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RESEARCH NOTE

CAPABILITY INDICES FOR NON-NORMAL DISTRIBUTIONS WITH AN APPLICATION IN ELECTROLYTIC CAPACITOR MANUFACTURING

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Abstract—Process capability indices $C_p(u, v)$, which include the four basic indices C_p, C_{pk}, C_{pm} and C_{pmk} as special cases, have been proposed to measure process potential and performance. $C_p(u, v)$ are appropriate indices for processes with normal distributions, but have been shown to be inappropriate for processes with non-normal distributions. In this paper, we first consider two generalizations of $C_p(u, v)$, which we refer to as $C_{Np}(u, v)$ and $C'_{Np}(u, v)$, to cover cases where the underlying distributions may not be normal. Comparisons between $C_{Np}(u, v)$ and $C'_{Np}(u, v)$ are provided. The results indicated that the generalizations $C_{Np}(u, v)$ are superior to $C'_{Np}(u, v)$ in measuring process capability. We then present a case study on an aluminum electrolytic-capacitor manufacturing process to illustrate how the generalizations $C_{Np}(u, v)$ may be applied to actual data collected from the factories. © 1997 Elsevier Science Ltd.

1. INTRODUCTION

Numerous process capability indices, including C_p, C_{pk}, C_{pm} and C_{pmk}, have been proposed to provide a unitless measure on whether a process is capable of producing items meeting the quality requirement preset by the product designer. Some of these indices, particularly C_p and C_{pk}, have been widely used in various manufacturing industries providing measures on process potential and performance. Examples include the manufacturing of semi-conductor products [1], head/gimbal assembly for memory storage systems [2], jet-turbine engine components [3], flip-chips and chip-on-board [4], audio-speaker drivers [5], wood products [6], and many others.

Vännman [7] constructed a superstructure to include the four basic indices, C_p, C_{pk}, C_{pm} and C_{pmk} as special cases. The superstructure has been referred to as $C_p(u, v)$, which can be defined as the following:

$$C_p(u, v) = \frac{d - u|\mu - m|}{3\sqrt{\sigma^2 + v(\mu - T)^2}}, \quad (1)$$

where μ is the process mean, σ is the process standard deviation, $d = (\text{USL} - \text{LSL})/2$ which is half of the length of the specification interval, $m = (\text{USL} + \text{LSL})/2$ is the mid-point between the upper and the lower specification limits, T is the target value, and $u, v \geq 0$. It is easy to verify that $C_p(0, 0) = C_p$, $C_p(1, 0) = C_{pk}$, $C_p(0, 1) = C_{pm}$ and $C_p(1, 1) = C_{pmk}$ which have been defined explicitly as the following [8–10]:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma},$$

$$C_{pk} = \min\left\{\frac{\text{USL} - \mu}{3\sigma}, \frac{\mu - \text{LSL}}{3\sigma}\right\},$$

$$C_{pm} = \frac{\text{USL} - \text{LSL}}{6\sqrt{\sigma^2 + (\mu - T)^2}},$$

$$C_{pmk} = \min\left\{\frac{\text{USL} - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - \text{LSL}}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\}.$$

In general, the relationships among the four indices can be established as the following: $C_{pm} = C_p\{1 + [(\mu - T)/\sigma]^2\}^{-1/2}$, and $C_{pmk} = C_{pk}\{1 + [(\mu - T)/\sigma]^2\}^{-1/2}$. The ranking of the four indices in terms of sensitivity to departure of the process mean from the target value, from the most sensitive up to the least sensitive are: (1) C_{pmk}, (2) C_{pm}, (3) C_{pk} and (4) C_p. For symmetric tolerances, we can show that $C_{pk} = (1 - k)$ C_p and $C_{pmk} = (1 - k)$ C_{pm}, where $k = |\mu - T|/d$ is the departure ratio. If the process is on-target, then clearly $k = 0$ ($\mu = T$) and $C_p = C_{pk} = C_{pm} = C_{pmk} = d/3\sigma$.

Estimators of the indices $C_p(u, v)$ may be obtained by replacing μ by the sample mean

$\bar{X} = (\Sigma_{i=1}^{n} X_i)/n$, and σ^2 by the sample variance $S^2 = (n-1)^{-1}\Sigma_{i=1}^{n}(X_i - \bar{X})^2$ in definition (1). For normal distributions, those estimators based on $\bar{X}$ and S^2 are quite stable and reliable. However, for non-normal distributions, those estimators become highly unstable since the distribution of the sample variance, S^2, is sensitive to departures from normality, and estimators of those indices are calculated using S^2, as pointed out by Chan, Cheng and Spiring [11]. In fact, Gunter [12] demonstrated the strong impact this has on the sampling distribution of C_{pk}. Thus, the basic indices $C_p(u, v)$ are inappropriate for processes with non-normal distributions.

In this paper, we consider two generalizations of $C_p(u, v)$, which we refer to as $C_{Np}(u, v)$ and $C'_{Np}(u, v)$, to cover cases where the underlying distributions may not be normal. Comparisons between the two generalizations $C_{Np}(u, v)$ and $C'_p(u, v)$ are provided. The results indicated that the generalizations $C_{Np}(u, v)$ are more accurate than $C'_p(u, v)$ in measuring process capability. We also present a case study on a non-polarized capacitor manufacturing process to illustrate how the generalizations $C_{Np}(u, v)$ may be applied to actual data collected from the factories.

2. CAPABILITY INDICES FOR NON-NORMAL DISTRIBUTIONS

To accommodate cases where the underlying distributions may not be normal, we consider the following generalizations of $C_p(u, v)$, which we refer to as $C_{Np}(u, v)$. The generalizations $C_{Np}(u, v)$ can be defined as the following (in superstructure form), where F_α is the αth percentile, M is the median of the distribution, $m = (\text{USL} - \text{LSL})/2$, μ and σ are process mean and process standard deviation, and $u, v \geq 0$.

$$C_{Np}(u, v) = \frac{d - u|M - m|}{3\sqrt{\left[\dfrac{F_{99.865} - F_{0.135}}{6}\right]^2 + v(M - T)^2}}. \quad (2)$$

Thus, in developing the generalizations $C_{Np}(u, v)$ we have replaced the process standard deviation, σ, by $(F_{99.865} - F_{0.135})/6$ in the definition of $C_p(u, v)$. The idea behind such replacements is to mimic the property of the normal distribution for which the tail probability outside the $\pm 3\sigma$ limits from μ is 0.27%, thus assuring that if the calculated value of $C_{Np}(u, v) = 1$ (assuming the process is well-centered) the probability that process is outside the specification interval (LSL, USL) will be negligibly small. We also have replaced the process mean, μ, by the process median M since the process median is a more robust measure of central tendency than the process mean, particularly for skew distributions with long tails. By setting the parameter values $(u, v) = (0, 0)$, (0, 1), (1, 0) and (1, 1), we obtain the four generalizations of the basic indices for non-normal distributions, which can be expressed explicitly as the following. We refer to these four generalizations as C_{Np}, C_{Npk}, C_{Npm} and C_{Npmk}.

$$C_{Np} = \frac{\text{USL} - \text{LSL}}{F_{99.865} - F_{0.135}},$$

$$C_{Npk} = \min\left\{\frac{\text{USL} - M}{\left[\dfrac{F_{99.865} - F_{0.135}}{2}\right]}, \frac{M - \text{LSL}}{\left[\dfrac{F_{99.865} - F_{0.135}}{2}\right]}\right\}$$

$$C_{Npm} = \frac{\text{USL} - \text{LSL}}{6\sqrt{\left[\dfrac{F_{99.865} - F_{0.135}}{6}\right]^2 + (M - T)^2}},$$

$$C_{Npmk} = \min\left\{\frac{\text{USL} - M}{3\sqrt{\left[\dfrac{F_{99.865} - F_{0.135}}{6}\right]^2 + (M - T)^2}}, \frac{M - \text{LSL}}{3\sqrt{\left[\dfrac{F_{99.865} - F_{0.135}}{6}\right]^2 + (M - T)^2}}\right\}.$$

In general, the relationships among the four generalizations can be established as the following: $C_{Npm} = C_{Np}\{1 + [(M - T)/\sigma'']^2\}^{-1/2}$, and $C_{Npmk}\{1 + [(M - T)/\sigma'']^2\}^{-1/2}$, where $\sigma'' = (F_{99.865} - F_{0.135})/6$. The ranking of the four generalizations in terms of sensitivity to departure of the process median from the target value, from the most sensitive to the least sensitive are: (1) C_{Npmk}, (2) C_{Npm}, (3) C_{Npk} and (4) C_{Np}. For symmetric tolerances, we can show that $C_{Npk} = (1 - k)\, C_{Np}$ and $C_{Npmk} = (1 - k)\, C_{Npm}$, where $k = |M - T|/d$ is the departure ratio. If the process is on-target, then clearly $k = 0$ $(M = T)$ and $C_{Np} = C_{Npk} = C_{Npm} = C_{Npmk} = d/3\sigma''$. On the other hand, if the underlying distribution is normal, then $M = \mu$, and $\sigma'' = \sigma$. Clearly, the generalizations $C_{Np}(u, v)$ reduce to the basic indices $C_p(u, v)$ and $C_{Np} = C_p$, $C_{Npk} = C_{pk}$, $C_{Npm} = C_{pm}$ and $C_{Npmk} = C_{pmk}$.

Pearn and Kotz [5], and Pearn and Chen [13] applied Clements' method [14], and proposed estimators for calculating $C_p(u, v)$ for non-normal Pearsonian distributions. The proposed estimators are essentially based on the estimates U_p and L_p for the two percentiles $F_{99.865}$ and $F_{0.135}$, utilizing estimates of the mean, standard deviation, skewness and kurtosis. The indices in which those estimators correspond to can be expressed as:

$$C'_{\mathrm{Np}}(u,v) = (1-u) \times \frac{\mathrm{USL}-\mathrm{LSL}}{6\sqrt{\left[\frac{F_{99.865}-F_{0.135}}{6}\right]^2 + v(M-T)^2}} + u \times \min\left\{\frac{\mathrm{USL}-M}{3\sqrt{\left[\frac{F_{99.865}-M}{3}\right]^2 + v(M-T)^2}}, \frac{M-\mathrm{LSL}}{3\sqrt{\left[\frac{M-F_{0.135}}{3}\right]^2 + v(M-T)^2}}\right\}. \quad (3)$$

By setting $(u, v) = (0, 0), (0, 1), (1, 0)$ and $(1, 1)$, we obtain the following four generalizations of the basic indices for non-normal distributions, which we refer to as C'_{Np}, C'_{Npk}, C'_{Npm} and C'_{Npmk}. It is easy to verify that $C'_{\mathrm{Np}} = C_{\mathrm{Np}}$ and $C'_{\mathrm{Npm}} = C_{\mathrm{Npm}}$. If the underlying distribution is normal, then $M = \mu$, $F_{99.865} - F_{0.135} = 6\sigma$, $F_{99.865} - M = 3\sigma$, and $M - F_{0.135} = 3\sigma$. Clearly, the generalizations $C'_{\mathrm{Np}}(u, v)$ reduce to the basic indices $C_{\mathrm{p}}(u, v)$, $C'_{\mathrm{Np}} = C_{\mathrm{p}}$, $C'_{\mathrm{Npk}} = C_{\mathrm{pk}}$, $C'_{\mathrm{Npm}} = C_{\mathrm{pm}}$ and $C'_{\mathrm{Npmk}} = C_{\mathrm{pmk}}$.

$$C'_{\mathrm{Np}} = \frac{\mathrm{USL}-\mathrm{LSL}}{F_{99.865}-F_{0.135}},$$

$$C'_{\mathrm{Npk}} = \min\left\{\frac{\mathrm{USL}-M}{F_{99.865}-M}, \frac{M-\mathrm{LSL}}{M-F_{0.135}}\right\},$$

$$C'_{\mathrm{Npm}} = \frac{\mathrm{USL}-\mathrm{LSL}}{6\sqrt{\left[\frac{F_{99.865}-F_{0.135}}{6}\right]^2 + (M-T)^2}},$$

$$C'_{\mathrm{Npmk}} = \min\left\{\frac{\mathrm{USL}-M}{3\sqrt{\left[\frac{F_{99.865}-M}{3}\right]^2 + (M-T)^2}}, \frac{M-\mathrm{LSL}}{3\sqrt{\left[\frac{M-F_{0.135}}{3}\right]^2 + (M-T)^2}}\right\}.$$

3. COMPARISONS

To compare the two generalizations $C_{\mathrm{Np}}(u, v)$ and $C'_{\mathrm{Np}}(u, v)$ with the basic indices $C_{\mathrm{p}}(u, v)$, we consider the following example consisting of three processes A, B and C (heavily skewed with long tails) (Fig. 1). The distributions of processes A, B and C are χ^2_2 (chi-square distribution with degrees of freedom 2). The process characteristics are summarized in Table 1 ($\sigma_{\mathrm{A}} = \sigma_{\mathrm{B}} = \sigma_{\mathrm{C}} = 2.00$). We note that process B is on-target, whereas processes A and C are severely off-target. In fact, for process A the process mean $\mu_{\mathrm{A}} = \mathrm{LSL} = 30$ and for process C the process mean $\mu_{\mathrm{C}} = \mathrm{USL} = 44$. Process capabilities for A and C are far below the minimum requirements (capable) set in the electronic industries.

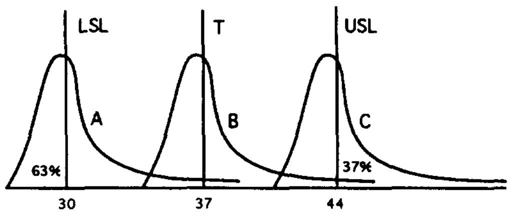

Fig. 1. Distributions of processes A, B and C.

Table 2 is a comparison between $C_{\mathrm{p}}(u, v)$ and $C_{\mathrm{Np}}(u, v)$ on the three processes A, B and C depicted in Fig. 1. The index values of C_{p}, C_{pk}, C_{pm} and C_{pmk} given to processes A and C are the same [(1.17, 0.00, 0.32, 0.00) for A and C]. Both processes A and C are severely off-target. However, the proportion of non-conforming for process A is 63%, which is significantly greater than that for process C (37%). Obviously, the basic indices $C_{\mathrm{p}}(u, v)$ inconsistently measure process capabilities of processes A and C in this case. On the other hand, the proposed generalizations $C_{\mathrm{Np}}(u, v)$ clearly differentiate processes A and C by giving smaller values to A and larger values to C (excluding C_{Np} which never considers process median and, hence, provides no sensitivity to process departure at all).

In the following, we compare the two generalizations $C_{\mathrm{Np}}(u, v)$, and $C'_{\mathrm{Np}}(u, v)$ based on a process characteristic discussed in Choi and Owen [15], which is related to loss functions. As we pointed out earlier, the generalizations $C_{\mathrm{Np}}(u, v)$ obtain the maximal values when the process is on-target ($M = T$). On the other hand, the generalizations $C'_{\mathrm{Np}}(u, v)$ obtain the maximal values when the process is off-target ($M < T$). To illustrate this point, we consider the following example with manufacturing specifications $\mathrm{LSL} = T - d$, $\mathrm{USL} = T + d$.

Table 3 displays the values of $C_{\mathrm{Np}}(u, v)$ and $C'_{\mathrm{Np}}(u, v)$ for various values of M, with fixed percentile deviation $F_{99.865} - F_{0.135} = 2d$. We note that in Table 3, the generalizations $C_{\mathrm{Np}}(u, v)$ are maximized by $M = T$ ($C_{\mathrm{Np}} = C_{\mathrm{Npk}} = C_{\mathrm{Npm}} = C_{\mathrm{Npmk}} = 1.00$). On the other hand, the generalizations $C'_{\mathrm{Np}}(u, v)$ are maximized by $M = T - 0.5\,d$ for C'_{Npk} and by $M = T - 0.25\,d$ for C'_{Npmk} (the process is off-target).

Table 1. Characteristics of processes A, B and C

Process	μ	M	σ	$F_{0.135}$	$F_{99.865}$
A	30.00	29.39	2.00	28.00	41.22
B	37.00	36.39	2.00	35.00	48.22
C	44.00	43.39	2.00	42.00	55.22

Table 2. A comparison between C_{Np} (u, v) and C_p (u, v)

Process	C_p	C_{pk}	C_{pm}	C_{pmk}	C_{Np}	C_{Npk}	C_{Npm}	C_{Npmk}
A	1.17	0.00	0.32	0.00	1.06	−0.09	0.29	−0.03
B	1.17	1.17	1.17	1.17	1.06	0.97	1.02	0.93
C	1.17	0.00	0.32	0.00	1.06	0.09	0.35	0.03

4. ELECTROLYTIC CAPACITOR MANUFACTURING

To illustrate how the generalizations $C_{Np}(u, v)$ may be applied to actual data collected from the factories, we present a case study on an aluminum electrolytic capacitor manufacturing process. The case which we studied was taken from an electronics company (located in Taiwan) who is a manufacturer and supplier of aluminum electrolytic capacitors supplying various kinds of capacitors including non-polarized capacitors (with radial or axial leads), and bi-polarized capacitors (with radial or axial leads).

Non-polarized capacitors are designed to be used in crossover networks with high-pitch, median-pitch and low-pitch sounds in high-fidelity audio speaker systems. The manufacturing specifications set on the performance characteristics for non-polarized capacitors are described in the following: the operating temperature range is between −40 and +85°C; the voltage range is between 50 and 100 V; the capacitance specified by the customers can be any value between 1 and 1000 μF; the leakage current is no greater than max $\{0.03\,CV,\ 3\,\mu A\}$ after 5 min application of the rated voltage, where C = the rated capacitance (in μF); and V = DC working voltage at 20°C.

Bi-polarized capacitors are designed for horizontal defection current in TV sets with high frequency and high ripple current flows, which have the advantage of small dissipation factors at high frequency. The manufacturing specifications set on the performance characteristics for bi-polarized capacitors are described in the following: the

Table 3. A comparison between C_{Np} (u, v) and C'_{Np} (u, v) (with production specifications USL = $T + d$, LSL = $T - d$)

Median	C'_{Np}	C'_{Npk}	C'_{Npm}	C'_{Npmk}	C_{Np}	C_{Npk}	C_{Npm}	C_{Npmk}
LSL	1.000	0.000	0.316	0.000	1.000	0.000	0.316	0.000
$T - 0.95d$	1.000	0.100	0.331	0.017	1.000	0.050	0.331	0.017
$T - 0.90d$	1.000	0.200	0.347	0.036	1.000	0.100	0.347	0.035
$T - 0.85d$	1.000	0.300	0.365	0.058	1.000	0.150	0.365	0.055
$T - 0.80d$	1.000	0.400	0.385	0.082	1.000	0.200	0.385	0.077
$T - 0.75d$	1.000	0.500	0.406	0.108	1.000	0.250	0.406	0.102
$T - 0.70d$	1.000	0.600	0.430	0.139	1.000	0.300	0.430	0.129
$T - 0.65d$	1.000	0.700	0.456	0.174	1.000	0.350	0.456	0.160
$T - 0.60d$	1.000	0.800	0.486	0.214	1.000	0.400	0.486	0.194
$T - 0.55d$	1.000	0.900	0.518	0.261	1.000	0.450	0.518	0.233
$T - 0.50d$	1.000	1.000	0.555	0.316	1.000	0.500	0.555	0.277
$T - 0.45d$	1.000	0.967	0.595	0.382	1.000	0.550	0.595	0.327
$T - 0.40d$	1.000	0.933	0.640	0.462	1.000	0.600	0.640	0.384
$T - 0.35d$	1.000	0.900	0.690	0.559	1.000	0.650	0.690	0.448
$T - 0.30d$	1.000	0.867	0.743	0.680	1.000	0.700	0.743	0.520
$T - 0.25d$	1.000	0.833	0.800	0.745	1.000	0.750	0.800	0.600
$T - 0.20d$	1.000	0.800	0.857	0.743	1.000	0.800	0.857	0.686
$T - 0.15d$	1.000	0.767	0.912	0.734	1.000	0.850	0.912	0.775
$T - 0.10d$	1.000	0.733	0.958	0.719	1.000	0.900	0.958	0.862
$T - 0.05d$	1.000	0.700	0.989	0.697	1.000	0.950	0.989	0.939
T	1.000	0.667	1.000	0.667	1.000	1.000	1.000	1.000
$T + 0.05d$	1.000	0.633	0.989	0.630	1.000	0.950	0.989	0.939
$T + 0.10d$	1.000	0.600	0.958	0.588	1.000	0.900	0.958	0.862
$T + 0.15d$	1.000	0.567	0.912	0.543	1.000	0.850	0.912	0.775
$T + 0.20d$	1.000	0.533	0.857	0.495	1.000	0.800	0.857	0.686
$T + 0.25d$	1.000	0.500	0.800	0.447	1.000	0.750	0.800	0.600
$T + 0.30d$	1.000	0.467	0.743	0.400	1.000	0.700	0.743	0.520
$T + 0.35d$	1.000	0.433	0.690	0.355	1.000	0.650	0.690	0.448
$T + 0.40d$	1.000	0.400	0.640	0.312	1.000	0.600	0.640	0.384
$T + 0.45d$	1.000	0.367	0.595	0.273	1.000	0.550	0.595	0.327
$T + 0.50d$	1.000	0.333	0.555	0.236	1.000	0.500	0.555	0.277
$T + 0.55d$	1.000	0.300	0.518	0.202	1.000	0.450	0.518	0.233
$T + 0.60d$	1.000	0.267	0.486	0.171	1.000	0.400	0.486	0.194
$T + 0.65d$	1.000	0.233	0.456	0.142	1.000	0.350	0.456	0.160
$T + 0.70d$	1.000	0.200	0.430	0.116	1.000	0.300	0.430	0.129
$T + 0.75d$	1.000	0.167	0.406	0.092	1.000	0.250	0.406	0.102
$T + 0.80d$	1.000	0.133	0.385	0.071	1.000	0.200	0.385	0.077
$T + 0.85d$	1.000	0.100	0.365	0.051	1.000	0.150	0.365	0.055
$T + 0.90d$	1.000	0.067	0.347	0.032	1.000	0.100	0.347	0.035
$T + 0.95d$	1.000	0.033	0.331	0.016	1.000	0.050	0.331	0.017
USL	1.000	0.000	0.316	0.000	1.000	0.000	0.316	0.000

Table 4. Non-polarized (NP) with radial leads; capacitance $T = 300\ \mu F$, LSL = 285 μF and USL = 315 μF

292	293	294	294	294	294	294	294	295	295
295	295	295	295	295	296	296	296	297	297
297	297	297	298	298	298	298	298	298	298
299	299	299	300	300	300	300	300	301	301
301	301	302	302	302	302	302	302	302	303
303	303	303	303	303	304	304	304	304	304
305	305	305	305	305	305	306	306	306	306
306	306	307	307	307	308	308	308	308	309
309	309	309	309	309	310	310	310	311	312
312	313	313	313	313	315	316	319	320	324

operating temperature range is between -40 and $+85$°C; the voltage range (specified by the customers) should not be greater than 50 V; the capacitance specified by the customers can be any value between 1 and 1000 μF, leakage current is no greater than max $\{0.03\ CV, 3\ \mu A\}$ after 5 min application of the rated voltage, where C = the rated capacitance (in μF) and V = DC working voltage at 20°C; and the capacitance at -40°C should not be less than 80% of the capacitance at 20°C.

In the manufacturing factory, the raw material aluminum-foil rolls shipped directly from the supplier are first cut into narrow aluminum-foil rolls with appropriate width (depending on the sizes of the capacitors to be made). The lead wire (aluminum for the head and copper for the legs) is then cleaned and stitched on to the aluminum foil sheet. Two piles of aluminum-foil sheet (with the lead wire stitched on) are rolled with two piles of paper to form the capacitor interior. The capacitor interior is then soaked into the electrolytic solution, loaded on to the automatic assembly machine, and assembled with the aluminum case, rubber end seal and PVC sleeve. The drying and cleaning work for the rubber end seal and copper-lead are processed by automatic machines before the assemblies. Finally, the assembled capacitor is loaded on to the shelf, and completed by the aging process to produce the capacitors.

The upper and lower specification limits, USL and LSL, for a particular model of aluminum non-polarized capacitor (with radial leads) were set to 285 and 315 (in μF). The target value is the mid-point between the two specification limits, which is 300. The collected sample data (a total of 100 observations) are displayed in Table 4. This is a non-normal distribution (based on the 100 observations).

5. CAPABILITY CALCULATIONS

Chang and Lu [16] considered a percentile method to calculate $F_{99.865}$ and $F_{0.135}$, and the median M, and applied Clements' method to obtain the percentile estimators for the three indices C'_{Np}, C'_{Npk} and C'_{Npm}. Extending their method to the generalizations $C_{Np}(u, v)$, we can construct a superstructure for the estimators of $C_{Np}(u, v)$, which may be expressed as the following:

$$\hat{C}_{Np}(u, v) = \frac{d - u|\hat{M} - m|}{3\sqrt{\left[\dfrac{\hat{F}_{99.865} - \hat{F}_{0.135}}{6}\right]^2 + v(\hat{M} - T)^2}},$$

$$\hat{F}_{99.865} = X_{(R_1)} + \left(\left[\frac{(99.865)n + 0.135}{100}\right] - R_1\right) \times (X_{(R_1+1)} - X_{(R_1)}),$$

$$\hat{F}_{0.135} = X_{(R_2)} + \left(\left[\frac{(0.135)n + 99.865}{100}\right] - R_2\right) \times (X_{(R_2+1)} - X_{(R_2)}),$$

$$\hat{M} = X_{(R_3)} + \left(\left[\frac{n+1}{2}\right] - R_3\right)(X_{(R_3+1)} - X_{(R_3)}),$$

where $R_1 = [(99.865\,n + 0.135)/100]$, $R_2 = [(0.135\,n + 99.865)/100]$ and $R_3 = [(n + 1)/2]$. In this setting, the notation $[R]$ is defined as the greatest integer less than or equal to the number R, and $x_{(i)}$ is defined as the ith order statistic.

For the 100 observations, $X_{(1)} = 292$, $X_{(99)} = 320$ and $X_{(100)} = 324$. To obtain the values of the estimators $\hat{C}_{Np}(u, v)$ for the proposed generalizations $C_{Np}(u, v)$, we first calculate the three sample percentiles obtaining $\hat{F}_{0.135} = 292.1$, $F_{99.865} = 323.5$ and $\hat{M} = 303$. Then, we substitute these values into the definition of $\hat{C}_{Np}(u, v)$ obtaining $\hat{C}_{Np} = 0.96$, $\hat{C}_{Npk} = 0.76$, $\hat{C}_{Npm} = 0.83$ and $\hat{C}_{Npmk} = 0.66$.

We note that the $\hat{C}_{Np}$ value is less than 1.00 and the process is "inadequate"; it indicates that the process is not adequate with respect to the given manufacturing specifications, either the process variation needs to be reduced or the process median needs to be shifted closer to the target value. In fact, there are four observations (316, 319, 320 and 324) falling outside the specification interval (LSL, USL) and the proportion of non-conforming is 4%.

The quality condition of such a process was considered to be unsatisfactory in the company. Some quality improvement activities, involving Taguchi's parameter designs, were initiated to identify the significant factors causing the process failing to meet the company's requirement. Consequently, machine settings for the aging process, as well as other process parameters were adjusted. To check whether the adjusted process was satisfactory, a new sample of 100 from the adjusted process was collected yielding the following measurements (Table 5). Specifications, process capability requirements remained the same. We performed the same calculations over the new sample of 100 observations. We obtained $\hat{C}_{Np} = 1.39$, $\hat{C}_{Npk} = 1.30$, $\hat{C}_{Npm} = 1.34$, and $\hat{C}_{Npmk} = 1.25$. We note that the new (adjusted) process has zero defectives. As a result, problems were successfully resolved and the quality of this manufacturing process improved significantly.

Table 5. Non-polarized (NP) with radial leads; capacitance $T = 300\ \mu F$, LSL $= 285\ \mu F$ and USL $= 315\ \mu F$

291	291	292	293	293	294	294	294	294	294
294	295	295	295	295	295	295	296	296	296
296	296	296	296	296	296	297	297	297	297
297	297	297	297	297	297	298	298	298	298
298	298	298	298	298	298	298	298	298	299
299	299	299	299	299	299	299	299	300	300
300	300	300	300	300	300	301	301	301	301
301	301	301	301	302	302	302	302	302	302
302	302	303	303	303	303	304	304	304	304
304	304	305	305	305	306	307	307	310	313

6. CONCLUSIONS

In this paper, we considered two generalizations of the basic indices $C_p(u, v)$, which we referred to as $C_{Np}(u, v)$ and $C'_{Np}(u, v)$, to cover non-normal distributions. If the underlying distribution is normal, then both generalizations $C_{Np}(u, v)$ and $C'_{Np}(u, v)$ reduce to the basic indices $C_p(u, v)$. The generalizations $C_{Np}(u, v)$ are compared with the basic indices $C_p(u, v)$ and $C'_{Np}(u, v)$. The results indicated that the proposed generalizations $C_{Np}(u, v)$ are more consistent and accurate than $C_p(u, v)$ and $C'_{Np}(u, v)$ indices in measuring process capability. In addition, we considered an estimation method based on sample percentiles to calculate the proposed generalizations $C_{Np}(u, v)$. Computations for obtaining the estimators do not require any assumptions on the underlying distributions or statistical tables.

We also presented a case study on an aluminum non-polarized capacitor manufacturing process to illustrate how the generalizations $C_{Np}(u, v)$ may be applied to actual data collected from the factories. The calculations are easy to understand, straightforward to apply and should be encouraged for in-plant applications.

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