

Adiabatically slow and adiabatically fast driven ratchetsV. M. Rozenbaum,^{1,2,3,*} Yu. A. Makhnovskii,^{1,4} I. V. Shapochkina,⁵ S.-Y. Sheu,⁶ D.-Y. Yang,^{1,†} and S. H. Lin^{1,2}¹*Institute of Atomic and Molecular Sciences, Academia Sinica, Taipei 106, Taiwan*²*Department of Applied Chemistry, National Chiao Tung University, 1001 Ta Hsuen Road, Hsinchu, Taiwan*³*Chuiko Institute of Surface Chemistry, National Academy of Sciences of Ukraine, Generala Naumova Street, 17, Kiev, 03164, Ukraine*⁴*Topchiev Institute of Petrochemical Synthesis, Russian Academy of Sciences, Leninsky Prospect 29, 119991 Moscow, Russia*⁵*Belorussian State University, Prospekt Nezavisimosti 4, 220050 Minsk, Belarus*⁶*Department of Life Sciences and Institute of Genome Sciences, Institute of Biomedical Informatics, National Yang-Ming University, Taipei 112, Taiwan*

(Received 22 January 2012; published 12 April 2012)

We revisit two known models of deterministically driven ratchets, which exhibit high energetic efficiency, with the goal to uncover similarities and differences in the principles of their operation. Both the models rely on adiabaticity of the potential change process, however, the adiabaticity that we deal with in the two cases is of different types, slow and fast. It is shown that in the former (latter) case the drift velocity is an even (odd) functional of the potential, with the notable consequence that for the adiabatically slow driven ratchet the necessary symmetry breaking occurs only due to time-dependent parametric perturbations, while the spatial asymmetry of the potential is a mandatory condition for the adiabatically fast driven ratchet to operate. To treat energetic characteristics, the models are restated in terms of traveling potential ratchets. With such an approach, we find that in these cases (i) the conditions of high energetic efficiency to be reached are similar, and (ii) the symmetry properties of the kinetic coefficients are different. Based on our results, a strategy for designing efficient Brownian motors is suggested.

DOI: [10.1103/PhysRevE.85.041116](https://doi.org/10.1103/PhysRevE.85.041116)

PACS number(s): 05.40.-a, 05.60.Cd, 82.20.-w, 87.16.Nn

I. INTRODUCTION

Physical models shedding light on efficient mechanisms of energy conversion on the nanoscale are claimed to handle a wide range of biologically valuable processes [1–3] and can be applied in nanotechnology [3–5]. A promising way to address this challenge is provided by the concept of ratchets or Brownian motors, according to which Brownian motion in a periodic potential combined with external unbiased perturbations, either deterministic or stochastic, can exhibit a net drift [3,6,7], thus enabling to convert the energy coming from a source of nonequilibrium into useful work [8,9]. The time-dependent potential models the combined effect of environment and the external perturbations on the motor dynamics and hence determines the efficiency of energy conversion [10,11].

In the present paper, we discuss two known examples of deterministically driven ratchets, which exhibit high energetic efficiency, with the goal to uncover similarities and differences in the principles of their operation. The first model is the so-called reversible ratchet, which functions due to adiabatically slow time-periodic variation of a set of parameters determining the periodic potential. An elegant formulation of this mechanism, where the entropy production vanishes in the quasistatic limit, was given in Refs. [12,13] (some further development of this idea can be found in Refs. [14,15]). The second model is the flashing ratchet, which, in contrast, is based on an intrinsically irreversible mechanism: The directed motion arises due to periodically repeated fast transitions

between two long-living spatially periodic potential profiles. Provided the transitions occur instantaneously, the working mechanism is also adiabatic in the sense that there is no thermal exchange between the system and environment. It can also be efficient if the potential profiles are mutually shifted so that Brownian motion is not involved in the transport generation [16,17]. A detailed analysis of the highly efficient flashing ratchet with half-period-shifted potentials was given in Refs. [18,19]. Thus, both models rely on adiabaticity of the potential change process, which is crucial to avoid energetic losses. However, the adiabaticity that we deal with in the two cases is of a different type, which is manifested, in particular, in distinctly different properties of the quantities of interest [20]. To distinguish between these models, the first (second) one is called hereafter the adiabatically slow (fast) driven ratchet.

In what follows, first (in Sec. II), we show that the symmetry properties of the known solutions for the drift velocity of the adiabatically slow and adiabatically fast driven ratchets are different. In Sec. III, to better understand the origin of this difference, we also invoke the high-temperature expansion of the velocity. In Sec. IV, we restate the models in terms of traveling potential ratchets, which makes the analysis of their energetic characteristics particularly transparent. Finally (Sec. V), based on the results of this paper, we formulate conditions for designing Brownian motors with high efficiency.

II. SYMMETRY PROPERTIES

Consider an overdamped Brownian particle $x(t)$ moving in the spatially periodic potential $V(x; \mathbf{R})$ with period L . The

*vik-roz@mail.ru, vrozen@pub.iams.sinica.edu.tw

†dyyang@pub.iams.sinica.edu.tw

potential depends on a number of parameters $\{R_1, R_2, \dots, R_n\}$ collected in a vector \mathbf{R} . If the parameters vary periodically in time (with period τ), $\mathbf{R}(t + \tau) = \mathbf{R}(t)$, the net drift of the particle arises even in the adiabatic limit, $\tau \rightarrow \infty$. The drift velocity of such adiabatically slow driven Brownian motor reads [13]

$$\mathfrak{A} = \frac{L}{\tau} \oint d\mathbf{R} \cdot \int_0^L dx \rho_+(x; \mathbf{R}) \int_0^x dx' \nabla_{\mathbf{R}} \rho_-(x'; \mathbf{R}), \quad (1)$$

where

$$\begin{aligned} \rho_{\pm}(x; \mathbf{R}) &= Z_{\pm}^{-1}(x; \mathbf{R}) e^{\pm \beta V(x; \mathbf{R})}, \\ Z_{\pm}(x; \mathbf{R}) &= \int_0^L dx e^{\pm \beta V(x; \mathbf{R})}, \end{aligned} \quad (2)$$

and $\beta = (k_B T)^{-1}$ (k_B is the Boltzmann constant and T is the absolute temperature). Note that the time-reversal transformation changes $d\mathbf{R}$ to $-d\mathbf{R}$ and hence inverts the direction of the drift velocity, as it should be for the reversible ratchet.

The velocity \mathfrak{A} in Eq. (1) possesses an additional remarkable symmetry property. To exhibit it, first note that using the identity $\oint d\mathbf{R} \cdot \nabla_{\mathbf{R}} \Phi(\mathbf{R}) = 0$ [with an arbitrary function $\Phi(\mathbf{R})$], Eq. (1) can be written in the form

$$\mathfrak{A} = -\frac{L}{\tau} \oint d\mathbf{R} \cdot \int_0^L dx \nabla_{\mathbf{R}} \rho_+(x; \mathbf{R}) \int_0^x dx' \rho_-(x'; \mathbf{R}). \quad (3)$$

Then changing the order of integration in Eq. (3) and taking into account that $\int_{x'}^L dx \nabla_{\mathbf{R}} \rho_+(x; \mathbf{R}) = -\int_0^{x'} dx \nabla_{\mathbf{R}} \rho_+(x; \mathbf{R})$, we arrive at the alternative presentation of the velocity:

$$\mathfrak{A} = \frac{L}{\tau} \oint d\mathbf{R} \cdot \int_0^L dx \rho_-(x; \mathbf{R}) \int_0^x dx' \nabla_{\mathbf{R}} \rho_+(x'; \mathbf{R}). \quad (4)$$

The quantities $\rho_+(x; \mathbf{R})$ and $\rho_-(x; \mathbf{R})$ are readily interconverted by the inversion of the sign of $V(x; \mathbf{R})$, as their definitions in Eq. (2) indicate. With this fact, a comparison of Eqs. (1) and (4) shows that the drift velocity is an even functional of the potential. So the inversion of the sign of the potential leaves the velocity unchanged, though the asymmetry of the potential (if it is present) is inverted by this transformation. This observation leads to important conclusions: (i) Asymmetry of the static potential $V(x; \mathbf{R})$ contributes nothing to the working mechanism of the adiabatically slow driven ratchet and (ii) the only parametric perturbations $\mathbf{R}(t)$ break the spatial inversion symmetry, allowing a directed motion set in even in an *a priori* symmetric potential.

As an example of an adiabatically fast driven ratchet, consider a flashing ratchet in which the potential switches periodically in time between two spatially periodic profiles $V_a(x)$ and $V_b(x)$. The adiabaticity condition implies that (i) transitions from $V_a(x)$ to $V_b(x)$ and vice versa are fast, and (ii) the particle residence times in states a and b , τ_a and τ_b , are large as compared to all characteristic times of the system. With these conditions satisfied, the solution for the

drift velocity is known [13,21]:

$$\mathfrak{A} = \frac{L}{\tau} \int_0^L dx [\rho_+^{(a)}(x) - \rho_+^{(b)}(x)] \int_0^x dx' [\rho_-^{(a)}(x') - \rho_-^{(b)}(x')], \quad (5)$$

where $\rho_{\pm}^{(a)}(x)$ and $\rho_{\pm}^{(b)}(x)$ are defined by Eq. (2) in which the potential $V(x; \mathbf{R})$ is replaced by $V_a(x)$ and $V_b(x)$, respectively. As for the adiabatically slow driven ratchet [see Eq. (1)], the drift velocity of an adiabatically fast driven ratchet is proportional to τ^{-1} . Note that Eq. (5) is invariant under the interchange $a \leftrightarrow b$ (which is equivalent to the time reversal), as it should be for this intrinsically irreversible mechanism.

To exhibit other symmetry properties, it is instructive to change the order of integration and take advantage of the relation $\int_{x'}^L dx [\rho_+^{(a)}(x) - \rho_+^{(b)}(x)] = -\int_0^{x'} dx [\rho_+^{(a)}(x) - \rho_+^{(b)}(x)]$. As a result we get

$$\begin{aligned} \mathfrak{A} &= -\frac{L}{\tau} \int_0^L dx [\rho_-^{(a)}(x) - \rho_-^{(b)}(x)] \\ &\quad \times \int_0^x dx' [\rho_+^{(a)}(x') - \rho_+^{(b)}(x')]. \end{aligned} \quad (6)$$

A comparison of Eqs. (5) and (6) shows that the average velocity is an odd functional of the potential: The simultaneous sign change of $V_a(x)$ and $V_b(x)$ inverts the velocity direction. Then, instead of $V_a(x)$ and $V_b(x)$, introduce $u(x) = \frac{1}{2}[V_a(x) + V_b(x)]$ and $w(x) = \frac{1}{2}[V_a(x) - V_b(x)]$, so that $V_{a,b}(x) = u(x) \pm w(x)$. The function $u(x)$ plays the role of an ‘‘average’’ potential, while $w(x)$ characterizes the deviation from the average. The invariance of \mathfrak{A} under the interchange $a \leftrightarrow b$ implies the velocity to be an even functional of $w(x)$. Additionally, the fact that the simultaneous sign change of $u(x)$ and $w(x)$ inverts \mathfrak{A} implies the drift velocity to be an odd functional of $u(x)$. Since the inversion of $u(x)$ implies the inversion of its reflection asymmetry, nonzero values of \mathfrak{A} are possible only when the function $u(x)$ is asymmetric. Thus, the broken reflection symmetry of the average potential $u(x)$ is an indispensable prerequisite for the adiabatically fast driven Brownian motor to operate. This is in sharp contrast to what we have seen for the adiabatically slow driven motor, where the necessary symmetry breaking occurs without the assistance of the spatial asymmetry of the potential.

Note that even if the potentials $V_a(x)$ and $V_b(x)$ alone are symmetric, their average, $u(x)$, can be asymmetric, which results in a nonzero drift velocity (a simple example is given in Ref. [21]). On the other hand, both $V_a(x)$ and $V_b(x)$ can be asymmetric, but nevertheless $\mathfrak{A} = 0$ as this occurs in the case where $V_a(x) = -V_b(x)$ [i.e., $u(x) \equiv 0$].

III. HIGH-TEMPERATURE EXPANSION

Consider the case of high temperatures $\beta V_0 \ll 1$, where V_0 is the characteristic amplitude of the potential. In this way, we illustrate the results obtained above and, moreover, reveal additional properties of the adiabatically driven models. Following the approach suggested in Refs. [22,23], one can write down the first two terms of the high-temperature

expansion of the drift velocity:

$$\mathfrak{A} = \frac{D}{L} \left\{ \beta^2 \sum_{qj \neq 0} \frac{(\omega_j \tau_D)(k_q L)^3}{(k_q L)^4 + (\omega_j \tau_D)^2} |V_{qj}|^2 + i\beta^3 \right. \\ \left. \times \sum_{qj, q'j' (\neq 0)} \frac{(k_q L)^2 (k_{q+q'} L)^2 (k_{q'} L) [(k_q L)^2 (k_{q+q'} L)^2 - (\omega_j \tau_D)(\omega_{j'} \tau_D)]}{[(k_q L)^4 + (\omega_j \tau_D)^2][(k_{q+q'} L)^4 + (\omega_{j'} \tau_D)^2]} V_{qj} V_{q'j'} V_{-q-q', -j-j'} + O(\beta^4) \right\}, \quad (7)$$

where D is the (potential free) diffusion coefficient, $\tau_D = L^2/D$ is the time step of diffusion over the spatial period, and V_{qj} are the Fourier components of an arbitrary potential $V(x, t) = V(x + L, t + \tau)$, with wave numbers $k_q = (2\pi/L)q$ and frequencies $\omega_j = (2\pi/\tau)j$ (q and j are integers). The first term in Eq. (7) is nonzero only when

$$|V_{qj}|^2 \neq |V_{-q, -j}|^2. \quad (8)$$

This condition is met for the parametric excitation $V[x; \mathbf{R}(t)]$, with at least two different time-dependent parameters $R_i(t)$ ($i = 1, 2$) such that the integral in Eq. (1) does not equal zero. To make sure this is the case, consider the potential $V[x; \mathbf{R}(t)]$ written in the form

$$V[x; \mathbf{R}(t)] = u(x)R_1(t) + w(x)R_2(t), \quad (9)$$

which corresponds, for example [20], to a two-well periodic potential profile, where the time periodic changes of the wells and the barriers are governed by the functions $R_1(t)$ and $R_2(t)$. Decomposing the functions in Eq. (9) into even and odd parts [designated as (s) and (a), respectively], we arrive at the following relation:

$$|V_{qj}|^2 - |V_{-q, -j}|^2 = 4 \begin{vmatrix} u_q^{(s)} & w_q^{(s)} \\ u_q^{(a)} & w_q^{(a)} \end{vmatrix} \begin{vmatrix} R_{1,j}^{(s)} & R_{2,j}^{(s)} \\ R_{1,j}^{(a)} & R_{2,j}^{(a)} \end{vmatrix}, \quad (10)$$

where the vertical bars on the right-hand side designate the determinants. As Eq. (10) indicates, the condition (8) is satisfied if all functions in Eq. (9) are nonzero and both even and odd parts of these functions contribute to Eq. (10). Note that ω_j is proportional to τ^{-1} and for large τ the second term in Eq. (7) goes to zero as τ^{-2} if the function $\mathbf{R}(t)$ is smooth. Therefore, for the adiabatically slow driven ratchet ($\tau \gg \tau_D$), Eq. (7) is reduced to

$$\mathfrak{A} \simeq (L/\tau)\beta^2 \left[\sum_{qj (\neq 0)} (j/q) |V_{qj}|^2 + O(\beta^2) \right], \quad (11)$$

so that the inversion of the potential sign leaves the velocity unchanged, in accordance with the conclusion of our general analysis.

If the condition (8) breaks down, i.e., $|V_{qj}|^2 = |V_{-q, -j}|^2$, the leading term in the expansion (7) is proportional to β^3 . This term behaves as τ^{-1} as $\tau \rightarrow \infty$ only if the function $\mathbf{R}(t)$ is a piecewise smooth function with jump discontinuities. The simplest example of such a situation is a dichotomous variation of the potential given by Eq. (9), in which $R_1(t) = 1$ and $R_2(t) = \sigma(t)$, with $\sigma(t)$ taking the values $+1$ and -1 during the time intervals τ_a and τ_b . So the potential switches between two profiles $V_{a,b}(x) = u(x) \pm w(x)$. Taking into account that

the Fourier components σ_j of $\sigma(t) = \pm 1$ decrease slowly with increasing j (as j^{-1}), the jj' summation of the series in the second term of Eq. (7) for the case of the adiabatically fast driven ratchet ($\tau_{a,b} \gg \tau_D$) leads to

$$\mathfrak{A} \simeq \frac{4iL}{\pi\tau} \beta^3 \left[\sum_{qq' (\neq 0)} q^{-1} u_{q'} w_q w_{-q-q'} + O(\beta^2) \right]. \quad (12)$$

As Eq. (12) indicates, the velocity is the linear form in u_q and the quadratic form in w_q , in accordance with our general analysis.

IV. ENERGETIC EFFICIENCY

In the remainder of the paper, we discuss the energetics of the adiabatically driven ratchets. To make the analysis particularly transparent, we restate the models in terms of traveling potential ratchets, which is a specific subclass of ratchets with periodic time-dependent potentials having the form $V(x - f(t))$, where the function $f(t)$ is associated with the nonequilibrium perturbation [6]. For such ratchets, the potential extrema shift in time so that diffusion-free directed motion is produced, and the condition of high efficiency mentioned in Refs. [16, 17] is satisfied. The overdamped dynamics of the traveling potential ratchets in the presence of a load force F is governed by the Langevin equation

$$\zeta \dot{y} = -V'(y) - \zeta \dot{f}(t) - F + \xi(t), \quad (13)$$

where $y = x - f(t)$ is the auxiliary variable, $\zeta = k_B T / D$ is the friction coefficient, the dotted and primed symbols stand for the respective time and coordinate derivatives, and $\xi(t)$ is unbiased Gaussian white noise with the correlation function $\langle \xi(t)\xi(s) \rangle = 2\zeta k_B T \delta(t - s)$.

First, consider a genuine traveling potential ratchet with $f(t) = ut$ [24–26]. We assume that the potential varies adiabatically slow with the traveling velocity $u = L/\tau$, $\tau \rightarrow \infty$, which implies the stopping force F_s (the value of the load force that nullifies the ratchet effect, $F \leq F_s$) is also small. Note that the periodic function $V(x - ut)$ can be presented as a definite function (not necessarily periodic) of a periodic argument, e.g., $\cos 2\pi[(x - ut)/L]$. Then its spatial time dependence is determined by a function of the right-hand side of Eq. (9) with $u(x) = \cos 2\pi x/L$, $w(x) = \sin 2\pi x/L$, $R_1(t) = \cos 2\pi t/\tau$, and $R_2(t) = \sin 2\pi t/\tau$. Thus in the adiabatic limit, the genuine traveling potential ratchet is equivalent to the adiabatically slow driven ratchet.

In this case the effective force $\tilde{F} = \zeta u + F$, entering in Eq. (13), is time independent, so the problem is reduced to the well-known exactly solvable problem of Brownian motion in a

periodic tilted potential [27–29]. In view of $\tilde{F} \rightarrow 0$, the motor operates in the linear response regime, so that $\langle \dot{y} \rangle = -\mu_{\text{eff}} \tilde{F}$, where μ_{eff} is the effective mobility of the Brownian particle in the periodic potential $V(y)$ [30]:

$$\mu_{\text{eff}} = \frac{L^2}{\int_0^L dy e^{\beta V(y)} \int_0^L dy e^{-\beta V(y)}} \zeta^{-1}. \quad (14)$$

Thus the desirable drift velocity $\mathfrak{A} = \langle \dot{y} \rangle + u$ reads

$$\mathfrak{A} = (1 - \zeta \mu_{\text{eff}}) u - \mu_{\text{eff}} F, \quad (15)$$

and it is an even functional of the potential as follows from Eq. (14). Note that for a sawtooth potential, the effective mobility $\mu_{\text{eff}} = \zeta^{-1} v^2 / \sinh^2 v$ (where $v \equiv \beta V_0/2$) is independent of the potential asymmetry, as it should be for the adiabatically slow ratchets.

The useful work done by the particle against the load force per unit time (power output) equals $P_{\text{out}} = F\mathfrak{A}$, whereas the average energy transferred to the particle per unit time due to variation of the potential (power input) is given by $P_{\text{in}} = u(\zeta \mathfrak{A} + F)$ [31]. The quantities $-F$ and u can be considered as generalized forces so in the vicinity of equilibrium P_{out} and P_{in} can be written as the quadratic forms of the forces:

$$P_{\text{out}} = -\lambda_{11} F^2 + \lambda_{12} Fu, \quad P_{\text{in}} = -\lambda_{21} Fu + \lambda_{22} u^2, \quad (16)$$

where λ_{ik} ($i, k = 1, 2$) are the kinetic coefficients defined by the relations

$$\lambda_{11} = \mu_{\text{eff}}, \quad \lambda_{12} = -\lambda_{21} = 1 - \zeta \mu_{\text{eff}}, \quad \lambda_{22} = \zeta \lambda_{12}. \quad (17)$$

We call attention to antisymmetry of the kinetic coefficients, which is a distinctive feature of the adiabatically slow driven ratchets (see also Ref. [20]).

From Eqs. (16) and (17), it follows that the maximum of the efficiency $\eta = P_{\text{out}}/P_{\text{in}}$ as a function of the load force is determined by the single parameter $Z = \lambda_{11} \lambda_{22} / \lambda_{12}^2 = \zeta \mu_{\text{eff}} / (1 - \zeta \mu_{\text{eff}})$: $\eta_m = (\sqrt{1+Z} - \sqrt{Z})^2$. Since the effective mobility, Eq. (14), and hence Z are exponentially small when the characteristic amplitude of the potential $V_0 \gg k_B T$, the efficiency maximum approaches unity as $\eta_m \simeq 1 - 2\sqrt{\zeta \mu_{\text{eff}}}$.

To treat the energetics of the adiabatically fast driven motor, consider another type of traveling potential ratchet scheme, characterized by a time-periodic driving function $f(t + \tau) = f(t)$, defined on the interval $[0, \tau]$ as follows: $f(t) = (L/2)\theta(t - \tau_a)$, where $\tau_a < \tau$ and $\theta(t) = 1$ for $t > 0$ and 0 for $t < 0$. With such driving, the potential $V(x - f(t))$ switches between two static profiles $V_a(x)$ and $V_b(x) = V_a(x + L/2)$, spending time τ_a in the former and time $\tau_b = \tau - \tau_a$ in the latter. In terms of the probability density $\rho(x, t)$ and the probability current $J(x, t)$, the ratchet dynamics can be described in the form of a continuity equation $\partial_t \rho(x, t) = -\partial_x J(x, t)$. The current $J(x, t) = \hat{J}(x, t)\rho(x, t)$ on the intervals $0 < t < \tau_a$ and $\tau_a < t < \tau$ can be determined by the corresponding explicit forms of the current operator: $\hat{J}_a(x) \equiv -\zeta^{-1}[V'_a(x) + F] - D\partial_x$ and $\hat{J}_b(x) = \hat{J}_a(x + L/2)$. It is easy to connect the currents at the points x and $x + L/2$ with the probability densities at the moments τ_a

and τ :

$$\hat{J}_a(x + L/2)\varphi_a(x + L/2) - \hat{J}_a(x)\varphi_a(x) = R(x), \quad (18)$$

$$R(x) \equiv \int_x^{x+L/2} dx' [\rho(x', 0) - \rho(x', \tau_a)],$$

where we have introduced the function $\varphi_a(x) \equiv \int_0^{\tau_a} dt \rho(x, t)$. This function is related to the similarly defined function $\varphi_b(x) \equiv \int_{\tau_a}^{\tau} dt \rho(x, t)$ by the relation $\varphi_b(x) = \varphi_a(x + L/2)$. The average velocity of the motor $\mathfrak{A} = (L/\tau)[\hat{J}_a(x)\varphi_a(x) + \hat{J}_b(x)\varphi_b(x)]$ is position independent and can be written in the form

$$\mathfrak{A} = \frac{L}{\tau} [2\hat{J}_a(x)\varphi_a(x) + R(x)] \quad (19)$$

by using Eq. (18). The power output, as usual, reads $P_{\text{out}} = F\mathfrak{A}$. The power input in this case is found to be [17–19]

$$P_{\text{in}} = \tau^{-1} \int_0^L dx [V_a(x) - V_b(x)] [\rho(x, 0) - \rho(x, \tau_a)]. \quad (20)$$

Let us now assume that in the interval $[0, L]$ the potential profile $V_a(x)$ has a high barrier V_0 , $\beta V_0 \gg 1$, at the point $x = 0$, so that the current $\hat{J}_a(0)\varphi_a(0)$ is small enough. Then \mathfrak{A} is approximately determined by the quantity $R(0)$ and hence $P_{\text{out}} \simeq (FL/\tau)R(0)$. Let us also assume that, in the interval $[0, L/2]$, the difference $\Delta V \equiv V_a(x) - V_a(x + L/2)$ in the vicinity of the potential minima varies only slightly with x . Then we obtain the approximate relation $P_{\text{in}} \simeq 2\tau^{-1}\Delta V R(0)$ and the estimate for the efficiency $\eta \simeq FL/2\Delta V$. Thus with these assumptions, the efficiency is close to unity when F is near the stopping force $F_s = 2\Delta V/L$. Further, consider the adiabatic limit $\tau_{a,b} \rightarrow \infty$, and neglect the contribution from the small current $\hat{J}_a(0)\varphi_a(0)$ to the probability density. Then the distribution can be approximated by the equilibrium one with the potential energy $U_a(x) = V_a(x) + Fx$ that allows us to write

$$\rho(x, 0) - \rho(x, \tau_a) \simeq [e^{\beta(\Delta V - FL/2)} - 1] \rho(x, \tau_a),$$

$$\rho(x, \tau_a) \simeq \left\{ [e^{\beta(\Delta V - FL/2)} + 1] \int_0^{L/2} dx e^{-\beta U_a(x)} \right\}^{-1} \times e^{-\beta U_a(x)}. \quad (21)$$

From these expressions it follows that $R(0)$ defined in Eq. (18) can be estimated as $R(0) \approx \tanh[\beta(\Delta V - FL/2)/2]$. The quasiequilibrium conditions imply the smallness of ΔV and F , so that $P_{\text{out}} \simeq \beta FL(\Delta V - FL/2)/2\tau$ and $P_{\text{in}} \simeq \beta \Delta V(\Delta V - FL/2)/\tau$. Considering these expressions for P_{out} and P_{in} as quadratic forms of the generalized forces $-F$ and ΔV [cf. Eq. (16)], we arrive at the conclusion that $\lambda_{12} = \lambda_{21}$, thus asserting symmetry of the kinetic coefficients in the case of adiabatically fast ratchets.

A more detailed and rigorous consideration of efficient flashing ratchets is presented in Refs. [18–20], where it is shown that the deviation of the maximum efficiency from unity decays exponentially with increasing V_0 at the instantaneous switching of the potential profiles and decays much slower (according to a power law at large V_0) when this switching occurs for a finite time interval.

V. CONCLUSIONS

To summarize, in the present paper, using different approaches, we have analyzed and compared the adiabatically slow and adiabatically fast driven ratchets, both exhibiting high efficiency of energy conversion. We have found that these intrinsically different models share common traits, which lead to the lowering of the energy loss: adiabaticity of the potential change process, diffusion-free directed motion generation (provided the potential extrema shift in time), and an effective rectification mechanism (taking place when the characteristic amplitude of the potential $V_0 \gg k_B T$). These observations suggest a reasonable framework for designing efficient Brownian motors. We have also found striking differences between the properties of the adiabatically slow and adiabatically fast driven ratchets: (i) the reversibility and irreversibility with respect to time reversal of the driving forces; (ii) the evenness and oddness of the motor velocity as the functional of the potential; (iii) the occurrence of

the necessary symmetry breaking without and with the assistance of the spatial asymmetry of the potential; and (iv) antisymmetry and symmetry of the kinetic coefficients in the quasiequilibrium regime. Additionally, we have shown that the models of adiabatically slow and adiabatically fast driven ratchets are equivalent to different types of traveling potential ratchets under some additional assumptions.

ACKNOWLEDGMENTS

This work was supported by National Chiao Tung University and Academia Sinica. V.M.R thanks the Complex Program for Basic Research of the National Academy of Sciences of Ukraine for partial support (Grant No. 3/12-H). Y.A.M. thanks the Russian Foundation for Basic Research for partial support (Grant No. 10-03-00393). V.M.R. and Y.A.M. gratefully acknowledge the kind hospitality received from the Institute of Atomic and Molecular Sciences.

-
- [1] H. Qian, *Biophys. Chem.* **67**, 263 (1997).
 - [2] J. Howard, *Mechanics of Motor Proteins and the Cytoskeleton* (Sinauer Associates, Sunderland, MA, 2001).
 - [3] R. D. Astumian, *Science* **276**, 917 (1997).
 - [4] K. E. Drexler, *Nanosystems: Molecular Machinery, Manufacturing and Computation* (Wiley, New York, 1992).
 - [5] G. S. Kottas, L. I. Clarke, D. Horinek, and J. Michl, *Chem. Rev.* **105**, 1281 (2005).
 - [6] P. Reimann, *Phys. Rep.* **361**, 57 (2002).
 - [7] P. Hänggi and F. Marchesoni, *Rev. Mod. Phys.* **81**, 387 (2009).
 - [8] U. Seifert, *Eur. Phys. J. B* **64**, 423 (2008).
 - [9] K. Sekimoto, *Stochastic Energetics* (Springer, Berlin, 2010).
 - [10] F. Slanina, *Phys. Rev. E* **80**, 061135 (2009).
 - [11] Y. X. Zhang, *J. Stat. Phys.* **134**, 669 (2009).
 - [12] J. M. R. Parrondo, J. M. Blanco, F. J. Chao, and R. Brito, *Europhys. Lett.* **43**, 248 (1998).
 - [13] J. M. R. Parrondo, *Phys. Rev. E* **57**, 7297 (1998).
 - [14] J. M. Horowitz and C. Jarzynski, *J. Stat. Phys.* **136**, 917 (2009).
 - [15] H. J. Chen, J. L. Huang, C. Y. Wang, and H. C. Tseng, *Phys. Rev. E* **82**, 052103 (2010).
 - [16] J.-F. Chauwin, A. Ajdari, and J. Prost, *Europhys. Lett.* **27**, 421 (1994).
 - [17] A. Parmeggiani, F. Jülicher, A. Ajdari, and J. Prost, *Phys. Rev. E* **60**, 2127 (1999).
 - [18] Yu. A. Makhnovskii, V. M. Rozenbaum, D.-Y. Yang, S. H. Lin, and T. Y. Tsong, *Phys. Rev. E* **69**, 021102 (2004).
 - [19] V. M. Rozenbaum, T. Ye. Korochkova, D.-Y. Yang, S. H. Lin, and T. Y. Tsong, *Phys. Rev. E* **71**, 041102 (2005).
 - [20] V. M. Rozenbaum and I. V. Shapochkina, *Phys. Rev. E* **84**, 051101 (2011).
 - [21] V. M. Rozenbaum, T. Ye. Korochkova, A. A. Chernova, and M. L. Dekhtyar, *Phys. Rev. E* **83**, 051120 (2011).
 - [22] V. M. Rozenbaum, *Pis'ma Zh. Eksp. Teor. Fiz.* **88**, 391 (2008) [*JETP Lett.* **88**, 342 (2008)].
 - [23] V. M. Rozenbaum, *Zh. Eksp. Teor. Fiz.* **137**, 740 (2010) [*JETP* **110**, 653 (2010)].
 - [24] K. M. Jansons and G. D. Lythe, *Phys. Rev. Lett.* **81**, 3136 (1998).
 - [25] M. Borromeo and F. Marchesoni, *Phys. Lett. A* **249**, 199 (1998).
 - [26] C. Van den Broeck, *Europhys. Lett.* **46**, 1 (1999).
 - [27] R. L. Stratonovich, *Radiotekh. Elektron.* **3**, 497 (1958).
 - [28] H. Risken, *The Fokker-Planck Equation* (Springer, Berlin, 1984).
 - [29] P. Reimann, C. VandenBroeck, H. Linke, P. Hänggi, J. M. Rubi, and A. Perez-Madrid, *Phys. Rev. Lett.* **87**, 010602 (2001).
 - [30] S. Lifson and J. L. Jackson, *J. Chem. Phys.* **36**, 2410 (1962).
 - [31] Y. X. Li, X. Z. Wu, and Y. Z. Zhuo, *Physica A* **286**, 147 (2000).