

# Nonuniversality of the intrinsic inverse spin-Hall effect in diffusive systems

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We studied the electric current induced in a two-dimensional electron gas by the spin current, in the presence of Rashba and cubic Dresselhaus spin-orbit interactions. We found out that the factor relating the electric and spin currents is not universal, but rather depends on the origin of the spin current. Drastic distinction has been found between two cases: the spin current created by diffusion of an inhomogeneous spin density, and the pure homogeneous spin current. We found that in the former case the inverse spin-Hall effect electric current is finite, while it turns to zero in the latter case, if the spin-orbit coupling is represented by Rashba interaction.

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## I. INTRODUCTION

The spin-Hall effect (SHE) and the inverse spin-Hall effect (ISHE) can be observed in two- and three-dimensional electron systems with a strong enough spin-orbit interaction (SOI).<sup>1,2</sup> Via this interaction the electric current induces a flux of spin polarization flowing in the perpendicular direction and vice versa. These effects take place in metals and semiconductors, where the spin-orbit interaction arises from impurity scattering, or band structure effects. Nowadays they are being intensively studied theoretically (for a review see Ref. 3) and experimentally.<sup>4,5</sup> These phenomena establish an important connection between spin and charge degrees of freedom that can be employed in spintronic applications.

Here we will focus on ISHE. This effect is driven by the spin current which can be produced in different ways. It can be created by diffusion of an inhomogeneous spin polarization, or it can be induced directly by a motive force of various natures.<sup>6,7</sup> In experimental studies the former method was used in Refs. 5,8, while the latter was employed in Refs. 9,10. From the theoretical point of view there are two quite distinct mechanisms of ISHE, depending on the extrinsic or intrinsic nature of SOI in an electron system. The extrinsic effect is promoted by the spin-orbit scattering of electrons from impurities.<sup>2</sup> The intrinsic effect is associated with the spin-orbit splitting of electron energy bands. This effect has been studied in Ref. 11 together with the extrinsic mechanism. A surprising result of this study is that the finite inverse SHE takes place even in the case of a pure intrinsic Rashba<sup>12</sup> SOI, while the direct effect has been shown to vanish in the considered case of a diffusive system.<sup>13</sup> A reasonable explanation is that the Onsager relation between direct (SHE) and reciprocal (ISHE) effects should not be satisfied, because the spin-current is not conserving. This argument also means that for the ISHE effect the coefficient in the local linear dependence  $I_c = CI_s$  of the charge current density  $I_c$  from the spin current density  $I_s$  can depend on the source that originally excites  $I_s$ . In this sense ISHE is not universal. At the same time, SHE is a universal effect, because the coefficient relating  $I_s$  to  $I_c$  does not depend on how the electric current is produced. It can be created, for example, by electron diffusion, as well as by an external electric field. The result will be the same. It follows from the gauge invariance of the electromagnetic

field. Formally, one obtains the same spin current, independent of whether it is induced by the scalar electric potential or time-dependent vector potential.

In order to demonstrate the nonuniversality of ISHE we will consider two kinds of spin-current sources. In the first case, an inhomogeneous spin polarization parallel to the  $z$  axis creates the spin flux due to spin diffusion. In the second case, the spin current is driven by a spatially uniform “electric” field, such that the fields acting on up and down spins have opposite signs. The latter situation corresponds to spin current generation mechanisms suggested in Refs. 6,7. Our goal is to show that the factors  $C$  are different in these two situations. Since our analysis has shown that in the case of the Rashba spin-orbit interaction  $C = 0$  for the source of the second kind, we will consider the cubic Dresselhaus interaction, as well, and demonstrate that the Onsager relation holds in this case.

The outline of the paper is as follows. In Sec. II the linear response equations relating the spin and charge currents to the auxiliary fields will be written for a disordered two-dimensional degenerate electron gas (2DEG). From this pair of equations the auxiliary fields can be excluded and linear relations between the electric and spin currents can be established. In Sec. III this theory will be applied to the cases with Rashba (Sec. III A) and Dresselhaus (Sec. III B) spin-orbit couplings. The discussion of results will be presented in Sec. IV.

## II. LINEAR RESPONSE THEORY

The Hamiltonian of the electron system has the form

$$H = H_0 + V, \quad (1)$$

where  $H_0$  is the unperturbed Hamiltonian of the 2DEG, which includes the electrons' spin-orbit coupling and their scattering on randomly distributed spin-independent elastic scatterers. The spin-orbit coupling has the general form

$$H_{so} = \mathbf{h}_k \cdot \boldsymbol{\sigma}, \quad (2)$$

where the effective magnetic field  $\mathbf{h}_k$  is a function of the electron momentum  $\mathbf{k}$  and  $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$  is the vector of Pauli matrices. In general,  $\mathbf{h}_k$  can be generated by the bulk-inversion asymmetry in the bulk and structure-inversion asymmetry in a quantum well (QW).<sup>14</sup> The perturbation term  $V = V_1 + V_2$

represents interactions of electrons with the auxiliary fields. We will consider two types of fields. The first one is a slowly varying in time nonuniform Zeeman field  $B$  which is directed perpendicular to the 2DEG ( $z$  direction). The corresponding interaction Hamiltonian is

$$V_1 = \sigma_z B. \quad (3)$$

Another interaction is

$$V_2 = \sigma_z \mathbf{k} \cdot \mathbf{A}. \quad (4)$$

This Hamiltonian contains the uniform spin-dependent field  $\sigma_z \mathbf{A}$ , where  $\mathbf{A}$  slowly varies in time. Such a field induces the spin current by driving in opposite directions electrons having opposite spins. It can be created, for example, by

applying a time-dependent strain to a noncentrosymmetric semiconductor. Indeed, as known<sup>15</sup> the strain field  $u_{xz}$  gives rise to the spin-orbit interaction  $\alpha \sigma_z u_{xz} k_x$ . Hence, in this case  $A_x = \alpha u_{xz}$ . Other mechanisms<sup>6,7</sup> of creating homogeneous spin currents can also be presented in a form of a spin-dependent vector potential that is able to drive spins.

Within the linear response theory the spin  $I^s$  and charge  $I^c$  currents of noninteracting electrons can be written in terms of retarded  $G_{\mathbf{k},\mathbf{k}'}^r(\omega)$  and advanced  $G_{\mathbf{k},\mathbf{k}'}^a(\omega)$  single-particle Green's functions. Due to impurity scattering these functions are nondiagonal with respect to the wave vectors  $\mathbf{k}$  and  $\mathbf{k}'$ . The linear response expressions for the currents, as functions of the frequency  $\Omega$  and wave vector  $\mathbf{q}$ , at  $\Omega \rightarrow 0$  are given by

$$\begin{aligned} \mathbf{I}^{s/c}(\Omega, \mathbf{q}) = & -i \sum_{\mathbf{k}, \mathbf{k}'} \int \frac{d\omega}{2\pi} \langle \text{Tr} \{ [G_{\mathbf{k},\mathbf{k}'}^r(\omega) - G_{\mathbf{k},\mathbf{k}'}^a(\omega)] \mathbf{j}_{s/c} G_{\mathbf{k}'+\mathbf{q},\mathbf{k}+\mathbf{q}}^a(\omega + \Omega) V(\Omega, \mathbf{q}) n_F(\omega) \\ & + G_{\mathbf{k},\mathbf{k}'}^r(\omega) \mathbf{j}_{s/c} [G_{\mathbf{k}'+\mathbf{q},\mathbf{k}+\mathbf{q}}^r(\omega + \Omega) - G_{\mathbf{k}'+\mathbf{q},\mathbf{k}+\mathbf{q}}^a(\omega + \Omega)] V(\Omega, \mathbf{q}) n_F(\omega + \Omega) \} \rangle \\ & + \frac{1}{2} \sum_{\mathbf{k}} n_F(E_{\mathbf{k}}) \text{Tr} \left[ \tau_{s/c}, \frac{\partial V(\Omega, \mathbf{q})}{\partial \mathbf{k}} \right]_+, \end{aligned} \quad (5)$$

where the spin-current and charge-current operators have the conventional form<sup>16</sup>  $\mathbf{j}_{s/c} = (1/2)[\tau_{s/c}, \mathbf{v}]_+$ , with  $\mathbf{v} = \mathbf{k}/m^* + \partial(\mathbf{h}_{\mathbf{k}} \cdot \boldsymbol{\sigma})/\partial \mathbf{k}$  and  $\tau_s = \sigma_z$ ,  $\tau_c = e$ ;  $n_F(\omega)$  is the Fermi distribution. In the following the low-temperature case will be assumed, so that  $n_F(\omega + \Omega) \simeq n_F(\omega) - \Omega \delta(\omega)$ . The angular brackets denote averaging over disorder. This averaging will be performed within the semiclassical approximation, according to the standard procedure,<sup>17</sup> where we will neglect the weak-localization corrections. We will assume that the spatial variations of the external field are slow within the electron mean-free path  $l$ , so that  $lq \ll 1$ . This case corresponds to the diffusion approximation, implying the expansion of Eq. (5) in powers of  $q$ . Also the SOI field will be assumed weak enough that  $h_{k_F} \ll 1/\tau$ , where  $\tau$  is the mean electron scattering time.

### III. INVERSE SPIN-HALL EFFECT

#### A. Rashba SOI

Let us first consider ISHE in the case of Rashba spin-orbit interaction, where the spin-orbit field is linear in  $k$  and has the form  $\mathbf{h}_{\mathbf{k}} \equiv \mathbf{h}_{\mathbf{k}}^R = \alpha \mathbf{k} \times \hat{z}$ . If the auxiliary field is  $V_1$ , given by Eq. (3), it creates a nonequilibrium and nonuniform in space spin polarization  $S_z$ . This distribution of electron spins relaxes to the uniform state via diffusion that is accompanied by a pure spin current. When  $V(\Omega, \mathbf{q})$  in Eq. (5) is represented by  $V_1(\Omega, \mathbf{q})$ , the last term in this expression vanishes. Also, the terms containing the products  $G^r G^r$  and  $G^a G^a$  can be shown to vanish, at least up to linear in  $q$  terms. Since in the following the higher-order terms starting from  $q^2$  will be ignored, only the products of the form  $G^r G^a$  will be retained in Eq. (5). We assume that  $B$  in Eq. (3) varies in the  $x$  direction, so that  $I^c$  is expected to flow in the  $y$  direction. In Fig. 1 the Feynman diagrams contributing to Eq. (5), where  $V = V_1$ ,

are shown. The upper (lower) arms in the diagrams denote the impurity averaged functions  $G_{\mathbf{k}}^{r(a)}(\omega) = (\omega - E_{\mathbf{k}} - \mathbf{h}_{\mathbf{k}} \cdot \boldsymbol{\sigma} \pm i\Gamma)^{-1}$  and the dashed lines depict the random scattering potential correlator  $\langle |U_{\mathbf{k}}|^2 \rangle$ . For simplicity this correlator will be assumed short-ranged, i.e., independent of  $k$ , so that  $\Gamma = \pi N_F \langle |U_{\mathbf{k}}|^2 \rangle \equiv \pi N_F |U|^2 = 1/2\tau$  is simply a constant. The multiple scattering blocks in the diagrams shown in Figs. 1(b) and 1(d) represent processes where the initial electron spin density  $S_z$  evolves in the diffusion process to  $S_i$ . Since this process is accompanied by the spin precession due to Rashba SOI,  $i$  can be either  $z$  or  $x$ , as follows from the spin diffusion equation<sup>18</sup> for the spin polarization varying in space along the  $x$  coordinate. In general such a diffusion-precession dynamics

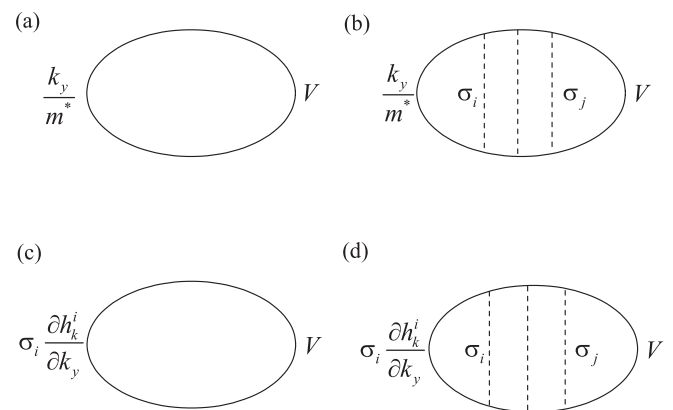


FIG. 1. The Feynman diagrams for the charge current generated by the intrinsic spin-Hall effect. The auxiliary field  $V$  can be either  $V_1$  or  $V_2$ , where  $V_1$  and  $V_2$  are defined by Eq. (3) and Eq. (4), respectively.

is represented by the diffusion propagator  $D_{ij}(\mathbf{q})$ . In the matrix form it can be represented as

$$D_{ij} = [(1 - |U|^2 \Psi/2)^{-1}]_{ij}, \quad (6)$$

$$\Psi_{ij} = \sum_{\mathbf{k}} \text{Tr}[\sigma_i G_{\mathbf{k}+\mathbf{q}}^r(\omega) \sigma_j G_{\mathbf{k}}^a(\omega)]. \quad (7)$$

Using the above definition, the contribution of all four types of diagrams in Fig. 1 can be written as

$$I_y^c = i \frac{\Omega}{2\pi} B \left( K_{yz} D_{zz} + K_{yx} D_{xz} + \alpha \frac{2\pi N_F}{\Gamma} D_{xz} \right), \quad (8)$$

where

$$K_{ij} = \sum_{\mathbf{k}} \frac{k_i}{m^*} \text{Tr}[G_{\mathbf{k}+\mathbf{q}}^r(\omega) \sigma_j G_{\mathbf{k}}^a(\omega)]. \quad (9)$$

It is easy to see that the diagonal components of  $D$  are finite at  $q \rightarrow 0$ , while the nondiagonal ones vanish as the first power of  $q$ . Therefore, in the leading approximation the correlator  $K$  in the second term of Eq. (8) must be calculated at  $q = 0$ . Up to the small semiclassical corrections of the order of  $(\alpha k_F/E_F)^3$  this correlator is  $K_{yx} = -2\pi\alpha N_F/\Gamma$ , and the last two terms cancel each other. At the same time, it is easy to see that  $K_{yy}$  is 0 at  $q = 0$ . Therefore, we did not include the corresponding term  $K_{yy} D_{yz}$  into Eq. (8). Further, as follows from Eq. (9), the correlator  $K_{yz}$  is proportional to  $\mathbf{h}_{\mathbf{k}} \times \mathbf{h}_{\mathbf{k}+\mathbf{q}}$ . Therefore, it turns to 0 at  $q = 0$ . In the leading approximation one finds from Eqs. (8) and (9) that  $K_{yz} = -i\pi q \alpha^2 N_F k_F^2 / 2m^* \Gamma^3$  and

$$I_y^c = \frac{\Omega}{2\pi} q B D_{zz} \frac{\alpha^2 k_F^2}{4\Gamma^3}. \quad (10)$$

Our goal is to get an expression of the charge current through the spin current  $I_x^s$ . Therefore, the next step is to calculate the spin current induced by the perturbation  $B\sigma_z$ . This current can be written in the form

$$I_x^s = i \frac{\Omega}{2\pi} B \left( R_{zx}^x D_{xz} + D_{zz} \sum_{\mathbf{k}} \frac{k_x}{m^*} \text{Tr}[\sigma_z G_{\mathbf{k}+\mathbf{q}}^r(\omega) \sigma_z G_{\mathbf{k}}^a(\omega)] \right), \quad (11)$$

where

$$R_{jk}^i = \sum_{\mathbf{k}} \frac{k_i}{m^*} \text{Tr}[\sigma_j G_{\mathbf{k}+\mathbf{q}}^r(\omega) \sigma_k G_{\mathbf{k}}^a(\omega)]. \quad (12)$$

The second term in the large parentheses of Eq. (11) is equal to  $-i\pi N_F q v_F^2 D_{zz} / 2\Gamma^2$ . This term represents the diffusion spin current. In its turn the first term is associated with spin precession caused by the Rashba field. It takes a simple form in the case when  $q \ll \alpha m^*$ , that is, when spatial variations of the Zeeman field are slower than spin-density variations caused by spin precession in the SOI field. In this case it follows from Eq. (6) that  $D_{xz} = |U|^2 \Psi_{xz} D_{xx} D_{zz} / 2$ . A straightforward calculation using Eqs. (6), (7), and (12) gives for the first term in the large parentheses of Eq. (11) the expression  $i\pi N_F q v_F^2 D_{zz} / \Gamma^2$ , which is twice larger and has opposite sign with respect to the second term. Finally, from Eqs. (10) and (11) the charge current becomes

$$I_y^c = -e \frac{\alpha^2 m^*}{\Gamma} I_x^s. \quad (13)$$

This result coincides with Ref. 11, taking into account that  $2\Gamma = 1/\tau$  and that the definition of  $I_x^s$  in Ref. 11 differs by the factor  $1/2$ .

The next example is the charge current induced in the  $y$  direction by the external perturbation given by Eq. (4), where  $\mathbf{A}$  is parallel to the  $x$  axis. In this case the last term in Eq. (5) turns to zero, along with the terms containing the products  $G^r G^r$  and  $G^a G^a$ . Further, a simple inspection of diagram (a) in Fig. 1 shows that it is zero at  $q = 0$ . The contribution of other diagrams to  $I_y^c$  can be expressed as

$$I_y^c = i \frac{\Omega}{2\pi} A \sum_i \left( \frac{|U|^2}{2} K_{yi} + \frac{\partial h_{\mathbf{k}}^i}{\partial k_y} \right) D_{ii} R_{iz}^x, \quad (14)$$

where the first term corresponds to Fig. 1(b), while the second one is given by Figs. 1(c) and 1(d). Since  $q = 0$ , only diagonal components of  $D$  enter in Eq. (14). Also, at  $q = 0$  only  $i = x$  must be retained in the sum. As a result, after calculation of  $K_{yx}$ , one can see that the sum in the large parentheses turns into zero, up to the small semiclassical corrections of the order of  $(\alpha k_F/E_F)^3$ . Therefore, within the semiclassical approximation the homogeneous pure spin current cannot induce ISHE. At the same time the spin current created by this source is finite and is given by the Drude formula

$$I_x^s = i m^* \Omega A \frac{v_F^2 N_F}{2\Gamma}. \quad (15)$$

This expression does not depend on the spin-orbit coupling. The latter enters as a small correction  $\sim h_k^2 \tau^2$ .

Our calculations in this subsection show that ISHE is not universal. The electric current induced by this effect is finite, or zero, depending on whether the spin-current is produced by diffusion of an inhomogeneous spin polarization or is a pure uniform spin flux created by an external force of the form Eq. (4). The driving force of this sort could be taken into account within the formalism employed in Ref. 19. We, however, cannot directly see whether their expressions for spin and charge currents give, as we expect, vanishing ISHE, because these equations are presented in a rather general form.

## B. Dresselhaus SOI

Although at  $V = V_2$  and for SOI given by the Rasha interaction the electric current is zero, we do not expect that the same takes place for a Dresselhaus SOI that is cubic in  $k$ . The reason is that the spin-Hall effect does not vanish in the latter case.<sup>20</sup> The Dresselhaus SOI field in a quantum well grown along the [001] direction is given by<sup>21</sup>

$$h_{\mathbf{k}}^x = \beta k_x (k_y^2 - \kappa_z^2), \quad h_{\mathbf{k}}^y = \beta k_y (\kappa_z^2 - k_x^2), \quad (16)$$

where  $\kappa_z^2$  denotes the operator  $-(\partial/\partial z)^2$  averaged over the lowest subband wave function. Since  $h_{\mathbf{k}}^i$  is a nonlinear function of  $\mathbf{k}$ ,  $\nabla_{\mathbf{k}} h_{\mathbf{k}}^i$  entering into Eq. (14) is not a constant. Therefore Eq. (14) has to be modified. Denoting by a bar the average  $\overline{\nabla_{\mathbf{k}} h_{\mathbf{k}}^i}$  over the Fermi surface, the modified expression for the

current can be written in the form

$$I_y^c = i \frac{\Omega}{2\pi} A \left[ \sum_i \left( \frac{|U|^2}{2} K_{yi} + \overline{\frac{\partial h_{\mathbf{k}}^i}{\partial k_y}} \right) D_{ii} R_{iz}^x + \frac{|U|^2}{2} \sum_i \Theta_i D_{ii} R_{iz}^x + \Phi \right], \quad (17)$$

where

$$\Theta_i = \sum_{j\mathbf{k}} \left( \frac{\partial h_{\mathbf{k}}^j}{\partial k_y} - \overline{\frac{\partial h_{\mathbf{k}}^j}{\partial k_y}} \right) \text{Tr}[\sigma_j G_{\mathbf{k}}^r(\omega) \sigma_i G_{\mathbf{k}}^a(\omega)] \quad (18)$$

and

$$\Phi = \sum_{i\mathbf{k}} k_x \left( \frac{\partial h_{\mathbf{k}}^i}{\partial k_y} - \overline{\frac{\partial h_{\mathbf{k}}^i}{\partial k_y}} \right) \text{Tr}[\sigma_j G_{\mathbf{k}}^r(\omega) \sigma_z G_{\mathbf{k}}^a(\omega)]. \quad (19)$$

It is easy to see that the first term in Eq. (17) turns to zero, similar to Eq. (14) in the Rashba case. However, other two terms are finite, while they vanish for Rashba SOI, as well as for any other SOI which depends linearly on  $\mathbf{k}$ . Taking SOI in the form of Eq. (16), from definitions (18), (19), (12), and (6)–(7) one obtains at  $q = 0$

$$\begin{aligned} R_{yz}^x &= -2\pi \frac{N_F}{\Gamma^2} \overline{h_{\mathbf{k}}^x k_x}, \quad R_{xz}^x = 0, \\ \Phi &= -2\pi \frac{N_F}{\Gamma^2} \left( \overline{\frac{\partial h_{\mathbf{k}}^y}{\partial k_y} h_{\mathbf{k}}^x k_x} - \overline{\frac{\partial h_{\mathbf{k}}^x}{\partial k_y} h_{\mathbf{k}}^y k_x} - \overline{\frac{\partial h_{\mathbf{k}}^y}{\partial k_y} h_{\mathbf{k}}^x k_x} \right), \\ \Psi_{xx} = \Psi_{yy} &= \frac{2\pi N_F}{\Gamma} - \frac{\pi N_F}{\Gamma^3} \overline{h_{\mathbf{k}}^2}, \\ D_{xx} = D_{yy} &= \frac{2\Gamma^2}{h_{\mathbf{k}}^2}. \end{aligned} \quad (20)$$

Since only  $R_{yz}^x$  is finite in Eq. (17), one has to calculate  $\Theta_y$ . From Eq. (18) it can be expressed as

$$\Theta_y = \pi \frac{N_F}{\Gamma^3} \left( 2 \overline{\frac{\partial h_{\mathbf{k}}^x}{\partial k_y} h_{\mathbf{k}}^x h_{\mathbf{k}}^y} - 2 \overline{\frac{\partial h_{\mathbf{k}}^y}{\partial k_y} h_{\mathbf{k}}^{x2}} + \overline{\frac{\partial h_{\mathbf{k}}^y}{\partial k_y} h_{\mathbf{k}}^2} \right). \quad (21)$$

Collecting all together one obtains from Eq. (17)

$$\begin{aligned} I_y^c &= ieA\Omega \frac{N_F}{\Gamma^2} \left[ 2 \overline{\frac{h_{\mathbf{k}}^x k_x}{h_{\mathbf{k}}^2}} \left( \overline{\frac{\partial h_{\mathbf{k}}^y}{\partial k_y} h_{\mathbf{k}}^{x2}} - \overline{\frac{\partial h_{\mathbf{k}}^x}{\partial k_y} h_{\mathbf{k}}^x h_{\mathbf{k}}^y} \right) \right. \\ &\quad \left. + \overline{\frac{\partial h_{\mathbf{k}}^x}{\partial k_y} h_{\mathbf{k}}^y k_x} - \overline{\frac{\partial h_{\mathbf{k}}^y}{\partial k_y} h_{\mathbf{k}}^x k_x} \right]. \quad (22) \end{aligned}$$

This electric current can now be expressed through the spin current. The latter is induced by the time-dependent ‘‘vector potential’’  $A$  in (4) and is given by (15). Denoting by  $Q$  the expression in the square brackets of Eq. (22), one obtains

$$I_y^c = \frac{2eQ}{\Gamma m^* v_F^2} I_x^s. \quad (23)$$

Taking into account that  $Q \propto h_{\mathbf{k}}^2$  it is easy to see that the charge-to-spin current ratio is of the same order of magnitude as in the case considered in Sec. III A, Eq. (13), provided that the Rashba and Dresselhaus interactions are comparable in their strengths. One more useful relation can be obtained by using the expression for the spin-Hall conductivity derived in Refs. 20,22. This conductivity can be written as  $\sigma_{SH} = eN_F Q / \Gamma^2$ . Expressing  $Q$  in Eq. (23) through  $\sigma_{SH}$ , and writing the electric conductivity in the form of the Einstein relation  $\sigma = 2N_F D$ , we find

$$I_y^c = \frac{\sigma_{SH}}{\sigma} I_x^s. \quad (24)$$

On the other hand, the spin current induced by the spin-Hall effect is given by  $I_x^s = \sigma_{SH} E$ , where  $E$  is the electric field in the  $y$  direction. Writing it as  $E = I_c^y / \sigma$  we arrive at  $I_x^s = \sigma_{SH} I_c^y / \sigma$ . This equation, together with Eq. (24), establishes Onsager relations between spin and charge currents.

#### IV. CONCLUSIONS

Our analysis shows that the proportionality coefficient in the linear relation between the electric and spin current densities in the inverse spin-Hall effect depends on the origin of the spin current. Therefore, it is not possible to introduce a universal parameter that determines a charge to spin current response. This nonuniversality is most clearly seen in the case of Rashba SOI, where a pure spin current produced by diffusion of an inhomogeneous spin polarization gives rise to the finite electric current, while the latter is zero when the spin current is induced by a force that is uniform in space. In this situation, however, the ISHE produces a finite charge current, if SOI is represented by a Dresselhaus SOI that is nonlinear in  $k$ . It is important that in such a case the spin-Hall effect and ISHE obey the Onsager relation for coefficients relating the spin and charge currents.

It should be noted that the expressions for the spin and charge currents calculated above are related to local current densities, while what is experimentally measured are total electric currents, or electric potentials that are responses not to local spin currents, but rather to currents that are integrated over some distance (in 2D transport). For example, due to SOI the spin-current density created by spin diffusion oscillates and decays when the distance  $x$  from the spin-injection source is increasing. One has to integrate this current over  $x$  to obtain the total electric current induced by ISHE. Since the relation Eq. (13) has the local form it will be preserved after such an integration.

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