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Fuzzy logic approach for removing noise

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Abstract

This paper shows how fuzzy reasoning techniques can be applied on smoothing filters design. It uses the fuzzy concept to decide whether a pixel in an image is a noise one or not in order to achieve maximum noise reduction in uniform areas and preserve details as well.

Keywords: Smoothing; Fuzzy reasoning; Uniform surface uncertainty; Fuzzy image

1. Introduction

To avoid distorting the visual information in an image, smoothing algorithms must preserve the details. However, some fine details are frequently treated as noise by general smoothing filters. In such situations,we encounter the problem of ambiguity or uncertainty between noise and fine details.

A new nonlinear smoothing method using fuzzy reasoning techniques is proposed. Fuzzy set theory [1,2] is a mathematical tool in modeling ambiguity or uncertainty and has been applied in image smoothing [3,4] and texture analysis [5]. In this paper, we use the definition of uniform surface uncertainty [3] as the fuzzification function. The uniform surface uncertainty, which ranges from 0 to 1, for a point p in the image is considered as the degree of belongingness of p to a uniform physical surface. Therefore, we can transform a grayscale image into a fuzzy image by using the fuzzification function. Then, we input the fuzzy image to fuzzy-rule-based system and produce a new fuzzy image. The processed fuzzy image is transformed back into a new grayscale smoothing image by the inverse of the fuzzification operator.

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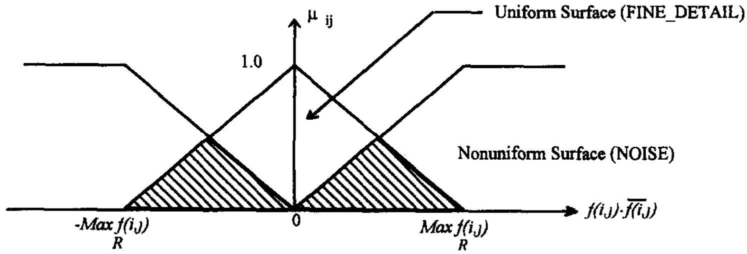


Fig. 1. Fuzzy membership function for a uniform surface.

The paper is organized as follows. In Section 2, we will introduce the definition of uniform surface uncertainty and fuzzy image. In Section 3, we will review fuzzy logic and approximate reasoning. The new proposed smoothing method and some experimental results will be shown and discussed in Section 4. Conclusions will be given in Section 5.

2. Uniform surface uncertainty and fuzzy image Π_γ

A grayscale image f can be transformed into a fuzzy image by a fuzzification function ϕ. A variety of fuzzification functions can be used to reflect the degree to which a pixel intensity represents a uniform physical surface. Here, a simplified triangular membership function illustrated in Fig. 1 is used to define a uniform surface γ. The uniform surface uncertainty is defined as

$$\mu_{ij} = 1 - \left[\frac{|f(i,j) - \bar{f}(i,j)|}{\max\limits_{R} f(i,j)} \right] \tag{1a}$$

$$= 1 - \left[\frac{1}{\max\limits_{R} f(i,j)} \frac{f(i,j) - \bar{f}(i,j)}{\operatorname{sgn}\big(f(i,j) - \bar{f}(i,j)\big)} \right] \tag{1b}$$

where sgn(·) is the sign function, $\max_R f(i,j)$ is the maximum intensity within the $(\omega \times \omega)$ surface region R centered at point (i, j) and the average intensity is given by

$$\bar{f}(i,j) = \frac{1}{\omega \times \omega - 1} \sum_{m,n \neq i,j}^{R} f(m,n). \tag{2}$$

Note that if $f(i, j)$ is equal to the average neighborhood intensity $\bar{f}(i, j)$ then $f(i, j)$ possesses 'full membership' to the surface region R, i.e. $\mu_{ij} = 1$. Alternatively, if $f(i, j)$ is significantly different from the average neighborhood intensity $\bar{f}(i, j)$, then $\mu_{ij} \to 0$. A large membership value is interpreted as a pixel that lies on a uniform surface region, and a small membership value reflects a pixel that maybe a noise one. All of these μ_{ij} form a fuzzy image Π_γ. Inputting the fuzzy image Π_γ

into the proposed fuzzy-rule-based system, we obtain a processed fuzzy image Π_Y^p. The processed fuzzy image is transformed back into a new gray scale image f^p by the inverse of the fuzzification operator, i.e. $f^p = \phi^{-1}(\Pi_Y^p)$. From Eq. (1), the defuzzification function, ϕ^{-1}, is given by:

$$f^p(i, j) = \bar{f}(i, j) + \max_R f(i, j) \operatorname{sgn}\big(f(i, j) - \bar{f}(i, j)\big)\big[1 - \mu_{ij}^p\big] \tag{3}$$

where $f^p(i, j)$ is the processed intensity value and μ_{ij}^p is the pixel of the processed fuzzy image Π_Y^p.

3. Fuzzy approximate reasoning

In our daily life, we often make inferences whose antecedents and consequences contain fuzzy concepts. Such an inference cannot be made adequately by the methods that are based on classical two-valued logic. In order to make such an inference, Zadeh [6] suggested an inference rule called the "compositional rule of inference " (CRI). Later, a new approach was proposed by Mamdani and Assilian [7]. This new form, fuzzy-if-then rules, was based on Zadeh's [8]. There are some other reasoning methods, such as Tsukamoto's and TSK fuzzy model can be found in the literal [9–12]. In this paper, we will use the TSK fuzzy model to perform image noise cleaning. The TSK fuzzy model is proposed by Takagi, Sugeno and Kang [9,10]. A typical fuzzy rule in a TSK fuzzy model can be described as the form:

$$\text{if } x \text{ is } A \text{ and } y \text{ is } B \text{ then } z = f(x, y) \tag{4}$$

where A and B are fuzzy sets in the antecedent part, and $z = f(x, y)$ is a crisp function in the consequent part. Usually $f(x, y)$ is a polynomial of the input variables x and y, but it can be any function as long as it can appropriately describe the output system around the fuzzy region specified by the antecedent of

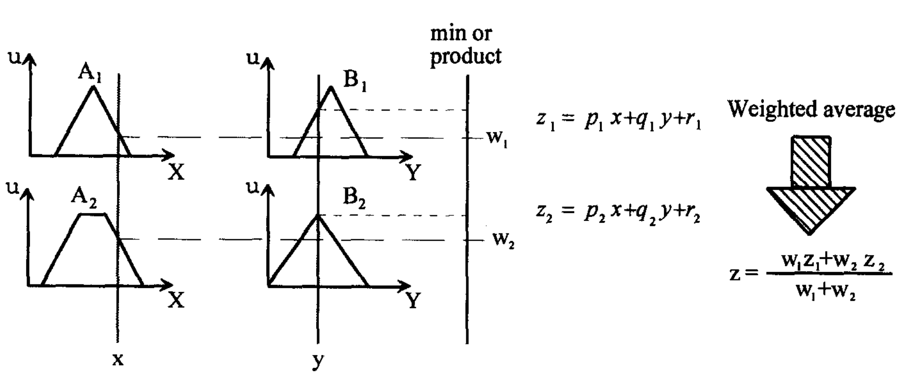


Fig. 2. TSK fuzzy inference model.

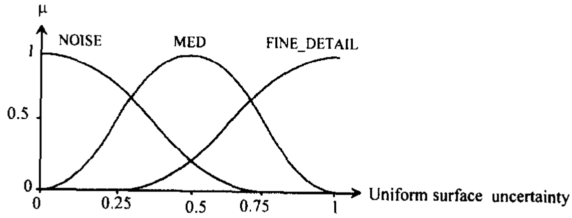


Fig. 3. The membership functions of FINE – DETIAL, MED and NOISE, respectively.

the rule. The fuzzy inference system with two-rule, two-input is illustrated in Fig. 2.

4. The smoothing algorithm and experimental results

Recall the definition of uniform surface uncertainty. If a point $f(i, j)$ possesses full membership to the surface region R, then $f(i, j)$ is considered as a point of fine details. On the other hand, if $\mu_{ij} \to 0$, then the point is considered as a noise one. As these opinions, we let FINE _DETAIL, MED and NOISE be three fuzzy sets on [0,1] (domain of uniform surface uncertainty). Their corresponding membership functions are described in Fig. 3. We then establish the following rules for image smoothing:

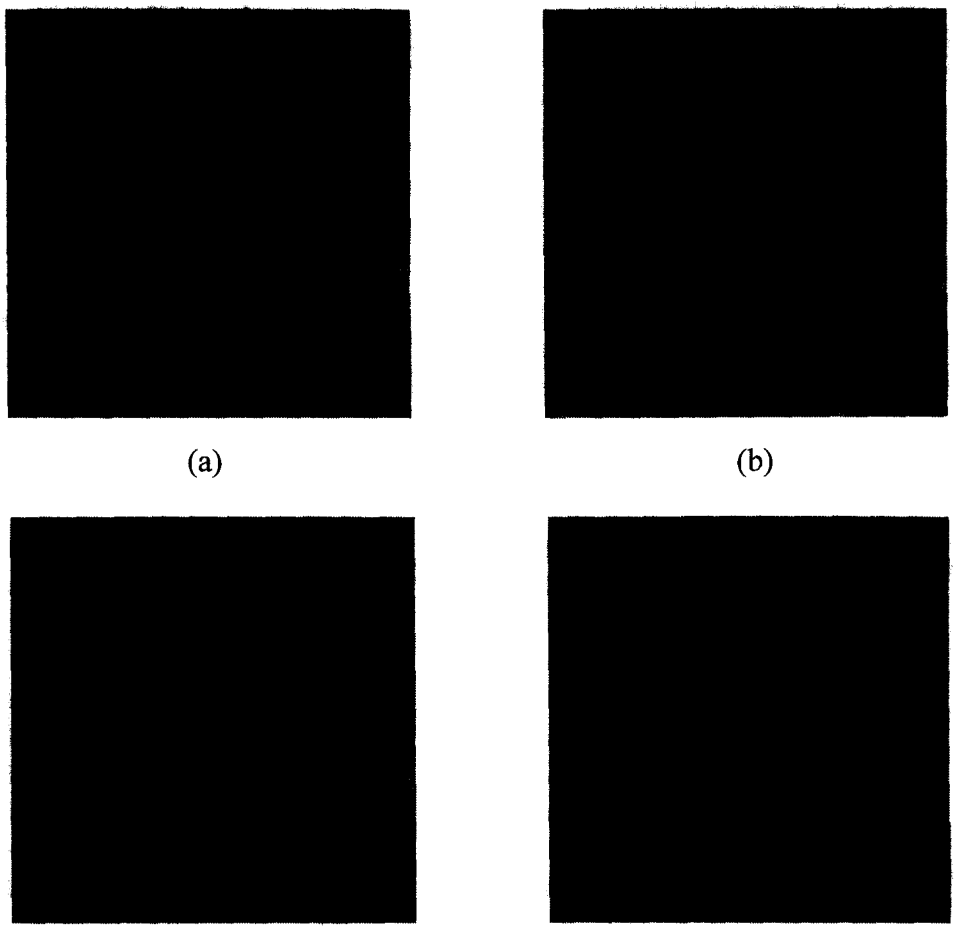
Rule 1: if x is FINE _DETAIL then y is x^{g1}
Rule 2: if x is MED then y is x^{g2}
Rule 3: if x is NOISE then y is 1

where x is the system observation μ_{ij} and y is the system output μ_{ij}^{p}. The coefficients' *g1* and *g2* are assigned with 0.5, 0.1 in this experiment, respectively. We can see that if *g1* and *g2* both are close to 0, this fuzzy filter becomes an averaged filter. That is,

$$f^{p}(i, j) = \bar{f}(i, j) \text{ if } (\mu_{ij}^{p} = 1) \text{ or } (g1 = g2 = 0) \tag{5}$$

Table 1
Mean square error for test image with Gaussian noise

Gaussian noise	5 * 5 mean filter	Fuzzy filter
$\sigma = 10$	164.79	141.84
$\sigma = 20$	175.01	153.18
$\sigma = 30$	193.83	174.12
$\sigma = 40$	221.27	205.63
$\sigma = 50$	257.70	246.68



(a) (b)

(c) (d)

Fig. 4. (a) Original Lena image. (b) Lena image with Gaussian noise $\sigma = 10$. (c) 5×5 mean filtering. (d) Fuzzy filtering with 5×5 region, $g1 = 0.5$, $g2 = 0.1$.

In order to show the practical effects on image processing of fuzzy filters, we demonstrate the image smoothing by a typical example of fuzzy filters and we evaluate the filters performance by a simple mean square error method (MSE).

Computer simulations have been carried out to compare the performance of the proposed filter with the mean filter. The original image is of size 256 by 256 with 256 gray levels, respectively corrupted by zero mean Gaussian noise with standard deviations $\sigma = 10$, 20, 30, 40 and 50. Table 1 shows the MSE of the results by various filtering. The proposed filter shows filter shows good result regardless of MSE. Visual quality can be observed in Figs. 4–5, in the cases of $\sigma = 10$ and 20.

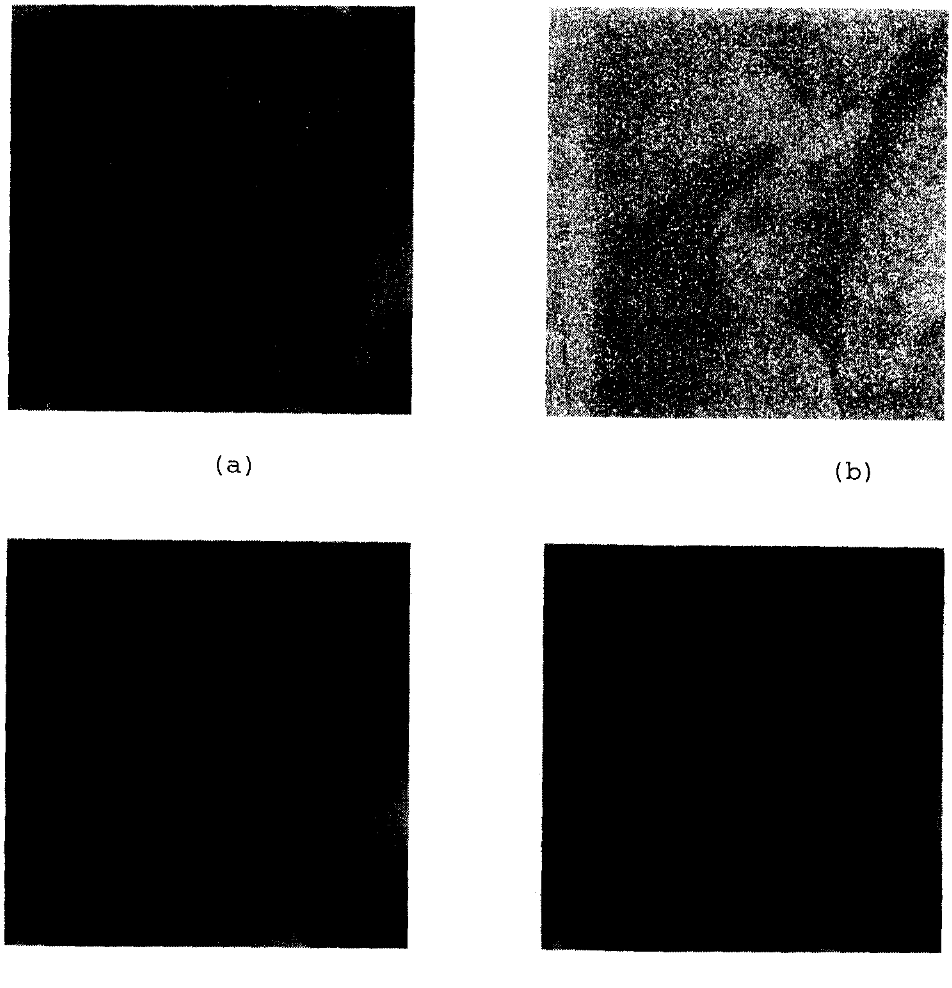


Fig. 5. (a) Lena image with Gaussian noise $\sigma = 20$. (b) Fuzzy image. (c) 5×5 mean filtering. (d) Fuzzy filtering with 5×5 region, $g1 = 0.5$, $g2 = 0.1$.

Clearly the mean filter can remove Gaussian noise but it blurs the details. On the other hand, the proposed filter can remove Gaussian noise and preserve details.

5. Conclusions

In this letter, a new nonlinear smoothing method using fuzzy reasoning techniques is proposed. The algorithm can remove Gaussian noise and preserve the fine details of the input image while the uniform regions are smoothed. Furthermore the proposed filter also can remove impulsive noises as well.

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