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The economic value of co-movement between oil price and exchange rate using copula-based GARCH models[☆]

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ABSTRACT

The US dollar is used as the primary currency of international crude oil trading; as such, the recent substantial depreciation in the US dollar has resulted in a corresponding increase in crude oil prices. In addition, oil price and exchange-rate returns have been shown to be skewed and leptokurtic, and to exhibit an asymmetric or tail dependence structure. Therefore, this study proposes dynamic copula-based GARCH models to explore the dependence structure between the oil price and the US dollar exchange rate. More importantly, an asset-allocation strategy is implemented to evaluate economic value and confirm the efficiency of the copula-based GARCH models. In terms of out-of-sample forecasting performance, a dynamic strategy based on the CGARCH model with the Student-t copula exhibits greater economic benefits than static and other dynamic strategies. In addition, the positive feedback trading activities are statistically significant within the oil market, but this information does not enhance the economic benefits from the perspective of an asset-allocation decision. Finally, a more risk-averse investor generates a higher fee for switching from a static strategy to a dynamic strategy based on copula-based GARCH models.

1. Introduction

Energy commodities differ from other trading products both in their uniqueness and their non-renewable nature. Due to the low number of oil-producing countries, most countries must rely on energy imports. As a result, the prices of energy commodities have been profoundly influenced by numerous factors, such as government policy, geopolitics, seasonal aspects, military conflicts, demand and supply. In particular, since the US dollar is commonly used as the invoicing currency in the international energy commodity market, changes in the value of the US dollar have knock-on effects on fluctuations of commodity prices and in turn affect the economic actions of energy commodity importing and exporting countries.[1] In addition, over the last few years, energy commodity prices have experienced an unprecedented high level of fluctuations. For example, the crude oil price rose steadily from $20 per barrel in January 2002 to a high of $147 per barrel in July 2008. It then fell sharply to $32 per barrel in January 2009. In the meantime, since 2002 the US dollar index (USDX[2]) has behaved in a markedly different manner to the way it behaved prior to 2002 in that it has tended to move in the opposite direction to the price of crude oil. As such, while the crude oil price has soared, the US dollar has depreciated to a historically low price, and vice versa. This negative relationship has resulted in diversification and hedging benefits between crude oil commodities and the US dollar. As a result, accurate modeling and forecasting of the volatility and dependence structures of oil and exchange-rate returns are of considerable interest to global energy-related researchers, financial institutions, and investors.

In recent years, a number of methods have been employed to explore the relationship between oil prices and the US dollar exchange rate. For example, using Hansen's GMM model, Yousefi

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[1] The relationship between oil and stock markets might also represent a pertinent issue in terms of energy market investigations; however, this needs to rely on the further theoretical foundations and is beyond the scope of the current study. Therefore, this study concentrates on the discussion and evaluation of the relationship between oil and exchange-rate markets, and the issue of the relationship between oil and stock markets is left for future research.

[2] The US Dollar Index (USDX®) is an average of six major world exchange rates: the Euro (57.6%), Japanese Yen (13.6%), UK Pound (11.9%), Canadian Dollar (9.1%), Swedish Krona (4.2%) and Swiss Franc (3.6%).

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and Wirjanto (2004) investigated the impact of fluctuations in the US dollar exchange rate on the formation of OPEC[3] and verified that the correlation of oil prices and the US dollar exchange rate is negative. Akram (2004) presented evidence of a non-linear negative relationship between oil prices and the Norwegian exchange rate, and pointed out that the nature of the relationship varies with the level and trend in oil prices. Cifarelli and Paladino (2010) used a multivariate CCC GARCH-M model to determine that oil price dynamics are associated with exchange rate behavior, and found strong evidence that oil price shifts are negatively related to exchange rate changes.

Further, additional studies have focused on discussing the lead–lag relationship between oil prices and the exchange rate, as well as their interactive influence. For example, Krichene (2005) used the vector error correction model (VECM) to demonstrate that the negative impact of the falling nominal effective exchange rate could lead to a surge in oil prices, and inversely either long-term or short-term effects. Sari et al. (2009) employed generalized forecast error variance decompositions and generalized impulse response functions to find evidence of a weak long-run equilibrium relationship but with strong feedback in the short run. Lizardo and Mollick (2010) used the cointegration analysis to reveal that oil prices significantly contribute to the explanation of movements in the value of the US dollar in the long-run: an increase in the real price of oil leads to a significant depreciation of the US dollar relative to net oil exporter countries. While these studies differ from the current study in terms of the ultimate purpose, they still support the negative relationship between oil prices and the exchange rate.

The majority of the existing literature points out the negative relationship between crude oil prices and the US dollar exchange rate. A number of possible explanations for this negative relationship are summarized as follows. First, oil-exporting countries want to stabilize the purchasing power of their export revenues (US dollar) in terms of their imports (non-US dollar), so in order to avoid losses they may adopt currencies pegged to the US dollar. Second, the depreciation of the US dollar makes oil cheaper for consumers in non-US dollar regions, thereby changing their crude oil demand, which eventually causes adjustments in the oil price as it is denominated in US dollars. Third, a falling US dollar reduces the returns on US dollar-denominated financial assets, increasing the attractiveness of oil and other commodities to foreign investors. Commodity assets are also regarded as a hedge against inflation, since US dollar depreciation increases the risk of inflationary pressures in the United States. Based on the above reasons, we must consider changes in the exchange rate and the oil price simultaneously.

The analysis of financial market movements and co-movements is important for effective diversification in portfolio management. Previous research, such as Bekiros and Diks (2008), and Chang et al. (forthcoming), has commonly used multivariate GARCH models as a means of estimating time-varying dependence structures, but this is often based on severe restrictions in order to guarantee a well-defined covariance matrix. In addition, the VAR model and multivariate GARCH models assume that the asset returns follow a multivariate normal or Student-t distribution with linear dependence. This assumption is at odds with numerous empirical research studies, which show that oil and exchange-rate returns are skewed, leptokurtic and fat-tailed, following very dissimilar marginal distributions as well as different degrees of freedom parameters.[4] Further, the actual relationship between oil prices and the exchange rate is possibly non-linear or asymmetrical. For example, crude oil returns appear to be more negatively associated with US dollar returns when the US dollar depreciates as compared to when the US dollar appreciates, especially after 2002. Thus, the linear correlation may fail to capture the potentially asymmetric dependence between oil and exchange-rate returns.

To address these drawbacks, we use copula-based GARCH models to capture the volatility and dependence structures of crude oil and exchange-rate returns. The copula-based GARCH models allow for better flexibility in joint distributions than bivariate normal or Student-t distributions. In addition, employing the heterogeneous agent model, Sentana and Wadhwani (1992) categorized investors into rational (i.e. expected utility maximizers) and positive feedback (i.e. trend chasing) investors and proposed a modified CAPM. By examining the role of positive feedback trading in the US stock market, they discovered that during low volatility periods, stock returns are positively autocorrelated, but during high volatility periods they tend to be negatively autocorrelated. Such a reversal relationship in stock return autocorrelation is consistent with the notion that some traders pursue positive feedback strategies, i.e. they buy (sell) when the price rises (falls). Recently, Cifarelli and Paladino (2010) employed this modified CAPM to investigate speculative behavior in the oil market, where they discovered evidence of positive feedback trading activities. Thus, the current study assumes that the impact of feedback trading activities will influence the dynamic behavior of oil prices. Moreover, three types of marginal models are employed to capture a variety of characteristics of oil and exchange-rate volatility processes, including volatility clustering, the leverage effect, and the long-run effect. Five types of copula functions are also used to provide a more general dependence structure, as opposed to treating it as simple linear correlation.

Furthermore, if a model performs better statistically this does not necessarily imply that the model performs well in practice; as such, we follow Fleming et al. (2001, 2003) in evaluating the out-of-sample forecast performance based on the copula-based GARCH models through the use of a strategic asset-allocation problem. We also take the transaction cost problem into consideration and compute the break-even transaction cost, as discussed in Han (2006): based on the relationship between the break-even cost and the real transaction cost, an investor decides whether or not to trade.

Our contribution to the literature is twofold. First, we propose the copula-based GARCH models to elastically describe the volatility and dependence structure of oil price and US dollar exchange-rate returns. The copula-based GARCH models can be used to capture the potential skewness and leptokurtosis of oil and exchange-rate returns, as well as the possibly asymmetric and tail dependence between oil and exchange-rate returns. We find that the symmetric copulas seem superior to the asymmetric copulas in terms of the description of a dependence structure between crude oil and exchange-rate returns, and the CGARCH model with the Gaussian copula exhibits a better explanatory ability. We also observe that the dependence structure between crude oil and US dollar exchange-rate returns is not very significant before 2003, but it becomes negative and descends continuously after 2003. Second, rather than using statistical criteria, we examine whether the copula-based GARCH models can benefit an investor by implementing an asset-allocation strategy. In terms of out-of-sample results, we find that the dynamic strategies based on the copula-based GARCH models outperform the static strategy and other dynamic strategies based on the CCC GARCH and DCC GARCH models; this demonstrates that skewness and leptokurtosis of crude oil and USDX futures returns are economically significant. Furthermore, the CGARCH model with the Student-t copula yields the highest performance fees and break-even transaction costs to attract investors to switch their trading strategy. In addition, positive feedback trading activities are statistically significant in the crude oil market, but this feedback trading information does not enhance investors' economic benefits. Finally, more risk-averse investors are willing to pay higher fees to switch from a static strategy to a dynamic strategy based on copula-based GARCH models.

[3] The Organization of the Petroleum Exporting Countries is a cartel of twelve countries. The principal goals are safeguarding the cartel's interests and securing a steady income to the producing countries.

[4] Examples include Giot and Laurent (2003), Patton (2006), and Fan et al. (2008).

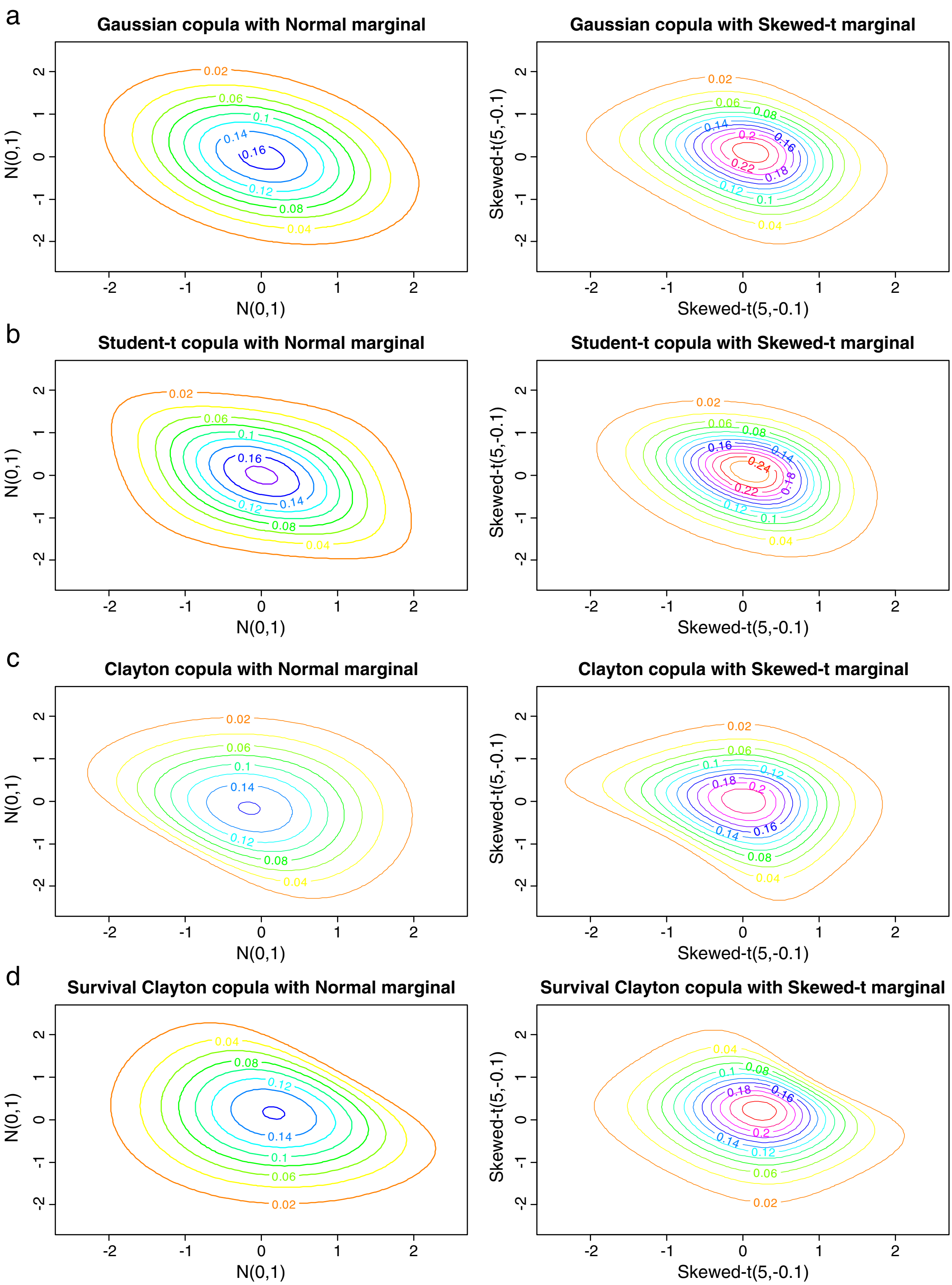

Fig. 1. Contour plot based on a. Gaussian copula, b. Student-t(5) copula, c. Clayton copula, and d. survival Clayton under the dependence parameter, $\tau = -0.2$, with two types of marginal distributions (Normal (0, 1) and Skewed-t (5, -0.1)).

The remainder of this paper is organized as follows. In the next section, we introduce the copula-based GARCH models in detail. Section 3 presents the empirical estimation results. Section 4 introduces an economic evaluation methodology and provides the results for the out-of-sample forecasts of the copula-based GARCH models. Finally, Section 5 offers conclusions.

2. Econometric model

2.1. Time-varying copula

In the past, multivariate normal distributions have been used to describe multiple asset returns across a broad range of financial and economic studies; the correlation is usually employed to describe co-movement between different asset returns. However, the correlation is only a simple measure of a dependence structure, and as a result it cannot express the relationship completely. In addition, empirical evidence has shown that the distributions of financial asset returns are usually skewed and leptokurtic, and differ from normality. Moreover, Fig. 1 demonstrates several copula contour plots under standard normal and skewed-t marginal distributions. Under the skewed-t marginal distribution, the axis of symmetry becomes a concave curve and the distribution becomes more centralized. These plots indicate that, even when using the same copula, the marginal difference causes great dissimilarity, while false assumptions of marginal distributions induce incorrect estimates of dependence structures. As such, this study employs the copula model to provide a flexible method of constructing multivariate distributions given the marginal distributions and the dependence structures separately. We briefly review the basic properties of a bivariate copula ($K=2$) below.[5]

According to Sklar's theorem, a joint distribution function can be separated into marginal distributions and a dependence structure. For any bivariate cumulative distribution function, $F(x_1, x_2)=P(X_1\leq x_1, X_2\leq x_2)$, which has continuous marginal cumulative functions, $F_i=P(X_i\leq x_i)$ for $1\leq i\leq 2$, there exists a unique copula function $C(u, v)$ such as $F(x_1, x_2)=C(F_1(x_1), F_2(x_2))$. Thus, different copula functions can be used to depict a flexible dependence structure between two random variables.

As previous studies indicated that comprehensive economic factors will induce a dependence structure to change over time, Patton (2006) extended Sklar's theorem and introduced the conditional copula function to model time-varying conditional dependence. Let $r_{o,t}$ and $r_{e,t}$ be random variables that denote oil price and exchange-rate returns at period t, respectively, with marginal conditional cumulative distribution functions $u_{o,\ t}=G_{o,t}(r_{o,t}|\Psi_{t-1})$ and $u_{e,t}=G_{e,t}(r_{e,t}|\Psi_{t-1})$, where Ψ_{t-1} denotes past information. Then, the conditional copula function $C_t(u_{o,t}, u_{e,t}|\Psi_{t-1})$ can be written using the two time-varying cumulative distribution functions. Extending Sklar's theorem, the bivariate conditional cumulative distribution functions of random variables $r_{o,t}$ and $r_{e,t}$ can be written as

$$F\left(r_{o,t}, r_{e,t}|\Psi_{t-1}\right) = C_t\left(u_{o,t}, u_{e,t}|\Psi_{t-1}\right) \tag{1}$$

Assume the cumulative distribution function is differentiable, and the conditional joint density can be expressed as

$$\begin{aligned} f\left(r_{o,t}, r_{e,t}|\Psi_{t-1}\right) &= \frac{\partial^2 F\left(r_{o,t}, r_{e,t}|\Psi_{t-1}\right)}{\partial r_{o,t}\partial r_{e,t}} \\ &= c_t\left(u_{o,t}, u_{e,t}|\Psi_{t-1}\right) \times g_{o,t}\left(r_{o,t}|\Psi_{t-1}\right) \times g_{e,t}\left(r_{e,t}|\Psi_{t-1}\right) \end{aligned} \tag{2}$$

where $c_t(u_t, v_t|\Psi_{t-1})=\partial^2 C_t(u_t, v_t|\Psi_{t-1})/\partial u_t\partial v_t$ is the conditional copula density function and $g_i(\cdot)$ is the density function corresponding to $G_i(\cdot)$.

From Eq. (2), the likelihood function can be expressed as:

$$L_{o,e}(\Theta) = L_o(\Theta_o) + L_e(\Theta_e) + L_c(\Theta_c), \tag{3}$$

where $\Theta_{\rm o}$ and $\Theta_{\rm e}$ are the parameter vectors of marginal distributions of oil and exchange-rate returns, respectively, and $\Theta_{\rm c}$ is the vector of parameters in the copula function, c_t. When the maximum likelihood method is implemented over a high dimension case, the optimization procedure will confront problems in terms of extensive computation and estimate accuracy. Consequently, we use the two-stage estimation method, known as inference functions for margins (IFM), to estimate the parameters of our copula-based GARCH models. Joe (1997, 2005) showed that this estimator is close to and asymptotically efficient to the maximum likelihood estimator under some regularity conditions. Hence, the two-stage estimation method can efficiently compute the estimator without losing any real information.

2.2. Marginal density

As indicated in the Introduction, this paper employs a feedback trading model to describe the short run dynamics of oil returns as well as exchange-rate returns. In addition, many financial time series have been shown to have a number of important features, including leptokurtosis, volatility clustering, long memory, volatility smile, and the leverage effect, among others. Therefore, we employ three kinds of ARCH-type models (GARCH, GJR-GARCH, and component GARCH) to capture the time-varying volatility structures of oil price and exchange-rate returns. The GJR-GARCH and component GARCH (CGARCH) models can be used to take the asymmetry effect into consideration, and to distinguish the difference in duration, respectively.

2.2.1. GARCH model

Following Cifarelli and Paladino (2010), the GARCH(1,1) model with feedback trading activities can be expressed as:

$$\begin{aligned} & r_{i,t} = \beta_{i,1} + \beta_{i,2}h_{i,t}^2 + \left(\beta_{i,3} + \beta_{i,4}h_{i,t}^2\right)r_{i,t-1} + \varepsilon_{i,t},\ \varepsilon_{i,t}|\Psi_{t-1} = h_{i,t}z_{i,t}, \\ & h_{i,t}^2 = c_i + a_i\varepsilon_{i,t-1}^2 + b_i h_{i,t-1}^2, \\ & z_{i,t}\sim skewed-t(z_i|\eta_i, \lambda_i), \quad i = o, e, \end{aligned} \tag{4}$$

where $\beta_{i,\ 2}h_{i,\ t}^2$ is the risk premium, $\beta_{i,3}$ captures the impact of nonsynchronous effects or market inefficiencies, $\beta_{i,4}$ captures the feedback trading activities, where the presence of positive feedback trading implies that $\beta_{i,4}$ is negative. The parameter restrictions in the variance equation are $c_i>0$, a_i, $b_i\geq 0$, and $a_i+b_i<1$. The error term $\varepsilon_{i,\ t}$ is assumed to be a skewed-t distribution, which can be used to describe the possibly asymmetric and heavy-tailed characteristics of oil price and exchange-rate returns. Following Hansen (1994), the density function is

$$skewed-t(z|\eta, \lambda) = \begin{cases} bc\left(1 + \dfrac{1}{\eta-2}\left(\dfrac{bz + a}{1-\lambda}\right)^2\right)^{-(\eta+1)/2}, z < -\dfrac{a}{b} \\ bc\left(1 + \dfrac{1}{\eta-2}\left(\dfrac{bz + a}{1+\lambda}\right)^2\right)^{-(\eta+1)/2}, z\geq -\dfrac{a}{b} \end{cases} \tag{5}$$

The values of a, b, and c are defined as

$$a\equiv 4\lambda c\frac{\eta-2}{\eta-1}, \quad b^2\equiv 1 + 2\lambda^2 - a^2 \quad \text{and} \quad c\equiv\frac{\Gamma(\eta+1/2)}{\sqrt{\pi(\eta-2)}\Gamma(\eta/2)}$$

where λ and η are the asymmetry and kurtosis parameters, respectively. These are restricted to $-1<\lambda<1$ and $2<\eta<\infty$. When $\lambda=0$, the skewed-t distribution will turn toward the Student-t distribution. If $\lambda=0$ and η diverge to infinity, it will be a normal distribution.

2.2.2. GJR–GARCH model

Another feature of the financial time series is the leverage effect,[6] whereby there is an asymmetric reaction of volatility changes in response to positive and negative shocks of the same magnitude. To

[5] See Cherubini et al. (2004) and Nelsen (2006) for more comprehensive introductions to the properties of a bivariate copula.

[6] Relative to the equities, this asymmetric reaction of volatilities for commodities can be explained by "risk aversion", which means that the negative shocks will oblige investors to sell commodities at times of stress. We thank an anonymous referee for raising this point.

this effect, we employ the GJR-GARCH model, proposed by Glosten et al. (1993), to take into account the asymmetric effect in the volatility structure, which is given by

$$r_{i,t} = \beta_{i,1} + \beta_{i,2}h_{i,t}^2 + \left(\beta_{i,3} + \beta_{i,4}h_{i,t}^2\right)r_{i,t-1} + \varepsilon_{i,t},\ \varepsilon_{i,t}|\Psi_{t-1} = h_{i,t}z_{i,t},$$
$$h_{i,t}^2 = c_i + a_i\varepsilon_{i,t-1}^2 + b_ih_{i,t-1}^2 + d_ik_{i,t-1}\varepsilon_{i,t-1}^2 \tag{6}$$

where $k_{i,t-1}=1$ if $\varepsilon_{i,\,t-1}$ is negative, otherwise $k_{i,\,t-1}=0$, and the parameter d_i is regarded as an asymmetric impact on the conditional volatility. If there is a leverage effect on the oil price or exchange-rate markets, the parameter d_i will be expected to be positive.

2.2.3. Component GARCH model

The component GARCH (CGARCH) model can be used to decompose conditional volatility into a long-run trend component and a short-run transitory component. Contrary to the traditional GARCH model, the component GARCH model allows the conditional volatility to revert to the time-varying long-run volatility level rather than the constant long-run volatility level. Engle and Lee (1999) replaced the constant unconditional variance with a time-varying permanent component, which represents the long-run volatility, to ensure that the volatility is not constant in the long-run, and proposed the following component GARCH model:

$$r_{i,t} = \beta_{i,1} + \beta_{i,2}h_{i,t}^2 + \left(\beta_{i,3} + \beta_{i,4}h_{i,t}^2\right)r_{i,t-1} + \varepsilon_{i,t},\ \varepsilon_{i,t}|\Psi_{t-1} = h_{i,t}z_{i,t},$$
$$h_{i,t}^2 = q_{i,t} + a_i\left(\varepsilon_{i,t-1}^2 - q_{i,t-1}\right) + b_i\left(h_{i,t-1}^2 - q_{i,t-1}\right)$$
$$q_{i,t} = \varpi_i + \phi_iq_{i,t-1} + \zeta_i\left(\varepsilon_{i,t-1}^2 - h_{i,t-1}^2\right) \tag{7}$$

where $\phi_i<1$ and $a_i+b_i<1$. The parameter ϕ_i measures the persistence in the permanent component and the forecast error $(\varepsilon_{i,\,t-1}^2 - h_{i,\,t-1}^2)$ serves as the driving factor for the time-dependent movement of the permanent component. The parameters ζ_i and a_i are regarded as the short-run shock effects of the permanent component and the transitory component, respectively.

2.3. Copula function and dynamic dependence structure

Here we use two families of copula function to describe the dependence structure between oil price and exchange-rate returns, in order to fit various phenomena. Two elliptical (Gaussian and Student-t copulas) and three Archimedean's copula functions (Clayton, survival Clayton, and mixture Clayton copulas) are employed to capture different dependence structures. The advantage of elliptical copulas is that one can specify different levels of correlation between the marginals; however, these copulas must possess radial symmetry. The property of the Student-t copula is symmetric and also implies symmetric dependence in the extreme tails. When the degree of freedom increases to infinity, the Student-t copula converges to the Gaussian one with zero dependence on the two side tails.

The families of Archimedean copulas were named by Ling (1965) and realized by Schweizer and Sklar (1961). In contrast to elliptical copulas, Archimedean copulas are characterized by their generator function, which has many useful properties. They can have upper tail dependence, lower tail dependence, or both; as such, they can better describe the reality of the behavior of financial markets. Here three types of Archimedean copula are used to integrate the marginal distributions into the joint distributions. In general, (survival) Gumbel and (survival) Clayton are commonly employed in the financial studies. Unfortunately, the Gumbel copula is limited to the description of a positive dependence structure. Hence, we tend to use the survival Clayton (SClayton) copula, which possesses similar properties to the Gumbel copula, but does not have a positive dependence restriction, where the density of the survival function can be written as:

$$\bar{c}_t\left(u_{o,t}, u_{e,t}\right) = c_t\left(1-u_{o,t}, 1-u_{e,t}\right). \tag{8}$$

Since the Clayton or survival Clayton copulas can only be used to capture one side of tail dependence, we also employ a mixture of Clayton and survival Clayton (MClayton) copulas to describe the possible lower and upper tail dependence structure between oil price and exchange-rate returns. The density of the MClayton copula can be expressed as:

$$c_t^{MClayton}\left(u_{o,t}, u_{e,t}\right) = \omega_c c_t^{Clayton}\left(u_{o,t}, u_{e,t}\right) + (1-\omega_c)c_t^{SClayton}\left(u_{o,t}, u_{e,t}\right), \tag{9}$$

where $\omega_c \in (0, 1)$ is the weighting parameter.

In the description of a dependence structure, Pearson's correlation coefficient (ρ) is commonly used in the Gaussian copula and the Student-t copula. On the other hand, we use Kendall's tau (τ) in Archimedean copulas. In addition, we follow the concept of Patton (2006) and Bartram et al. (2007) by assuming that the dependence parameters rely on past dependence and historical information $(u_{o,t-1}-0.5)(u_{e,t-1}-0.5)$. If both $u_{o,t-1}$ and $u_{e,t-1}$ are either bigger or smaller than 0.5, we infer that the dependence is higher than previously. Let ρ_t^* and τ_t^* be an appropriate logistic transformation[7] of dependence parameters ρ_t and τ_t, respectively, such that the time-varying parameters ρ_t^* and τ_t^* can be expressed as:

$$\rho_t^* = \alpha_c + \beta_c\rho_{t-1}^* + \gamma_c\left(u_{o,t-1}-0.5\right)\left(u_{e,t-1}-0.5\right)$$
$$\tau_t^* = \alpha_c + \beta_c\tau_{t-1}^* + \gamma_c\left(u_{o,t-1}-0.5\right)\left(u_{e,t-1}-0.5\right) \tag{10}$$

where $0 \le \beta_c < 1$.

3. Data and empirical results

3.1. Data and descriptive statistics

This study uses West Texas Intermediate (WTI) crude oil and US dollar index (USDX) futures data to represent oil price and exchange-rate markets. WTI crude oil, also known as light sweet oil, is the futures contract traded on the New York Mercantile Exchange (NYMEX). The USDX represents the trade-weighted value of the US dollar in terms of a basket of six major foreign currencies, which includes a futures contract and an option contract traded on the New York Board of Trade (NYBOT). Both WTI crude oil and USDX futures price data[8] with the nearest to maturity for the period from January 2, 1990 to December 28, 2009 are obtained from DATASTREAM, and 1045 weekly return observations[9] are generated for each asset. In addition, we use the three-month Treasury bill as the risk-free rate, obtained from the Federal Reserve Board. The weekly close prices, returns, and trading volumes of WTI crude oil and USDX futures over the sample period are graphed in Fig. 2. Fig. 2c shows that the trading volumes of both crude oil and USDX futures increase over time, especially after 2007. The reason for this phenomenon may be that some new investment or speculation opportunities are possibly derived by traders based on the linkage between the oil and US dollar exchange-rate markets.

[7] The appropriate logistic transformation is used to ensure the dependence parameters fall within the interval (−1,1), which can be written as $\rho_t^* = -\ln[(1-\rho_t)/(\rho_t+1)]$ and $\tau_t^* = -\ln[(1-\tau_t)/(\tau_t+1)]$.

[8] The futures price data are continuous series, as defined by DATASTREAM.

[9] Following Cifarelli and Paladino (2010), this study uses the Tuesday prices of the WTI crude oil and USDX futures; when a holiday occurs on Tuesday, Monday's observation is used in its place.

The descriptive statistics for crude oil and exchange-rate returns are reported in Table 1, which shows that the standard deviation of oil returns is higher than that of USDX returns, consistent with the general findings in the literature that commodities have higher volatilities. The skewness statistic of crude oil is negative and significant, thereby indicating that the oil returns are significantly skewed to the left. With respect to the excess kurtosis statistics, the values of both crude oil and USDX are significantly positive, thereby implying that the distribution of returns has larger, thicker tails than the normal distribution. Similarly, the Jarque–Bera statistics are large and significant, thereby implying that the assumption of skewed-t is more appropriate in our study.

Table 1
Summary statistics for crude oil and USDX futures returns.

	Crude oil futures	USDX futures
Mean(%)	0.122688	−0.017437
SD(%)	5.377643	1.234347
Skewness	−0.235948***	0.113900
Excess Kurtosis	3.109873***	1.520627***
Max(%)	31.35646	6.218743
Min(%)	−28.07634	−6.189992
JB	430.3888***	102.8427***

Note: This table reports the descriptive statistics for weekly crude oil and USDX futures returns for the sample period from January 2, 1990 to December 28, 2009. JB is the Jarque–Bera statistic, which is used to test for normality. The symbols *, **, and *** represent statistical significance at the 10%, 5%, and 1% levels, respectively.

3.2. Estimation results

Table 2 presents the estimated results for the three classes of copula-based GARCH models with feedback trading activities. Panel A reports the parameter estimates of marginal distributions with the GARCH, GJR-GARCH and CGARCH models. Overall, it can be concluded that an asymmetric effect does not add much to the explanatory ability of the model, and that the CGARCH model is the best performing model in terms of most information criteria.

The parameters of greatest interest in the mean equations are those governing the autocorrelation of returns, i.e., β_3 and β_4. The constant components of the autocorrelation, β_3, are all non-significant, suggesting slight autocorrelations resulted from non-synchronous trading or market inefficiencies in both crude oil and USDX markets. In addition, the parameters, β_4, are negative and statistically significant in the crude oil market, but insignificant in

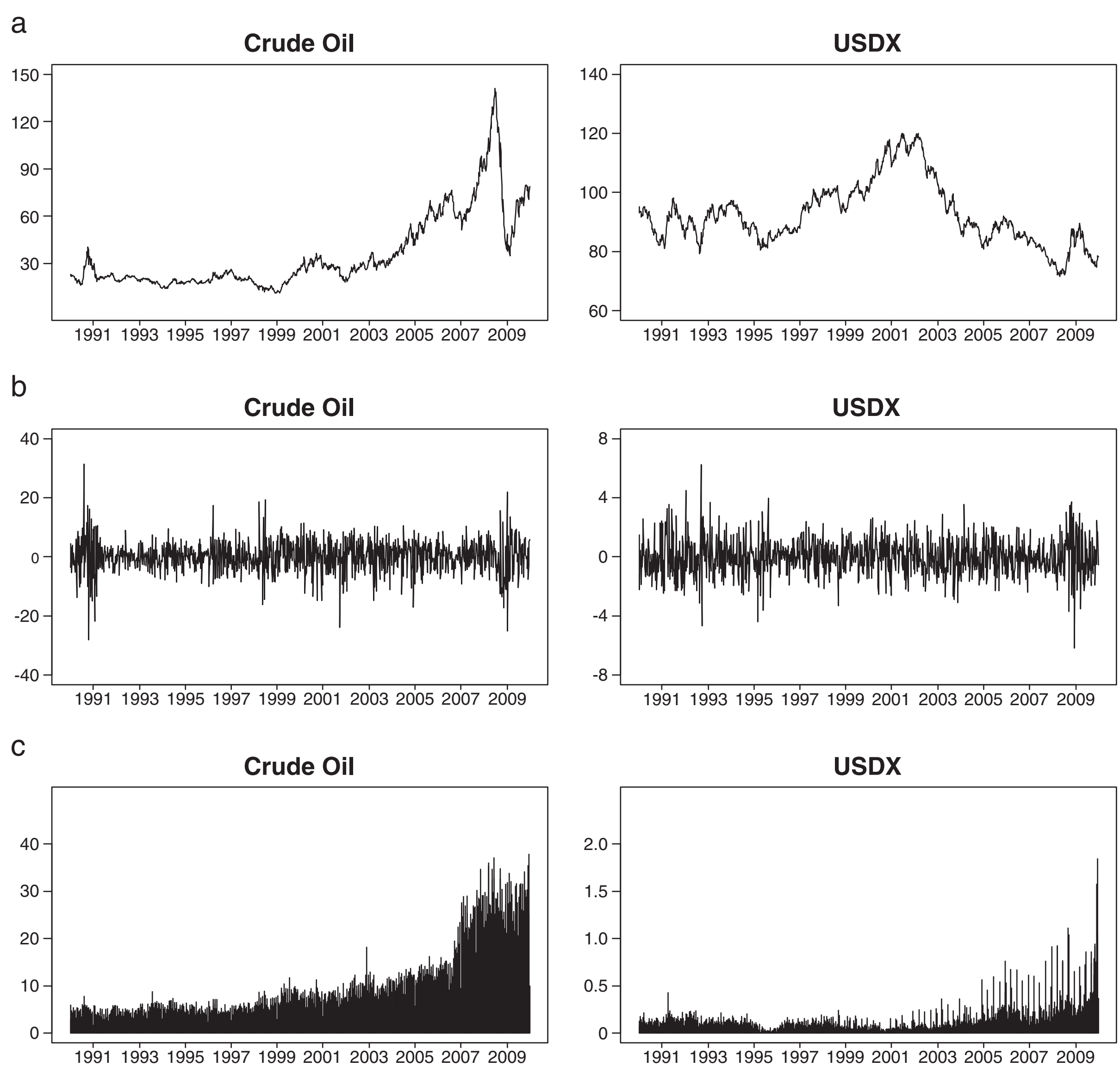

Fig. 2. a. Weekly close prices, b. Weekly returns, and c. Weekly trading volumes of crude oil and USDX futures, January 2, 1990–December 28, 2009.

Table 2
Estimation results of copula-based GARCH models.

	GARCH		GJR–GARCH		Component GARCH	
	Crude oil	USDX	Crude oil	USDX	Crude oil	USDX
Panel A: Estimation of marginals						
β_1	−0.03861	−0.14353	−0.04460	−0.14130	−0.05865	−0.14382
	(0.20174)	(0.10501)	(0.24193)	(0.10251)	(0.23638)	(0.11046)
β_2	0.00440	0.08835	0.00480	0.08440	0.00994	0.08854
	(0.00826)	(0.07328)	(0.00976)	(0.07201)	(0.01022)	(0.07679)
β_3	−0.04842	0.08662	−0.04804	0.08306	−0.05099	0.08650
	(0.05101)	(0.07603)	(0.05145)	(0.07641)	(0.04953)	(0.07827)
β_4	−0.00223**	−0.05489	−0.00216**	−0.05245	−0.00249**	−0.05473
	(0.00111)	(0.04122)	(0.00110)	(0.04117)	(0.00111)	(0.04229)
c_i	0.66673**	0.04805**	0.68935***	0.04912**		
	(0.26925)	(0.02083)	(0.29160)	(0.02111)		
a_i	0.09952***	0.05809***	0.09837***	0.05384***	0.08048***	0.00717
	(0.02088)	(0.01456)	(0.02582)	(0.01650)	(0.01819)	(0.03161)
b_i	0.88026***	0.90977***	0.87709***	0.90724***	0.86015***	0.95981***
	(0.02173)	(0.02201)	(0.02337)	(0.00229)	(0.03312)	(0.02861)
η_i	8.92101***	12.52421***	9.60491***	12.86263***	9.97846***	12.51314***
	(0.52824)	(0.70421)	(0.12871)	(0.48664)	(0.90967)	(0.89374)
λ_i	−0.17281***	0.06025	−0.17204**	0.05791	−0.16366**	0.06022
	(0.04496)	(0.04485)	(0.04512)	(0.04506)	(0.04459)	(0.04501)
d_i			0.00293	0.01238		
			(0.03073)	(0.02722)		
ϖ_i					0.05011***	0.04797*
					(0.00822)	(0.02492)
ϕ_i					0.99949***	0.96790***
					(0.00033)	(0.01967)
ζ_i					−0.01437***	0.05094
					(0.00073)	(0.03153)
Half life	33.932522	21.218035	29.690959	20.829237	11.324932	20.643225
Panel B: Estimation of Gaussian dependence structure						
α_c	−0.00039		−0.00038		−0.00036	
	(0.00088)		(0.00088)		(0.00083)	
β_c	0.98277***		0.98277***		0.98310***	
	(0.00795)		(0.00792)		(0.00771)	
γ_c	0.21852***		0.21936***		0.21100***	
	(0.07063)		(0.07054)		(0.06631)	
ln(L)	−4747.450		−4747.344		−4740.122	
AIC	9536.901		9540.688		9530.244	
BIC	9640.868		9654.557		9654.014	
Panel C: Estimation of Student-t dependence structure						
α_c	−0.00044		−0.00046		−0.00045	
	(0.00089)		(0.00090)		(0.00085)	
β_c	0.98325***		0.98328***		0.98380***	
	(0.00794)		(0.00795)		(0.00771)	
γ_c	0.22062***		0.22225***		0.21313***	
	(0.07298)		(0.07365)		(0.06920)	
υ	33.29138***		26.55568***		26.15277***	
	(0.21593)		(0.19956)		(0.26537)	
ln(L)	−4747.052		−4746.979		−4739.427	
AIC	9538.105		9541.959		9530.855	
BIC	9647.023		9660.778		9659.576	
Panel D: Estimation of Clayton dependence structure						
α_c	−0.00523		−0.01956		−0.00329	
	(0.00494)		(0.01426)		(0.00251)	
β_c	0.94356***		0.79598***		0.95721***	
	(0.04286)		(0.14448)		(0.02155)	
γ_c	0.17951*		0.49897		0.15261***	
	(0.09660)		(0.36001)		(0.05380)	
ln(L)	−4758.212		−4758.209		−4750.265	
AIC	9558.423		9562.580		9550.529	
BIC	9662.390		9676.448		9674.300	
Panel E: Estimation of survival Clayton dependence structure						
α_c	−0.02726**		−0.02675*		−0.02603*	
	(0.01386)		(0.01392)		(0.01385)	
β_c	0.76715***		0.77056***		0.77471***	
	(0.08202)		(0.08919)		(0.09283)	
γ_c	0.78126***		0.78198***		0.68003***	
	(0.23828)		(0.23630)		(0.26127)	
ln(L)	−4756.587		−4756.440		−4749.085	
AIC	9555.175		9558.880		9548.170	
BIC	9659.142		9672.749		9671.941	

Table 2 (*continued*)

	GARCH		GJR–GARCH		Component GARCH	
	Crude oil	USDX	Crude oil	USDX	Crude oil	USDX
Panel F: Estimation of mixture Clayton dependence structure						
α_c	−0.00132		−0.00131		−0.00169*	
	(0.00095)		(0.00097)		(0.00102)	
β_c	0.98458***		0.98453***		0.98114***	
	(0.00431)		(0.00413)		(0.00656)	
γ_c	0.21183***		0.21415***		0.18818***	
	(0.03742)		(0.03614)		(0.04733)	
ω_c	0.50977***		0.50832***		0.54861***	
	(0.10859)		(0.10805)		(0.13980)	
ln(L)	−4749.997		−4749.802		−4744.388	
AIC	9543.994		9547.603		9540.776	
BIC	9652.912		9666.423		9669.497	

Note: The table reports the maximum likelihood estimates of three classes of copula-based GARCH models, which are based on the weekly crude oil and USDX futures returns for the sample period from January 2, 1990 to December 28, 2009. Three types of marginal distributions (GARCH, GJR–GARCH and component GARCH models) and five types of copula functions (Gaussian, Student-t, Clayton, survival Clayton, and mixture Clayton copulas) are utilized to describe the volatility and dependence structures, respectively. The half lives are calculated by the formula: $\ln(0.5)/\ln(a_i+b_i+0.5^*d_i)$. The Akaike information criteria (AIC) and Bayesian information criteria (BIC) are used to evaluate the goodness of fit of the selected models. The numbers in parentheses are standard deviations.

* Indicates statistical significance at the 5% level.

** Indicates statistical significance at the 1% level.

*** Indicates statistical significance at the 10% level.

the USDX market. The implication is that positive feedback trading is an important determinant of short-term movements in the crude oil market in agreement with the findings of Cifarelli and Paladino (2010).

As can be seen in the variance equations, the asymmetry parameters, λ_i, are significant and negative for crude oil returns, but insignificant for USDX returns, exhibiting that crude oil returns are skewed to the left. In addition, in the GARCH model, the parameters a_i and b_i are significant and as such explain that crude oil and exchange-rate returns have volatility clustering. The fact that the volatility half lives[10] of about 34 and 21 weeks for crude oil and USDX markets, respectively, indicates that the shock to the volatility for crude oil lasts for a longer time period than the shock to USDX. Further, the asymmetric parameters d_i in the GJR-GARCH model are insignificant and exhibit no asymmetric effect on the volatility structures of crude oil and exchange-rate markets, which is consistent with Lanza et al. (2006) and Wang and Yang (2009). This result may indicate that the asymmetric reaction to equities markets does not apply to the crude oil and USDX futures markets. Turning to the CGARCH model in Table 2, the result demonstrates that the permanent volatility component decays very slowly and is highly persistent especially for the crude oil returns. In addition, the half life of crude oil dramatically changes from the GARCH model (34 weeks) to CGARCH model (11 weeks), thereby implying a less shock persistence in the transitory volatility component of crude oil, while the half life of USDX is quite similar based on each marginal model. This finding enables us to completely understand the influences of volatility shocks on various volatility components.

Panels B–F of Table 2 report the parameter estimates for different copula functions. In terms of the values of AIC and BIC, the Gaussian dependence structure exhibits better explanatory ability than other dependence structures despite the marginal models employed, while the Clayton and survival Clayton copulas have worse explanatory ability. These results imply that introducing the tail dependence between oil and exchange-rate returns does not add much to the explanatory ability of the models. In addition, the CGARCH model with the Gaussian copula exhibits superior performance to any other selected model. Moreover, we can see the autoregressive parameter β_c is close to 1, implying a high degree of persistence pertaining to the dependence structure between oil and exchange-rate returns. The latent parameter γ_c is also significant and displays that latest return information is a meaningful measure. Specially, γ_c in the survival Clayton copula is much larger than others, which means it has a greater short-run response than other copula functions.

Fig. 3 plots the volatility estimates of crude oil and USDX returns based on the GARCH, GJR-GARCH and CGARCH models. Crude oil underwent two periods of high volatility in our sample period. The first period began in August 1990, commonly known as “the third energy crisis”, due to the Gulf War. Because the oil demands of most countries rely on imports, wars involving oil-producing countries cause supply to diminish, thereby sending the price soaring. The second period began in July, 2008 due to the American subprime mortgage crisis: the oil price suffered a major depreciation from \$147 a barrel to \$32 a barrel. OPEC intervened by cutting oil output by more than 4 million barrels per day in order to aid the price recovery. By comparison, the USDX is very stable. The worst period in this regard followed the US government's intentional manipulation of the dollar value in order to prevent American economic decline following the financial meltdown. In addition, the volatility estimates of crude oil based on the GARCH and GJR-GARCH models are more persistent than those based on the CGARCH model, which is in line with the shorter half life of CGARCH model; in comparison, the volatility estimates of USDX based on three different marginal models are very similar. We also find that the circumstances in which crude oil and USDX volatilities usually rise at the same time imply a connection between crude oil prices and the USDX.

The dependence parameter estimates between oil price and exchange-rate returns over the sample period generated from different copula models are plotted in Fig. 4. We can observe that the dependence structure between crude oil and USDX returns maintains a lower level or zero dependence during the period 1990 to 2003. However, from 2003, the dependence begins to descend and continues to do so now. This may be due to the fact that US government policy caused the US dollar to decrease greatly in value relative to most other countries' currencies in order to support its exports as well as reduce the international trade deficit. Over the past few years the depreciation of the US dollar against other currencies has had the effect of driving up the oil price. Since the US dollar is the main invoicing currency of crude oil futures, its depreciation has motivated speculators to buy an abundance of crude oil futures contracts to secure greater profits, and in doing so resulted in the unusual rate of oil price increases.

[10] The half-life, which is defined as the time taken until half of the initial shock is absorbed in the variance, is a standard representation of the persistence of a volatility shock (Bollerslev et al., 1994).

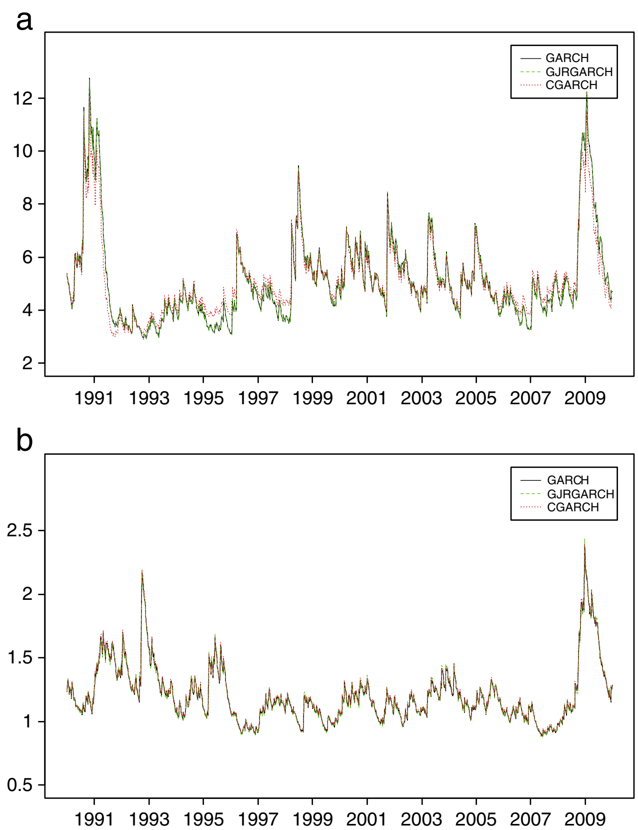

Fig. 3. Volatility estimates of a. crude oil and b. USDX futures returns based on the GARCH, GJR -GARCH, and component GARCH models for the sample period from January 2, 1990 to December 28, 2009.

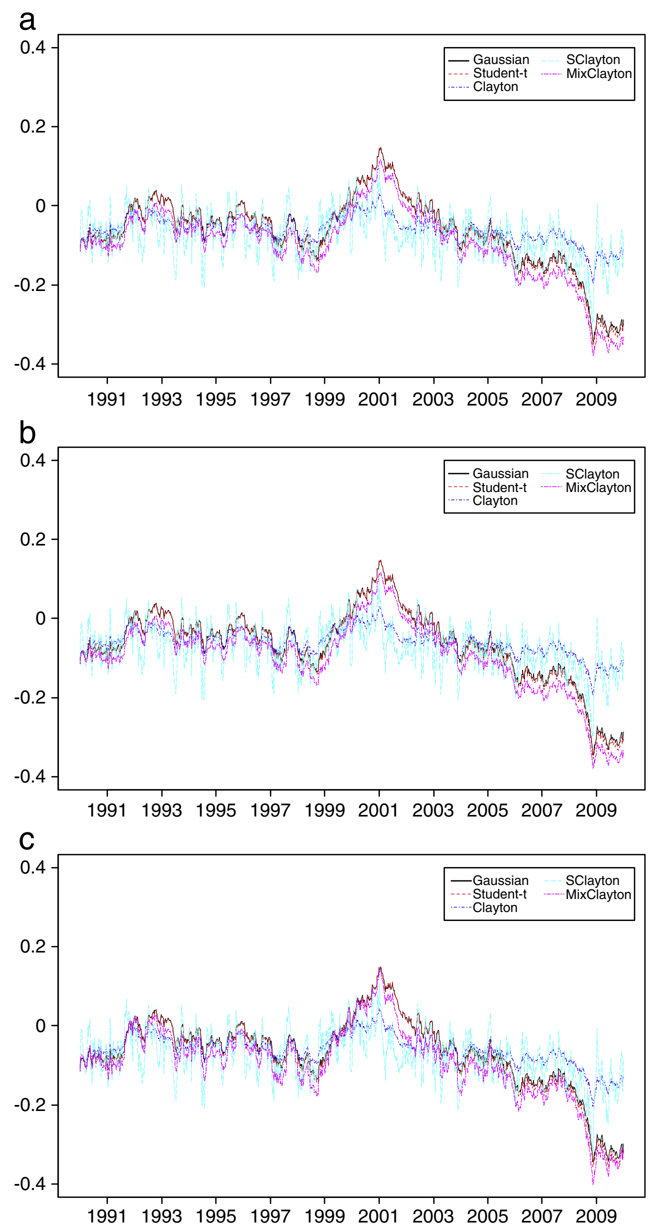

Fig. 4. Dependence estimates (Kendall's tau) between crude oil and USDX futures returns based on marginal distribution under a. the GARCH model, b. GJR -GARCH model, and c. the component GARCH model, January 2, 1990 -December 28, 2009.

In Fig. 4, the two paths from the Gaussian and Student-t copulas are very consistent with the results in Panel C of Table 2, which shows that the degree of freedom of the Student-t copula is considerable. The Clayton and survival Clayton copulas exhibit a similar dependence trend to one another, while displaying a low level of dependence relative to the symmetric copulas. Moreover, the main differences in dependence estimates between the Clayton and survival Clayton copulas are that the survival Clayton copula exhibits larger ripples. Finally, the dependence estimates based on the mixture Clayton copula are almost smaller than those based on the Gaussian and Student-t copulas.

4. An economic evaluation methodology

In the previous section, we note the explanatory ability of each selected model. However, the fact that estimation results perform well does not necessarily imply an economically useful application. Thus, in this section, we follow Fleming et al. (2001, 2003) to evaluate the economic value of copula-based GARCH models using a dynamic asset-allocation strategy. First, we use crude oil futures, USDX futures and three-month Treasury bills to construct our portfolio, where the optimal portfolio weights of selected assets are constructed under the mean–variance framework. Second, the quadratic utility function is employed to assess the performance of dynamic strategies based on different models and to quantify how personal opinion affects performance. Finally, this framework establishes a concise approach to assess the significance and robustness of the results.

4.1. Evaluation methodology

First we consider an investor who wants to minimize portfolio variance subject to achieving a particular expected return. Let r_t be $N\times 1$ vector of returns on the risky assets; the investor solves the following optimization at each period t,

$$\min_{\mathbf{w}_t} \ \mathbf{w}_t' \Sigma_{t+1} \mathbf{w}_t \qquad \text{s.t.} \ \mathbf{w}_t' \mu_{t+1} + \left(1-\mathbf{w}_t'\mathbf{1}\right) r_{f,t+1} = \mu_p^* \tag{11}$$

where $\mathbf{w_t}$ is an $N\times 1$ vector of portfolio weights on risky assets, μ_t and Σ_{t+1} are the vector of conditional expected returns and the conditional covariance matrix of risky assets, respectively, r_f is return on the riskless asset and μ_p^* is the target conditional expected return of the portfolio. The solution for the optimization problem is

$$\mathbf{w}_t = \frac{\left(\mu_p^* - r_{f,t+1}\right)\Sigma_{t+1}^{-1}\left(\mu_{t+1} - r_{f,t+1}\mathbf{1}\right)}{\left(\mu_{t+1} - r_{f,t+1}\mathbf{1}\right)'\Sigma_{t+1}^{-1}\left(\mu_{t+1} - r_{f,t+1}\mathbf{1}\right)}, \tag{12}$$

which is the optimal weights on risky assets, and the weight on the riskless asset is $1-\mathbf{w_t}'\mathbf{1}$.

In order to measure the value of our models, we compare the performance of the dynamic strategies based on copula-based GARCH models to that of the static strategy based on a sample mean and covariance matrix. Using the Taylor series, we can obtain the quadratic utility as a second-order approximation to the investor's true utility function. Under this specification, the investor's realized utility in period $t+1$ can be written as

$$U(W_{t+1}) = W_t r_{p,t+1} - \frac{aW_t^2}{2} r_{p,t+1}^2 \tag{13}$$

where W_{t+1} is the investor's wealth at $t+1$, a is his or her absolute risk aversion (ARA), and $r_{p,\,t+1} = r_f + \mathbf{w}_t' r_{t+1}$ is the portfolio return at period $t+1$. Under the assumption of constant relative risk aversion, which means $\gamma_t = {}^-U''/U' = aW_t/1\,{}^-aW_t = \gamma$, the average realized utility can be used to estimate the expected utility generated by a given level of initial wealth W_0, which is as follows

$$\overline{U}(\cdot) = W_0\left(\sum_{t=0}^{T-1} r_{p,t+1} - \frac{\gamma}{2(1+\gamma)} r_{p,t+1}^2\right). \tag{14}$$

For the purposes of comparison between the static strategy and the dynamic strategy based on the selected models, we estimate the switching fees by equating the two average utility equations as follows:

$$\sum_{t=0}^{T-1}\left(r_{p,t+1}^d - \Delta\right) - \frac{\gamma}{2(1+\gamma)}\left(r_{p,t+1}^d - \Delta\right)^2 = \sum_{t=0}^{T-1} r_{p,t+1}^s - \frac{\gamma}{2(1+\gamma)}\left(r_{p,t+1}^s\right)^2 \tag{15}$$

where $r_{p,\,t+1}^s$ and $r_{p,\,t+1}^d$ denote the portfolio returns based on the static and dynamic strategies, respectively, and Δ is explained as the maximum fee that an investor would be willing to pay to switch from the static strategy to the dynamic strategy.

In addition, transaction cost is an important consideration for any dynamic strategy and has a substantial impact on the profitability of trading strategies. However, making an accurate determination of the size of transaction costs is difficult because it involves many factors. According to Han (2006), we assume that transaction costs equal a fixed proportion tc of the value traded in each asset,

$$\text{cost} = tc\left|w_t - w_{t-1}\frac{1+r_t}{1+r_{d,t}}\right|. \tag{16}$$

Due to the lack of reliable estimates of suitable transaction costs, we consider the break-even transaction cost. In comparing the dynamic strategy with the static strategy, an investor will prefer the dynamic strategy when the break-even transaction cost is high enough. Furthermore, the fact that the break-even transaction cost is much higher makes it easier to implement the dynamic strategy.

4.2. Out-of-sample evaluation results

In this section, we explore how a constant relative risk-averse investor can allocate wealth between the risk-free asset, crude oil futures and USDX futures based on different models. The out-of-sample period covers five years ranging from January 4, 2005 to December 28, 2009 with 262 observations. The rolling window method is implemented to compute the one-period-ahead expected return and covariance forecasts and then to determine the series of optimal portfolio weights. We compare the out-of-sample performance of the dynamic strategies based on selected models with the static strategy based on the constant expected return and covariance matrix. In this part, our research focuses on the performance fees Δ that an investor is willing to pay for switching from the static strategy to the dynamic strategy. The fees display the economic value of each selected model relative to the static strategy, with a target return of 5%, 10% and 15%. We present the fees with the relative risk-aversion level of $\gamma = 1$, 5, and 10.

First, in order to abstract from the issues that would be posed by expected return predictability, we assume constant expected returns and concentrate on volatility and dependence timing. Table 3 presents the out-of-sample performance fees and break-even transaction costs for the dynamic strategies based on selected models versus the static strategy for three levels of risk-aversion and three target expected returns with a minimum variance strategy. With the exception of the CCC model, the dynamic strategy models have positive performance fees, which demonstrate that the dynamic strategy is superior to the static strategy. For instance, when using the copula-based GARCH models, the investor is willing to pay from 13 to 417 annualized basis points (bps) to use the dynamic strategy rather than the static strategy. Next we compare the different dynamic models to verify their merits. We find that GARCH$^{\text{Gaussian}}$ is uniformly better than DCC. The discrepancy between the two models is produced by their residual distributions; because crude oil and exchange-rate returns differ from normality, the skewed-t distribution is better able to describe the characterization, and therefore leads to higher economic value.

Furthermore, compared with the three different marginal distributions, we find that based on each copula function, the CGARCH model performs best. This phenomenon is also concordant with the previous estimate result. We conclude that the CGARCH model is the best volatility model to explain the variations in crude oil and the exchange rate. For example, using the copula-based CGARCH dynamic strategy instead of the static strategy, the performance fee is between 35 and 497 basis points. Among all the models, CGARCH$^{\text{Student-t}}$ achieves an excellent standard. In fact, of all the selected copula functions, the Student-t copula achieves a better rating in terms of economic value despite the marginal distributions.

The impact of transaction costs is an important consideration when constructing the profitability of trading strategies. In this study, we compute the break-even transaction costs tc^{be} as the minimum proportional cost. If the transaction costs are sufficiently high, the period-by-period changes in the dynamic weights of an optimal strategy will cause the strategy to be too costly to implement relative to the static model. Comparing the dynamic strategy with the static strategy, an investor prefers the dynamic strategy when paying transaction costs that are lower than the break-even transaction costs. The break-even transaction cost values are expressed in basis points per trade and are reported only when the performance fee Δ is positive. Further, we assume that the transaction costs of crude oil and USDX futures are at the same level.

Under different relative risk-aversion levels, a high level commonly accompanies high break-even transaction costs. The results demonstrate that tc^{be} values of copula-based models are generally positive and reasonably high; as such, we conclude that the reported performance fees for the dynamic strategy represent robust to reasonably high transaction costs. After examining the forecast performance of all models in terms of performance fee and break-even transaction costs, we find that the CGARCH marginal achieves an excellent standard in all respects, while the Student-t copula performs well in most situations.

In order to evaluate whether an investor will obtain an incremental benefit due to the feedback trading information, Table 4[11] assumes that the expected returns can be predicted by the

[11] The conclusions based on CCC, DCC and the copula models with other marginals are similar to those based on the copula model with the CGARCH marginal; therefore, in the interests of space, this study only reports the results based on the CGARCH marginal.

Table 3
Out-of-sample economic value for dynamic strategy based on selected models without the feedback trading information versus static strategy with a minimum variance strategy.

Panel A: Performance fee

μ_p^*	*CCC*			*DCC*		
	$\triangle_1$	$\triangle_5$	$\triangle_{10}$	$\triangle_1$	$\triangle_5$	$\triangle_{10}$
5%	−27	−30	−34	6	16	30
10%	−56	−68	−84	16	60	115
15%	−87	−114	−149	33	131	257

μ_p^*	*GARCH*^{Gaussian}			*GARCH*$^{Student\text{-}t}$			*GARCH*^{Clayton}			*GARCH*^{SClayton}			*GARCH*^{MixClayton}		
	$\triangle_1$	$\triangle_5$	$\triangle_{10}$	$\triangle_1$	$\triangle_5$	$\triangle_{10}$	$\triangle_1$	$\triangle_5$	$\triangle_{10}$	$\triangle_1$	$\triangle_5$	$\triangle_{10}$	$\triangle_1$	$\triangle_5$	$\triangle_{10}$
5%	30	39	50	35	45	57	13	23	36	16	27	40	23	39	59
10%	64	100	147	75	113	162	31	71	121	38	79	132	53	117	197
15%	102	184	290	120	206	316	53	143	259	64	158	278	92	234	417

μ_p^*	*GJR-GARCH*^{Gaussian}			*GJR-GARCH*$^{Student\text{-}t}$			*GJR-GARCH*^{Clayton}			*GJR-GARCH*^{SClayton}			*GJR-GARCH*^{MixClayton}		
	$\triangle_1$	$\triangle_5$	$\triangle_{10}$	$\triangle_1$	$\triangle_5$	$\triangle_{10}$	$\triangle_1$	$\triangle_5$	$\triangle_{10}$	$\triangle_1$	$\triangle_5$	$\triangle_{10}$	$\triangle_1$	$\triangle_5$	$\triangle_{10}$
5%	26	35	46	32	41	53	12	22	35	15	25	38	18	34	54
10%	56	92	138	68	106	154	29	69	120	35	76	129	45	107	186
15%	91	172	276	109	194	303	51	141	256	60	154	274	78	219	399

μ_p^*	*CGARCH*^{Gaussian}			*CGARCH*$^{Student\text{-}t}$			*CGARCH*^{Clayton}			*CGARCH*^{SClayton}			*CGARCH*^{MixClayton}		
	$\triangle_1$	$\triangle_5$	$\triangle_{10}$	$\triangle_1$	$\triangle_5$	$\triangle_{10}$	$\triangle_1$	$\triangle_5$	$\triangle_{10}$	$\triangle_1$	$\triangle_5$	$\triangle_{10}$	$\triangle_1$	$\triangle_5$	$\triangle_{10}$
5%	45	58	73	50	63	79	37	50	67	37	50	67	35	53	75
10%	97	146	209	107	158	222	80	133	200	81	134	202	79	150	241
15%	154	266	409	170	284	431	130	249	401	131	252	406	132	292	497

Panel B: Break-even transaction costs

μ_p^*	*CCC*			*DCC*		
	tc_1^{be}	tc_5^{be}	tc_{10}^{be}	tc_1^{be}	tc_5^{be}	tc_{10}^{be}
5%	–	–	–	2	6	12
10%	–	–	–	3	12	23
15%	–	–	–	4	17	34

μ_p^*	*GARCH*^{Gaussian}			*GARCH*$^{Student\text{-}t}$			*GARCH*^{Clayton}			*GARCH*^{SClayton}			*GARCH*^{MixClayton}		
	tc_1^{be}	tc_5^{be}	tc_{10}^{be}	tc_1^{be}	tc_5^{be}	tc_{10}^{be}	tc_1^{be}	tc_5^{be}	tc_{10}^{be}	tc_1^{be}	tc_5^{be}	tc_{10}^{be}	tc_1^{be}	tc_5^{be}	tc_{10}^{be}
5%	11	14	19	13	17	21	6	10	15	7	11	17	8	13	20
10%	12	18	27	14	21	30	7	15	27	8	17	29	9	20	33
15%	13	23	36	15	26	40	8	21	38	9	23	40	10	26	47

μ_p^*	*GJR-GARCH*^{Gaussian}			*GJR-GARCH*$^{Student\text{-}t}$			*GJR-GARCH*^{Clayton}			*GJR-GARCH*^{SClayton}			*GJR-GARCH*^{MixClayton}		
	tc_1^{be}	tc_5^{be}	tc_{10}^{be}	tc_1^{be}	tc_5^{be}	tc_{10}^{be}	tc_1^{be}	tc_5^{be}	tc_{10}^{be}	tc_1^{be}	tc_5^{be}	tc_{10}^{be}	tc_1^{be}	tc_5^{be}	tc_{10}^{be}
5%	9	13	17	12	15	20	5	9	15	6	11	16	6	11	18
10%	10	17	25	13	20	29	6	15	26	7	16	28	7	18	31
15%	11	21	34	13	24	38	7	20	37	8	22	39	9	24	45

μ_p^*	*CGARCH*^{Gaussian}			*CGARCH*$^{Student\text{-}t}$			*CGARCH*^{Clayton}			*CGARCH*^{SClayton}			*CGARCH*^{MixClayton}		
	tc_1^{be}	tc_5^{be}	tc_{10}^{be}	tc_1^{be}	tc_5^{be}	tc_{10}^{be}	tc_1^{be}	tc_5^{be}	tc_{10}^{be}	tc_1^{be}	tc_5^{be}	tc_{10}^{be}	tc_1^{be}	tc_5^{be}	tc_{10}^{be}
5%	18	23	29	20	25	32	17	23	31	17	23	31	13	19	27
10%	19	29	42	21	32	45	19	31	48	18	31	47	14	27	44
15%	20	35	55	23	38	59	20	39	65	20	39	64	16	35	61

Note: The table presents the out-of-sample performance fees (Panel A) and break-even transaction costs (Panel B) for a dynamic strategy based on selected models with constant expected returns versus the static strategy for three target returns (5%, 10% and 15%) with a minimum variance strategy. Each minimum variance strategy builds an efficient portfolio by investing in the weekly returns of crude oil futures, USDX futures, and a risk-free asset. The fees are denoted as the amount which an investor is willing to pay for switching from the static strategy to a dynamic strategy with the relative risk aversion level $\gamma = 1$, 5 and 10. The performance fee ($\triangle$) is expressed in annualized basis points. The break-even transaction cost (tc^{be}) is defined as the minimum proportional cost per trade for which the dynamic strategies would have the same utility as the static strategy. In addition, (tc^{be}) values are reported only when $\triangle$ is positive. The out-of-sample period runs from January 2, 2005 to December 28, 2009.

feedback trading information and computes the performance fees and break-even transaction costs under the CGARCH marginal. With respect to the performance fee, the values for a less risk-averse investor are smaller than those associated with the CGARCH models that lack feedback trading information. In contrast, the values for a more risk-averse investor are larger and can be as high as 2353 basis points per year. However, after considering the impact of transaction costs, we find that the break-even transaction costs with the feedback trading information are apparently smaller than those without the feedback trading information, suggesting that incorporating the feedback trading information into the investment strategies does not enhance the economic value.

5. Conclusions

In recent years, both oil commodity prices and the US dollar currency have experienced unprecedented high fluctuations while exhibiting significantly opposite trends. This negative relationship has enabled the

Table 4
Out-of-sample economic value for dynamic strategy based on the component GARCH model with the feedback trading information versus static strategy with a minimum variance strategy.

Panel A: Performance fee

μ_p^*	$CGARCH^{Gaussian}$			$CGARCH^{Student\text{-}t}$			$CGARCH^{Clayton}$			$CGARCH^{SClayton}$			$CGARCH^{MixClayton}$		
	$\triangle_1$	$\triangle_5$	$\triangle_{10}$	$\triangle_1$	$\triangle_5$	$\triangle_{10}$	$\triangle_1$	$\triangle_5$	$\triangle_{10}$	$\triangle_1$	$\triangle_5$	$\triangle_{10}$	$\triangle_1$	$\triangle_5$	$\triangle_{10}$
5%	−161	−32	130	−153	−24	139	−158	−29	135	−170	−41	123	−163	−33	132
10%	−257	260	908	−241	277	927	−252	268	922	−276	245	898	−262	264	923
15%	−289	876	2316	−265	902	2347	−281	892	2342	−317	857	2308	−294	890	2353

Panel B: Break-even transaction costs

μ_p^*	$CGARCH^{Gaussian}$			$CGARCH^{Student\text{-}t}$			$CGARCH^{Clayton}$			$CGARCH^{SClayton}$			$CGARCH^{MixClayton}$		
	tc_1^{be}	tc_5^{be}	tc_{10}^{be}	tc_1^{be}	tc_5^{be}	tc_{10}^{be}	tc_1^{be}	tc_5^{be}	tc_{10}^{be}	tc_1^{be}	tc_5^{be}	tc_{10}^{be}	tc_1^{be}	tc_5^{be}	tc_{10}^{be}
5%	–	–	4	–	–	4	–	–	4	–	–	4	–	–	4
10%	–	4	14	–	4	14	–	4	14	–	4	14	–	4	14
15%	–	9	23	–	9	24	–	9	24	–	9	23	–	9	24

Note: The table presents the out-of-sample performance fees (Panel A) and break-even transaction costs (Panel B) for a dynamic strategy based on the component GARCH model with feedback trading information versus the static strategy for three target returns (5%, 10% and 15%) with a minimum variance strategy. Each minimum variance strategy builds an efficient portfolio by investing in the weekly returns of crude oil futures, USDX futures, and a risk-free asset. The fees are denoted as the amount which an investor is willing to pay for switching from the static strategy to a dynamic strategy with the relative risk aversion level $\gamma = 1$, 5 and 10. The performance fee ($\triangle$) is expressed in annualized basis points. The break-even transaction cost (tc^{be}) is defined as the minimum proportional cost per trade for which the dynamic strategies would have the same utility as the static strategy. In addition, (tc^{be}) values are reported only when $\triangle$ is positive. The out-of-sample period runs from January 2, 2005 to December 28, 2009.

oil commodity and the US dollar currency to serve as useful tools for strategic asset allocation and risk management. For these reasons, forecasts of the volatility and co-movement structures of oil price and exchange-rate returns have attracted much attention among academics and institutional investors.

However, it has been demonstrated that oil price and exchange-rate returns are skewed and leptokurtic, and may follow extremely dissimilar marginal distributions as well as different degrees of freedom parameters. The relationship structure between oil price and exchange-rate returns may also exhibit an asymmetric or tail dependence structure. Therefore, in order to address the drawbacks of the conventional multivariate GARCH model, this paper proposes three classes of copula-based GARCH models to elastically describe the volatility and dependence structure of oil price and US dollar exchange-rate returns. In addition, a modified CAPM is employed to explore the speculative trading behaviors in the oil and exchange-rate markets; the results reveal that feedback trading activities are significant in the crude oil market but insignificant in the USDX market under all marginal models. The CGARCH model with the Gaussian copula possesses better explanatory ability for crude oil and USDX futures returns, suggesting that the tail dependence structure between crude oil price and USDX futures returns is not apparent. In addition, the leverage effects are demonstrated to be insignificant for both crude oil and USDX futures. Based on the marginal distribution with the component GARCH model, we find that the persistence of short-run volatility is apparently smaller than that of long-run volatility for crude oil futures, while it is not significant for USDX futures. We also observe that the dependence structure between crude oil and US dollar exchange-rate returns becomes negative and decreases continuously after 2003, unlike the pattern of the preceding period.

In addition, in order to examine whether copula-based GARCH models can benefit an investor, we evaluate the economic value of our models by implementing a strategic asset-allocation problem. In terms of out-of-sample results, we find that the dynamic strategies based on the copula-based GARCH models outperform the static strategy and other dynamic strategies based on the CCC GARCH and DCC GARCH models, which demonstrates that the skewness and leptokurtosis of crude oil and USDX futures returns are economically significant. Furthermore, the CGARCH model with the Student-t copula yields the highest performance fees and break-even transaction costs to attract investors to switch their trading strategy; it also performs the best among all selected models. In addition, although the feedback trading information is statistically significant in the crude oil market, it does not help investors to enhance their economic benefits pertaining to an asset-allocation decision. Finally, more risk-averse investors are willing to pay higher fees to switch their strategy from a static strategy to a dynamic strategy based on copula-based GARCH models.

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