity loss due to quantized feedback is thoroughly analyzed in [4]. In multimode precoding [5], the number of substreams M can vary with the channel. In addition, a precoder codebook is designed for each possible M. If, in addition to quantized precoder, power allocation and/or bit allocation (BA) is also available to the transmitter, the performance can be further improved [6]–[8]. In these works, bit loading is not quantized, and a large feedback rate may be needed.

A particular useful class of spatial multiplexing transceivers is the VBLAST system, which employs successive interference cancellation at the receiver [9]. The conventional VBLAST system uses uniform bit and power allocation, and thus, no feedback is needed. It has been extended by incorporating power allocation or BA when there is feedback [10]–[16]. In [10], approximate minimum BER power allocation is derived, and the feedback is the power allocation information. An efficient algorithm for per-antenna power and rate control of VBLAST system is developed in [11]. Successive quantization of power and BA is proposed in [12]. In [13], the receiver feedbacks only the ordering of detection to the transmitter, and a low feedback rate is needed. Average error probability is analyzed in [14] when power and BA are taken into consideration. The optimal BA is obtained by exhausting all possible constellations subject to a sum rate constraint. A joint design of optimal BA and precoder for minimizing the p-norm of a performance-related vector is proposed in [15]. A number of optimal designs of MIMO transceivers with decision feedback and bit loading have been developed in [16]. For the case of limited feedback, the use of an identity precoder combined with feedback of only BA, which is called QR-based system therein, is suggested as it intuitively requires less feedback. In earlier works of VBLAST systems with BA and a sum rate constraint, an exhaustive listing of all possible constellation combinations is generally used [11], [14], [16], and thus, a moderate feedback rate may be needed.

When the channel statistics are available to the transmitter but not the current state of the channel, the precoder can be designed according to the statistics. For example, optimal beamforming for maximizing the average capacity of correlated channels has been designed in [17]. Precoders for minimizing error probability are derived in [18] and [19]. The optimal

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precoder that minimizes the sum of the mean-squared error is given in [20]. A unified framework for solving a number of transceiver design problems for correlated channels is presented in [21]. In these works, a uniform BA is assumed. Optimization of precoders for a fixed BA vector has been considered in [22].

In this paper, we consider statistical precoding and feedback of BA for correlated MIMO channels. The feedback of BA, as suggested by [16], requires few feedback bits, by intuition. For a given channel, a BA vector is chosen from a codebook whose codewords satisfy the target transmission rate. Assuming that the transmitter also knows the statistics of the channel, which requires only infrequent feedback, we optimize the precoder to minimize bounds of BER for both linear and decision feedback receivers.[1] With the aid of the statistical precoder, the distribution of the BA is highly skewed. We propose the use of vector quantization (VQ) to exploit the nonuniform distribution. With the proposed statistical precoding and VQ-based quantization for BA, satisfactory performance can be achieved at a very low feedback rate, as will be shown in simulation examples. The main contributions of our paper are given as follows: 1) We jointly consider statistical precoding and feedback of BA to minimize bounds of average BER. Statistical precoders have been designed in the past for uniform BA or for a given fixed BA, but not for the case of channel-dependent BA. 2) We propose a VQ approach to the design of codebooks for BA. BA is treated as a vector signal and quantized. The codebook can be tailored to the distribution of BA, which can then be quantized using very few bits. Thus, good performance can be achieved at a low feedback rate.

The sections are organized as follows: In Section II, we give the MIMO system model of the proposed BA system. In Section III, unconstrained BA is derived, and statistical precoders for both linear and decision feedback receivers are designed. The feedback of BA using a codebook is presented in Section IV. Simulation examples are given in Section V, and a conclusion is given in Section VI.

Notation: 1) Boldfaced lower case letters represent vectors, and boldfaced upper case letters are reserved for matrices. Notation $\mathbf{A}\dagger$ denotes the transpose-conjugate of $\mathbf{A}$. 2) Function $E[y]$ denotes the expected value of a random variable y. 3) Notation $\mathbf{I}_m$ is used to represent the $m \times m$ identity matrix.

II. System Model of the Bit Allocation System

Consider the wireless system with M_t transmit antennas and M_r receive antennas in Fig. 1. The channel is modeled by an $M_r \times M_t$ memoryless matrix $\mathbf{H}$ with $M_r \times 1$ channel noise $\mathbf{q}$. We assume that the channel is slow fading so that the channel does not change during each channel use. The noise vector $\mathbf{q}$ is assumed to be additive white Gaussian with zero mean and variance N_0. The channel considered in this paper is of the form

$$\mathbf{H} = \mathbf{H}_w \mathbf{R}_t^{1/2} \tag{1}$$

[1]The receivers are of the zero-forcing type for convenience of analysis as well as for the reason that zero forcing and minimum mean squared error receivers have similar performance when there is bit allocation [23].

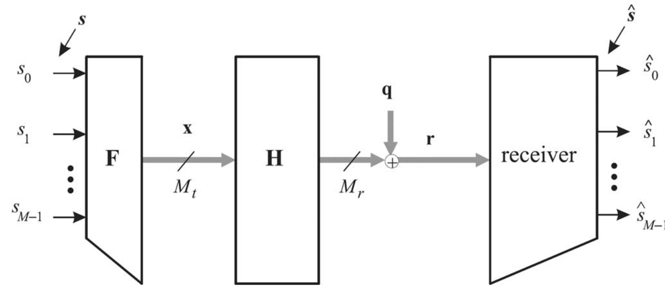


Fig. 1. MIMO system with M_t transmit antennas and M_r receive antennas.

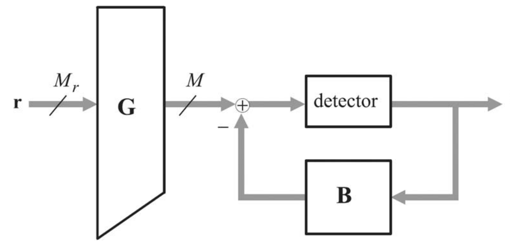


Fig. 2. Block diagram of the decision feedback receiver based on QR decomposition.

where $\mathbf{H}_w$ is an $M_r \times M_t$ matrix whose elements are independent Gaussian random variables with unit variance. The matrix $\mathbf{R}_t$, of dimensions $M_t \times M_t$, is called the transmit correlation matrix. In this case, the rows of $\mathbf{H}$ are independent, and each has an autocorrelation matrix equal to $\mathbf{R}_t$. This model is useful for downlink transmission [24] when the receiving antennas are well separated. Suppose that the transmitter and receiver can process M substreams of symbols, where $M \leq \min(M_t, M_r)$. Spatial multiplexing precoder $\mathbf{F}$ is an $M_t \times M$ matrix. Input vector $\mathbf{s}$ is an $M \times 1$ vector consisting of modulation symbols s_k, for $k = 0, 1, \ldots, M-1$. The symbols s_k are assumed to be uncorrelated with zero mean and unit variance. The total transmission power $E[\mathbf{x}^\dagger \mathbf{x}]$ can also be written as $trace(E[\mathbf{x}\mathbf{x}^\dagger]) = trace(E[\mathbf{F}\mathbf{s}\mathbf{s}^\dagger \mathbf{F}^\dagger]) = trace(\mathbf{F}\mathbf{F}^\dagger)$. Assuming that the total transmission power is P_t, then $trace(\mathbf{F}\mathbf{F}^\dagger) = P_t$. The actual number of symbols transmitted can be smaller than M if one or more subchannels are loaded with zero bits.

We will consider two types of zero-forcing receivers, i.e., linear and decision feedback receivers. Define the error vector at the output of the receiver as $\mathbf{e} = \widehat{\mathbf{s}} - \mathbf{s}$. When the receiver is linear and zero forcing (see Fig. 1), the receiver output $\widehat{\mathbf{s}} = \mathbf{G}\mathbf{r}$, where $M \times M_r$ receive matrix $\mathbf{G}$ is $(\mathbf{F}^\dagger \mathbf{H}^\dagger \mathbf{H}\mathbf{F})^{-1} \mathbf{F}^\dagger \mathbf{H}^\dagger$ [25]. In this case, $\mathbf{e}$ has autocorrelation matrix $\mathbf{R}_e = E[\mathbf{e}\mathbf{e}^\dagger]$ given by [25]

$$\mathbf{R}_e = N_0 (\mathbf{F}^\dagger \mathbf{H}^\dagger \mathbf{H}\mathbf{F})^{-1}. \tag{2}$$

To consider a decision feedback receiver, we can use the receiver structure in Fig. 2 based on the QR decomposition of $\mathbf{HF}$ [16], [20]. This corresponds to the case of a reverse detection ordering. Let the QR decomposition of $\mathbf{HF}$ be $\mathbf{QR}$, where $\mathbf{Q}$ is an $M_r \times M$ matrix with orthonormal columns and $\mathbf{R}$ is an $M \times M$ upper triangular matrix, with $[\mathbf{R}]_{ii} = r_{ii}$.

The feedforward matrix $\mathbf{G}$ and feedback matrix $\mathbf{B}$ are given by [16]

$$\mathbf{G} = \text{diag}\left(r_{00}^{-1} \quad r_{11}^{-1} \quad \cdots \quad r_{M-1,M-1}^{-1}\right)\mathbf{Q}^\dagger$$
$$\mathbf{B} = \text{diag}\left(r_{00}^{-1} \quad r_{11}^{-1} \quad \cdots \quad r_{M-1,M-1}^{-1}\right)\mathbf{R} - \mathbf{I}_M$$

respectively. Assuming that there is no error propagation, the kth subchannel error $e_k = \widehat{s}_k - s_k$ has variance $\sigma_{e_k}^2 = N_0 r_{kk}^{-2}$, for $k = 0, 1, \ldots, M-1$. The error variance averaged over the random channel is $\overline{\sigma}_{e_k}^2 = E[\sigma_{e_k}^2] = N_0 E[r_{kk}^{-2}]$. The value $E[r_{kk}^{-2}]$ has been shown to be related to the Cholesky decomposition of $\mathbf{F}^\dagger\mathbf{R}_t\mathbf{F}$ in [20] when $M_r > M$. Let the Cholesky decomposition of $\mathbf{F}^\dagger\mathbf{R}_t\mathbf{F}$ be $\mathbf{LDL}^\dagger$, where $\mathbf{L}$ is a lower triangular matrix with unity diagonal elements and $\mathbf{D}$ is diagonal. Then

$$E\left[r_{kk}^{-2}\right] = d_{kk}^{-1}/(M_r - k - 1), \quad k = 0, 1, \ldots, M-1 \tag{3}$$

where d_{kk} is the kth diagonal element of $\mathbf{D}$.

Assuming that inputs s_k are b_k-bit QAM symbols, the number of bits transmitted per channel use R_b is thus $\sum_{k=0}^{M-1} b_k$. The kth symbol error rate is well approximated by [26]

$$SER_k = 4(1 - 2^{-b_k/2})Q\left(\sqrt{\frac{3}{(2^{b_k} - 1)\sigma_{e_k}^2}}\right) \tag{4}$$

where $Q(y) = 1/\sqrt{2\pi}\int_y^\infty e^{-t^2/2}dt$, $y \geq 0$. Note that, for the decision feedback receiver, $1/\sigma_{e_k}^2$ is the postdetection SNR and (4) is the error rate, assuming that there is no error in detecting previous symbols. When Gray code is used, the BER can be approximated by $BER_k \approx SER_k/b_k$. Using this approximation, the BER can be computed using

$$BER \approx \frac{1}{R_b}\sum_{k=0}^{M-1} b_k BER_k \approx \frac{1}{R_b}\sum_{k=0}^{M-1} SER_k. \tag{5}$$

III. Design of Unconstrained Bit Allocation and Statistical Precoder

In this section, we will consider the BA system when there is no integer constraint on BA. For a given precoder, we will derive a BA formula that minimizes the BER under a high-bit-rate assumption. The resulting BER will then be used to design statistical precoders.

A. Design of Unconstrained BA

Assume that the transmission rate is high and the number of bits loaded on the kth subchannel, b_k, is large enough, so that $1 - 2^{-b_k/2} \approx 1$ and $1 - 2^{-b_k} \approx 1$; then, the symbol error rate expression in (4) can be approximated by

$$SER_k \approx 4Q\left(\sqrt{3 \cdot 2^{-b_k}\sigma_{e_k}^{-2}}\right). \tag{6}$$

For convenience of derivation, we define the function

$$f(y) = Q(1/\sqrt{y}), y > 0. \tag{7}$$

The function $f(y)$ is monotone increasing, and it can be verified that it is convex for $y \leq 1/3$ and concave for $y > 1/3$. Using $f(\cdot)$, we have $SER_k \approx 4f(2^{b_k}\sigma_{e_k}^2/3)$. Therefore, the BER in (5) can be written as

$$BER \approx \frac{4}{R_b}\sum_{k=0}^{M-1} f\left(2^{b_k}\sigma_{e_k}^2/3\right). \tag{8}$$

Let us consider the high-SNR case in which the convexity of $f(\cdot)$ holds and the low SNR case in which the concavity of $f(\cdot)$ holds.

Assume that the SNR is large enough, so that the convexity of $f(\cdot)$ holds; then, we have

$$\begin{aligned} BER &\gtrsim \frac{4}{(R_b/M)} f\left(\frac{1}{3M}\sum_{k=0}^{M-1} 2^{b_k}\sigma_{e_k}^2\right) \\ &\geq \frac{4}{(R_b/M)} f\left(\frac{2^{R_b/M}}{3}\prod_{k=0}^{M-1}\sigma_{e_k}^{2/M}\right) \\ &\triangleq BER_0. \end{aligned} \tag{9}$$

The second inequality is obtained by using $\sum_{k=0}^{M-1} b_k = R_b$ and the arithmetic-mean–geometric-mean (AM-GM) inequality and also using the monotone increasing property of $f(\cdot)$. Notice that the lower bound BER_0 in (9) is independent of the BA. The best BA is such that the two inequalities in (9) become equalities. Due to the convexity of $f(\cdot)$, the first inequality in (9) holds if and only if $2^{b_k}\sigma_{e_k}^2$ are of the same value for all k's. The same set of conditions is also necessary and sufficient for equality to hold in the second inequality as $f(\cdot)$ is monotone increasing. The BA that achieves BER_0 is thus

$$b_k = \frac{1}{M}\sum_{\ell=0}^{M-1}\log_2\left(\sigma_{e_\ell}^2\right) - \log_2\left(\sigma_{e_k}^2\right) + R_b/M. \tag{10}$$

We can see that the symbols with smaller error variances are allocated with more bits. When bits are allocated as in (10), $2^{b_k}\sigma_{e_k}^2$ are equalized, and so are the subchannel symbol error rates. The aforementioned BA formula, which was derived using the criterion of minimum BER, has the same form as that designed for minimum transmission power in [16].

On the other hand, when the SNR is low enough so that the concavity of $f(\cdot)$ holds, we have

$$BER \lesssim \frac{4}{(R_b/M)} f\left(\frac{1}{3M}\sum_{k=0}^{M-1} 2^{b_k}\sigma_{e_k}^2\right). \tag{11}$$

The inequality follows from the concavity of $f(\cdot)$. Similar to the high-SNR case, the quantity on the right-hand side is minimized if the BA is chosen according to (10). In this case, the upper bound on the right-hand side is equal to BER_0, and at the same time, the inequality in (11) becomes an equality, i.e., $BER \approx BER_0$. Therefore, for both high- and low-SNR regions, the BER with BA in (10) is approximately BER_0. The results can be used for both linear and decision feedback receivers.

B. Design of Statistical Precoders

To consider the average BER performance, we average BER_0 computed in (9) over the random channel $\mathbf{H}$

$$\overline{BER}_0 = E[BER_0] = E\left[\frac{4}{(R_b/M)} f\left(c\prod_{k=0}^{M-1}\sigma_{e_k}^{2/M}\right)\right]$$

where we have used $c = (1/3)2^{R_b/M}$. To further simplify the expression, we define the geometric mean function

$$h(\mathbf{y}) = \left(\prod_{i=0}^{M-1} y_i\right)^{1/M} \tag{12}$$

where $\mathbf{y} = [y_0 \quad y_1 \quad \cdots \quad y_{M-1}]^T$, and $y_i > 0$. Let $y_i = c\sigma_{e_i}^2$; then, $BER_0 = (4/(R_b/M))f(h(\mathbf{y}))$. To analyze $\overline{BER}_0$, we first observe the Hessian matrix of $f(h(\mathbf{y}))$, which is an $M \times M$ matrix with the (i, j)th entry given by $[\mathbf{H}_{\text{ess}}]_{ij} = \partial^2 f(h(\mathbf{y}))/\partial y_i \partial y_j$, for $0 \le i, j < M$. We can verify that

$$[\mathbf{H}_{\text{ess}}]_{ij} = \begin{cases} 0.5/M^2 f'(h(\mathbf{y}))\, y_i^{-1} y_j^{-1}(1-h(\mathbf{y}))\,, & i \neq j \\ 0.5/M^2 f'(h(\mathbf{y}))\, y_i^{-2}(1-(1+2M)h(\mathbf{y}))\,, & i = j \end{cases} \tag{13}$$

where $f'(x)$ denotes the first derivative of $f(x)$. It is known that a function is convex (concave) if and only if the Hessian matrix is positive (negative) semi-definite [31]. In the following, we discuss the behavior of $\overline{BER}_0$ for the high- and low-SNR cases.

High-SNR Case: Consider $P_t/N_0 \gg 1$ such that $h(\mathbf{y}) \ll 1$. We can approximate the diagonal elements of the Hessian matrix in (13) as $0.5/M^2 f'(h(\mathbf{y}))y_i^{-2}$. Defining the $M \times 1$ vector $\mathbf{u}$ with ith element $u_i = 1/y_i$, we have $\mathbf{H}_{\text{ess}} \approx 0.5/M^2 f'(h(\mathbf{y}))\mathbf{u}\mathbf{u}^T$, which is a positive semidefinite matrix. Applying Jensen's inequality $E[f(h(\mathbf{y}))] \gtrsim f(h(E[\mathbf{y}]))$, we have

$$E[BER_0] \gtrsim \frac{4}{(R_b/M)} f\left(c\prod_{k=0}^{M-1}\overline{\sigma}_{e_k}^{2/M}\right) \triangleq \underline{BER}_{bd} \tag{14}$$

where $\overline{\sigma}_{e_k}^2 = E[\sigma_{e_k}^2]$ is the kth error variance averaged over channel $\mathbf{H}$. The right-hand side $\underline{BER}_{bd}$ is a lower bound of $\overline{BER}_0$.

Low-SNR Case: A property of $f(h(\mathbf{y}))$ that is useful for studying $E[BER_0]$ in the low-SNR region is presented in the following lemma:

Lemma 1: Let $f(x)$ and $h(\mathbf{y})$ be as defined in (7) *and* (12)*, respectively. Then, the composite function $f(h(\mathbf{y}))$ for $y_i > 0$ is concave when $h(\mathbf{y}) \ge 1/3$.*

A proof is given in Appendix A. The aforementioned lemma means that BER_0 is concave in $\sigma_{e_k}^2$ when $h(\mathbf{y}) = (1/3)2^{R_b/M}\prod_{k=0}^{M-1}\sigma_{e_k}^{2/M} \ge 1/3$, which holds in the low-SNR case, i.e., small P_t/N_0. When $f(h(\mathbf{y}))$ is concave, we can apply Jensen's inequality $E[f(h(\mathbf{y}))] \le f(h(E[\mathbf{y}]))$ to obtain

$$\overline{BER}_0 \le \frac{4}{(R_b/M)} f\left(c\prod_{k=0}^{M-1}\overline{\sigma}_{e_k}^{2/M}\right) = \overline{BER}_{bd}. \tag{15}$$

Now, $\overline{BER}_{bd}$ becomes an upper bound of $\overline{BER}_0$. In the low-SNR region, we would like to have the upper bound $\overline{BER}_{bd}$ minimized. In the high-SNR region, we would also like to have the lower bound $\underline{BER}_{bd}$ minimized because, if the lower bound is large, then $\overline{BER}_0$ will be large as well. We have found it difficult to directly minimize $\overline{BER}_0$, whereas minimizing the bound leads to one tractable solution, as we will see next. This is separately discussed for linear receivers and decision feedback receivers.

Linear Receiver: The bound $\overline{BER}_{bd}$ in (14) can be minimized if the geometric mean $\prod_{k=0}^{M-1}\overline{\sigma}_{e_k}^{2/M}$ is minimized. We can obtain $\overline{\sigma}_{e_k}^2$ by averaging the error correlation matrix $\mathbf{R}_e = N_0(\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F})^{-1}$ over the channel. Let the ith column of $\mathbf{H}^\dagger$ be $\mathbf{g}_i$; then, the autocorrelation matrix of $\mathbf{g}_i$ is equal to $\mathbf{R}_t$. It is known that $\mathbf{H}^\dagger\mathbf{H} = \sum_{i=0}^{M_r-1}\mathbf{g}_i\mathbf{g}_i^\dagger$ has a complex Wishart distribution with M_r degrees of freedom, which are denoted as $\mathcal{W}_{M_t}(\mathbf{R}_t, M_r)$ [29]. Furthermore, $\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F}$ is $\mathcal{W}_M(\mathbf{F}^\dagger\mathbf{R}_t\mathbf{F}, M_r)$, and so, $\mathbf{R}_e^{-1} = (1/N_0)\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F}$ has a Wishart distribution $\mathcal{W}_M(N_0^{-1}\mathbf{F}^\dagger\mathbf{R}_t\mathbf{F}, M_r)$. Then, $\mathbf{R}_e$ has an inverse Wishart distribution. It has been shown in [30] that, when a matrix $\mathbf{B}$ is of Wishart distribution $\mathcal{W}_p(\mathbf{A}, r)$ with $r > p$, then $E[\mathbf{B}^{-1}] = 1/(r-p)\mathbf{A}^{-1}$. Using this result, $\overline{\mathbf{R}}_e = E[\mathbf{R}_e]$ is given by

$$\overline{\mathbf{R}}_e = \frac{N_0}{M_r - M}(\mathbf{F}^\dagger\mathbf{R}_t\mathbf{F})^{-1} \tag{16}$$

assuming that $M_r > M$. Let the eigendecomposition of $\mathbf{R}_t$ be $\mathbf{U}_t\mathbf{\Lambda}_t\mathbf{U}_t^\dagger$, where $\mathbf{\Lambda}_t$ is a diagonal matrix and the diagonal elements $\lambda_{t,i}$ are the eigenvalues of $\mathbf{R}_t$. Let $\lambda_{t,i}$ be ordered such that $\lambda_{t,0} \ge \lambda_{t,1} \ge \cdots \lambda_{t,M_t-1}$, and assume that $\lambda_{t,M-1} > 0$.

Theorem 1: For the linear receiver with $M_r > M$, the BER bound $\overline{BER}_{bd}$ in (14) *satisfies $\overline{BER}_{bd} \ge \overline{BER}_{bd,lin}$, where*

$$\overline{BER}_{bd,lin} = \frac{4M}{R_b}Q\left(\sqrt{\frac{3P_t/M}{2^{R_b/M}N_0}(M_r - M)\prod_{k=0}^{M-1}\lambda_{t,k}^{1/M}}\right). \tag{17}$$

The inequality becomes an equality when $\mathbf{F} = \sqrt{P_t/M}\mathbf{U}_{t,M}$, where $\mathbf{U}_{t,M}$ is the submatrix of $\mathbf{U}_t$ that consists of the first M columns of $\mathbf{U}_t$.

A proof is given in Appendix B. We can conclude that, to minimize the BER bound $\overline{BER}_{bd}$, the optimal precoder is $\mathbf{F} = \sqrt{P_t/M}\mathbf{U}_{t,M}$. Such a precoder has also been shown to be optimal for beamforming in [17].

Decision Feedback Receiver:

Theorem 2: For the decision feedback receiver with $M_r > M$, the BER bound $\overline{BER}_{bd}$ in (14) *satisfies $\overline{BER}_{bd} \ge \overline{BER}_{bd,df}$, where*

$$\overline{BER}_{bd,df} = \frac{4M}{R_b}Q\left(\sqrt{\frac{3P_t/M}{2^{R_b/M}N_0}\prod_{k=0}^{M-1}(M_r-k-1)^{1/M}\lambda_{t,k}^{1/M}}\right). \tag{18}$$

The inequality becomes an equality when $\mathbf{F} = \sqrt{P_t/M}\mathbf{U}_{t,M}$.

Proof: Using (3), we can readily obtain $\overline{\sigma}_{e_k}^2 = N_0 d_{kk}^{-1}/(M_r - k - 1)$. It follows that $\prod_{k=0}^{M-1}\overline{\sigma}_{e_k}^2 = \prod_{k=0}^{M-1} N_0 d_{kk}^{-1}/$

$(M_r - k - 1)$. Note that $\prod_{k=0}^{M-1} d_{kk} = \det(\mathbf{F}^\dagger \mathbf{R}_t \mathbf{F})$. From the proof of Theorem 1, we know that $\det(\mathbf{F}^\dagger \mathbf{R}_t \mathbf{F}) \le (P_t/M)^M \prod_{k=0}^{M-1} \lambda_{t,k}$. Equality holds when $\mathbf{F} = \sqrt{P_t/M}\mathbf{U}_{t,M}$. Using this inequality and the monotone increasing property of $f(\cdot)$, we arrive at (18).

As in the case of linear receiver, to minimize the BER bound $\overline{BER}_{bd}$, the optimal precoder is $\mathbf{F} = \sqrt{P_t/M}\mathbf{U}_{t,M}$. Comparing $\overline{BER}_{bd,df}$ to $\overline{BER}_{bd,lin}$ in (17), we see that $\overline{BER}_{bd,bd} \le \overline{BER}_{bd,lin}$, and the two are equal only when $M = 1$. In the special case where the channel is uncorrelated, $\mathbf{R}_t = \mathbf{I}_{M_t}$, and we have $\mathbf{F} = \sqrt{P_t/M}[\mathbf{I}_M \quad \mathbf{0}]^T$. In this case, the BA system with a decision feedback receiver becomes the QR-based system with BA proposed in [16]. However, the performances of these two systems will be different when the feedback rate is limited because the BA codebooks are different, as we will see in the simulations. Note that the results in Theorems 1 and 2 hold for $M_r > M$, as the average error variances in (3) and (16) are valid for $M_r > M$. For $M_r = M$, closed-form expressions of the average error variances are not known, and the optimal precoder cannot be obtained this way. For the implementation of the BA system for $M_r = M$, we will use the precoder derived for $M_r > M$. Implementation issues will be discussed in the next section.

Connection With Optimal Precoder Systems: It turns out that BER_0 for a linear receiver is related to the BER of the optimal precoder system in a very nice manner. In particular, when the receiver is linear and zero forcing, and $M = M_t$ ($M_t \le M_r$), we will show that BER_0 in (9) is always smaller than the BER of the BER-minimizing system [3] that has an optimized precoder $\mathbf{F}$ but has no BA. Suppose that R_b bits are transmitted using total power P_t for each channel use, and each s_k is an R_b/M-bit QAM symbol with unit variance. The optimal unitary precoder is such that the subchannel variances are equalized [3], i.e., $\sigma_{e_k}^2 = \mathcal{E}_{rr}$, for $k = 0, 1, \ldots, M-1$, where $\mathcal{E}_{rr} = (N_0/P_t) trace(\mathbf{H}^\dagger \mathbf{H})^{-1}$. Note that $\mathcal{E}_{rr}$ is also the average subchannel error variance for the BA system when the precoder is $\mathbf{F} = \sqrt{P_t/M}\mathbf{V}$, where $\mathbf{V}$ is an arbitrary $M \times M$ unitary matrix. That is, $\mathcal{E}_{rr} = (1/M)\sum_{\ell=0}^{M-1} \sigma_{e_{\ell,BA}}^2$, where $\sigma_{e_{\ell,BA}}^2$ denotes the ℓth subchannel error variance of the BA system. Using the approximation in (6), the minimized BER of the precoder system can be expressed as

$$\begin{aligned} BER &\approx \frac{4}{(R_b/M)} Q\left(\sqrt{3 \cdot 2^{-R_b/M} \mathcal{E}_{rr}^{-1}}\right) \\ &\ge \frac{4}{(R_b/M)} Q\left(\sqrt{3 \cdot 2^{-R_b/M} \prod_{\ell=0}^{M-1} \sigma_{e_{\ell,BA}}^{-2/M}}\right) = BER_0. \end{aligned}$$

The inequality is obtained by applying the AM-GM inequality $(1/M)\sum_{\ell=0}^{M-1} \sigma_{e_{\ell,BA}}^2 \ge \prod_{\ell=0}^{M-1} \sigma_{e_{\ell,BA}}^{2/M}$. This implies that the BA system with unconstrained BA and an arbitrary fixed unitary precoder has a smaller BER than that of the optimal precoder system that uses the optimal precoder but no BA. Therefore, when we have the choice of feeding back either the precoder or BA, the feedback of the BA leads to a smaller BER.

IV. Feedback of Bit Allocation

In the proposed BA system, BA is adapted according to the varying random channel. In this section, we consider the design of codebooks and codeword selection. We will also propose the augmentation the precoding matrix when $M < M_t$. We will show that the use of augmented precoding allows the BA system to achieve full diversity order $M_r M_t$.

Based on the results in the previous section, we uniformly distribute the transmission power among the subchannels loaded with nonzero bits. The precoder is thus chosen as $\mathbf{F} = \mathbf{U}_{t,M}\mathbf{\Lambda}_f$, where $\mathbf{\Lambda}_f$ is a diagonal matrix with $[\mathbf{\Lambda}_f]_{kk} = \sqrt{P_t/M_0}$ if $b_k > 0$, where M_0 is the number of subchannels loaded with nonzero bits. For subchannels not loaded with bits, $[\mathbf{\Lambda}_f]_{kk} = 0$. Such a precoder depends only on the channel statistics and BA. The channel statistics need not be frequently fed back to the transmitter. When we consider BA in practical applications, the bits assigned to the symbols are typically integer-valued. The components of the BA vector $\mathbf{b}$ satisfy the sum rate constraint $b_0 + b_1 + \cdots + b_{M-1} = R_b$, where $b_i \in \mathcal{Z}^+$ and $\mathcal{Z}^+$ denotes the set of nonnegative integers. The number of such nonnegative integer BA vectors is [28] $C(R_b + M - 1, R_b)$, where $C(\cdot, \cdot)$ denotes the choose function. This requires a large feedback rate when R_b is large. In the following, we will treat the BA as a vector and use a VQ approach to design codebooks and select codewords from the codebook:

Codeword Selection: Suppose that we are given B feedback bits and a codebook $\mathcal{C}_b$ of 2^B BA vectors. The vectors in $\mathcal{C}_b$ satisfy the sum rate constraint so that the number of bits transmitted for each channel use is R_b. The BER in (5) is a function of the BA vector. For a given channel $\mathbf{H}$, we can choose the best BA vector $\widehat{\mathbf{b}} \in \mathcal{C}_b$ that minimizes the BER, $\widehat{\mathbf{b}} = \arg\min_{\mathbf{b}\in\mathcal{C}_b} BER(\mathbf{b}, \mathbf{H})$, where $BER(\mathbf{b}, \mathbf{H})$ denotes the BER when the channel is $\mathbf{H}$ and the BA vector is $\mathbf{b}$. To make codeword selection more efficient, we can choose (suboptimal) codewords based on the BA given in (10). The criterion of minimizing the largest subchannel error rate will be considered. Suppose that the unconstrained BA vector computed from (10) is $\mathbf{b}^*$. Given a BA vector $\mathbf{b} \in \mathcal{C}_b$, the kth subchannel symbol error rate associated with $\mathbf{b}$ is

$$SER_k \approx 4Q\left(\sqrt{3\sigma_{e_k}^{-2} 2^{-b_k}}\right) = 4Q\left(\sqrt{3\sigma_{e_k}^{-2} 2^{-b_k^*} 2^{(b_k^* - b_k)}.}\right).$$

As shown in Section III, the BA $\mathbf{b}^*$ equalizes the quantity $3\sigma_{e_k}^{-2} 2^{-b_k^*}$. Let us call this subchannel independent quantity A. Then, we have $SER_k \approx 4Q(\sqrt{A 2^{(b_k^* - b_k)}})$. Therefore, the largest subchannel error rate can be minimized by choosing the BA vector $\mathbf{b} \in \mathcal{C}_b$ that has the largest $\min_k (b_k^* - b_k)$. The BA in (10) is derived under the assumption that all M subchannels are loaded with nonzero bits. To remove the assumption, we can compute BER_0 in (9) for each M_0, with $0 < M_0 \le M$, where M_0 is the number of subchannels used, and choose the M_0 that has the smallest BER_0. We can then apply quantization on the corresponding unconstrained BA using the aforementioned maximin criterion. Such a suboptimal selection criterion does not require the computation of BER for each BA in the codebook. Simulations in Section V will demonstrate that the use of the suboptimal maximin criterion leads to only minor degradation, compared with the optimal BER criterion.

Design of $\mathcal{C}_b$: The BA codebook $\mathcal{C}_b$ can be designed using the generalized Lloyd algorithm for VQ codebooks [33]. Suppose that we are given J training channels $\mathbf{H}_0, \mathbf{H}_1, \ldots, \mathbf{H}_{J-1}$. First, we initialize the codebook $\mathcal{C}_b = \{\mathbf{b}_i, i = 0, 1, \ldots, 2^B - 1\}$. The generalized Lloyd algorithm repeats the following two steps: 1) Partition the training set into clusters R_i using

$$R_i = \{k : BER(\mathbf{b}_i, \mathbf{H}_k) \leq BER(\mathbf{b}_j, \mathbf{H}_k), \text{for all } j \neq i\}.$$

2) Find the BA vector $\mathbf{a}_k$ for $\mathbf{H}_k$ using (10), and compute $\mathbf{b}_i = 1/|R_i| \sum_{k \in R_i} \mathbf{a}_k$, where $|R_i|$ is the number of elements in R_i. The iterations can be ended when there is little improvement in the average BER. The resulting BA vectors $\mathbf{b}_i$ can be quantized to have integer entries subject to the sum rate constraint using the method in [34]. Note that the derivation of BER bounds in the previous section needs the assumption $M_r > M$. When we use the BA system in practice, M need not be smaller than M_r; it can be equal to M_r.

Augmented Precoding: In the aforementioned discussion, we have used a fixed $M_t \times M$ matrix $\mathbf{F}$ as the precoder. When $M < M_t$ and the channel matrix is such that the column space of $\mathbf{F}$ is contained in the null space of $\mathbf{H}$, there is zero signal power at the receiver. This can be avoided by starting off with an $M_t \times M_t$ augmented precoder $\mathbf{F}' = \mathbf{U}_t \mathbf{\Lambda}'_f$, where $\mathbf{\Lambda}'_f$ is an $M_t \times M_t$ diagonal matrix with at most M nonzero diagonal entries and the nonzero entries of $\mathbf{\Lambda}'_f$ are of the same value. Equivalently, we are choosing M columns out of $\mathbf{F}'$ to form the actual $M_t \times M$ precoder $\mathbf{F}$, i.e., $(M_t - M)$ columns of $\mathbf{F}'$ are removed. The corresponding augmented input vector $\mathbf{s}'$ and BA vector $\mathbf{b}'$ are of size $M_t \times 1$. The entries of $\mathbf{s}'$ and $\mathbf{b}'$ corresponding to the removed columns of $\mathbf{F}'$ are all equal to zero so that the transmitter output $\mathbf{F}'\mathbf{s}'$ is equal to $\mathbf{F}\mathbf{s}$. As we choose M columns from $\mathbf{F}'$, there are $C(M_t, M)$ possible choices for precoders. The augmented $\mathbf{b}'$ satisfies $\sum_{k=0}^{M_t-1} b'_k = R_b$, $b'_i \in \mathcal{Z}^+$, with the additional constraint that $\mathbf{b}'$ has at most M nonzero components as it is assumed that the transmitter and receiver can process at most M substreams. It can be verified that the total number of possible integer BA vectors satisfying the sum rate constraint is $\sum_{k=M_t-M}^{M_t-1} C(M_t, k) C(R_b - 1, M_t - 1 - k)$. As in the nonaugmented case, we can design a smaller codebook $\mathcal{C}'_b$ to have a smaller feedback rate. There is no need to feedback the information of the actual precoder $\mathbf{F}$ used. The information is embedded in the augmented $\mathbf{b}'$. For $i = 0, 1, \ldots, M_t - 1$, the transmitter removes the ith column from $\mathbf{F}'$ if $b'_i = 0$. The transmitter can then use the resulting $M_t \times M_0$ submatrix as the precoder, where M_0 is the number of nonzero entries in $\mathbf{b}'$. The simulations given in Section V will demonstrate that, when $M < M_t$, the system with augmented precoding outperforms that with a fixed precoder for the same number of feedback bits. Furthermore, the use of augmented precoding leads to full diversity order, as shown next.

Diversity Gain: In the following, we show that, with augmented precoding, the BA system can achieve diversity order $M_r M_t$ for a system with M_r receive antennas and M_t transmit antennas if the codebook is properly chosen and has at least M_t codewords. Assume that the BA codebook $\mathcal{C}'_b$ contains the subset of codewords $\mathcal{A} = \{R_b \mathbf{e}_0, R_b \mathbf{e}_1, \ldots, R_b \mathbf{e}_{M_t-1}\}$, where $\mathbf{e}_i$ are standard vectors of size $M_t \times 1$, i.e., $[\mathbf{e}_i]_i = 1$ and $[\mathbf{e}_i]_j = 0$ for $j \neq i$. As $\mathcal{A}$ is a subset of $\mathcal{C}'_b$, we have $\min_{\mathbf{b}' \in \mathcal{C}'_b} BER(\mathbf{b}', \mathbf{H}) \leq \min_{\mathbf{b}' \in \mathcal{A}} BER(\mathbf{b}', \mathbf{H})$. The BER averaged over the channel is thus bounded above by $E[\min_{\mathbf{b}' \in \mathcal{A}} BER(\mathbf{b}', \mathbf{H})]$. When the BA is chosen from $\mathcal{A}$, all the R_b bits are allocated to the same symbol, and this becomes a beamforming system, in which the best beamforming vector is chosen among the columns of $\mathbf{F}'$ to maximize the received SNR. In other words, the equivalent codebook of beamforming vectors is $\mathcal{C}_f = \{\mathbf{f}'_0, \mathbf{f}'_1, \ldots, \mathbf{f}'_{M_t-1}\}$, where $\mathbf{f}'_i$ is the ith column of $\mathbf{F}'$. From [27], we know that such a beamforming system has diversity order equal to $M_r M_t$ if $\mathbf{F}'$ has a rank equal to M_t. Therefore, the BA system has diversity order $M_r M_t$ as well when the codebook contains subset $\mathcal{A}$.

Note on Low SNR Case: In some applications, it is desirable to maintain the quality of service, even in the low-SNR case. One possible way to do this in the BA system is to design the codebook to contain two subcodebooks, one with a higher transmission rate than the other. When the BER goes above a certain threshold using the high-rate subcodebook, we can switch to the low-rate subcodebook. The size of the low-rate subcodebook can be smaller as the transmission rate is lower. In this case, quality of service can be incorporated at the expense of a somewhat higher feedback rate.

V. Simulation Examples

In the following examples, the channel is of the form $\mathbf{H} = \mathbf{H}_w \mathbf{R}_t^{1/2}$ as in (1). The exponential correlation model [35] is used to generate the random channel. The elements of Hermitian matrix $\mathbf{R}_t$ are given by $[\mathbf{R}_t]_{mn} = \gamma^{n-m}$, for $n \geq m$, where γ is the correlation coefficient between neighboring antennas. When $\gamma = 0$, the channel becomes uncorrelated, and the entries of the channel matrix are independent and identically distributed (i.i.d) Gaussian random variables. The error rates are computed using (5) for both linear and decision feedback receivers. We have used 10^5 channel realizations in the training of the BA codebook and 10^6 channel realizations in the Monte Carlo simulations of BER performance.

Example 1—Distribution of BA Vectors: In this example, we compute the empirical distribution of BA vectors for $M_r = 5$ and $M_t = 4$. The number of bits transmitted per channel use is $R_b = 12$, and the number of substreams that the transmitter and receiver can process is $M = 4$. The channel is i.i.d, and the corresponding optimal precoder is $\mathbf{\Lambda}_f$. The receiver is linear. The number of possible integer BA vectors is 455. We include in the codebook all 455 vectors. For a given channel realization, the best BA in the codebook is chosen using the BER criterion. Fig. 3 shows the cumulative distribution function of the BA vectors, where the indexes of the vectors are ordered so that the probabilities are in decreasing order. We can see that some BA vectors are far more probable than others. The probability of the 52 most probable BA vectors is more than 99%. The distribution of the BA vectors is highly skewed rather than uniform. A properly designed codebook will allow more efficient use of the available feedback bits, which will be demonstrated later.

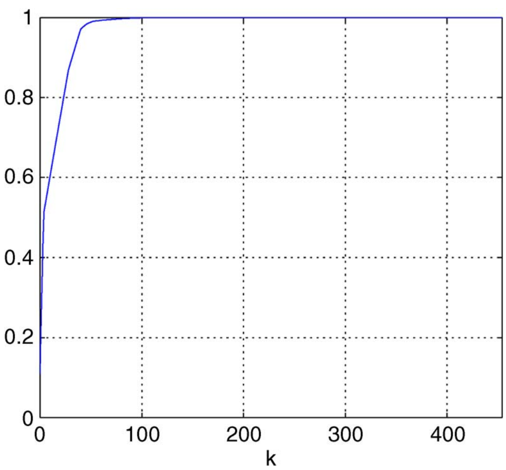
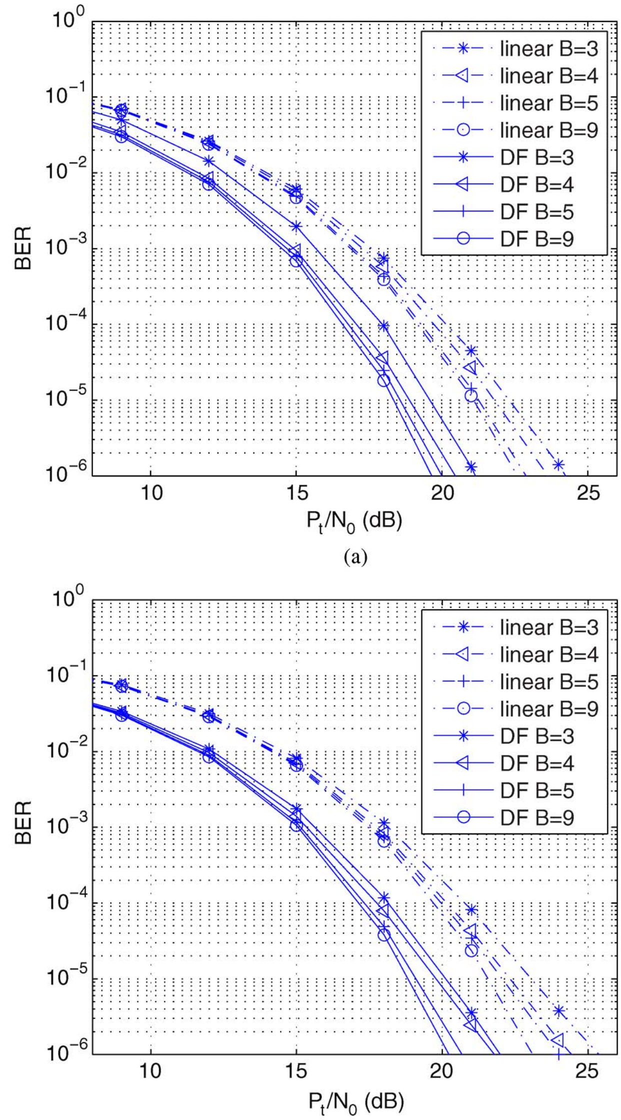


Fig. 3. Cumulative distribution function of the BA vectors, where the indexes of the vectors are ordered, so that the probabilities are in nonincreasing order for $M_r = 5$, $M_t = 4$, $M = 4$, and $R_b = 12$.

Example 2—BER of the BA System: In this example, $M_r = 5$, $M_t = 4$, $M = 4$, and $R_b = 12$, as in the previous example. In this case, the number of all possible integer BAs is 455, which requires 9 bits of feedback. We can design the BA codebook with fewer bits using the generalized Lloyd algorithm described in Section IV. Fig. 4(a) shows the BER of the BA system over an uncorrelated channel for different numbers of feedback bits. The codewords are selected to minimize the BER. The performance is shown for both linear and decision feedback receivers for different numbers of feedback bits. We can see that, with $B = 4$, the performance comes close to that of $B = 9$, in which case all the integer BA codewords are used. The gain of the decision feedback receiver over the linear receiver is around 3 dB. Fig. 4(b) shows the BER of the BA system when the channel is correlated with correlation coefficient $\gamma = 0.5$. The gain of the decision feedback receiver over the linear receiver is similar to that in Fig. 4(a).

Example 3—BER for Different Precoders: In this example, $M_r = 3$, $M_t = 4$, $M = 3$, $R_b = 12$, the number of feedback bits $B = 3$, and a decision feedback receiver is used. The BER plots are given for two types of $M_t \times M_t$ augmented precoders: 1) $\mathbf{F}'_1 = \mathbf{U}_t\mathbf{\Lambda}'_f$; and 2) $\mathbf{F}'_2 = \mathbf{\Lambda}_f$. Fig. 5(a) shows the BER for the case when the channel is correlated with $\gamma = 0.2$. The plots for $\gamma = 0.7$ are given in Fig. 5(b). We can see that $\mathbf{F}'_1$ enjoys smaller transmission power for the same error rate. We have also shown the BER when augmented precoding is not applied. In each of the two nonaugmented cases, the fixed $M_t \times M$ precoder is obtained by keeping the first M columns of the augmented precoder, i.e., $\mathbf{F}_1 = \mathbf{U}_{t,M}\mathbf{\Lambda}_f$ and $\mathbf{F}_1 = [\mathbf{I}_M \quad \mathbf{0}]^T\mathbf{\Lambda}_f$, respectively. The use of augmented precoding reduces the transmission power by around 3 dB at BER $= 10^{-4}$. Note that, for $B = 3$, both Fig. 5(a) and (b) demonstrate that $\mathbf{F}_1$ is better than $\mathbf{F}_2$, although the later was shown in [16] to be a transmission power minimizing precoder. This is because the result in [16] is obtained under the assumption of optimal BA. When the feedback rate is small and there are only a few BA codewords in the codebook, $\mathbf{F}_1$ usually gives better performance for a correlated channel. The reason is that the equivalent channel seen by the inputs of the transmitter is



(a)

(b)

Fig. 4. BER performance of the BA system for $M_r = 5$, $M_t = 4$, $M = 4$, and $R_b = 12$ over (a) an uncorrelated channel and (b) a correlated channel with correlation coefficient $\gamma = 0.5$.

$\mathbf{H}\mathbf{F}_1 = \mathbf{H}_w\mathbf{U}_t\mathbf{\Lambda}_t^{1/2}[\mathbf{I}_M \quad \mathbf{0}]^T\mathbf{\Lambda}_f$. As $\mathbf{U}_t$ is unitary, $\mathbf{H}_w\mathbf{U}_t$ has the same distribution as $\mathbf{H}_w$ [36]. Thus, $\mathbf{H}\mathbf{F}_1$ has the same distribution as $\mathbf{H}_w\mathbf{\Lambda}_t^{1/2}[\mathbf{I}_M \quad \mathbf{0}]^T\mathbf{\Lambda}_f$. That is, the inputs of the transmitter are first scaled and then passed through an uncorrelated channel $\mathbf{H}_w$. Therefore, the distribution of BA vectors (like the one shown in Example 1) for $\mathbf{F}_1$ is more skewed. As a result, when BA is quantized, the precoder $\mathbf{F}_1$ will give better performance.

Example 4—Codeword Selection Criterion: In this example, we use $M_r = 5$, $M_t = 4$, $M = 4$, $R_b = 12$, and the receiver is linear. We compare the results using the BER criterion and the maximin criterion. In the first case, the codeword that has the minimum BER is chosen. In the second case, a suboptimal codeword is chosen by quantizing the optimal BA vector using the maximin criterion described in Section IV. The results for $B = 9$ are shown in Fig. 6(a) for an uncorrelated channel and in Fig. 6(b) for a correlated channel with $\gamma = 0.6$. In both cases,

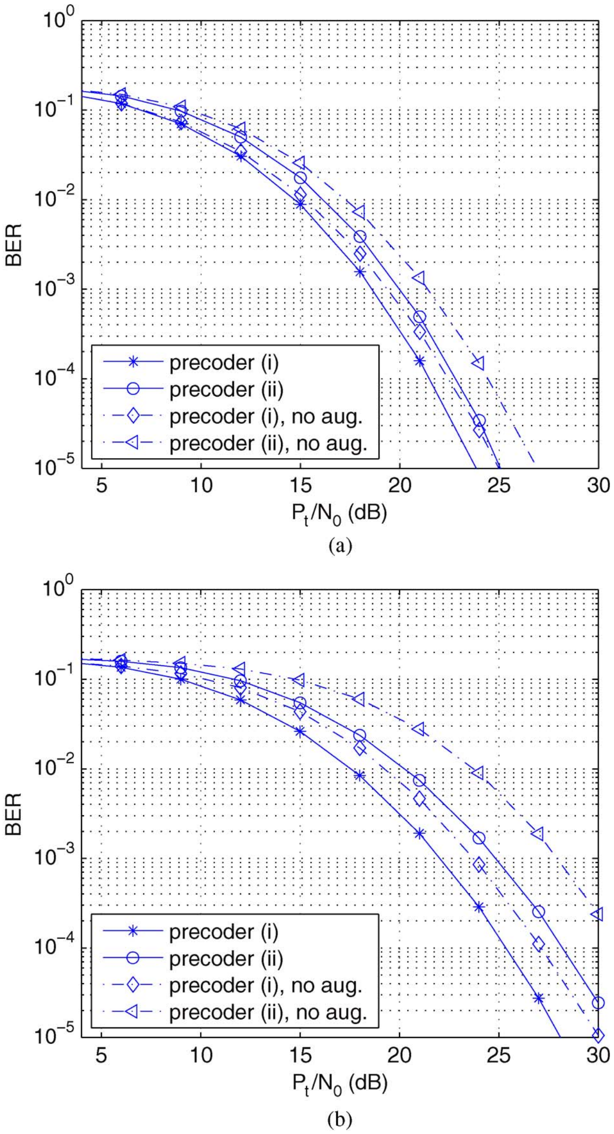


Fig. 5. BER performance with different precoders for $M_r = 3$, $M_t = 4$, $M = 3$, and $R_b = 12$ over correlated channels. (a) $\gamma = 0.2$. (b) $\gamma = 0.7$.

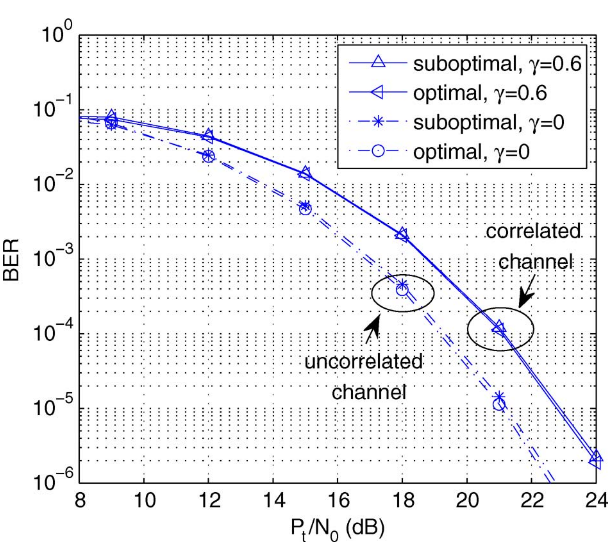


Fig. 6. BERs of the BA system for $M_r = 5$, $M_t = 4$, $M = 4$, and $R_b = 12$ using the optimal BER criterion and suboptimal maximin criterion.

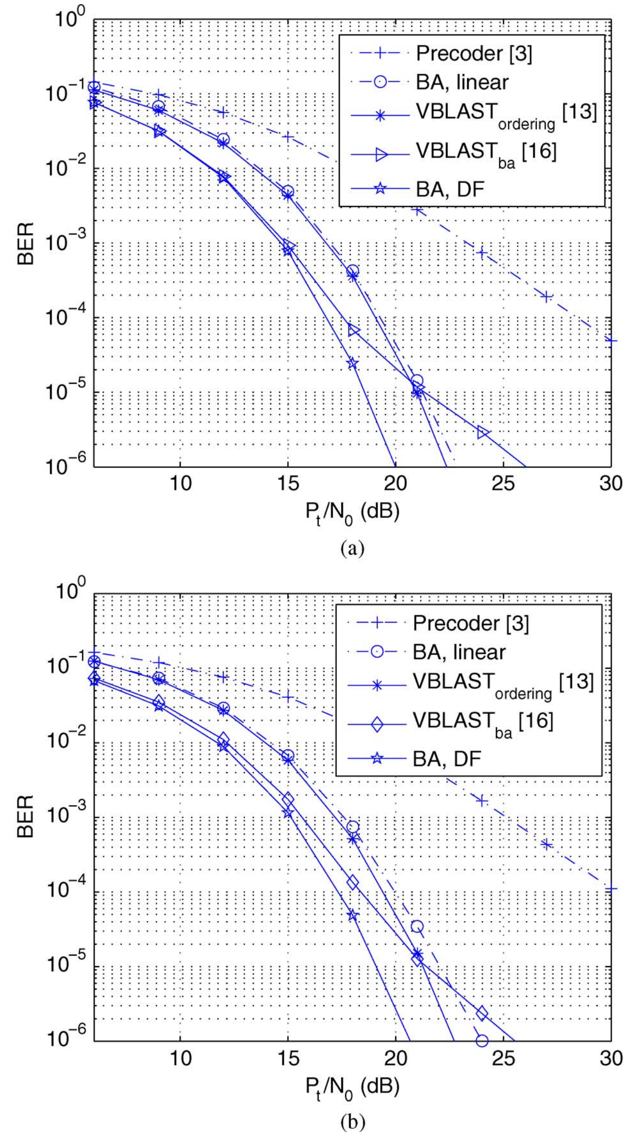


Fig. 7. Comparisons of BER for (a) an uncorrelated channel and (b) a correlated channel with a correlation coefficient of 0.5.

the BER using the suboptimal maximin criterion is close to that using the minimum BER criterion.

Example 5—Comparison for $M_r = 5$, $M_t = 4$, $M = 4$, and $R_b = 12$: In this example, we will compare the BA system with the precoding system [3], in which the feedback is the index of the optimal precoder in the codebook and bits are uniformly loaded on all M symbols transmitted. In addition, we will compare with the QR-based system with BA (VBLAST_{ba}) [16] and the VBLAST system with feedback of ordering ($\text{VBLAST}_{\text{ordering}}$) [13]. The $\text{VBLAST}_{\text{ordering}}$ system in [13] feedbacks detection ordering for a fixed BA, and this is equivalent to having a codebook of all permutations of a single BA vector. With $M_t = M = 4$, the required number of feedback bits is $\log_2(4!) \approx 5$ [13]. The number of feedback bits is made as close to 5 as possible for other systems. For VBLAST_{ba}, the original codebook containing all integer vectors satisfying the sum rate constraint is trimmed by setting $b_i \geq 2$ as in [16], which results in a codebook of 35 codewords. For the precoder and BA systems, the codebook size is 32.

The results are shown in Fig. 7(a) for an uncorrelated channel and in Fig. 7(b) for a correlated channel with $\gamma = 0.5$. The performance of the BA system with a linear receiver is much better than that of the precoder system [3] and is comparable to that of $\text{VBLAST}_{\text{ordering}}$ with a decision feedback receiver. The VBLAST_{ba} system has BER similar to the BA system with a decision feedback receiver in low SNR. For higher SNR, the BER of VBLAST_{ba} is dominated by the worst subchannels. We can see that the BA system achieves a good performance due to the flexibility in codebook design.

VI. Conclusion

In this paper, we have considered the combination of statistical precoding and feedback of BA for correlated MIMO channels. We have derived the optimal statistical precoder that minimizes bounds of average BER for a correlated MIMO channel with feedback of BA. Due to statistical precoding, the distribution of BA is highly skewed, which allows the BA to be efficiently quantized using VQ. Furthermore, when the number of transmit antenna is larger than the number of symbols transmitted, augmented precoding has been shown to achieve full diversity and significantly improve the performance. The use of augmented precoding does not require additional feedback. Simulations have demonstrated that the proposed BA system achieves a good tradeoff between performance and feedback rate.

Appendix A
Proof of Lemma 1

The Hessian matrix in (13) can be rewritten as

$$\mathbf{H}_{\text{ess}} = 1/M^2 f'\left(h(\mathbf{y})\right) h(\mathbf{y}) \left[0.5\left(1/h(\mathbf{y}) - 1\right)\mathbf{u}\mathbf{u}^T - M\mathbf{D}\right]$$

where $\mathbf{u}$ is $M \times 1$ with the ith element $u_i = 1/y_i$, and $\mathbf{D}$ is a diagonal matrix with $[\mathbf{D}]_{ii} = 1/y_i^2$. We examine the quadratic form $\mathbf{v}^T\mathbf{H}\mathbf{v}$ for an arbitrary $M \times 1$ vector $\mathbf{v}$. It can be rearranged as

$$\mathbf{v}^T\mathbf{H}\mathbf{v} = \frac{1}{M^2} f'\left(h(\mathbf{y})\right) h(\mathbf{y}) \times \left[(\mathbf{v}^T\mathbf{u}\mathbf{u}^T\mathbf{v} - M\mathbf{v}^T\mathbf{D}\mathbf{v}) + 0.5\left(1/h(\mathbf{y}) - 3\right)\mathbf{v}^T\mathbf{u}\mathbf{u}^T\mathbf{v}\right].$$

The first term in the bracket $\mathbf{v}^T\mathbf{u}\mathbf{u}^T\mathbf{v} - M\mathbf{v}^T\mathbf{D}\mathbf{v}$ is equal to $(\sum_{k=0}^{M-1} v_k u_k)^2 - M\sum_{k=0}^{M-1} v_k^2 u_k^2$, which is always nonpositive due to Cauchy-Schwartz inequality. The second term in the bracket $0.5(1/h(\mathbf{y}) - 3)\mathbf{v}^T\mathbf{u}\mathbf{u}^T\mathbf{v}$, is nonpositive if $h(\mathbf{y}) \geq 1/3$. Therefore, we can conclude that, when $h(\mathbf{y}) \geq 1/3$, the Hessian matrix is negative semidefinite, and thus, $f(h(\mathbf{y}))$ is concave.

Appendix B
Proof of Lemma 2

Majorization theorem [32] will be used to prove the theorem. For completeness, some related definitions are given here. 1) Given a sequence $a_0, a_1, \ldots, a_{M-1}$, notation $a_{[k]}$ refers to the permuted sequence such that $a_{[0]} \geq a_{[1]} \geq \cdots \geq a_{[M-1]}$. 2) Given two real vectors $\mathbf{a} = [a_0 \quad a_1 \quad \cdots \quad a_{M-1}]^T$ and $\mathbf{b} = [b_0 \quad b_1 \quad \cdots \quad b_{M-1}]^T$, we say that $\mathbf{a}$ majorizes $\mathbf{b}$ if the following two conditions are satisfied: $\sum_{k=0}^{M-1} a_k = \sum_{k=0}^{M-1} b_k$ and $\sum_{k=0}^{n} a_{[k]} \geq \sum_{k=0}^{n} b_{[k]}$, $0 \leq n \leq M-2$. 3) Let $g(\mathbf{y})$ be a real-valued function of a real vector $\mathbf{y}$. We say that $g(\mathbf{y})$ is Schur-concave if $g(\mathbf{a}) \leq g(\mathbf{b})$ whenever $\mathbf{a}$ majorizes $\mathbf{b}$.

The function $g(\mathbf{x}) = \prod_{i=0}^{M-1} x_i$, for $x_i > 0$, is known to be Schur concave [32]. As $\overline{\sigma}_{e_i}^2$ are the diagonal elements of $\overline{\mathbf{R}}_e$, the sequence $\{\overline{\sigma}_{e_i}^2\}_{i=0}^{M-1}$ is majorized by $\{\lambda_i(\overline{\mathbf{R}}_e)\}_{i=0}^{M-1}$, where we have used $\lambda_i(\mathbf{A})$ to denote the ith largest eigenvalue of $\mathbf{A}$. Thus, $\prod_{i=0}^{M-1} \overline{\sigma}_{e_i}^2 \geq \prod_{i=0}^{M-1} \lambda_i(\overline{\mathbf{R}}_e) = 1/\det(\overline{\mathbf{R}}_e^{-1})$, and the equality holds when $\overline{\mathbf{R}}_e$ is a diagonal matrix. Note that $\overline{\mathbf{R}}_e^{-1} = ((M_r - M)/N_0)\mathbf{F}^\dagger\mathbf{R}_t\mathbf{F}$. Let the singular value decomposition of $\mathbf{F}$ be $\mathbf{U}_f\mathbf{\Sigma}_f\mathbf{V}_f^\dagger$, where $\mathbf{U}_f$ is $M_t \times M$ with $\mathbf{U}_f^\dagger\mathbf{U}_f = \mathbf{I}_M$, $\mathbf{V}_f$ is $M \times M$ unitary, and $\mathbf{\Sigma}_f$ is a diagonal matrix. Then, $\det(\mathbf{F}^\dagger\mathbf{R}_t\mathbf{F}) = \det(\mathbf{\Sigma}_f^2)\det(\mathbf{U}_f^\dagger\mathbf{R}_t\mathbf{U}_f)$. As $\mathbf{U}_f$ has orthonormal columns, we can apply the Poincare separation theorem [31] to bound $\det(\mathbf{U}_f^\dagger\mathbf{R}_t\mathbf{U}_f)$ using the eigenvalues of $\mathbf{R}_t$. Poincare separation theorem says $\lambda_i(\mathbf{B}) \geq \lambda_i(\mathbf{C}^\dagger\mathbf{B}\mathbf{C})$, $i = 0, 1, \ldots, r-1$, for any $n \times n$ Hermitian matrix $\mathbf{B}$ and any $n \times r$ matrix $\mathbf{C}$ with orthonormal columns. Using this theorem, we have $\det(\mathbf{U}_f^\dagger\mathbf{R}_t\mathbf{U}_f) \leq \prod_{i=0}^{M-1} \lambda_i(\mathbf{R}_t)$. On the other hand $\det(\mathbf{\Sigma}_f^2) = \prod_{i=0}^{M-1}[\mathbf{\Sigma}_f^2]_{ii} \leq (trace(\mathbf{\Sigma}_f^2/M))^M = (trace(\mathbf{F}^\dagger\mathbf{F}/M))^M = (P_t/M)^M$. Thus, $\det(\mathbf{F}^\dagger\mathbf{R}_t\mathbf{F}) \leq (P_t/M)^M \prod_{i=0}^{M-1} \lambda_i(\mathbf{R}_t)$. Equality holds if $\mathbf{U}_f = \mathbf{U}_{t,M}$ and the diagonal elements of $\mathbf{\Sigma}_f$ are identical. It follows that

$$\prod_{i=0}^{M-1} \overline{\sigma}_{e_i}^2 \geq \prod_{i=0}^{M-1} \lambda_i(\overline{\mathbf{R}}_e) \geq \prod_{i=0}^{M-1} \frac{N_0}{M_r - M}\frac{1}{P_t/M}\frac{1}{\lambda_i(\mathbf{R}_t)}.$$

The first inequality can be satisfied by choosing $\mathbf{V}_f = \mathbf{I}_M$. Therefore, the lower bound of the above equation can be achieved by choosing $\mathbf{F} = \sqrt{P_t/M}\mathbf{U}_{t,M}$. Using the aforementioned inequality and the monotone increasing property of $f(\cdot)$, we arrive at the result of the theorem.

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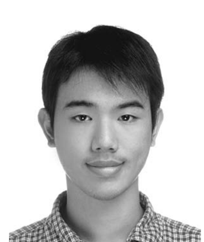

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