Experimental characterization of the vibrational behavior of a fluid-filled thin shell for the purpose of response estimation

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Experimental techniques are proposed for developing a model to estimate structural vibrations induced by internal acoustic pressure at low frequencies. In these methods, the acoustic field and the structure are treated as a coupled system with the acoustic loading effect taken into account. The response model between the internal pressure and the surface vibration is established by a modified modal test on the coupled system. Experiments are performed to investigate the surface vibrations of a closed steel cylindrical shell excited by a loudspeaker inside. The experimental results show that developed techniques provide accurate estimation for the acoustically induced vibrations. © *1995 Acoustical Society of America.*

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INTRODUCTION

Sound–structure interaction phenomena exist in many engineering applications. Examples include chimney stacks, heat exchangers, combustion chambers,[1,2] window–room systems,[3] vehicle cabins,[4,5] gas circulators in nuclear reactors,[6] and piping systems.[7] In these examples, a feedback loop exists between the fluid and the structure to cause dynamic instability. For instance, interior pressure oscillations could induce severe vibration of a combustion chamber, which is very damaging to engine performance. Acoustically induced vibration may cause failure of structural components. It is then highly desirable to estimate the responses at the design stage.

Experimental techniques are proposed for developing a model to estimate structural vibrations induced by internal acoustic pressure at low frequencies (below 400 Hz, where modal density is sufficiently low to admit modal analysis). In the analysis, the response of the structure is described with displacements, while the state of the acoustic volume is described with pressures. For simple systems, the modal characteristics of the individual subsystems can be obtained either analytically or numerically, which can then be combined into a coupled system by using modal synthesis techniques.[8–12] For complex systems where analytical and numerical methods may not always be possible, one has to resort to experimental approaches. Yet this poses another difficulty of being unable to completely remove the coupling between the acoustical and structural subsystems. This paper presents two experimental methods entitled the resonant system technique and the coupled system technique for estimation of acoustically induced vibrations. These two methods are based on the reformulated orthogonality condition proposed by Ma.[13,14] The required modal properties of the structure are identified in advance by an experimental modal analysis procedure.[15–19]

The experimental techniques are well suited for the applications where acoustically induced vibrations are desired, but direct measurement of vibrations may not always be feasible. In the methods, the acoustic field and the structure are treated as a coupled system, with fluid loading taken into account. The response model between the internal pressure and the surface vibration is established by a modified modal test on the coupled system. The vibration of the structure is then estimated by the response model of the coupled system. The difference between the resonant system technique and the coupled system technique lies in the fact that the former applies to only resonant frequencies, whereas the latter applies to bandlimited Gaussian signals. It should be pointed out that the developed methods are intended mainly for low-frequency analysis (even for bandlimited Gaussian signals) since the low-frequency responses are generally predominant in the overall response. Should one be concerned with high-frequency responses (e.g., above the Schroeder's cutoff) where high modal density precludes a meaningful use of modal analysis, *statistical energy analysis*[8] (SEA) is probably more adequate for this purpose. Another note is concerned with the rotational degrees of freedom (DOF) which are not considered in the following presentation. This is because not only are they difficult to measure but they also have negligible effects on the structural response induced by inviscid acoustic media.

Experiments are conducted to investigate the surface vibrations of a closed steel cylindrical shell excited by a loudspeaker inside. The performance and the limitations of the estimation methods are compared and discussed.

I. THEORY AND METHODS

A. The resonant system technique

Fluid loading is present between a vibrating surface (excited either mechanically or acoustically) and the medium in contact with the structure. If the fluid volume is unbounded, the fluid simply mass-loads the structure and provides acoustic radiation damping. On the other hand, if the fluid volume is bounded, the problem becomes more complex because both the structure and the fluid can sustain standing waves, and there is interaction between the structure and the fluid.

The energy is not dissipated, but is constantly circulating between the acoustic volume and the containing structure. This mechanism leads to the gyrostatic effect in the coupled system.[8]

Consider the equation of motion of a fluid-filled *planar* structure subjected to acoustic excitation. The relation between the acoustic pressure and the surface velocity of the structure can be expressed in temporal frequency and spatial frequency domains as

$$[\tilde{Z}_m(\mathbf{k},\omega)+\tilde{Z}_r(\mathbf{k},\omega)]\tilde{v}(\mathbf{k},\omega)=\tilde{p}_{\mathrm{bl}}(\mathbf{k},\omega) \tag{1a}$$

or

$$\tilde{Z}_m(\mathbf{k},\omega)\tilde{v}(\mathbf{k},\omega)=\tilde{p}(\mathbf{k},\omega), \tag{1b}$$

where ω is the temporal frequency, $\mathbf{k}=(k_x,k_y)$ is the wave-number vector associated with the (x,y) plane, $\tilde{v}(\mathbf{k},\omega)=j\omega\tilde{w}(\mathbf{k},\omega)$ is the transformed surface velocity, $\tilde{p}_{\mathrm{bl}}$ denotes the *blocked pressure*,[8] $\tilde{p}=\tilde{p}_{\mathrm{bl}}-\tilde{p}_r$ is the total acoustic pressure, $\tilde{Z}_m$ is the mechanical impedance of the structure, and $\tilde{Z}_r(\mathbf{k},\omega)=\tilde{p}_r(\mathbf{k},\omega)/\ \tilde{v}(\mathbf{k},\omega)$ is the acoustic radiation impedance.

In order to estimate the structural vibration $\tilde{v}$, one way is to use Eq. (1a). This approach, however, poses difficulty because determination of $\tilde{p}_{\mathrm{bl}}$ requires the structure to be perfectly clamped. Another alternative is to use Eq. (1b). This results in yet another problem with measuring $\tilde{Z}_m$ in vacuum. A crude remedy to this dilemma is to assume $\tilde{Z}_m\approx\tilde{Z}$ in Eq. (1b):

$$\tilde{Z}(\mathbf{k},\omega)\tilde{v}(\mathbf{k},\omega)\approx\tilde{p}(\mathbf{k},\omega). \tag{2}$$

Direct application of Eq. (2) generally gives rise to significant errors when the coupling between the structure and the sound field is not negligible. Although the above analysis has been developed here specifically for an infinite plane, in which case both the scattered and radiated fields have a unique wave-number component, the fact applies to the other types of flexible surfaces in contact with fluids.[8] This motivated the development of the following experimental method entitled the resonant system technique to estimate the vibrations induced by acoustic excitations.

In this method, the sound field and the structure are treated as a coupled system. The subsystems share common natural modes pertaining to the coupled system. A mode shape *per se* represents the relative amplitudes of motion of all DOFs of a system at a particular resonant frequency. For the problem considered herein, the response variables involve *both* displacements of the structure and sound pressures of the acoustic volume. The (displacement) amplitudes of all points at a resonant frequency can then be determined from any measured pressure amplitude of a certain point, based on the knowledge of the mode shape at that frequency. To extract the required mode shapes of a coupled system, an experimental modal analysis technique can be employed. Consider a coupled system represented by the following discrete model:[3,13]

$$\mathbf{M}\ddot{\mathbf{d}}+\mathbf{K}\mathbf{d}=\mathbf{f}_c, \tag{3}$$

where

$$\mathbf{d}=\begin{Bmatrix}\mathbf{x}\\ \mathbf{p}\end{Bmatrix},\quad \mathbf{f}_c=\begin{Bmatrix}\mathbf{f}\\ \mathbf{u}\end{Bmatrix},$$

$$\mathbf{M}=\begin{bmatrix}\mathbf{M}_{ss} & \mathbf{o}\\ \mathbf{M}_{as} & \mathbf{M}_{aa}\end{bmatrix},\quad \mathbf{K}=\begin{bmatrix}\mathbf{K}_{ss} & \mathbf{K}_{sa}\\ \mathbf{o} & \mathbf{K}_{aa}\end{bmatrix}.$$

$\mathbf{M}$ and $\mathbf{K}$ are the mass and stiffness matrices, respectively. The subscripts s and a denote the structure and the acoustic field, respectively. It can be shown that the coupling matrices $\mathbf{K}_{sa}$ and $\mathbf{M}_{as}$ satisfy $\mathbf{K}_{sa}=-\mathbf{M}_{as}^T$. Here, $\mathbf{x}$ is the nodal displacement vector of the structure, $\mathbf{p}$ is the sound pressure vector of the acoustic field, $\mathbf{f}$ is the mechanical force vector applied to the structure, and $\mathbf{u}$ is the volume acceleration vector of the acoustic field. It follows that the eigenvalue problem associated with Eq. (3) is

$$(\mathbf{K}-\omega_r^2\mathbf{M})\phi_r=\mathbf{0}, \tag{4}$$

where ω_r is the rth resonant frequency of the coupled system, ϕ_r is the corresponding *right eigenvector* of the coupled system, and $\phi_r=[\phi_{sr}^T\quad \phi_{ar}^T]^T$, where ϕ_{sr} and ϕ_{ar} denote the structural and acoustical elements of the eigenvector.

It should be noted that the structural modes are not orthogonal because of radiation damping. Nevertheless, the *coupled* modes are orthogonal since the coupling mechanism in Eq. (3) only exchanges (but does not dissipate) energy between two subsystems. Because the matrices $\mathbf{K}$ and $\mathbf{M}$ in Eq. (8) are not symmetric, the orthogonality condition of the coupled system is not as straightforward as that of the structure alone. This condition can be derived by means of *left eigenvectors* that satisfy the *left eigenvalue problem*

$$\bar{\phi}_r^T(\mathbf{K}-\omega_r^2\mathbf{M})=\mathbf{0}. \tag{5}$$

It can be shown in Ref. 14 that the following three properties hold for the general eigenvalue problem of a coupled sound–structure system: (i) All eigenvalues and eigenvectors in Eq. (4) are real; (ii) the left eigenvectors can be related to the right eigenvectors by

$$\bar{\phi}_r=\begin{Bmatrix}\phi_{sr}\\ \dfrac{1}{\omega_r^2}\,\phi_{ar}\end{Bmatrix}\quad(\omega_r\neq0), \tag{6}$$

and (iii) the orthogonality condition for the coupled systems is given as

$$\bar{\phi}_i^T\mathbf{M}\phi_j=\phi_{si}^T\mathbf{M}_{ss}\phi_{sj}+(1/\omega_i^2)(\phi_{ai}^T\mathbf{M}_{as}\phi_{sj}+\phi_{ai}^T\mathbf{M}_{aa}\phi_{aj})$$
$$=\delta_{ij}, \tag{7}$$

$$\bar{\phi}_i^T\mathbf{K}\phi_j=\phi_{si}^T\mathbf{K}_{ss}\phi_{sj}+\phi_{si}^T\mathbf{K}_{sa}\phi_{aj}+(1/\omega_i^2)\phi_{ai}^T\mathbf{K}_{aa}\phi_{aj}$$
$$=\omega_i^2\delta_{ij}, \tag{8}$$

where δ_{ij} is the Kronecker delta. The orthogonality and mass-normalization condition for the coupled system can be expressed as

$$\bar{\Phi}^T\mathbf{K}\Phi=[{}^{\backslash}\omega_r^2{}_{\backslash}]\quad\text{and}\quad\bar{\Phi}^T\mathbf{M}\Phi=\mathbf{I}. \tag{9}$$

Using Eq. (9), the receptance matrix of the coupled system can then be written as

$$\mathbf{Y}=(\mathbf{K}-\omega^2\mathbf{M})^{-1}=\Phi[{}^{\backslash}1/(\omega_r^2-\omega^2){}_{\backslash}]\bar{\Phi}^{\mathbf{T}}. \tag{10}$$

Up to this point, no allowance can be made for dissipation mechanisms in the structure or the external field. It is then common practice to employ *ad hoc* damping terms[8] into Eq. (9):

$$\mathbf{Y}=\boldsymbol{\Phi}[{}^{\backslash}1/(\omega_r^2+j2\zeta_r\omega_r\omega-\omega^2)_{\backslash}]\bar{\boldsymbol{\Phi}}^T, \tag{11}$$

where ζ_r is the generalized modal damping ratio of the rth mode. For the modal expression inside the bracket, it is assumed that the damping coupling between modes is neglected. This is generally a reasonable approximation (at least for lightly damped systems) and is adopted to simplify modal tests.[8,16] Hence, the receptance matrix can be written explicitly as

$$y_{ij}(\omega)=\frac{-1}{\omega^2 M_{ij}}+\sum_{r=m_1}^{m_2}\frac{{}_r a_{ij}}{\omega_r^2+j2\zeta_r\omega_r\omega-\omega^2}+\frac{1}{K_{ij}}, \tag{12}$$

where m_1 and m_2 are the first and the last modes, respectively, in the frequency range of curve fitting, ${}_r a_{ij}={}_r\phi_i\,{}_r\bar{\phi}_j$ is the residue number for the rth mode, and M_{ij} and K_{ij} denote the lumped mass and stiffness effect due to the residual modes.[16]

The modal parameters $\omega_r, \zeta_r, {}_r a_{ij;\ r=1\sim N}$ can be extracted from the measured frequency response functions (FRF) through a curve-fitting procedure. Similar to a conventional modal analysis, it requires the measurement of only one row (or one column) of FRF matrix $\mathbf{Y}$ (which is the receptance matrix in our case) to extract mode shapes and other parameters.

Once the modal model is established, the dynamic response of each DOF at any resonant frequency can in principle be calculated based on the mode shape that represents the relative magnitudes at that particular frequency. Note that this is not the case for nonresonant frequencies because at any of these frequencies all modes contribute to the total response and the relative amplitude is not a simple ratio.

B. The coupled system technique

Similar to the resonant system technique, this method treats the system as a coupled system. Let the matrix $\mathbf{Y}_{xp}$ be the FRF matrix between the pressure vector $\tilde{\mathbf{p}}$ and the displacement vector $\tilde{\mathbf{x}}$. The response model in the frequency domain takes the form

$$\tilde{\mathbf{x}}=\mathbf{Y}_{xp}\tilde{\mathbf{p}}. \tag{13}$$

The simplicity masks the subtlety of the multiple-input–output model. Before the estimation of acoustically induced vibrations, the FRF matrix $\mathbf{Y}_{xp}$ must be provided. This poses a problem since direct measurement of $\mathbf{Y}_{xp}$ requires the excitation pressure to be applied only at a single DOF, which is impossible in reality. To circumvent this difficulty, an indirect approach is developed to obtain $\mathbf{Y}_{xp}$. For harmonic excitations, i.e., $\mathbf{f}_c=\tilde{\mathbf{f}}_c\exp(j\omega t)$ and $\mathbf{d}=\tilde{\mathbf{d}}\exp(j\omega t)$, Eq. (3) can be written as

$$\tilde{\mathbf{d}}=\mathbf{Y}\tilde{\mathbf{f}}_c, \tag{14}$$

where $\mathbf{Y}$ is the FRF matrix of the coupled system. If there are n and m DOFs for the structure and the acoustic field, respectively, then the dimension of $\mathbf{Y}$ is $(n+m)\times(n+m)$. Equation (14) may be written in a partitioned form as

$$\begin{Bmatrix}\tilde{\mathbf{x}}\\ \tilde{\mathbf{p}}\end{Bmatrix}=\begin{bmatrix}\mathbf{Y}_{xf} & \mathbf{Y}_{xu}\\ \mathbf{Y}_{pf} & \mathbf{Y}_{pu}\end{bmatrix}\begin{Bmatrix}\tilde{\mathbf{f}}\\ \tilde{\mathbf{u}}\end{Bmatrix}, \tag{15}$$

where $\mathbf{Y}_{xf}$ is an $n\times n$ FRF submatrix associated with $\tilde{x}_i/\tilde{f}_j$, $\mathbf{Y}_{pf}$ is an $m\times n$ FRF submatrix associated with $\tilde{p}_i/\tilde{f}_j$, $\mathbf{Y}_{xu}$ is an $n\times m$ FRF submatrix associated with $\tilde{x}_i/\tilde{u}_j$, and $\mathbf{Y}_{pu}$ is an $m\times m$ FRF submatrix associated with $\tilde{p}_i/\tilde{u}_j$. If only the acoustic excitation is present in the system, i.e., $\tilde{\mathbf{f}}=\mathbf{0}$, then Eq. (15) leads to

$$\tilde{\mathbf{x}}=\mathbf{Y}_{xu}\tilde{\mathbf{u}} \tag{16}$$

and

$$\tilde{\mathbf{p}}=\mathbf{Y}_{pu}\tilde{\mathbf{u}}. \tag{17}$$

Inverting Eq. (17) gives

$$\tilde{\mathbf{u}}_{\text{eq}}=\mathbf{Y}_{pu}^{-1}\tilde{\mathbf{p}}, \tag{18}$$

where $\mathbf{Y}_{pu}$ is assumed nonsingular and $\tilde{\mathbf{u}}_{\text{eq}}$ can be regarded as *equivalent volume acceleration sources*. Substituting Eq. (18) into Eq. (16) yields

$$\tilde{\mathbf{x}}=\mathbf{Y}_{xu}\tilde{\mathbf{u}}_{\text{eq}}=\mathbf{Y}_{xu}\mathbf{Y}_{pu}^{-1}\tilde{\mathbf{p}}. \tag{19}$$

As a result, the FRF submatrix $\mathbf{Y}_{xp}$ in Eq. (15) can be obtained indirectly as

$$\mathbf{Y}_{xp}=\mathbf{Y}_{xu}\mathbf{Y}_{pu}^{-1}. \tag{20}$$

The experimental modal analysis employed in the resonant system technique can equally be used here to construct the modal model. The full FRF matrix $\mathbf{Y}$ in Eq. (14) can be computed from the modal model by using Eq. (11). Then, the FRF submatrices $\mathbf{Y}_{xu}$ and $\mathbf{Y}_{pu}$ can be obtained by partitioning the matrix $\mathbf{Y}$. By substituting the curve-fitted FRF matrices and the measured excitation pressures $\tilde{\mathbf{p}}$ into Eq. (19), one may calculate the structural displacements $\tilde{\mathbf{x}}$. However, there is a pitfall in this process. The curve-fitting procedure in general suffers from ill-conditioned problems, as will become clear in the experimental investigations. Direct inversion of the submatrix $\mathbf{Y}_{pu}$ in Eq. (19) usually results in significant errors due to rank deficiency. This inherent problem stems from the modal truncation and spatial truncation involved in using a discrete model to represent a continuous system. If the number of spatial coordinates is greater than the number of modes in the frequency range of interest, the curve-fitted modal model will lead to singular matrices. Hence, two methods are proposed to alleviate the ill-conditioned problem. One alternative is to pseudoinvert the curve-fitted FRF by singular-value decomposition (SVD).[20] Another approach is to directly measure $\mathbf{Y}_{pu}$ by successive application of an acoustic source to each DOF instead of curve-fitting it. However, a problem will arise because it is difficult to construct a small and movable source of calibrated acoustic volume acceleration $\tilde{u}$. Thus we replace the conceptual volume acceleration source by the electrical voltage source $\tilde{v}$ that drives the loudspeaker, i.e.,

$$\tilde{u}(\omega)=\tilde{t}(\omega)\tilde{v}(\omega), \tag{21}$$

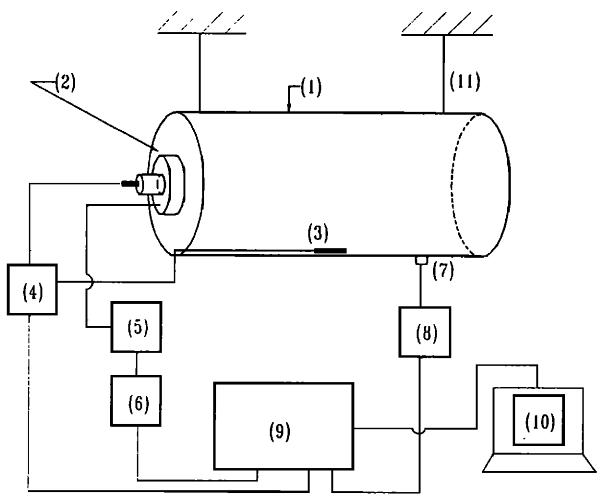

FIG. 1. Experimental setup. (1) cylindrical shell, (2) acoustic driver (loudspeaker), (3) microphone, (4) microphone power supply, (5) power supply, (6) white-noise source, (7) accelerometer, (8) charge amplifier, (9) FFT analyzer, (10) personal computer, (11) elastic string.

where $\tilde{t}(\omega)$ is the FRF between the signals $\tilde{u}$ and $\tilde{v}$. If the same loudspeaker is used for all measurements of the FRFs, the overall FRF matrix between the pressure and displacement is then

$$\mathbf{Y}_{xp}=\mathbf{Y}_{xu}\mathbf{Y}_{pu}^{-1}=(1/\tilde{t}\mathbf{Y}_{xv})(\tilde{t}\mathbf{Y}_{pv}^{-1})=\mathbf{Y}_{xv}\mathbf{Y}_{pv}^{-1}, \tag{22}$$

where the submatrix $\mathbf{Y}_{xv}$ is associated with $\tilde{x}_i/\tilde{v}_j$ and the submatrix $\mathbf{Y}_{pv}$ is associated with $\tilde{p}_i/\tilde{v}_j$, and each entry of the FRFs can easily be measured by the voltage source. Substituting Eq. (22) into Eq. (19) gives

$$\tilde{\mathbf{x}}=\mathbf{Y}_{xv}\mathbf{Y}_{pv}^{-1}\tilde{\mathbf{p}}. \tag{23}$$

From Eq. (23), the FRF submatrices $\mathbf{Y}_{xv}$ and $\mathbf{Y}_{pv}$ can be used to estimate the acoustically induced vibrations. Note that inversion of $\mathbf{Y}_{pv}$ does not have ill-conditioned problems since it is directly measured and is generally full rank. In addition, the coupled system technique does not require the system of interest to be weakly coupled nor in resonance.

II. EXPERIMENTAL INVESTIGATION

Figure 1 shows the experimental setup and instrumentation. A steel cylindrical shell of length 1 m, radius 0.2 m, and thickness 2 mm is selected to verify the developed techniques. The ends of the cylindrical shell are covered with steel plates and the closure is done by screws and tapes to ensure air tightness and vibration suppression. The shell is freely suspended by elastic strings. A 4-Ω and 30-W loudspeaker is attached on the top cover of the shell to serve as an acoustic source. Sound pressure and surface acceleration are measured with a 1/2-in. condenser microphone and an 11-g accelerometer, respectively. It should be noted that in the following experimental results the source points used to generate the response curves were all different from the points used to measure the acoustic pressure under point force mechanical excitation.

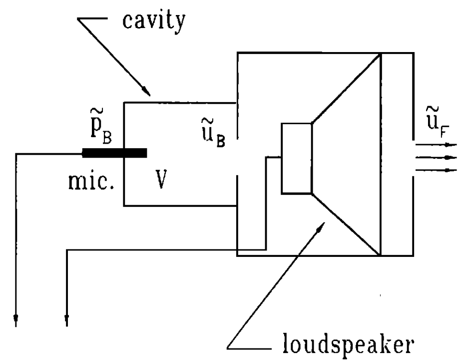

FIG. 2. The convertible acoustic driver.

A. The resonant system technique

This method requires the measurement of volume acceleration of the acoustic source to determine $\mathbf{Y}_{xu}$ or $\mathbf{Y}_{pu}$. To this end, a convertible acoustic driver[21,22] is mounted on the top of the cylindrical shell to excite the internal acoustic field (Fig. 2).

To calculate the mode shapes of the coupled system, it suffices to measure only one column or one row of the FRF matrix. Figure 3 shows 32 DOFs for the FRF measurement, including the structural DOFs 1–30 and the acoustic DOFs 31 and 32 located on the external and the internal surfaces of the shell, respectively. The volume acceleration excitation is applied to the acoustic DOF 31 so that a set of FRFs y_{i31}, $i=1-32$ can be measured (y_{i31}, $i=1-30$ correspond to $\mathbf{Y}_{xu}$ and y_{i31}, $i=31$, 32 correspond to $\mathbf{Y}_{pu}$). From the measured FRFs, the natural frequencies and mode shapes of the coupled system can be calculated by the forgoing curve-fitting procedure.

An experiment is performed to verify this method. The cylindrical shell is subjected to sinusoidal acoustic excitations at the resonant frequencies 323 and 385 Hz, by using a loudspeaker located inside the shell at a position different

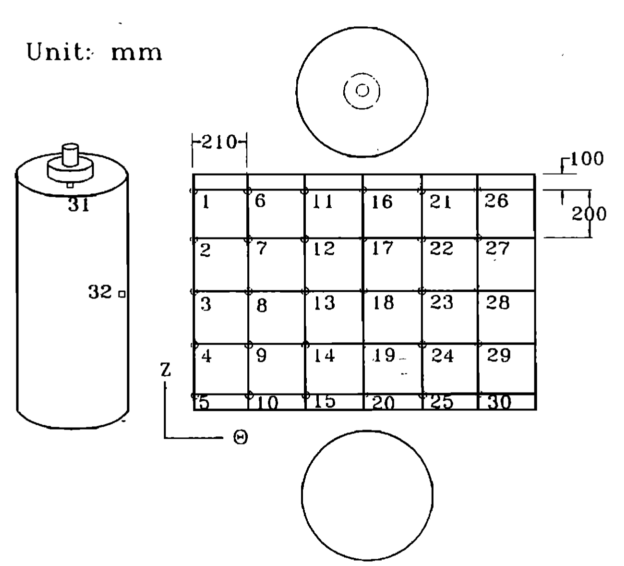

FIG. 3. The DOFs on the cylindrical shell defined for the resonant system technique. O: structural DOF, □: acoustic DOF.

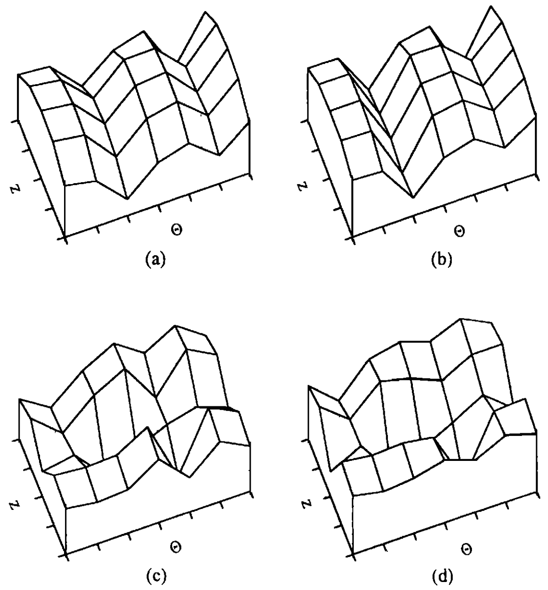

FIG. 4. Displacement amplitudes of all structural DOFs for the resonant frequencies 323 and 385 Hz estimated by the resonant system technique in comparison with the measured data. (a) Estimated data at 323 Hz; (b) measured data at 323 Hz; (c) estimated data at 385 Hz; (d) measured data at 385 Hz.

from the original one. Then, based on the extracted mode shapes, the surface displacements at the resonant frequencies are estimated by the measured pressure at the acoustic DOF 32. Figure 4 compares the estimated displacements with the measured data for each structural DOF. The results appear to be in good agreement except for some discrepancies because of numerical errors of curve fitting.

B. The coupled system technique

As mentioned earlier, two methods are available for obtaining the FRF submatrices $\mathbf{Y}_{xu}$ and $\mathbf{Y}_{pu}$ required by this method. One may construct the modal model of the coupled system, based on only one column of FRF measurements, and partition the resulting FRF matrix. Alternatively, one may directly measure the full FRF matrix without curve-fitting. In the following case of FRF measurement, six DOFs (including the structural DOFs 1 and 2 and the acoustic DOFs 3–6) are defined for the cylindrical shell, as shown in Fig. 5. Hence, the $\mathbf{Y}_{xu}$ and $\mathbf{Y}_{pu}$ are 2×4 and 4×4 matrices, respectively.

When the first method is employed, one column of the FRF matrix is measured by using the volume acceleration source acting on DOF 3. By curve-fitting the FRFs in the frequency range 70–390 Hz, the modal parameters are calculated. Then, the modal model of the coupled system is constructed from the extracted modal parameters.

Alternatively, when the full FRF matrix is directly measured by using Eq. (22), the voltage signal driving the loudspeaker is taken as the excitation. The FRFs y_{ij} between the excitation position j (voltage signal) and the response position i (displacement or pressure) are measured.

In this experiment, a loudspeaker is located at the acoustic DOF 3. The sound pressures $\tilde{\mathbf{p}}$ are measured at each

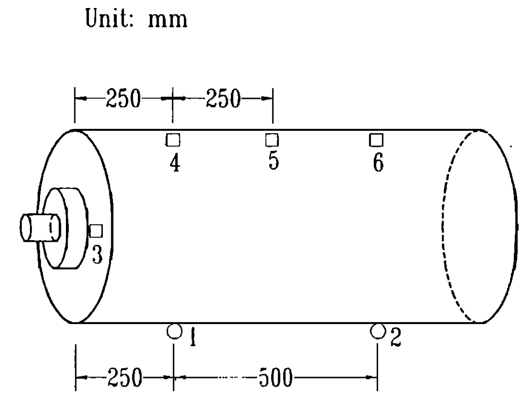

FIG. 5. The DOFs on the cylindrical shell defined for the coupled system technique. ○: structural DOF, □ acoustic DOF.

acoustic DOF. The sound pressures $\tilde{\mathbf{p}}$ and the FRF submatrices $\mathbf{Y}_{xu}$ and $\mathbf{Y}_{pu}$ are substituted into Eq. (19) to calculate the displacements $\tilde{\mathbf{x}}$ at each frequency.

Figure 6 compares the estimated and the measured displacement power spectra of the structural DOF 1. It can be seen from the result that significant error occurs due to rank deficiency of $\mathbf{Y}_{pu}$. This can be alleviated by SVD pseudoinversion. The calculated result in Fig. 7 appears better than that obtained from direct inversion of $\mathbf{Y}_{pu}$ in Fig. 6.

Next, the method of direct FRF measurement is applied to the same test case. As can be observed in the experimental result of Fig. 8, the estimated displacement power spectrum is in excellent agreement with the measured spectrum because this approach essentially eliminates the rank-deficiency problem of $\mathbf{Y}_{pu}$.

III. CONCLUSION

Two experimental techniques have been developed to estimate shell vibrations induced by internal acoustic pressure. Experiments are conducted on a steel cylindrical shell to verify the proposed methods. The surface vibrations estimated by these methods are found to be in good agreement with the measured data.

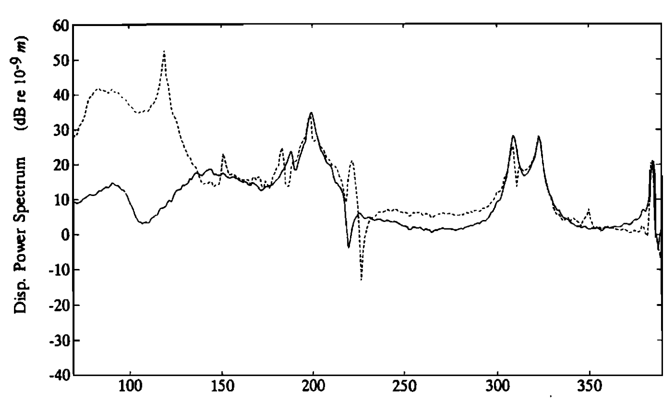

FIG. 6. Displacement power spectrum of DOF 1 estimated by the coupled system technique based on the derived modal model with four acoustic DOFs. ———, estimated data; --- measured data.

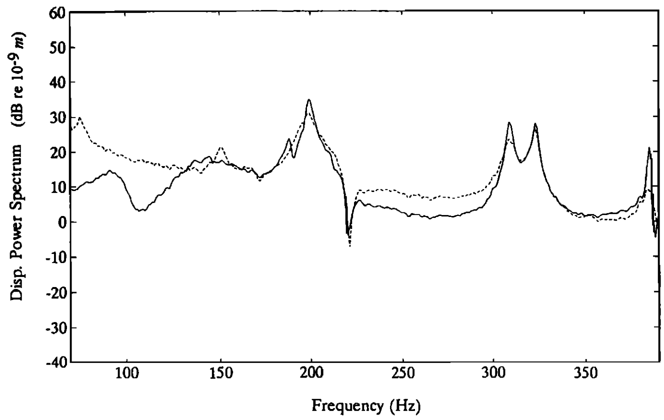

FIG. 7. Displacement power spectrum of DOF 1 estimated by the coupled system technique based on the derived modal model with four acoustic DOFs. Pseudoinversion of $\mathbf{Y}_{pu}$ is calculated by SVD. ———, estimated data; --- measured data.

From the experimental results, some remarks are in order. The advantage of the resonant system technique is that it provides satisfactory estimations for the acoustically induced vibration at resonant frequencies. Its disadvantages, on the other hand, are that (i) it is not applicable to nonresonant frequencies and (ii) one need construct a volume acceleration source for FRF measurement. The advantages of the coupled system technique are that (i) it provides accurate estimations for the acoustically induced vibration, (ii) it has less frequency limitation than the other two methods, and (iii) its experimental procedure is simple to apply. The disadvantage of this method lies in the fact that the data-acquisition procedure appears somewhat tedious if direct FRF measurement is employed.

The experimental techniques are well suited for the applications where structural vibrations induced by internal acoustic fields are desired, but direct measurement of vibrations is not feasible. For complex problems where analytical or numerical approaches are prohibitive, the proposed experimental techniques serve as simple but useful alternatives.

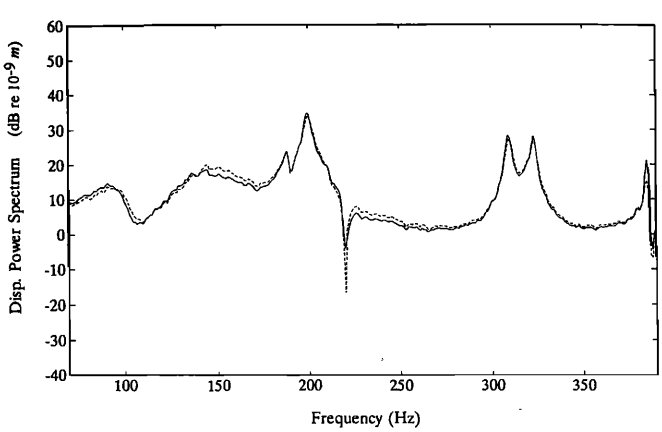

FIG. 8. Displacement power spectrum of DOF 1 estimated by the coupled system technique based on direct FRF measurement with four acoustic DOFs. ———, estimated data; --- measured data.

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