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Heat transport in partially saturated heterogeneous aquifers

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SUMMARY

This paper presents a stochastic analysis of the behaviour of heat transfer by soil moisture in a partially saturated heterogeneous aquifer. A spectral approach based on Fourier–Stieltjes representations for the perturbed quantities is used to develop closed-form expressions that describe the field-scale heat advection and the variability of temperature profile. It is shown that the field-scale heat advection is related to the variability of the specific discharge, which, in turn, is related to the variation of hydraulic conductivity. Our analysis is, therefore, focused on the impact of the correlation scale of the log saturated hydraulic conductivity field on these analytical results. It is found that the correlation scale of log saturated hydraulic conductivity has an influence on enhancing the heat advection and the variability of the mean temperature field.

Key words: Fourier analysis; Hydrology; Heat flow.

1 INTRODUCTION

The temperature of the land surface is influenced by seasonal heating and cooling. Vertical water seepage in the vadose zone near the land atmosphere interface results in a heat transport that modifies the temperature profile and, in turn, affects most reactions occurring in the vadose zone (e.g. Parlange *et al*. 1998; Sung *et al*. 2002). Therefore, it is of great importance in characterizing and predicting the heat transport processes in the vadose zone.

It is well known that the spatial variability of flow velocities attributed primarily to the spatial variations of hydraulic conductivity is responsible for the field-scale spreading of non-reactive solutes in aquifers. One possible approach to this field-scale prediction problem is to treat the aquifer properties as spatial random fields characterized by a limited number of statistical parameters in a stochastic sense. The stochastic framework, then, allows the effects of heterogeneity incorporated into the prediction and the uncertainty associated with its characterization.

Due to the analogy between the contaminant and heat transports, it is expected that the heterogeneity of natural formations also plays an important role in influencing the heat advection at field scale. Stochastic analysis of solute transport in heterogeneous aquifers has been employed in numerous studies (e.g. Gelhar & Axness 1983; Dagan 1984, 1987; Neuman *et al*. 1987; Neuman & Zhang 1990; Vomvoris & Gelhar 1990; Rehfeldt & Gelhar 1992, Russo 1993; Chang & Yeh 2007). However, the application of stochastic methods to the analysis of heat transport in partially saturated heterogeneous aquifers by taking into account the heterogeneity effect and uncertainty has so far not been attempted, and this is the task undertaken here. It is hoped that our findings will be useful in stimulating further research in this area.

2 MATHEMATICAL FORMULATION OF THE PROBLEM

Analogous to the transport of solutes by hydrodynamic dispersion in solute transport theory, a temperature-based equation describing the heat transport in a variably saturated porous medium at the local scale can be written as

$$\frac{\partial}{\partial X_i}\left(K_{\mathrm{e}}\frac{\partial T}{\partial X_i}\right) - \rho_w C_w \frac{\partial}{\partial X_i}(q_i\, T) = \rho C \frac{\partial T}{\partial t} \qquad i = 1,2,3, \tag{1}$$

where T is the temperature, the coefficient K_e includes effects of conduction through the rock–fluid matrix as well as effects of thermal dispersion, ρ_w and C_w are density and specific heat of fluid, respectively, q_i is the specific discharge in the principal coordinate directions, and ρ and C are density and specific heat of rock-fluid matrix, respectively. Eq. (1) assumes that ρ_w, C_w, ρ and C are constants. Note that the coefficients K_e and C as well as the parameter q_i are all functions of moisture content.

Under natural conditions, air pressure in the vadose zone is generally controlled at the soil-atmosphere boundary by atmospheric pressure, which normally varies over a small range. In other words, the gas pressure gradients in the vadose zone are small under natural conditions (e.g. Parker 1997). Furthermore, even if there is considerable gas flow in the vadose zone, it will not translate into much heat transport because the

$\rho_w C_w$ factor for the water phase is much larger than the $\rho_g C_g$ factor for the air phase. The heat transport by the airflow is therefore neglected in this study.

The effect of thermal dispersion is very small and negligible when compared with that of conduction (Bear 1972; Hopmans *et al*. 2002). This simplifies eq. (1) to

$$\frac{K_T}{\rho C}\frac{\partial^2 T}{\partial X_i^2} - \mu\frac{\partial}{\partial X_i}(q_i T) = \frac{\partial T}{\partial t}, \tag{2}$$

where K_T is the effective thermal conductivity (analogous to the molecular diffusion coefficient in the advection-dispersion equation for conservative solutes) at local scale and $\mu = (\rho_w C_w)/(\rho C)$. The effects of varying the thermal conductivity are minor in relation to the effects of varying the hydraulic conductivity (e.g. Anderson 2005) so that the thermal conductivity in eq. (2) is treated as a constant.

We start with considering the case of steady state flow through a variably saturated, heterogeneous porous medium, where the subsurface is dominated by the vertical flow. Within the unsaturated zone, the temperature profile is subject to seasonal variations. The 1-D vertical flow assumption reduces the heat transport eq. (2) considerable to

$$\frac{K_T}{\rho C}\frac{\partial^2 T}{\partial X_1^2} - \mu q\frac{\partial T}{\partial X_1} = \frac{\partial T}{\partial t}, \tag{3}$$

where q is the specific discharge in the vertical direction. Due to aquifer heterogeneity, a difficulty with eq. (3) is thus a matter of scale. One promising way to deal with this scale effect is to treat the specific discharge as a second-order stationary stochastic process. Then $T(X_1, t)$ in eq. (3) becomes a scale-dependent random function. Note that the assumption of a second-order stationary stochastic process for the specific discharge field is applicable, at least far enough from the soil surface and the water table.

3 STOCHASTIC ANALYSIS IN ONE DIMENSION

The stochastic partial differential eq. (3) may be approximated by expanding the input (the specific discharge) and output (temperature field) variables in terms of ensemble means and small perturbations around the mean, respectively,

$$q = Q + q', \tag{4}$$

$$T = \Gamma + T', \tag{5}$$

The equation governing the mean T field is then obtained after substituting eqs (4) and (5) into eq. (3) and subsequently taking the expected value in the resulting equation

$$\frac{K_T}{\rho C}\frac{\partial^2 \Gamma}{\partial X_1^2} - \mu Q\frac{\partial \Gamma}{\partial X_1} - \mu\frac{\partial}{\partial X_1}\langle q' T'\rangle = \frac{\partial \Gamma}{\partial t}, \tag{6}$$

where $\langle\rangle$ stands for the ensemble average, while the equation for the perturbation is developed by subtracting eq. (6) from eq. (3)

$$\frac{K_T}{\rho C}\frac{\partial^2 T'}{\partial X_1^2} - \mu Q\frac{\partial T'}{\partial X_1} - \mu q'\frac{\partial \Gamma}{\partial X_1} = \frac{\partial T'}{\partial t}. \tag{7}$$

The last term on the left-hand side of eq. (6) is referred to as macrodispersive flux in the work of Gelhar & Axness (1983) for the case of the solute transport in a saturated heterogeneous aquifer. It reflects the additional heat advection produced as the result of the correlation between specific discharge and temperature fluctuations. Note that the field-scale effect has been shown to be produced by the cross-correlation term appearing in the mean transport equation for conservative solutes (Gelhar & Axness 1983). Eq. (7) describes the temperature perturbation due to the variation of specific discharge, which provides the stochastic differential equation required to develop second moment of the temperature field in terms of the statistics of the input hydraulic parameters.

4 SPECTRAL SOLUTIONS

In analysing changes in the temperature field in time, it is convenient to introduce a moving coordinate system, $\xi = X_1 - \mu Q t$, that follows the mean advective movement of the heat distribution. As such, eqs (6) and (7) take the following forms

$$\frac{K_T}{\rho C}\frac{\partial^2 \Gamma}{\partial \xi^2} - \mu\frac{\partial}{\partial \xi}\langle q' T'\rangle = \frac{\partial \Gamma}{\partial t}, \tag{8}$$

$$\frac{K_T}{\rho C}\frac{\partial^2 T'}{\partial \xi^2} - \mu q'\frac{\partial \Gamma}{\partial \xi} = \frac{\partial T'}{\partial t}. \tag{9}$$

Under the assumption of stationary-increment random field for the perturbed quantities, the Fourier–Stieltjes integral representations is one of the most efficient approaches in solving the small-perturbation expansion of the solute transport equation (Gelhar & Axness 1983). We now proceed to solve eq. (9) in Fourier space by applying the spectral representations of the random fluctuations T' and q' into eq. (9) in terms of Fourier–Stieltjes integrals

$$T' = \int_{-\infty}^{\infty} \exp[\mathrm{i}R(\xi + \mu Qt] \, \mathrm{d}Z_T(R, t), \tag{10}$$

$$q' = \int_{-\infty}^{\infty} \exp[\mathrm{i}R(\xi + \mu Qt] \, \mathrm{d}Z_q(R), \tag{11}$$

where R is the wave number, and $\mathrm{d}Z_T(R,t)$ and $\mathrm{d}Z_q(R)$ are the complex Fourier–Stieltjes increments. The transient-state spectral relation follows from eq. (9) through the application of eqs (10) and (11) and the use of uniqueness of the representations

$$\frac{\partial}{\partial t} \mathrm{d}Z_T(R, t) + (\mathrm{i}\mu R + \alpha_T R^2) Q \, \mathrm{d}Z_T(R, t) = -\mu \frac{\partial \Gamma}{\partial \xi} \mathrm{d}Z_q(R), \tag{12}$$

where $\alpha_T = K_T / (\rho C Q)$.

It is assumed that the known initial temperature distribution in the vadose zone is uniform, that is, there is no heat advection at $t = 0$. Therefore, $\mathrm{d}Z_T = 0$ at $t = 0$. The solution of eq. (12) with $\mathrm{d}Z_T = 0$ at $t = 0$ is

$$\mathrm{d}Z_T(R, t) = \mu G \frac{1 - \exp[-(\mathrm{i}\mu R + \alpha_T R^2) Qt]}{\alpha_T R^2 + \mathrm{i}\mu R} \frac{\mathrm{d}Z_q(R)}{Q}, \tag{13}$$

where $G = -\partial \Gamma / \partial \xi$. Eq. (13) assumes that G is approximated as a constant; that is, the mean temperature field is a linear function of the spatial coordinate.

Note that in solving the perturbation eq. (9), the mean temperature gradient, $\partial \Gamma / \partial \xi$, which represents as the source term in eq. (9) must be known. It is recognized that there is a disparity in scale between the mean temperature field and the temperature perturbations. In general, the mean temperature field is a smooth function of space and time, and the perturbations fluctuate on a much smaller scale than that associated with variations in the mean. It is then possible to simplify eq. (9) by approximating the mean temperature gradient, the coefficient in eq. (9), as a constant when solving eq. (9). It is expected that the assumption of a constant mean temperature gradient will not be valid near the heat source (or the boundary) where a large temperature gradient and sharp curvature occur. This implies that the domain of the problem (eq. 9) is far enough from the boundary and within this framework the problem is simplified to an initial value problem.

Note that according to the studies on solute transport in heterogeneous unsaturated porous formations (Russo 1996, 1998; Harter & Zhang 1999), a constant mean concentration gradient will reach if the solute plume travels at least several to tens of correlation lengths of formation heterogeneity away from the source. In addition, the literature on saturated contaminant transport (e.g. Gelhar 1993; Rubin 2003) also implies that the constant mean gradient assumption is however not restrictive if the transport process is taking place away from the source of contamination.

4.1 Field-scale macrodispersion coefficient

The cross-correlation term in eq. (8) is then found by multiplying both sides of eq. (13) by the complex conjugate of $\mathrm{d}Z_q(R)$, taking the expected value, using the spectral representation theorem, and integrating over the wave number domain

$$\langle T' q' \rangle = \beta Q G, \tag{14}$$

where β is referred to as the macrodispersivity, a description of the heat advection, and in the form

$$\beta = \mu \int_{-\infty}^{\infty} \frac{1 - \exp[-(\mathrm{i}\mu R + \alpha_T R^2) Qt]}{\alpha_T R^2 + \mathrm{i}\mu R} \frac{S_{qq}(R)}{Q^2} \mathrm{d}R. \tag{15}$$

In eq. (15), $S_{qq}(R)$ denotes the spectrum of the specific discharge.

Eq. (15) relates the spectra of specific discharge variation to that of the dispersive heat flux. In other words, the heat advection at the field scale depends on the variability of the specific discharge, which, in turn, is related to the variation of the hydraulic conductivity. Note that from eqs (8) and (14), the resulting ensemble average of temperature distribution is thus governed by the following advection operator

$$(\upsilon + \beta) \mu Q \frac{\partial^2 \Gamma}{\partial \xi^2} = \frac{\partial \Gamma}{\partial t}, \tag{16}$$

where $\upsilon = \alpha_T / \mu$.

The determination of the spectrum of the specific discharge in eq. (15) can be done using the Fourier–Stieltjes representations of the random fields in the first-order perturbation approximation of Darcy's law (Yeh *et al*. 1985) expressed the specific discharge spectrum as

$$S_{qq}(R) = \frac{\alpha^2 J^2 Q^2}{R^2 + \alpha^2 (2J + 1)^2} S_{ff}(R), \tag{17}$$

where J is the gradient of mean capillary pressure head, α is a soil pore-size distribution parameter, and $S_{ff}(R)$ denotes the spectrum of the local log saturated hydraulic conductivity field. In the development of eq. (17), they assumed that the local unsaturated hydraulic conductivity K is related to the capillary pressure head ψ by $K = K_s \exp(-\alpha\psi)$, where K_s is the local saturated hydraulic conductivity.

4.2 Variation of temperature field

The amplitude relationship of eq. (13) can also be used to find the spectrum of temperature perturbation field and, in turn, the temperature variance within the following spectral framework:

$$\sigma_T^2 = \int_{-\infty}^{\infty} S_{TT}(R)\mathrm{d}R, \tag{18}$$

where $S_{TT}(R)$ is the spectrum of temperature perturbation field. Multiplying both sides of eq. (13) by its complex conjugate, taking the expected value, and using the spectral representation theorem results in the spectrum of temperature perturbation field

$$S_{TT}(R) = G^2\,\mu^2\,\frac{1-2\cos(\mu Qt R)\exp(-\alpha_T\,Qt\,R^2)+\exp(-2\,\alpha_T\,Qt\,R^2)}{\alpha_T^2\,R^4+\mu^2\,R^2}\,\frac{S_{qq}(R)}{Q^2}. \tag{19}$$

5 CLOSED-FORM SOLUTIONS

To proceed with the development of macrodispersivity eq. (15) and variance of temperature field eq. (19), one must select the form of the ln$K$s spectrum. The random ln$K$s perturbation field under consideration is characterized by the following spectral density function (Bakr *et al*. 1978)

$$S_{ff}(R) = \frac{2\,\lambda^3\,R^2}{\pi(1+\lambda^2\,R^2)^2}\,\sigma_f^2, \tag{20}$$

where λ is the correlation length of lnK_s and σ_f^2 is the variance of lnK_s.

Combining eqs (17) and (20) with eq. (15) gives the macrodispersivity as

$$\beta = \sigma_f^2\,\lambda\frac{J^2}{(2J+1)^2}\,\frac{\varsigma^2\,\eta^2}{\varsigma^2-\eta^2}$$
$$\times\left\{\begin{aligned}&\frac{\varsigma-\eta}{\eta}\,\frac{2+\eta+\varsigma}{(1+\eta)^2(1+\varsigma)^2}+\left[2\frac{\Psi(0.5\sqrt{\tau\eta})}{(1-\eta^2)^2}-\frac{\Lambda_1}{(1-\varsigma^2)^2}\right]\\&+\left[\frac{\Lambda_2}{(1-\eta^2)^2}-\frac{\Lambda_3}{(1-\varsigma^2)^2}\right]\exp\left[\tau\left(\frac{\eta+1}{\eta}\right)\right]\Psi(\varepsilon_1)+\left[\frac{\Lambda_4}{(1-\varsigma^2)^2}-\frac{\Lambda_5}{(1-\eta^2)^2}\right]\exp\left[\tau\left(\frac{\eta-1}{\eta}\right)\right]\Psi(\varepsilon_2)\\&+2\sqrt{\frac{\tau\eta}{\pi}}\left[\frac{\varsigma^2}{\eta^2}\,\frac{1}{1-\varsigma^2}-\frac{1}{1-\eta^2}\right]\exp(-\tau\eta/4)\end{aligned}\right\}, \tag{21}$$

where $\Psi(-)$ denotes the complementary error function, $\eta = \lambda\mu/\alpha_T$, $\varsigma = \lambda\alpha(2J+1)$ and $\tau = \mu Qt/\lambda$,

$$\varepsilon_1 = \left(\frac{\tau}{\eta}\right)^{0.5}+\frac{1}{2}(\tau\eta)^{0.5}, \tag{22a}$$

$$\varepsilon_2 = \left(\frac{\tau}{\eta}\right)^{0.5}-\frac{1}{2}(\tau\eta)^{0.5}, \tag{22b}$$

$$\varepsilon_3 = \left(\frac{\tau\,\varsigma^2}{\eta}\right)^{0.5}-\frac{1}{2}(\tau\eta)^{0.5}, \tag{22c}$$

$$\varepsilon_4 = \left(\frac{\tau\,\varsigma^2}{\eta}\right)^{0.5}+\frac{1}{2}(\tau\eta)^{0.5}, \tag{22d}$$

$$\Lambda_1 = \left(1+\frac{\varsigma}{\eta}\right)\exp\left[\tau\varsigma\left(\frac{\varsigma}{\eta}-1\right)\right]\Psi(\varepsilon_3)-\left(1-\frac{\varsigma}{\eta}\right)\exp\left[\tau\varsigma\left(\frac{\varsigma}{\eta}+1\right)\right]\Psi(\varepsilon_4), \tag{22e}$$

$$\Lambda_2 = -\frac{\tau}{\eta}+1+\frac{\tau}{2}+\frac{3}{2}(\tau-1)\eta-\frac{1}{2}\tau\,\eta^2+\frac{1}{2}(1-\tau)\,\eta^3, \tag{22f}$$

$$\Lambda_3 = 1-\frac{1}{2}\tau+\frac{1}{2}\tau\,\varsigma^2+\frac{1}{2\eta}[\varsigma^2(1-\tau)(\varsigma^2-3)-2\tau]+\tau\frac{\varsigma^2}{\eta^2}(1-\varsigma^2), \tag{22g}$$

$$\Lambda_4 = 1+\frac{1}{2}\tau-\frac{1}{2}\tau\,\varsigma^2+\frac{1}{2\eta}[3\,\varsigma^2(1+\tau)-\varsigma^4(1+\tau)-2\tau]+\tau\frac{\varsigma^2}{\eta^2}(\varsigma^2-1), \tag{22h}$$

$$\Lambda_5 = -\frac{\tau}{\eta}+1-\frac{\tau}{2}+\frac{3}{2}(\tau+1)\eta+\frac{1}{2}\tau\,\eta^2-\frac{1}{2}(1+\tau)\,\eta^3. \tag{22i}$$

The result of eq. (21) is presented graphically in terms of a function of time for various correlation scales of lnK_s in Fig. 1. It is clear that the correlation scale of log saturated hydraulic conductivity has a positive effect on the field-scale heat advection. An increase in the correlation

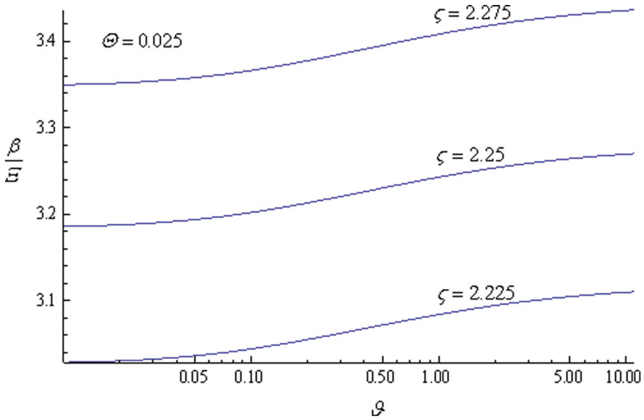

Figure 1. Dimensionless macrodispersivity versus dimensionless time for various ς, where $\Xi = \frac{\sigma_f^2 J^2}{\alpha(2J+1)^2}$, $\vartheta = \mu Q\alpha(2J+1)t$, and $\Theta = (\mu/\alpha_T)/[\alpha(2J+1)]$.

scale of $\ln K_s$ produces more persistence of the specific discharge fluctuation, which leads to larger deviations of specific discharge from the mean specific discharge. It is clear from eq. (14) that the fluctuations in the temperature field are positively correlated to those in the specific discharge. Therefore, the field-scale heat advection increases with the correlation scale of $\ln K_s$.

From eqs (17), (19) and (20), the variance of temperature field in eq. (18) can be expressed as

$$\sigma_T^2 = G^2\sigma_f^2\lambda^2\frac{J^2}{(2J+1)^2}\frac{\varsigma^2\eta^2}{\varsigma^2-\eta^2} \times \left\{ \begin{array}{l} \dfrac{\varsigma-\eta}{\varsigma\eta}\dfrac{2+(\eta+\varsigma)(2+\eta)(2+\varsigma)}{(1+\eta)^2(1+\varsigma)^2} + 2\left[\dfrac{\Omega_1}{\varsigma(1-\varsigma^2)^2} - \dfrac{\Omega_2}{\eta(1-\eta^2)^2}\right] \\ + 2\left[\dfrac{\Omega_3}{(1-\varsigma^2)^2} - \dfrac{\Omega_4}{(1-\eta^2)^2}\right]\exp\left(\tau\dfrac{1+\eta}{\eta}\right)\Psi(\varepsilon_1) + 2\left[\dfrac{\Omega_5}{(1-\varsigma^2)^2} - \dfrac{\Omega_6}{(1-\eta^2)^2}\right]\exp\left(\tau\dfrac{1-\eta}{\eta}\right)\Psi(\varepsilon_2) \\ + 4\sqrt{\dfrac{\tau}{\eta\pi}}\left(\dfrac{1}{1-\eta^2} - \dfrac{1}{1-\varsigma^2}\right)\exp\left(-\dfrac{\tau\eta}{4}\right) + \left[\dfrac{\Omega_7}{(1-\eta^2)^2} - \dfrac{\Omega_8}{(1-\varsigma^2)^2}\right]\exp\left(2\dfrac{\tau}{\eta}\right)\Psi\left(\sqrt{2\dfrac{\tau}{\eta}}\right) \\ + 2\sqrt{\dfrac{2\tau}{\eta\pi}}\left(\dfrac{1}{1-\varsigma^2} - \dfrac{1}{1-\eta^2}\right) + 2\left[\dfrac{\exp(2\tau\eta)}{\eta(1-\eta^2)^2}\Psi(\sqrt{2\tau\eta}) - \dfrac{\exp\left(2\dfrac{\tau\varsigma^2}{\eta}\right)}{\varsigma(1-\varsigma^2)^2}\Psi\left(\sqrt{2\dfrac{\tau\varsigma^2}{\eta}}\right)\right] \end{array} \right\}, \tag{23}$$

where

$$\Omega_1 = \exp\left(\tau\varsigma\frac{\varsigma+\eta}{\eta}\right)\Psi(\varepsilon_4) + \exp\left(\tau\varsigma\frac{\varsigma-\eta}{\eta}\right)\Psi(\varepsilon_3), \tag{24a}$$

$$\Omega_2 = \exp(2\tau\eta)\Psi(1.5\sqrt{\tau\eta}) + \Psi(0.5\sqrt{\tau\eta}), \tag{24b}$$

$$\Omega_3 = \frac{\tau}{\eta}(1-\varsigma^2) + \frac{1}{2}(\tau-3) + \frac{1}{2}\varsigma^2(1-\tau), \tag{24b}$$

$$\Omega_4 = \frac{\tau}{\eta} + \frac{1}{2}(\tau-3) - \tau\eta + \frac{1}{2}\eta^2(1-\tau), \tag{24c}$$

$$\Omega_5 = \frac{\tau}{\eta}(1-\varsigma^2) - \frac{1}{2}(\tau+3) + \frac{1}{2}\varsigma^2(1+\tau), \tag{24d}$$

$$\Omega_6 = \frac{\tau}{\eta} - \frac{1}{2}(\tau+3) - \tau\eta + \frac{1}{2}\eta^2(1+\tau), \tag{24e}$$

$$\Omega_7 = 4\frac{\tau}{\eta} - 3 - 4\tau\eta + \eta^2, \tag{24f}$$

$$\Omega_8 = 4\frac{\tau}{\eta}(1-\tau) - 3 + \varsigma^2. \tag{24g}$$

Fig. 2 illustrate the behaviour of the temperature variance as a function of time for various correlation scales of $\ln K_s$. It indicates that the variation of the temperature field around the mean is proportional to the correlation scale. The larger the correlation scale, the higher the variability for the specific discharge, and consequently, the higher the variation of the temperature field around the mean. This is consistent with the results of the analysis of macrodispersivity (Fig. 1).

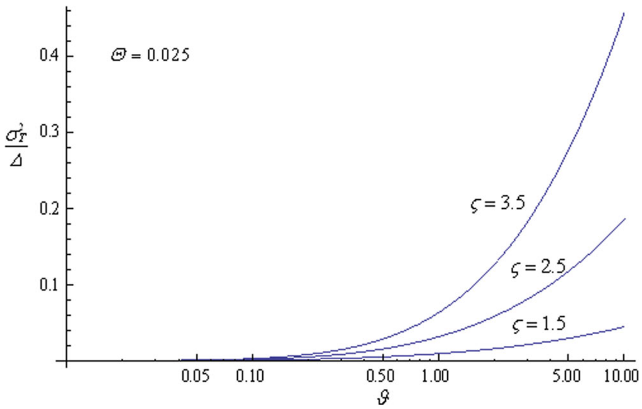

Figure 2. Dimensionless variance of temperature field versus dimensionless time for various ς, where $\Delta = \frac{G^2 \sigma_f^2 J^2}{\alpha^2 (2J+1)^4}$, $\vartheta = \mu Q \alpha (2J+1) t$, and $\Theta = (\mu/\alpha_T)/[\alpha(2J+1)]$.

The stochastic approach in the paper may provide a rational basis for extrapolating to space scales at which direct observations of dependent variables are not feasible. In addition, the variance can be used to characterize the reliability to be anticipated in applying the deterministic model. It is hoped that our findings will be useful in stimulating further research in this area.

6 CONCLUSIONS

Within the framework of stochastic theory and the spectral perturbation techniques, closed-form expressions are developed to describe the field-scale heat advection and the variability of temperature profile in a partially saturated porous heterogeneous aquifer. These results expressed in terms of the statistical properties of log saturated hydraulic conductivity and the soil parameters are allowed to investigate the impact of the correlation scale of log saturated hydraulic conductivity on the field-scale heat advection and the variability of mean temperature field.

Our results indicate that the macrodispersive heat flux is dependent of the variability of the specific discharge, which, in turn, is dependent of the variation of hydraulic conductivity. The correlation scale of log saturated hydraulic conductivity is important in enhancing the heat advection and the variability of the mean temperature field.

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