Measuring the Manufacturing Yield for Processes With Multiple Manufacturing Lines

Y. T. Tai, W. L. Pearn, and Chun-Min Kao

***Abstract*—Process yield is the most common criterion used in the semiconductor manufacturing industry for measuring process performance. In the globally competitive manufacturing environment, photolithography processes involving multiple manufacturing lines are quite common in the Science-Based Industrial Park in Hsinchu, Taiwan, due to economic scale considerations. In this paper, we develop an effective method for measuring the manufacturing yield for photolithography processes with multiple manufacturing lines. Exact distribution of the estimated measure is analytically intractable. We obtain a rather accurate approximation to the distribution. In addition, we tabulate the lower conference bounds based on the obtained approximated distributions for the convenience of industry applications. We also develop a decision-making method for process precision testing to determine whether a process meets the process yield requirement preset in the factory. For illustration purposes, an application example is included.**

***Index Terms*—Capability index, lower confidence bound, multiple manufacturing lines, process yield.**

I. INTRODUCTION

THE PROGRESS in semiconductor manufacturing technologies has led to the wide use of electronic devices applications. Due to competition from the global semiconductor manufacturing industry, smaller die size should be achieved to reduce manufacturing costs. In a wafer fabrication, the photolithography process is considered a bottleneck and has the greatest impact on manufacturing yield [1], particularly, for small-die-size product types. Photolithography is the process of transferring geometric shapes on a mask to the surface of a silicon wafer. Photolithography process results are crucial to the final functions of one product in the semiconductor manufacturing process. A typical photolithography process consists of seven major steps [2]: pretreatment or priming, spin coating, prebake, exposure, post-exposure bake, development, and metrology. Fig. 1 depicts the exposure operation, which is the operation of transferring circuit patterns on a mask to a wafer. In the photolithography process, the most essential specification is critical dimensions (CDs), that is defined as the linewidth of the photo resist line printed on a wafer and reflects whether the exposure and development are proper to produce geometers of the correct size [3]. In wafer fabrications, it is essential to assess the manufacturing yield on the photolithography process; it could provide feedback to engineers on what actions need to take for manufacturing yield control and improvement.

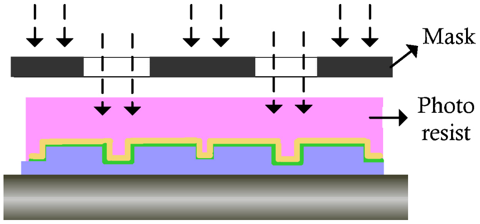

Fig. 1. Exposure operation.

Since the photolithography process requires very low fraction of defectives in parts per million (ppm), process capability indices (PCIs) have been widely and popularly applied in quality assurance in recent years. Capability measures for processes with a single line have been investigated extensively [4]–[9]. However, in most globally competitive wafer fabrications, a process with multiple manufacturing lines is common, particularly, for the wafer fabs in the Science-Based Industrial Park in Hsinchu, Taiwan. In those wafer fabs, a photolithography process with multiple manufacturing lines consists of multiple parallel independent manufacturing lines, with each manufacturing line having a machine or a group of machines performing necessary identical job operations. As the manufacturing lines have various process averages and standard deviations, the values of PCIs will be different for each manufacturing line. The combined output of all manufacturing lines leads to inaccurate yield measures of the photolithography process.

In this paper, to assess the manufacturing yield for photolithography process with multiple manufacturing lines, we present a new capability index (S_{pk}^{M}) method for exact manufacturing yield calculation. This paper is organized as follows. Section II presents the manufacturing yield problem of the photolithography process with multiple manufacturing lines. Section III shows the new process capability index S_{pk}^{M} for processes with multiple manufacturing lines and the distribution of $\hat{S}_{pk}^{M}$. Section IV provides the lower confidence bound of S_{pk}^{M} and decision making on whether a process

---

Manuscript received June 12, 2011; revised September 20, 2011; accepted November 27, 2011. Date of publication December 13, 2011; date of current version May 4, 2012.

Y. T. Tai is with the Department of Information Management, Kainan University, Taoyuan 33857, Taiwan (e-mail: yttai@mail.knu.edu.tw).

W. L. Pearn is with the Department of Industrial Engineering and Management, National Chiao Tung University, Hsinchu 300, Taiwan (e-mail: wlpearn@mail.nctu.edu.tw).

C.-M. Kao is with the Institute of Statistics, National Chiao Tung University, Hsinchu 406, Taiwan (e-mail: cmkao1031@gmail.com).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSM.2011.2179568

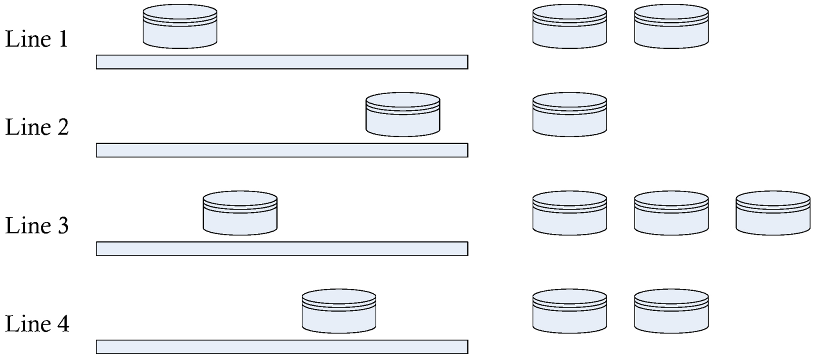

Fig. 2. Photolithography process with multiple manufacturing lines.

with multiple manufacturing lines is capable. For illustration purposes, an application example is provided in Section V. Finally, Section VI includes the conclusions.

II. MANUFACTURING YIELD OF THE WAFER PHOTOLITHOGRAPHY PROCESS

In semiconductor manufacturing processes, the wafer photolithography process is the most essential process; it requires high resolution, precise alignment, and low defect density. In general, a common thin-film transistor liquid crystal display (TFT-LCD) driver integrated circuit (IC) chip used in smart phone usually needs more than 30 masks for the exposure and developing steps which is a complex process and needs precise process yield assessment and control. In the photolithography process, CDs are the most important specification. To control the photolithography process effectively, CD measurements must be made on each layer in the manufacturing process [10]. For example, in CMOS semiconductor manufacturing process, poly layer is mainly used to fabricate the gate terminal of CMOS devices. Poly width is a CD in photolithography process that affects the manufacturing yield in two ways. First, thinner poly width of a random access memory (RAM) cell used in TFT-LCD driver IC may cause device leakage current in the order of several tens of times. It is noted that a TFT-LCD driver IC of high definition 720 (HD720) (720RGB × 1280) product type involves $720 \times 1280 \times 24$ RAM cells. That means total leakage current would be 100 million times when comparing original requirement. It is unexpected since more power consumption resulting from greater leakage current could shorten battery duration on portable devices.

In addition, wider poly width would consume larger device layout area, especially, for a high-definition application. Take HD720 TFT-LCD driver IC for example, if poly width on each RAM cell has been increased, it would cause same proportion area increasing. It is uncompetitive and cannot satisfy the trend of smaller portable devices applications. In the shop floor, if the process parameter is out of control, some yield improvement actions must be initiated immediately. Consequently, obtaining the accurate process yield is very essential which could feedback to internal photo process engineers or external practitioners in IC design houses and make yield improvements.

It should be noted that a photolithography process usually consists of multiple manufacturing lines in the Science-Based Industrial Park. Fig. 2 depicts four manufacturing lines involved in a photolithography process. Since the manufacturing lines have various process averages and standard deviations, the values of process capability indices will be different for each manufacturing line. To access the manufacturing yield of the combined output of all manufacturing lines for the photolithography process, we provide a new overall index S_{pk}^M to obtain the exact yield measure in the subsequent section.

III. OVERALL MANUFACTURING YIELD MEASURES FOR PHOTOLITHOGRAPHY PROCESSES WITH MULTIPLE MANUFACTURING LINES

Process yield, the percentage of processed product unit passing the inspection, has been the most basic and common criterion used in the semiconductor manufacturing industry for measuring process performance. Due to competition in wafer fabrications, the manufacturing yield of a photolithography process demands very low fraction of defective, normally measured by ppm or parts per billion. Bolyes [6] considered the yield measurement index S_{pk} for normal processes with single manufacturing line. It provides exact measure of the process yield for a normally distributed process with a fixed value of S_{pk}. To assess the photolithography process with multiple manufacturing lines, we present the new yield measure index S_{pk}^M and show its distribution.

A. Yield Measure Index S_{pk}^M

For the process with multiple manufacturing lines, we consider a k-lines process with k yield measures P_1, P_2, …, and P_k. We propose the overall capability index for the multiple manufacturing lines process, referred to as S_{pk}^M, as follows:

$$S_{pk}^M = \frac{1}{3}\Phi^{-1}\left\{\left[\frac{1}{k}\sum_{j=1}^{k}(2\Phi(3S_{pkj})-1)+1\right]/2\right\} \tag{1}$$

where S_{pkj} is the S_{pk} value of the jth line for $j = 1, 2, \ldots, k$. The new index, S_{pk}^M, can be viewed as a generalization of multiple manufacturing lines yield index. From (1), given $S_{pk}^M = c$, we have

$$\left\{\frac{1}{k}\sum_{j=1}^{k}(2\Phi(3S_{pkj})-1)\right\} = 2\Phi(3S_{pk}^M) - 1 = 2\Phi(3c) - 1. \tag{2}$$

TABLE I

NONCONFORMITIES (IN PPM) FOR VARIOUS VALUES OF S_{pk}^M AND THE NUMBER OF MANUFACTURING LINES

S_{pk}^M	No. of Manufacturing Lines									
	1	2	3	4	5	6	7	8	9	10
1.00	2699.8	2699.8	2699.8	2699.8	2699.8	2699.8	2699.8	2699.8	2699.8	2699.8
1.25	176.8	176.8	176.8	176.8	176.8	176.8	176.8	176.8	176.8	176.8
1.33	66.1	66.1	66.1	66.1	66.1	66.1	66.1	66.1	66.1	66.1
1.45	13.6	13.6	13.6	13.6	13.6	13.6	13.6	13.6	13.6	13.6
1.50	6.8	6.8	6.8	6.8	6.8	6.8	6.8	6.8	6.8	6.8
1.60	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6
1.67	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
2.00	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002

A one-to-one correspondence relationship between the index S_{pk}^M and the overall process yield P can be established as

$$P = \frac{1}{k}\sum_{j=1}^{k} P_j = \frac{1}{k}\sum_{j=1}^{k}[2\Phi(3S_{pkj}) - 1] = 2\Phi(3c) - 1. \quad (3)$$

Consequently, the new index S_{pk}^M provides an exact measure of the manufacturing lines yield. It is noted that if $S_{pk}^M = 1.00$, the overall process yield would be exactly 99.73%. The number of nonconformities in parts per million (NCPPM) is 2699.8. We tabulated various commonly capability requirements and corresponding overall process yield in Table I. It can be seen from the definition that the weighted average is, indeed, independent of the number of manufacturing lines.

B. Distribution of $\hat{S}_{pk}^M$

To evaluate the overall yield measure index S_{pk}^M, we consider the following $\hat{S}_{pk}^M$ that can be expressed as

$$\hat{S}_{pk}^M = \frac{1}{3}\Phi^{-1}\left\{\left[\frac{1}{k}\sum_{j=1}^{k}(2\Phi(3\hat{S}_{pkj}) - 1) + 1\right]/2\right\} \quad (4)$$

where $\hat{S}_{pkj}$ denotes the estimator of S_{pkj}. It should be noted that the exact distribution of the overall yield index S_{pk}^M is analytically intractable. Thus, we apply the Taylor expansion of k-variate and taking the first order, the asymptotic distribution of $\hat{S}_{pk}^M$ can be expressed as

$$\hat{S}_{pk}^M \sim N\left(S_{pk}^M, \frac{1}{36k^2 n\left[\phi(3S_{pk}^M)\right]^2}\sum_{j=1}^{k}\left(a_j^2 + b_j^2\right)\right) \quad (5)$$

where

$$a_j = \{[(\mathrm{USL} - \mu_j)/\sqrt{2}\sigma_j]\phi[(\mathrm{USL} - \mu_j)/\sigma_j]$$

$$+[(\mu_j - \mathrm{LSL})/\sqrt{2}\sigma_j]\phi[(\mu_j - \mathrm{LSL})/\sigma_j]\},\ b_j$$

$$= \{\phi[(\mathrm{USL} - \mu_j)/\sigma_j] - \phi[(\mu_j - \mathrm{LSL})/\sigma_j].$$

Variables a_j and b_j are the corresponding parameters of the jth manufacturing line (see Appendix).

C. Accuracy of Yield Measure

To measure how accurate the normal approximation is, we conduct a simulation study using the statistical package as follows. For convenience to present our new development, we consider two manufacturing lines of a process. In the simulation scenario, the value of S_{pk}^M is 1.00 and process mean of the two investigated manufacturing lines are unequal. We simulate 10 000 000 random sample of size $n = 60, 100, 500, 1000$ from $N_2(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, 0)$, a normal process with two independent lines. In addition, we compute 10 000 000 $\hat{S}_{pk}^M$ using (4). Figs. 3 and 4 depict approximate and exact distributions, it is clear that as the sample size $n = 1000$, the approximate and simulated distributions are almost indistinguishable. In fact, even with $n = 60$, the approximation is quite reasonable for practical purposes.

IV. DECISION MAKING ON PHOTO PROCESSES WITH MULTIPLE MANUFACTURING LINES

For in-plant applications, to calculate the yield index with multiple manufacturing lines, sample data must be collected. It is noted that a great degree of uncertainty may be introduced into capability assessments due to sampling errors. Consequently, conclusions based on the calculated values will lead to make unreliable decisions as the sampling errors have been ignored. A reliable approach for assessing the true value of process index is to construct the lower confidence bound. The lower confidence bound is not only essential to manufacturing yield assurance but also can be used in capability testing for decision making.

A. Lower Confidence Bound

It should be noted that the variance of $\hat{S}_{pk}^M$ is not identical for a fixed value of S_{pk}^M. Since the sampling distribution of $\hat{S}_{pk}^M$ can be presented in $2k+1$ parameters, a_j, b_j, $j = 1, \ldots, k$, and S_{pk}^M (5), the lower confidence bound ($S_{pk}^{M\,\mathrm{LB}}$) is also a function of those parameters. Thus, we have to consider the effect of all the parameters in the calculation of lower confidence bound to ensure that the lower bounds obtained are reliable. The word "reliable" here means that the probability that the obtained lower bound (subject to the sample estimate) is smaller than the actual capability index S_{pk}^M and is greater than the desired confidence level. For a process with two manufacturing lines, i.e., $k = 2$, we note that for an identical value of S_{pk}^M, there are numerous combinations of S_{pk1} and S_{pk2}. Fig. 5 shows the relationship between S_{pk1} and S_{pk2} under different values of S_{pk}^M.

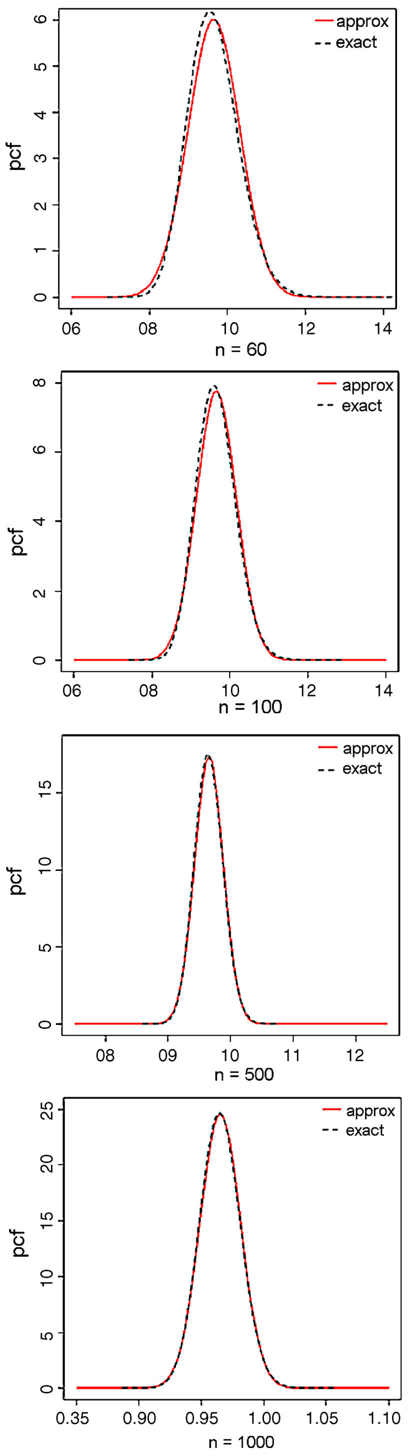

Fig. 3. Comparison of normal approximate and exact densities via simulations.

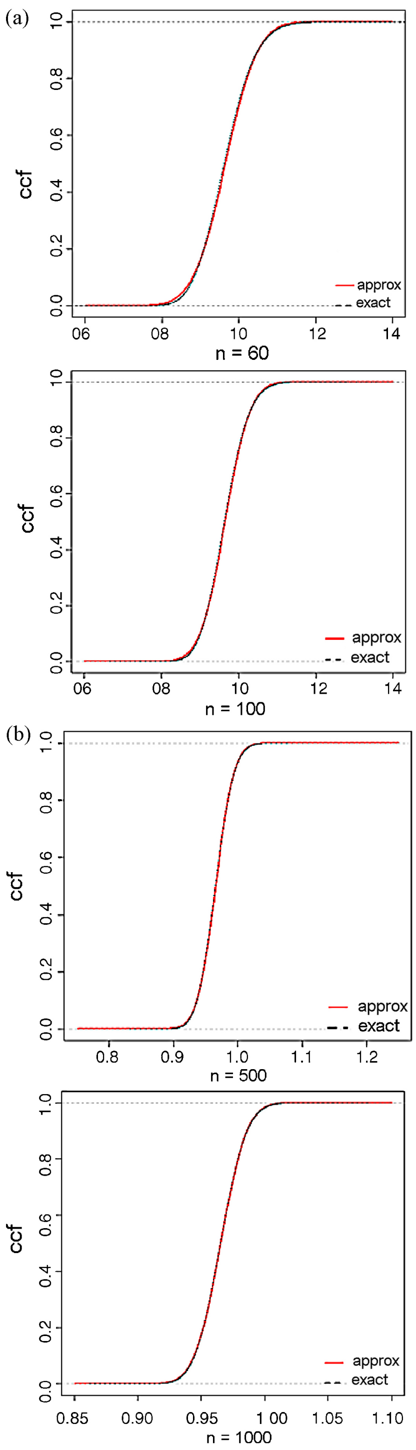

Fig. 4. Comparison of (a) approximate and (b) exact cumulative distribution function.

For a fixed value of S_{pkj}, there are numerous combinations of a_j and b_j. Table II displays some combinations of the parameters S_{pk1}, S_{pk2}, a_1, a_2, b_1, and b_2 under $S_{pk}^M = 1$, and the corresponding lower confidence bounds of S_{pk}^M. Our extensive calculation results show that: 1) the lower confidence bound of S_{pk}^M obtains its maximum at $S_{pk1}=S_{pk2}$, and minimum at $S_{pk1} \geq 2.5$ and $S_{pk2} = (1/3)\Phi^{-1}[2\Phi\ (3S_{pk}^M) - 1]$ (or $S_{pk1} = (1/3)\Phi^{-1}[2\Phi(3S_{pk}^M) - 1]$ and $S_{pk2} \geq 2.5$); and 2) for fixed values of S_{pk1} and S_{pk2}, the lower confidence bound of S_{pk}^M reaches its minimum at $b_1 = b_2 = 0$. Figs. 6 and 7 show lower confidence bounds plot of a two manufacturing-line process and plane curve for $S_{pk}^M = 1.00, 1.33, 1.50, 1.67, 2.00$. Consequently, to obtain the reliable lower bound, we set $S_{pk1} = 2.5, S_{pk2} = (1/3)\Phi^{-1}(2\Phi\ (3S_{pk}^M) - 1)$, $a_1 = \sqrt{2}(3S_{pk1})\phi(3S_{pk1})$, $a_2 = \sqrt{2}(3S_{pk2})\phi(3S_{pk2})$, and $b_1 = b_2 = 0$.

For general cases, two essential results are listed as follows: 1) for an identical value of S_{pk}^M, the lower confidence bound of S_{pk}^M is minimum at $S_{pki} = (1/3)\Phi^{-1}\{[k(2\Phi(3S_{pk}^M)$

TABLE II
VARIOUS COMBINATIONS OF THE PARAMETERS UNDER $S_{pk}^M = 1.00$, $n = 10$, AND CORRESPONDING LOWER CONFIDENCE BOUNDS OF S_{pk}^M

S_{pk1}	S_{pk2}	a_1	a_2	b_1	b_2	$S_{pk}^{M\ \mathrm{LB}}$
1.00	1.00	0.0188027	0.0188027	0.0000000	0.0000000	0.9750000
1.00	1.00	0.0080391	0.0080391	0.0000000	0.0000000	0.9954301
1.00	1.00	0.0031153	0.0031153	0.0000000	0.0000000	0.9993137
1.00	1.00	0.0010957	0.0010957	0.0000000	0.0000000	0.9999151
1.00	1.00	0.0163703	0.0163703	−0.0083214	−0.0083214	0.9761533
1.00	1.00	0.0163701	0.0163701	−0.0083214	−0.0083214	0.9761538
1.00	1.00	0.0163711	0.0163711	−0.0083220	−0.0083220	0.9761508
1.00	1.00	0.0163711	0.0163711	−0.0083220	−0.0083220	0.9761508
1.00	1.00	0.0163711	0.0163711	−0.0083220	−0.0083220	0.9761508
1.00	1.00	0.0163711	0.0163711	−0.0083220	−0.0083220	0.9761508
1.00	1.00	0.0163711	0.0163711	−0.0083220	−0.0083220	0.9761508
1.33	0.9287227	0.0007859	0.0324173	0.0000000	0.0000000	0.9628229
1.33	0.9287227	0.0007314	0.0316824	0.0002593	−0.0067986	0.9628548
1.33	0.9287227	0.0007249	0.0283878	−0.0002676	−0.0145563	0.9639949
1.33	0.9287227	0.0007245	0.0277405	−0.0002680	−0.0152509	0.9645474
1.33	0.9287227	0.0007245	0.0276590	−0.0002680	−0.0153112	0.9646420
1.33	0.9287227	0.0007245	0.0276524	−0.0002680	−0.0153149	0.9646509
1.33	0.9287227	0.0007245	0.0276520	−0.0002680	−0.0153151	0.9646514
1.33	0.9287227	0.0007245	0.0276520	−0.0002680	−0.0153151	0.9646514

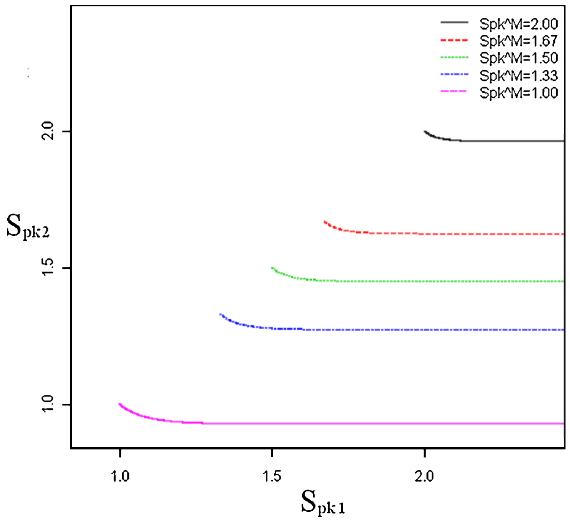

Fig. 5. Combinations of S_{pk1} and S_{pk2} for $S_{pk}^M = 1.00, 1.33, 1.50, 1.67, 2.00$ (from bottom to top).

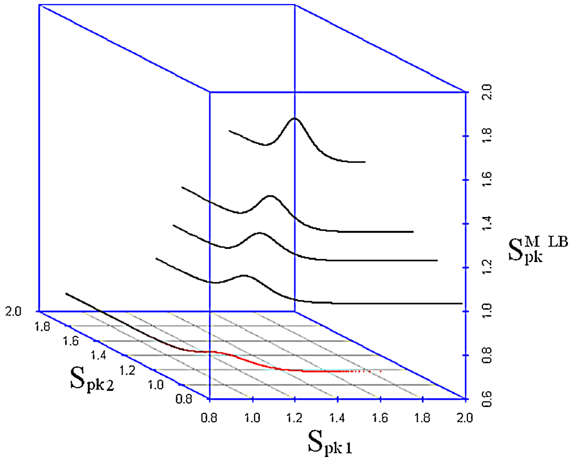

Fig. 6. Lower confidence bounds plot of a two manufacturing-line process for $S_{pk}^M = 1.00, 1.33, 1.50, 1.67, 2.00$ (from bottom to top).

$-1) - (k-2)]/2\}$ and $S_{pkj} \geq 2.5$, where $j \neq i$; and 2) for fixed value of S_{pkj}, the lower confidence bound of S_{pk}^M reaches its minimum at $b_j = 0$, i.e., the mean is on-center. Consequently, in the calculation of lower confidence bound of S_{pk}^M, we set $S_{pki} = (1/3)\Phi^{-1}\{[k(2\Phi(3S_{pk}^M) - 1) - (k-2)]/2\}$ and $S_{pkj} \geq 2.5$, for all $j \neq i$, $a_j = \sqrt{2}\{3S_{pkj}\phi(3S_{pkj})\}$, and $b_j = 0$ for all $j = 1, \ldots, k$. In this way, the level of confidence can be ensured, and the decisions (lower confidence bound) made based on such an approach are indeed more reliable.

We note that with the above parameters setting, the sampling distribution of S_{pk}^M can be written in a shorter and simpler form, that is

$$\hat{S}_{pk}^M \sim N\left(S_{pk}^M, \frac{D^2\phi^2(3D)}{2k^2 n\phi^2(3S_{pk}^M)}\right) \tag{6}$$

where $D = (1/3)\Phi^{-1}\{[k(2\Phi(3S_{pk}^M) - 1) - (k-2)]/2\}$. Thus, given an estimated value of S_{pk}^M, a sample size n, the number of manufacturing line k, a confidence level $1-\alpha$, and the lower confidence bound of S_{pk}^M (denoted as S_{pk}^{MLB}) can be obtained as follows:

$$S_{pk}^{M\mathrm{LB}} = \hat{S}_{pk}^M - Z_\alpha \frac{\hat{D}\phi(3\hat{D})}{k\sqrt{2n}\phi(3\hat{S}_{pk}^M)} \tag{7}$$

where Z_α is the upper 100α percentile of the standard normal distribution.

B. Test for Decision Making on Photolithography Process Yield Measure

Using the index S_{pk}^M, engineers can access the process performance and monitor the manufacturing process on a routine basis. To test whether a given process with multiple manufacturing lines is capable, we consider that

$$H_0 : S_{pk}^M \leq c \text{ versus } H_1 : S_{pk}^M > c \tag{8}$$

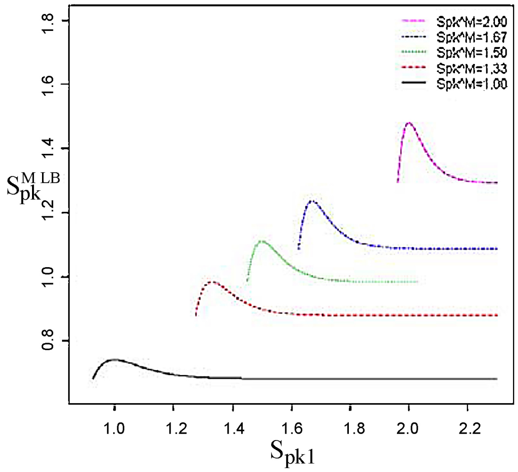

Fig. 7. Plane curves of $S_{pk}^{M\,\mathrm{LB}}$ versus S_{pk1} for S_{pk}^{M} = 1.00, 1.33, 1.50, 1.67, 2.00 (from bottom to top).

can be executed by considering the testing statistic

$$T = \frac{(\hat{S}_{pk}^{M} - c)k\sqrt{2n}\phi(3\hat{S}_{pk}^{M})}{\hat{D}\phi(3\hat{D})} \tag{9}$$

where $\hat{D} = (1/3)\Phi^{-1}\left\{\left[k(2\Phi(3\hat{S}_{pk}^{M}) - 1) - (k-2)\right]/2\right\}$. The null hypothesis H_0 is rejected at α level if $T > z_\alpha$, where z_α is the upper $100\alpha\%$ point of the standard normal distribution.

For more general cases with unequal production quantity for each line, our method can be easily extended as follows. We first take weighted average of the overall yield. We then convert the average of the overall yield into the corresponding value of S_{pk}^{M}.

V. MANUFACTURING YIELD ASSESSMENT FOR PHOTOLITHOGRAPHY PROCESSES

In this section, to demonstrate the applicability of the proposed method, we consider a real-world application taken from a wafer fab located in the Science-Based Industrial Park. For the case investigated, a high-voltage manufacturing process is applied on the product type that includes 32 various masks. To control the photolithography process effectively, CD measurements are made on each layer in the manufacturing process. In the case, we focus on the specification of poly width on RAM cells in the essential poly layer for TFT-LCD driver IC product series. It is noted that three manufacturing lines are involved in the process. The upper and lower specification limits of the critical dimension parameter for the high-density product type are set to 118 nm and 102 nm, respectively. Measurement data are collected from the shop floor via the designate sampling plans. In the case, 100 sample observations of the critical dimension are collected from each manufacturing line in the photolithography process. Sample mean, sample standard deviation, and the values of $\hat{S}_{pkj}$ are shown in Table III.

TABLE III
CALCULATED STATISTICS OF THE THREE MANUFACTURING LINES (UNIT: nm)

Lines	$\bar{x}_j$	s_j	$\hat{S}_{pkj}$
1	112.5494	1.7383	1.1112
2	108.1011	1.3645	1.5391
3	111.9718	0.9383	2.1764

Using the proposed method in this paper, we can assess the value of $\hat{S}_{pk}^{M}$ that is equal to 1.2089. The process yield of the photolithography process is 99.9713% and the number of NCPPM is 287.066. For testing the null hypothesis H_0 with $c = 1$ against the alternative hypothesis as given in (8), the testing statistic T, as given in (9), can be calculated as 2.864724. Since 2.864724 is greater than the value of $Z_{0.05}$, the null hypothesis H_0 is rejected at α=0.05. We may conclude that the process satisfies the capability requirement $S_{pk}^{M} \geq 1.00$.

A. Discussion

In existing literature [11], [12], the authors considered a yield index with multiple process streams. In the application and calculation in Wang *et al.* [12], they calculated the estimated yield using $\hat{S}_{pk}^{M} = (1/k)\sum_{j=1}^{k}\hat{S}_{pkj}$. The estimator of S_{pk}^{M}, considered by Wang *et al.* [12], would certainly overestimate the process yield as stated in Wang *et al.* [11]. Further, the variance of $\hat{S}_{pk}^{M}$ considered in Wang *et al.* [12] remained random variables as $\hat{a}$ and $\hat{b}$. This problem can be resolved by analyzing the combination of values of S_{pkj}^{M}, $j = 1, \ldots, k$, which is accomplished in this paper.

VI. CONCLUSION

Process yield is the most common and standard criterion for use in the manufacturing industry for measuring process performance. Capability measures for processes with a single manufacturing line have been investigated extensively. As many industry cases today, it is common that a process simultaneously involves more than one manufacturing line. In this paper, the yield index S_{pk}^{M} provided an exact measure on the manufacturing yield for photolithography processes with the multiple manufacturing lines. Unfortunately, the distribution properties of the estimated S_{pk}^{M} were mathematically intractable. We used Taylor expansion technique, taking the first order, to obtain rather accurate approximate distribution. Consequently, the lower confidence bounds based on the developed distribution can be obtained. In addition, hypothesis testing was performed for implementing the method we proposed. Using the provided yield measure method, the practitioners can determine whether a process with multiple manufacturing lines meets the process yield requirement preset in the factory. The results obtained could help the practitioners to make more reliable decisions on what yield improvement actions need to be initiated in controlling the photolithography process with multiple manufacturing lines.

APPENDIX
TAYLOR EXPANSION OF $\hat{S}_{pk}^{M}$

$$S_{pk}^{M} = \frac{1}{3}\Phi^{-1}\left\{\left[\frac{1}{k}\sum_{j=1}^{k}(2\Phi(3S_{pkj}) - 1) + 1\right]/2\right\}$$

where $\Phi(\cdot)$ is the cumulative distribution function of $N(0, 1)$. We can obtain $\hat{S}_{pk}^{M} = f(\hat{\mu}_1, \ldots, \hat{\mu}_k, \hat{\sigma}_1^2 \ldots, \hat{\sigma}_k^2)$, where $\hat{\mu}_j = \overline{X}_j$ and $\hat{\sigma}_j^2 = S_j^2 = \sum_{l=1}^{n}(X_{jl} - \bar{X}_{jl})^2/(n-1)$.

Applying the first-order expansion of k-variate Taylor series, we can obtain

$$\begin{aligned}\hat{S}_{pk}^{M} &\approx f(\mu_1, \ldots, \mu_k; \sigma_1^2, \ldots, \sigma_k^2)\\ &+\sum_{i=1}^{k} \frac{\partial f(\mu_1, \ldots, \mu_k; \sigma_1^2, \ldots, \sigma_k^2)}{\partial \hat{\mu}_i}(\hat{\mu}_i - \mu_i)\\ &+\sum_{i=1}^{k} \frac{\partial f_7(\mu_1, \ldots, \mu_k; \sigma_1^2, \ldots, \sigma_k^2)}{\partial \hat{\sigma}_i^2}(\hat{\sigma}_i^2 - \sigma_i^2).\end{aligned}$$

Differentiating implicitly with respect to $\hat{\mu}_j$ and $\hat{\sigma}_j^2$, $j = 1, \ldots, k$ gives

$$\begin{aligned}&\frac{\partial f(\mu_1, \ldots, \mu_k, \sigma_1^2, \ldots, \sigma_k^2)}{\partial \hat{\mu}_k} = \frac{-1}{6k\phi(3S_{pk}^{M})}\\ &\times\frac{1}{\sigma_j}\left\{\phi\left(\frac{\mathrm{USL} - \mu_j}{\sigma_j}\right) - \phi\left(\frac{\mu_j - \mathrm{LSL}}{\sigma_j}\right)\right\},\ j=1, \ldots, k\\ &\frac{\partial f(\mu_1, \ldots, \mu_k, \sigma_1^2, \ldots, \sigma_k^2)}{\partial \hat{\sigma}_k^2} = \frac{-1}{6k\phi(3S_{pk}^{M})}\frac{1}{2\sigma_j^2}\\ &\times\left\{\left(\frac{\mathrm{USL} - \mu_j}{\sigma_j^2}\right)\phi\left(\frac{\mathrm{USL} - \mu_j}{\sigma_j^2}\right)\right.\\ &\quad\left.+\left(\frac{\mu_j - \mathrm{LSL}}{\sigma_j^2}\right)\phi\left(\frac{\mu_j - \mathrm{LSL}}{\sigma_j^2}\right)\right\}.\end{aligned}$$

Consequently, we can obtain

$$\hat{S}_{pk}^{M} \approx S_{pk}^{M} + \frac{-1}{6k\phi(3S_{pk}^{M})}\sum_{j=1}^{k}\left(W_j + G_j\right)$$

where

$$S_{pk}^{M} = \frac{1}{3}\Phi^{-1}\left\{[\frac{1}{k}\sum_{j=1}^{k}(2\Phi(3S_{pkj}) - 1) + 1]/2\right\}$$

$$W_j = \frac{\left(\overline{\mathrm{X}}_j - \mu_j\right)}{\sigma_j}b_j \text{ and } G_j = \frac{\left(S_j^2 - \sigma_j^2\right)}{\sqrt{2}\sigma_j^2}a_j$$

$$a_j = \left[\frac{\mathrm{USL} - \mu_j}{\sqrt{2}\sigma_j}\phi\left(\frac{\mathrm{USL} - \mu_j}{\sigma_j}\right) + \frac{\mu_j - \mathrm{LSL}}{\sqrt{2}\sigma_j}\ \phi\left(\frac{\mu_j - \mathrm{LSL}}{\sigma_j}\right)\right]$$

$$b_j = \left[\phi\left(\frac{\mathrm{USL} - \mu_j}{\sigma_j}\right) - \phi\left(\frac{\mu_j - \mathrm{LSL}}{\sigma_j}\right)\right],\ j = 1, \ldots, k.$$

Let $Z_j = \sqrt{n}\left(\overline{X}_j - \mu_j\right)$ and $Y_j = \sqrt{n}\left(S_j^2 - \sigma_j^2\right)$, $j = 1, \ldots, k$. Then, Z_j and Y_j are independent. Since the first two moments of $\overline{X}_j$ and S_j^2 exist, by the central limit theorem, Z_j and Y_j converge to $N\left(0, \sigma_j^2\right)$ and $N\left(0, 2\sigma_j^4\right)$, respectively, where $j = 1, \ldots, k$. Consequently, we can obtain $E(\hat{S}_{pk}^{M}) \approx E(S_{pk}^{M}) = S_{pk}^{M}$ and

$$\begin{aligned}Var(\hat{S}_{pk}^{M}) &\approx Var\left\{S_{pk}^{M} + \frac{-1}{6k\phi(3S_{pk}^{M})}\sum_{j=1}^{k}\left(W_j + G_j\right)\right\}\\ &\approx \frac{1}{36k^2n\left[\phi(3S_{pk}^{M})\right]^2}\sum_{j=1}^{k}\left(a_j^2 + b_j^2\right).\end{aligned}$$

REFERENCES

[1] W. L. Pearn, H. Y. Kang, A. H. I. Lee, and M. Y. Liao, "Photolithography control in wafer fabrication based on process capability indices with multiple characteristics," *IEEE Trans. Semicond. Manuf.*, vol. 22, no. 3, pp. 351–356, Aug. 2009.

[2] B. Grosman, S. Lachman-Shalem, R. Swissa, and D. R. Lewin, "Yield enhancement in photolithography through model-based process control: Average mode control," *IEEE Trans. Semicond. Manuf.*, vol. 18, no. 1, pp. 86–93, Feb. 2005.

[3] C. E. Chemali, J. Freudenberg, M. Hankinson, and J. J. Bendik, "Run-to-run critical dimension and sidewall angle lithography control using the PROLITH simulator," *IEEE Trans. Semicond. Manuf.*, vol. 17, no. 3, pp. 388–401, Aug. 2004.

[4] F. K. Wang, "A general procedure for process yield with multiple characteristics," *IEEE Trans. Semicond. Manuf.*, vol. 23, no. 4, pp. 503–508, Nov. 2010.

[5] J. C. Lee, H. N. Hung, W. L. Pearn, and T. L. Kueng, "On the distribution of the estimated process yield index S_{pk}," *Qual. Reliab. Eng. Int.*, vol. 18, no. 1, pp. 111–116, 2002.

[6] R. A. Boyles, "Process capability with asymmetric tolerances," *Commun. Statist.: Simul. Computat.*, vol. 23, no. 3, pp. 615–643, 1994.

[7] S. Kotz and N. L. Johnson, "Process capability indices-review, 1992–2000," *J. Qual. Technol.*, vol. 34, no. 1, pp. 1–19, 2002.

[8] V. E. Kane, "Process capability indices," *J. Qual. Technol.*, vol. 18, no. 1, pp. 41–52, 1986.

[9] W. L. Pearn, S. Kotz, and N. L. Johnson, "Distributional and inferential properties of process capability indices," *J. Qual. Technol.*, vol. 24, no. 4, pp. 216–231, 1992.

[10] H. Y. Kang and A. H. I. Lee, "Critical dimension control in photolithography based on the yield by a simulation program," *Microelectron. Reliab.*, vol. 46, nos. 5–6, pp. 1006–1012, 2006.

[11] D. S. Wang, T. Y. Koo, and C. Y. Chou, "Yield measure for the process with multiple streams," *Qual. Quantity*, vol. 43, no. 4, pp. 661–668, 2009.

[12] D. S. Wang, T. Y. Koo, and C. Y. Chou, "Statistical inference on the multiple-stream yield index S_{pk}^{m}," *J. Statist. Manag. Syst.*, vol. 12, no. 4, pp. 671–678, 2009.

Y. T. Tai received the Ph.D. degree in industrial engineering and management from National Chiao-Tung University, Hsinchu, Taiwan.

She is currently an Assistant Professor with the Department of Information Management, Kainan University, Taoyuan, Taiwan. Her current research interests include process capability indices, scheduling, and semiconductor manufacturing management.

W. L. Pearn received the Ph.D. degree in operations research from the University of Maryland, College Park.

He is currently a Professor of operations research and quality assurance with National Chiao-Tung University (NCTU), Hsinchu, Taiwan. He was with Bell Laboratories, Holmdel, NJ, as a Quality Research Scientist before joining NCTU. His current research interests include process capability, network optimization, and production management. His publications have appeared in the *Journal of the Royal Statistical Society, Series C*, the *Journal of Quality Technology*, the *European Journal of Operational Research*, the *Journal of the Operational Research Society*, *Operations Research Letters*, *Omega*, *Networks*, and the *International Journal of Productions Research*.

Chun-Min Kao received the Masters degree in statistics from the National Chiao-Tung University, Hsinchu, Taiwan.

She is currently a Product Engineer with the Video Solutions Integration Division, AU Optronics Corporation, Taichung, Taiwan. Her current research interests include statistical process control, experimental design, and applied statistics.