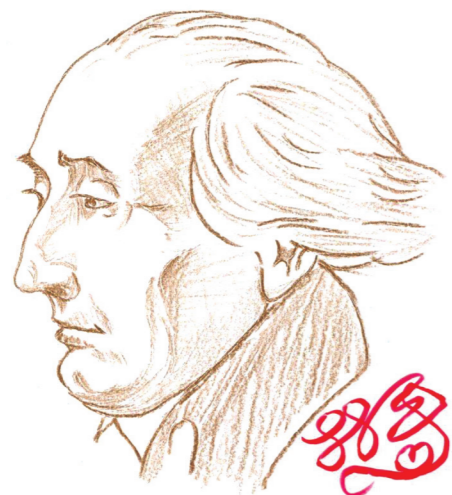# 泰勒展開式的傳奇

文／林一平 講座教授

拉格朗日（Joseph Lagrange；1736-1813）。
林一平／繪

最近我以 ChatGPT 加持的 Wolfram 數學平台進行一些泰勒展開式（Taylor expansion）的變形運算。ＡＩ可以通過自動計算來操作泰勒展開式的係數，給我不少相當有意思的答案，令我喜不自勝。接下來反思，我是不是被ＡＩ取代了。我早期進行無線通訊研究，分析無線電通道時，會用到泰勒展開式，只要問題能套入泰勒展開式，總是探驪得珠般的精神一振，因為複雜的數學式子可藉此清理乾淨。泰勒展開也運用於ＡＩ，神經網路的表現取決於其參數值，在訓練過程中會進行調整。泰勒展開可用於調整ＡＩ參數值，實現更有效的優化。

是誰想出這個好方法？顧名思義，自然是泰勒這位學者。展開式的確是以英國數學家泰勒（Sir Brook Taylor）命名。然而法國學者拉格朗日（Joseph Lagrange）早在一七九七年之前，就提出帶有餘項的泰勒定理。泰勒級數有無限多項，實際應用中，需要截斷，只取有限項，再利用泰勒定理估算這個近似值的誤差。我靠計算誤差，發表了好幾篇期刊論文。

泰勒展開式最神奇的故事發生在俄羅斯革命期間。約一九一七年，廿三歲的塔姆（Igor Tamm; 1895-1971）是烏克蘭奧德薩（Odessa）大學的物理學教授。當時兵荒馬亂，食物短缺，所以餓著肚皮的他去附近的一個村莊尋找食物。

當他來到村子時，運氣不好，遇到一群反對共產黨的暴徒，包圍了整個城鎮。塔姆穿著城市服裝，不像一般村民，讓暴徒們相當起疑，當場將他拿下。

暴徒首領問他以什麼為生。他解釋，自己是一名大學教授，在尋找食物。「你在大學教啥學科？」塔姆結結巴巴地回答說：「我教數學。」暴徒首領說：「好吧。我出個數學題目讓你解解看。你將麥克勞林（Maclaurin）系列展開到第 n 項，然後算出其誤差。算對了饒你一命，算錯了，我就開槍打你，讓你蒙主寵召。」麥克勞林級數是函數在 x=0 處的泰勒級數，是牛頓的學生麥克勞林於一七四二年推導出來的。

塔姆聽完後，可不是一般的驚嚇。在槍口下，握筆發抖，硬著頭皮，以泰勒展開式，戰戰兢兢地慢慢計算，設法找出了答案。他把結果展示給暴徒首領。首領細讀了一下，宣布道：「正確！回家吧。」塔姆屁滾尿流的逃離，也不知道土匪的名字。

塔姆後來發現契忍可夫輻射（Cherenkov radiation）而獲得一九五八年諾貝爾物理學獎。今日的學生養尊處優，常常讓我艷羨的想起暴徒首領，很想學他，拿槍逼學生，或許可以逼出一點研究成果。那麼ＡＩ是否能取代數學家？當暴徒拿槍逼您當場推導泰勒展開式時，ChatGPT 是幫不上忙的。（作者為前科技部代理部長）

林一平
國立陽明交通大學資工系終身講座教授暨華邦電子講座

現為國立陽明交通大學資工系終身講座 教授暨華邦電子講座，曾任科技部次長，為 ACM Fellow、IEEE Fellow、AAAS Fellow 及 IET Fellow。研究興趣為物聯網、行動計算及系統模擬，發展出一套物聯網系統 IoTtalk，廣泛應用於智慧農業、智慧教育、智慧校園等領域／場域。興趣多元，喜好藝術、繪畫、寫作，遨遊於科技與人文間自得其樂，著有＜閃文集＞、＜大橋驟雨＞。

The Legend of Taylor expansion

Recently, I've been exploring various manipulations of Taylor expansion on the Wolfram mathematics platform with the assistance of ChatGPT. AI demonstrates the capability to automatically compute coefficients within Taylor expansions, resulting in intriguing and genuinely delightful answers. This leads me to contemplate whether AI is gradually replacing me. In my earlier years, I immersed myself in wireless communications research, employing Taylor expansion during the analysis of radio channels. Each time a problem fit into a Taylor expansion, it felt like a moment of enlightenment, given its ability to organize intricate mathematical expressions neatly. Moreover, Taylor expansion finds application within the realm of AI. The effectiveness of neural networks relies on adjusting their parameter values during the training process. Taylor expansion can be leveraged to fine-tune AI parameter values, enabling more efficient optimization.

Who devised this effective method? As the name implies, it was none other than the scholar Taylor himself. The expansion is, indeed, attributed to the British mathematician Taylor (Sir Brook Taylor). However, before 1797, the French scholar Lagrange (Joseph Lagrange) introduced the concept of the Taylor theorem with remainders. The Taylor series consists of an infinite number of terms, but in practical applications, truncation becomes necessary, involving the selection of a limited number of terms. Subsequently, the Taylor theorem is used to estimate the error in this approximation. I have published multiple journal papers focused on the computation of errors.

The most remarkable story involving the Taylor expansion occurred during the Russian Revolution. Around 1917, a 23-year-old named Igor Tamm (1895-1971) held a position as a physics professor at Odessa University in Ukraine. This period was marked by chaos, disorder, and a scarcity of food. With hunger gnawing at his stomach, Tamm ventured into a nearby village in search of food. Unfortunately, luck did not favor him upon his arrival. He found himself face-to-face with a group of anti-communist rioters who had surrounded the entire town. Tamm's urban attire set him apart from the typical villagers, arousing suspicion among the rioters and ultimately leading to his instant capture.

The mob leader inquired about his means of livelihood. Tamm explained that he was a university professor and was searching for food. "What subject do you teach at the university?" Tamm stuttered in reply, "I teach mathematics." The mob leader said, "Very well. I shall present you with a mathematical problem to solve. Expand the Maclaurin series up to the nth term and calculate the error. Should you provide the correct answer, your life shall be spared. An incorrect response, however, will result in a gunshot, allowing you to meet your Creator." The Maclaurin series is, in fact, the Taylor series of a function at x=0, formulated by Newton's student Maclaurin in the year 1742.

Having listened, Tamm was more than just scared. With the gun's muzzle aimed at him and his hand trembling as he held the pen, he summoned his courage and, using Taylor's expansion, carefully and gradually worked out the solution. He then displayed the outcome to the mob leader. After scrutinizing it closely, the leader announced, "Correct! You're free to leave." In a state of panic, Tamm hurriedly escaped, not even aware of the bandit's name.

Tamm's subsequent discovery of Cherenkov radiation earned him the 1958 Nobel Prize in Physics. Today's students, who are familiar with a comfortable and privileged lifestyle, often evoke within me a sense of envy for the mob leader. I find myself genuinely intrigued by the notion of adopting his tactics—employing intimidation with firearms against students—as it might potentially lead to research breakthroughs. This raises the question: Can AI truly replace mathematicians? Nevertheless, when faced with a scenario in which a gangster forces you to perform an on-the-spot derivation of a Taylor series at gunpoint, ChatGPT would be unable to provide any assistance.

Dr. Jason Yi-Bing Lin

Lifetime Chair Professor of the Department of Computer Science at National Yang Ming Chiao Tung University and Winbond Chair Professor

Dr. Lin is currently a lifetime chair professor of the Department of Computer Science at National Yang Ming Chiao Tung University and Winbond chair professor. He is an ACM Fellow, IEEE Fellow, AAAS Fellow and IET Fellow. His research interests include Internet of Things, mobile computing, and system simulation. He has developed an Internet of Things system called IoTtalk, which is widely used in smart agriculture, smart education, smart campus, and other fields. He has a variety of interests, such as art, painting, and writing, as well as voyaging through science, technology, and humanities.