

Identification of a Threshold Value for the DEMATEL Method: Using the Maximum Mean De-Entropy Algorithm

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Abstract. To deal with complex problems, structuring them through graphical representations and analyzing causal influences can aid in illuminating complex issues, systems, or concepts. The DEMATEL method is a methodology which can be used for researching and solving complicated and intertwined problem groups. The end product of the DEMATEL process is a visual representation—the impact-relations map—by which respondents organize their own actions in the world. The applicability of the DEMATEL method is widespread, ranging from analyzing world problematique decision making to industrial planning. The most important property of the DEMATEL method used in the multi-criteria decision making (MCDM) field is to construct interrelations between criteria. In order to obtain a suitable impact-relations map, an appropriate threshold value is needed to obtain adequate information for further analysis and decision-making. In this paper, we propose a method based on the entropy approach, the maximum mean de-entropy algorithm, to achieve this purpose. Using real cases to find the interrelationships between the criteria for evaluating effects in E-learning programs as an examples, we will compare the results obtained from the respondents and from our method, and discuss that the different impact-relations maps from these two methods.

Keywords: DEMATEL, Multiple Criteria Decision Making (MCDM), entropy, Maximum Mean De-Entropy (MMDE) Algorithm.

1 Introduction

The DEMATEL (Decision Making Trial and Evaluation Laboratory) method, developed by the Science and Human Affairs Program of the Battelle Memorial Institute of Geneva between 1972 and 1976, was used to research and solve complicated and intertwined problem groups [1, 2]. The applicability of the DEMATEL method is widespread, ranging from analyzing world problematique decision making to industrial planning [3-6]. The most important property of the DEMATEL method used in

the multi-criteria decision making (MCDM) field is to construct interrelations between criteria. After the interrelations between criteria were determined, the results derived from the DEMATEL method could be used for fuzzy integrals to measure the super-additive effectiveness value or for the Analytic Network Process method (ANP) [7, 8] to measure dependence and feedback relationships between certain criteria.

There are four steps in the DEMATEL method: (1) calculate the average matrix, (2) calculate the *normalized initial direct-influence matrix*, (3) derive the *total relation matrix*, and (4) set a threshold value and obtain the *impact-relations map*. In Step 4, an appropriate threshold value is necessary to obtain a suitable impact-relations map as well as adequate information for further analysis and decision-making. The traditional method followed to set a threshold value is conducting discussions with experts. The results of the threshold values may differ among different researchers.

In contrast to the traditional method, which confronts the loop from a “set a threshold value” to obtain “the needed impact-relations-map”, we propose the Maximum Mean De-Entropy (MMDE) algorithm to obtain a unique threshold value for delineating the impact-relations map. In the numerical examples, real cases are used to discover and illustrate the intertwined effects of a structural multi-criteria decision making model.

The rest of this paper is organized as follows: Section 2 briefly describes the DEMATEL method. The steps of the maximum mean de-entropy algorithm will be described, explained, and discussed in Section 3. In Section 4, numerical examples are shown in order to explain the proposed algorithm and discuss the results. Finally, in Section 5, we draw conclusions.

2 DEMATEL Method

The end product of the DEMATEL process—the impact-relations map—is a visual representation of the mind by which the respondent organizes his or her own action in the world. This organizational process must occur for the respondent to keep internally coherent and to reach his or her personal goals. The steps of the DEMATEL method [6] are described as follows:

Step 1: Find the average matrix.

Suppose there are h experts available to solve a complex problem and there are n factors to be considered. The scores given by each expert give us a $n \times n$ non-negative answer matrix X^k , with $1 \leq k \leq h$. Thus X^1, X^2, \dots, X^h are the answer matrices for each of the h experts, and each element of X^k is an integer denoted by x_{ij}^k . The diagonal elements of each answer matrix X^k are all set to zero. We can then compute the $n \times n$ average matrix A by averaging the h experts’ score matrices. The (i, j) element of matrix A is denoted by a_{ij} ,

$$a_{ij} = \frac{1}{h} \sum_{k=1}^h x_{ij}^k \tag{1}$$

Step 2: Calculate the normalized initial direct-relation matrix.

We then create a matrix D by using a simple matrix operation on A . Suppose we create matrix D and $D = s \cdot A$ where

$$s = \text{Min} \left[\frac{1}{\max_{1 \leq i \leq n} \sum_{j=1}^n |a_{ij}|}, \frac{1}{\max_{1 \leq j \leq n} \sum_{i=1}^n |a_{ij}|} \right] \tag{2}$$

Matrix D is called the *normalized initial direct-relation matrix*. The (i, j) element d_{ij} denotes the direct influence from factor x_i to factor x_j . Suppose $d_{i\bullet}$ denotes the row sum of the i -th row of matrix D .

$$d_{i\bullet} = \sum_{j=1}^n d_{ij} \tag{3}$$

The $d_{i\bullet}$ shows the sum of influence directly exerted from factor x_i to the other factors. Suppose $d_{\bullet j}$ denotes the column sum of the j -th column of matrix D .

$$d_{\bullet j} = \sum_{i=1}^n d_{ij} \tag{4}$$

Then $d_{\bullet j}$ shows the sum of influence that factor x_j received from the other factors. We can normalize $d_{i\bullet}$ and $d_{\bullet j}$ as

$$w_i(d) = \frac{d_{i\bullet}}{\sum_{i=1}^n d_{i\bullet}} \quad \text{and} \quad v_j(d) = \frac{d_{\bullet j}}{\sum_{j=1}^n d_{\bullet j}}$$

Step 3: Compute the total relation matrix.

A continuous decrease of the indirect effects of problems along the powers of matrix D , e.g. $D^2, D^3, \dots, D^\infty$, guarantees convergent solutions to the matrix inversion, similar to an absorbing Markov chain matrix. Note that $\lim_{m \rightarrow \infty} D^m = [0]_{n \times n}$, where $[0]_{n \times n}$ is the $n \times n$ null matrix. The total relation matrix T is an $n \times n$ matrix and is defined as follows:

$$\begin{aligned} \sum_{m=1}^{\infty} D^m &= D + D^2 + D^3 + \dots + D^m = D(I + D + D^2 + D^3 + \dots + D^{m-1}) \\ &= D(I - D)^{-1}(I - D^m) \\ &= D(I - D)^{-1}, \end{aligned} \tag{5}$$

where I is the identity matrix and T is called the *total relation matrix*. The (i, j) element of the matrix T , t_{ij} , denotes the full direct and indirect influence exerted from factor x_i to factor x_j . Like the formula (3) — (4), we can obtain $t_{i\bullet}$, $t_{\bullet j}$, $w_i(t)$, and $v_j(t)$.

Step 4: Set a threshold value and obtain the impact-relations map.

In order to explain the structural relationship among the factors while keeping the complexity of the system to a manageable level, it is necessary to set a threshold

value p to filter out the negligible effects in matrix T . Only those factors that have an effect in matrix T greater than the threshold value should be chosen and shown in an impact-relations map. In this Step, the threshold value can be chosen by the decision maker or through discussions with experts. If the threshold value is too low, the map will be too complex to show the necessary information for decision-making. If the threshold value is too high, many factors will be presented as independent factors, without showing the relationships with other factors. An appropriate threshold value is necessary to obtain a suitable impact-relations map as well as adequate information for further analysis and decision-making.

3 Maximum Mean De-Entropy Algorithm (MMDE)

As we mentioned above, the threshold value is determined by asking experts or by the researcher (as a decision maker). Choosing a consistent threshold value is time-consuming if the impact-relations maps are similar when threshold values are changed slightly. We propose the Maximum Mean De-Entropy (MMDE) algorithm to find a threshold value for delineating the impact-relations map [9]. In this algorithm, we use the approach of entropy, which has been widely applied in information science, but define another two information measures: *de-entropy* and *mean de-entropy*. In addition, the proposed algorithm mainly serves to search for the threshold value by nodes (or vertices). This algorithm differs from the traditional methods through which the threshold value is decided by searching a suitable impact-relations map. In this section, we use the symbol ■ as the end of a definition or a step in the proposed algorithm.

3.1 Information Entropy

Entropy is a physical measurement of thermal-dynamics and has become an important concept in the social sciences [10, 11]. In information theory, entropy is used to measure the expected information content of certain messages, and is a criterion for the amount of "uncertainty" represented by a discrete probability distribution.

Definition 1: Let a random variable with n elements be denoted as $X =$, with a corresponding probability $P = \{p_1, p_2, \dots, p_n\}$, then we define the entropy, H , of X as follows:

$$H(p_1, p_2, \dots, p_n) = -\sum p_i \lg p_i$$

subject to constraints (8) and (9):

$$\sum_{i=1}^n p_i = 1 \tag{6}$$

$$p_i \lg p_i = 0 \quad \text{if} \quad p_i = 0 \tag{7} \blacksquare$$

Definition 2: For a given finite discrete scheme of X , the *de-entropy* of X is denoted as H^D and defined as:

$$H^D = H\left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right) - H(p_1, p_2, \dots, p_n) \quad \blacksquare$$

By Definition 2, the value of H^D is equal to or larger than 0. By Definition 1, Formula (8) can be proven (the proof can be found in [12]):

$$H_n = H\left(\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\right) \leq H\left(\frac{1}{n+1}, \frac{1}{n+1}, \dots, \frac{1}{n+1}\right) = H_{n+1} \tag{8}$$

Formula (8) explains that when adding a new variable to a system where all variables in the system have the same probability, the entropy of the system will increase. To delineate an impact-relations map, if adding a new factor to the impact-relations map can make the system less uncertain, or lead to more de-entropy, then the new factor provides worthwhile information for a decision maker. In other words, in an existing information system whose variables and corresponding probabilities have been fixed, adding a new variable to the system will change the probability distribution; if $H_{n+1}^D > H_n^D$ exists, then this new variable provides useful information to avoid uncertainty for the decision maker.

3.2 The Dispatch- and Receive-Nodes

In the DEMATEL method, the (i, j) element of the matrix T , t_{ij} , refers to the full direct and indirect influence exerted from factor x_i to factor x_j . Like the “vertices” and “edges” in graph theory [13], x_i and x_j are vertices in the directed graph impact-relations map, and t_{ij} can be considered as a directed edge which connects factors x_i and x_j with an influence value.

Definition 3: The (i, j) element of the matrix T is denoted as t_{ij} and refers to a directed influence relations from factor x_i to factor x_j . For each t_{ij} , the factor x_i is defined as a *dispatch-node* and factor x_j is defined as a *receive-node* with respect to t_{ij} . ■

By Definition 2, an $n \times n$ total relation matrix T can be considered as a set (set T) with n^2 pair ordered elements. Every subset of set T can be divided into two sets: an ordered dispatch-node set and an ordered receive-node set. For an ordered dispatch-node set (or an ordered receive-node set), we can count the frequency of the different elements of the set. If the finite cardinality of an order dispatch-node set (or an ordered receive-node set) is m and the frequency of element x_i is k , we assign the corresponding probability of x_i as $p_i = \frac{k}{m}$. In this way, for an ordered set, we can assign

each different element a probability and follow Definition 1 for $\sum_{i=1}^n p_i = 1$.

Notation: In this paper, $C(X)$ denotes the cardinal number of an ordered set X and $N(X)$ denotes the cardinal number of different elements in set X .

3.3 Maximum Mean De-Entropy Algorithm

Based on a calculated total relation matrix T , the steps of the proposed Maximum Mean De-Entropy algorithm for determining a threshold value are described as follows:

- Step 1:* Transforming the $n \times n$ total relation matrix T into an ordered set T , $\{t_{11}, t_{12}, \dots, t_{21}, t_{22}, \dots, t_{nn}\}$, rearranging the element order in set T from large to small, and transforming to a corresponding ordered triplets (t_{ij}, x_i, x_j) set denotes T^* . ■
- Step 2:* Taking the second element, the dispatch-node, from the ordered triplets of the set T^* and then obtaining a new ordered dispatch-node set, T^{Di} . ■
- Step 3:* Taking the first t elements of T^{Di} as a new set T_t^{Di} , assign the probability of different elements, and then calculate the H^D of the set T_t^{Di} , H_t^{Di} . We can calculate the *mean de-entropy* by $MDE_t^{Di} = \frac{H_t^{Di}}{N(T_t^{Di})}$. At first, the t is set as 1, then of value of t is determined by raising the value from 1 to $C(T^{Di})$ in increments of 1. ■
- Step 4:* In $C(T^{Di})$ mean de-entropy values, select the *maximum mean de-entropy* and its corresponding T_t^{Di} . This dispatch-node set, with the maximum mean de-entropy, is denoted as T_{max}^{Di} . ■
- Step 5:* Similar to Steps 2 to 4, an ordered receive-node set T^{Re} and a maximum mean de-entropy receive-node set T_{max}^{Re} can be derived. ■
- Step 6:* Taking the first u elements in T^* as the subset, T^{Th} , which includes all elements of T_{max}^{Di} in the dispatch-node and all elements of T_{max}^{Re} in the receive-node, the minimum influence value in T^{Th} is the threshold value, and Formula (9) and (10) hold.

$$1 < C(T^{Th}) < C(T^*) \tag{9} \blacksquare$$

$$G(T^{Th}) = G(T_{max}^{Di}) \cup G(T_{max}^{Re}) \tag{10}$$

4 Numerical Cases of Deciding Intertwined Criteria in Evaluating E-Learning Program

The empirical experiments of this paper are demonstrated with e-learning company-training programs [6]. In [6], they applied the DEMATEL method to illustrate the interrelations among criteria and found the central criteria to represent the effectiveness of factors/aspects. The principle component analysis was used to extract nine factors from fifty-eight criteria and nine factors were chosen. According to factor analysis results, some experts and professionals were invited to discuss and scored the relation between criteria of each factor based on the DEMATEL approach. The threshold value of each factor from factor 1 to factor 9 is 0.85, 0.47, 1.5, 2.1, 1.6, 6.5, 2.1, 3.8 and 3.5.

Based on the subset obtained in Step 6, the threshold value could be determined and then the impact-relations map can be structured. In the E-learning case, the impact-relations maps derived from the MMDE algorithm are similar as that shown in Table 1. We compare the results obtained from the respondents and from our method, and find that there are six factors will be structured by same components. Another three factors,

although the numbers of components are different, have similar threshold values. This means that MMDE is a suitable method to determine a threshold value in the first, or the final, step in order to discuss the adequacy of the impact-relations map.

The proposed MMDE algorithm has some properties that differ from the traditional method to make the threshold value, as discussed below.

Table 1. The results obtained from the respondents and from MMDE method

Factors	Number of Components	Threshold values by experts	Number of Components	Threshold values using MMDE	Number of Components using MMDE
1. Personal Characteristics and System Instruction	12	0.85	6	0.84	9
2. Participant Motivation and System Interaction	14	0.47	8	0.50	7
3. Range of Instruction Materials and Accuracy	4	1.5	4	1.19	4
4. Webpage Design and Display of Instruction Materials	6	2.1	5	2.09	6
5. E-Learning Environment	6	1.6	5	1.65	5
6. Webpage Connection	4	6.5	3	6.58	3
7. Course Quality and Work Influence	6	2.1	6	1.92	6
8. Learning Records	3	3.8	3	3.43	3
9. Instruction Materials	3	3.5	3	5.67	3

1. *The MMDE mainly serves to decide the “node” rather than the “map”*

In traditional methods, the researcher set a subject adequate threshold to draw the impact-relations map and discussed it with experts to obtain a consistent opinion. In the proposed MMDE, the main issue is about whether it is suitable to add a new “node”. If adding a new node can improve the “mean de-entropy”, then adding it can be helpful to understand a problematic by decreasing the uncertainty of information.

2. *The MMDE considers the properties of both the dispatch and receive influences of a factor*

In the DEMATEL method, after a suitable map is obtained, the focus of the problem can be shown by analyzing the values w_i and v_i , as Formulas (5)—(6), of the factors in the map. Using the proposed MMDE, we search the nodes, including dispatch- and receive-nodes, simultaneously. The MMDE not only considers the factors which strongly influence others, but also the factors which are easily influenced by other factors.

3. *The MMDE can obtain a unique threshold value*

In the traditional method, the researcher may determine the threshold value by decreasing the value (this will change the impact-relations map from simple to complex) or by increasing the value (this will change the impact-relations map from complex to simple), so the results of these two methods may differ. Using the MMDE, a researcher can obtain a unique threshold value, which is helpful to solve the problem a researcher confronts in regards to selecting a consistent threshold value.

5 Conclusions

In the DEMATEL process, an appropriate threshold value is important in order to obtain adequate information to delineate the impact-relations map for further analysis and decision-making. Until now, the threshold value has been determined through discussions with respondents or chosen subjectively by researchers. It is time-consuming to make a consistent decision on the threshold value, especially when the number of factors in the problematique makes it too difficult to discuss the adequacy of an impact-relations map. If the threshold is determined by the researcher alone, it is important to clarify how to choose the specific value. A theoretical method to aid in deciding the threshold value is necessary. This paper proposed an MMDE algorithm to determine the threshold value. With this method, a unique threshold value can be obtained, solving the problem of choosing the threshold value in the traditional way.

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