On Group Partition for Wireless Multicast Flow Control

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***Abstract*—In this paper, we propose partitioning group members into subgroups according to their instantaneous channel gains for single-hop wireless multicast flow control. We study the case in which there are a number of logical channels and the access point uses a logical channel to broadcast information to a subgroup. In particular, distinct logical channels could be allocated different levels of transmission power and fractions of time. In order to maximize the network throughput, we formulate and solve a discrete optimization problem. In addition, we use simulation results to show group partition is a promising approach for wireless multicast flow control.**

***Index Terms*—Cross-layer design, wireless multicast, group partition, discrete optimization.**

I. INTRODUCTION

IN a wireless local area network, if the access point has to choose a single downlink data transmission rate for all of the nodes in a multicast group, the node with the worst channel gain might dominate the network performance. To improve the network throughput, we propose using a number of logical channels with different levels of transmission power and partitioning group members into several subgroups according to the instantaneous channel gains. McCanne, Jacobson, and Vetterli [1] proposed layered multicast in which several layers of information are provided and each receiver subscribes to one specific layer. Bhattacharyya, Kurose, Towsley, and Nagarajan [2] studied the problem of finding the optimal rate at each layer to minimize the completion time of a fixed-size file. Gau, Haas, and Krishnamachari [3] studied the impacts of the distribution of the receiver capacities on the throughput of multicast flow control. Many related works on Internet multicast flow control can be found in [3] [4] [5] and the reference therein. They focused on wired networks in which the network throughput depends only on the partitioning of group members. In contrast, in this paper, we focus on wireless networks in which the network throughput depends on channel assignment and partitioning of group members. Ge, Zhang, and Shen [6] proposed a cross-layer wireless multicast scheme in which the access point transmits information only when enough receivers have channel gains greater than a predetermined threshold. Pantelidou and Ephremides [7] studied the problem of choosing at each time slot a power vector and a rate vector for multicast sessions to maximize the sum of utilities of all destinations, when the wireless channel is not known exactly. Rajawat, Gatsis, and Giannakis [8] proposed a cross-layer design and an optimal multicast resource allocation framework for wireless fading networks in which the nodes could perform network coding. They took an approach of continuous optimization, while we take an approach of discrete optimization in this paper. Recently, the multicast capacity of multihop wireless networks had been extensively studied [9] [10]. We focus on single-hop wireless networks and therefore routing [11] is beyond the scope of this paper. Our major technical contributions include proposing a discrete optimization approach for single-hop wireless multicast flow control.

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II. SYSTEM MODELS

As in [6], we study single-hop wireless multicast. In the wireless network, there is a multicast session from the access point to a group of n nodes. Time is partitioned into time slots. The access point executes the same algorithm in every time slot. Therefore, it is sufficient to consider a time slot. Let g_i be the channel gain from the access point to node i in the time slot, $\forall 1 \leq i \leq n$. We adopt the widely used IID (independent, identically distributed) Rayleigh fading model [12]. Thus, g_i's are IID exponential random variables [12]. For a node, the channel gains at different time slots are assumed to be IID random variables. Let σ^2 be the power spectral density of the additive white Gaussian noise. Let w be a given positive integer. The access point selects some nodes from the group to form w subgroups according to instantaneous channel gains. There are w logical channels and each logical channel is used to broadcast information to a subgroup. To avoid the problem of "listening to the slowest receiver", a node may not be selected. Let S_j be the set composed of the indexes of nodes that belong to the jth subgroup, $\forall 1 \leq j \leq w$. Then, $S_j \subset \{1, 2, .., n\}$, $\forall j$, and $S_u \cap S_v = \emptyset$, $\forall u \neq v$. Denote the cardinality of the set S by $|S|$. Then, $|S_j|$ is the total number of nodes in the jth subgroup. A node that is not selected by the access point is said to be assigned to the $(w+1)$th subgroup. Let P be the total power allocated to the access point to serve the multicast session. Let $P_j \geq 0$ be the power allocated to the jth logical channel and $\alpha_j \geq 0$ be the fraction of time allocated to the jth logical channel. Then, $\sum_{j=1}^{w} P_j = P$ and $\sum_{j=1}^{w} \alpha_j = 1$. In this paper, we study the case in which the values of n, g_i's, w, P_j's and α_j's are given in advance. Let θ be a permutation of $\{1, 2, .., w\}$ such that $\theta(j)$ is the index of the logical channel that is assigned to the jth subgroup, $\forall 1 \leq j \leq w$. Note that a a permutation of $\{1, 2, .., w\}$ is a one-to-one mapping from $\{1, 2, .., w\}$ to $\{1, 2, .., w\}$. The permutation function θ is called the channel assignment function. Let Π_w be the set composed of all the permutations of $\{1, 2, .., w\}$. Then, $\theta \in \Pi_w$.

III. Optimal Group Partition

Without loss of essential generality, it is assumed that $g_i \geq g_{i+1}$, $\forall 1 \leq i \leq n-1$. The access point uses the $\theta(j)$th logical channel to broadcast information to the jth subgroup. When node i is the unique node in the jth subgroup, based on information theory [12], if the access point transmits information to node i at rate smaller than $\alpha_{\theta(j)} \log_2(1+\frac{P_{\theta(j)} g_i}{\alpha_{\theta(j)}\sigma^2})$, node i could successfully receive the information. To assure that all the nodes in the jth subgroup could successfully receive the information broadcast from the access point, the data transmission rate for the jth subgroup equals $\alpha_{\theta(j)} \times \min_{i:i\in S_j} \log_2(1+\frac{P_{\theta(j)} g_i}{\alpha_{\theta(j)}\sigma^2})$. Recall that $|S_j|$ is the total number of nodes in the jth subgroup. Thus, the overall throughput for the jth subgroup equals $\alpha_{\theta(j)} \times \min_{i:i\in S_j} \log_2(1+\frac{P_{\theta(j)} g_i}{\alpha_{\theta(j)}\sigma^2}) \times |S_j|$. In order to maximize the network throughput, we formulate the following wth order optimal group partition problem.

$$\begin{aligned}
&\max \sum_{j=1}^{w} \alpha_{\theta(j)} \times \min_{i:i\in S_j} \log_2(1+\frac{P_{\theta(j)} g_i}{\alpha_{\theta(j)}\sigma^2}) \times |S_j| \\
&subject\ to \\
&S_j \subset \{1,2,..,n\}, \forall 1 \leq j \leq w \\
&S_u \cap S_v = \emptyset, \forall u \neq v \\
&\theta \in \Pi_w. \qquad (1)
\end{aligned}$$

To reflect that the information-theoretic capacity may not be reached in practice, one can multiply the above object function by $\zeta \in [0,1]$. To make the presentation concise, it is assumed that $\zeta = 1$.

Let $h[w,n]$ be the optimal value of the object function for the wth order optimal group partition problem in (1). The above discrete optimization problem can be solved by the brute-force approach. For a fixed channel assignment function θ, each of the n nodes could be assigned to one of the $(w+1)$ subgroups. Thus, for a fixed channel assignment function θ, there are $(w+1)^n$ different ways for the access point to select the n nodes to form $(w+1)$ subgroups. Although there are $w!$ different channel assignment functions, based on symmetry, it is sufficient to consider a channel assignment function. Therefore, the computational complexity of the brute-force approach is $O((w+1)^n)$. The definition of $O(\cdot)$ can be found in [13].

In order to develop fast algorithms for solving the optimal group partition problem, we first derive related analytical results. Let $\beta > 0$ be a positive real number. Define $f_\beta(x) = \log_2(1+\beta x)$, $\forall x > 0$. Since $\frac{df_\beta(x)}{dx} = \frac{\beta \log_2(e)}{1+\beta x} > 0$, $\forall x > 0$, $f_\beta(x)$ is an increasing function of x in $(0,\infty)$.

Theorem 1: Let $(S_1^*, S_2^*, .., S_w^*, \theta^*)$ be an optimal solution for the wth order optimal group partition problem. If $a \in \cup_{j=1}^{w} S_j^*$, $b \notin \cup_{j=1}^{w} S_j^*$, and $g_i \neq g_j$, $\forall i \neq j$, then $g_a \geq g_b$.

Proof:

1. Suppose $g_a < g_b$. Since $a \in \cup_{j=1}^{w} S_j^*$, there exists an integer $k \in \{1,2,..,w\}$ such that $a \in S_k^*$. Define $S_{w+1}^* = \{1,2,..,n\} - \cup_{j=1}^{w} S_j^*$. Let a' and b' be two integers such that $g_{a'} = \min_{i:i\in S_k^*} g_i$ and $g_{b'} = \max_{i:i\in S_{w+1}^*} g_i$. Then, $g_{a'} \leq g_a < g_b \leq g_{b'}$.

2. Define $S_k = S_k^* - \{a'\} + \{b'\}$ and $S_j = S_j^*$, $\forall j \in \{1,2,..,k-1,k+1,k+2,..,w\}$. Then, $|S_j| = |S_j^*|$, $\forall 1 \leq j \leq w$. In addition, $\min_{i:i\in S_j} \log_2(1+\frac{P_{\theta(j)} g_i}{\alpha_{\theta(j)}\sigma^2}) = \min_{i:i\in S_j^*} \log_2(1+\frac{P_{\theta(j)} g_i}{\alpha_{\theta(j)}\sigma^2})$, $\forall j \in \{1,2,..,k-1,k+1,k+2,..,w\}$. Furthermore, since $g_{b'} > g_{a'}$ and $g_i \neq g_j$, $\forall i \neq j$, $\min_{i:i\in S_k} \log_2(1+\frac{P_{\theta(j)} g_i}{\alpha_{\theta(j)}\sigma^2}) > \min_{i:i\in S_k^*} \log_2(1+\frac{P_{\theta(j)} g_i}{\alpha_{\theta(j)}\sigma^2})$.

3. Then,

$$\begin{aligned}
&\sum_{j=1}^{w} \alpha_{\theta(j)} \times \min_{i:i\in S_j} \log_2(1+\frac{P_{\theta(j)} g_i}{\alpha_{\theta(j)}\sigma^2}) \times |S_j| - \\
&\sum_{j=1}^{w} \alpha_{\theta(j)} \times \min_{i:i\in S_j^*} \log_2(1+\frac{P_{\theta(j)} g_i}{\alpha_{\theta(j)}\sigma^2}) \times |S_j^*| \\
= &\ [\min_{i:i\in S_k} \log_2(1+\frac{P_{\theta(j)} g_i}{\alpha_{\theta(j)}\sigma^2}) - \\
&\min_{i:i\in S_k^*} \log_2(1+\frac{P_{\theta(j)} g_i}{\alpha_{\theta(j)}\sigma^2})] \times |S_k| \\
> &\ 0.
\end{aligned}$$

4. However, the above result contradicts with the fact that $(S_1^*, S_2^*, .., S_w^*, \theta^*)$ is an optimal solution for the wth order optimal group partition problem. Thus, $g_a \geq g_b$.

QED.

Definition: For the wth order optimal group partition problem, a solution $(S_1, S_2, .., S_w, \theta)$ is said to be an ordered solution if there exists a permutation $\phi \in \Pi_w$ such that $\max_{k:k\in S_{\phi(j)}} g_k \leq \min_{k:k\in S_{\phi(j+1)}} g_k, \forall 1 \leq j \leq w-1$.

Theorem 2: For the wth order optimal group partition problem, if $g_i \neq g_j$, $\forall i \neq j$, there exists an ordered optimal solution.

Proof:

1. Let $(S_1, S_2, .., S_w, \theta)$ be a solution that is not ordered. Then, a better solution $(S_1^r, S_2^r, .., S_w^r, \theta)$ can be found as follows. Without loss of essential generality, it is assumed that $\min_{k:k\in S_1} g_k = \min_{k:k\in \cup_{j=1}^{w} S_j} g_k$. Let ϕ be a permutation of $\{1,2,..,n\}$ such that $g_{\phi(i)} < g_{\phi(i+1)}$, $\forall 1 \leq i \leq n-1$. Define $n_j = |S_j|$, $\forall 1 \leq j \leq w$. Define $t_j = \sum_{k=1}^{j} n_k$, $\forall 1 \leq j \leq w$. Define $t_0 = 0$. Initially, set $r = 1$. Define $S_j^0 = S_j$, $\forall 1 \leq j \leq w$.

2. If $S_r = \{\phi(t_{r-1}+1), \phi(t_{r-1}+2), .., \phi(t_r)\}$, define $S_j^r = S_j^{r-1}$, $\forall 1 \leq j \leq w$, increase the value of r by one, and then repeat step 2. Otherwise, let $x(r)$ be the minimum element in the set $\{x : t_{r-1}+1 \leq x \leq t_r, \phi(x) \notin S_r\}$. Let $u(r)$ be an integer such that $\phi(x(r)) \in S_{u(r)}$. By definitions of $x(r)$ and $u(r)$, $u(r) \neq r$ and $\min_{k:k\in S_{u(r)}} g_k = g_{\phi(x(r))}$. In addition, there exists an integer $y(r)$ such that $y(r) > x(r)$, $\phi(y(r)) \in S_r$, and $g_{\phi(y(r))} > g_{\phi(x(r))}$. If $x(r) \neq t_{r-1}+1$, go to step 3. Otherwise, swap S_r and S_u, recalculate the values of n_j's and t_j's, and then repeat step 2.

3. Define $S_r^r = S_r^{r-1} - \{\phi(y(r))\} + \{\phi(x(r))\}$ and $S_{u(r)}^r = S_{u(r)}^{r-1} + \{\phi(y(r))\} - \{\phi(x(r))\}$. Define $S_j^r = S_j^{r-1}$, $\forall j \notin$

$\{r, u(r)\}$. Then,

$$\begin{aligned}
& \sum_{j=1}^{w} \alpha_{\theta(j)} \times \min_{i:i\in S_j^r} \log_2(1+\frac{P_{\theta(j)}g_i}{\alpha_{\theta(j)}\sigma^2}) \times |S_j^r| - \\
& \sum_{j=1}^{w} \alpha_{\theta(j)} \times \min_{i:i\in S_j} \log_2(1+\frac{P_{\theta(j)}g_i}{\alpha_{\theta(j)}\sigma^2}) \times |S_j| \\
= \; & \sum_{j=1}^{w} \alpha_{\theta(j)} \times \min_{i:i\in S_j^r} \log_2(1+\frac{P_{\theta(j)}g_i}{\alpha_{\theta(j)}\sigma^2}) \times |S_j^r| - \\
& \sum_{j=1}^{w} \alpha_{\theta(j)} \times \min_{i:i\in S_j^{r-1}} \log_2(1+\frac{P_{\theta(j)}g_i}{\alpha_{\theta(j)}\sigma^2}) \times |S_j^{r-1}| \\
= \; & \alpha_{\theta(u(r))} \min_{i:i\in S_{u(r)}^r} \log_2(1+\frac{P_{\theta(u(r))}g_i}{\alpha_{\theta(u(r))}\sigma^2}) \cdot |S_{u(r)}^r| - \\
& \alpha_{\theta(u(r))} \min_{i:i\in S_{u(r)}^{r-1}} \log_2(1+\frac{P_{\theta(u(r))}g_i}{\alpha_{\theta(u(r))}\sigma^2}) \cdot |S_{u(r)}^{r-1}| \\
> \; & 0.
\end{aligned}$$

The second equality is due to that $S_j^r = S_j^{r-1}$, $\forall j \notin \{r, u(r)\}$, $|S_r^r| = |S_r^{r-1}|$, and $\min_{i:i\in S_r^r} g_i = g_{\phi(t_{r-1}+1)} = \min_{i:i\in S_r^{r-1}} g_i$. The inequality is due to that $|S_{u(r)}^r| = |S_{u(r)}^{r-1}|$, $\min_{i:i\in S_{u(r)}^r} g_i = g_{\phi(y(r))} > g_{\phi(x(r))} = \min_{i:i\in S_{u(r)}^{r-1}} g_i$, and $\log_2(1+\beta x)$ is an increasing function of x, when $\beta, x > 0$.

4. Since the total number of feasible solutions is finite, there exists an optimal solution. A solution is either ordered or not. Based on 3, a solution that is not ordered cannot be optimal. Thus, there exists an optimal ordered solution.

QED.

Note that when $w = 1$, $P_1 = P$ and $\alpha_1 = 1$. Recall that $g_i \geq g_{i+1}$. Based on the above two theorems, we solve the first-order optimal group partition problem as follows.

$$\begin{aligned}
& h[1,n] = \max_{d_1:1\le d_1\le n} \log_2(1+\frac{P\cdot g_{d_1}}{\sigma^2}) \times d_1 \\
& d_1^* = \arg\max_{d_1:1\le d_1\le n} \log_2(1+\frac{P\cdot g_{d_1}}{\sigma^2}) \times d_1 \\
& S_1^* = \{1,2,..,d_1^*\}. \qquad (2)
\end{aligned}$$

Based on equation (2), it takes $O(n)$ time to solve the first-order optimal group partition problem. Similarly, the second-order optimal group partition problem can be solved by **Procedure 1** in $O(2!n^2) = O(n^2)$ time.

It can be proved that the optimal group partition problem has optimal substructure [13]. In particular, an optimal solution for the optimal group partition problem can be constructed efficiently from optimal solutions to its subproblems. Thus, when $w \geq 2$, we propose using dynamic programming [13] to solve the problem as follows. For each triple (k, m, θ), where $k \in \{1,2,..,w\}$, $m \in \{1,2,..,n\}$, and $\theta \in \Pi_w$, define an optimization problem $\Psi(k, m, \theta)$ as follows.

$$\begin{aligned}
& \max \sum_{j=1}^{k} \alpha_{\theta(j)} \times \min_{i:i\in S_j} \log_2(1+\frac{P_{\theta(j)}\times g_i}{\alpha_{\theta(j)}\times\sigma^2}) \times |S_j| \\
& subject\ to \\
& S_j \subset \{1,2,..,m\}, \forall 1\le j\le k \\
& S_u \cap S_v = \emptyset, \forall u \neq v \\
& \cup_{j=1}^{k} S_j = \{1,2,..,m\}. \qquad (3)
\end{aligned}$$

Procedure 1 The Second-Order Group Partition Algorithm

Input: $n, (g_1, g_2, .., g_n), (P_1, P_2), (\alpha_1, \alpha_2), \sigma^2$.
Output: $(S_1^*, S_2^*), \theta^*, h[2,n]$.
$w \leftarrow 2$. $h[2,n] \leftarrow 0$. $d_0 \leftarrow 0$.
Let $\Pi_{w,k}$ be the kth element in Π_w.
for $k = 1$ to $w!$ **do**
  $\theta \leftarrow \Pi_{w,k}$.
  for $d_2 = 1$ to n **do**
    for $d_1 = 1$ to d_2 **do**
      $S_1 \leftarrow \{1,2,..,d_1\}$.
      $S_2 \leftarrow \{d_1+1, d_1+2, .., d_2\}$.
      $\mu \leftarrow \sum_{j=1}^{w} \alpha_{\theta(j)} \times \log_2(1+\frac{P_{\theta(j)}g_{d_j}}{\alpha_{\theta(j)}\sigma^2}) \times (d_j - d_{j-1})$.
      if $\mu > h[2,n]$ **then**
        $h[2,n] \leftarrow \mu$, $(S_1^*, S_2^*) \leftarrow (S_1, S_2)$, $\theta^* \leftarrow \theta$.
      end if
    end for
  end for
end for

Let $f[k, m, \theta]$ be the optimal value of the object function for the optimization problem in (3). Note that

$$h[w,n] = \max_{\theta:\theta\in\Pi_w} \max_{d_w:1\le d_w\le n} f[w, d_w, \theta]. \qquad (4)$$

For each fixed $\theta \in \Pi_w$, we derive the values of $f[k, m, \theta]$'s as follows. First, $\forall 1 \le m \le n$,

$$f[1,m,\theta] = \alpha_{\theta(1)} \times \log_2(1+\frac{P_{\theta(1)}g_m}{\alpha_{\theta(1)}\sigma^2}) \times m. \qquad (5)$$

In addition, $\forall 1 \le k \le w-1, 1 \le m \le n$,

$$\begin{aligned}
& f[k+1, m, \theta] \\
= \; & \max_{d_k:1\le d_k\le m} \{f[k, d_k, \theta] + \alpha_{\theta(k+1)} \times \\
& \log_2(1+\frac{P_{\theta(k+1)}g_m}{\alpha_{\theta(k+1)}\sigma^2}) \times (m - d_k)\}. \qquad (6)
\end{aligned}$$

Based on equations (4)-(6), when $w \geq 2$, the wth order optimal group partition problem can be solved in $O(w!(w-1)n^2)$ time. Note that in practice, the value of n could be very large, while the value of w is usually small. Therefore, the algorithm is efficient in practice.

IV. Simulation Results

We wrote a C program to perform discrete event-driven simulation. To the best of our knowledge, well-known simulation tools such as ns-2 and OPNET are also based on discrete event-driven simulation. For proof of concept, we only simulate selected key features at the transport layer and the physical layer. At the MAC layer, a perfect TDMA scheme is used. In a simulation instance, there are 100,000 time slots. We first study the case in which the mean of g_i equals one, $n \in \{10, 20\}$, $P = 100$, and $\sigma^2 = 1$. When there is no partition, the access point broadcasts information to all group members at rate $\log_2(1+\frac{P\cdot g_n}{\sigma^2})$. When $w = 2$, the access point allocates total signal power to two subgroups such that $\frac{P_1}{P} = \alpha_1$. In Figure 1, we show the network throughput. In terms of the network throughput, setting $w = 1$ is superior to setting $w = 2$. This is due to that the power

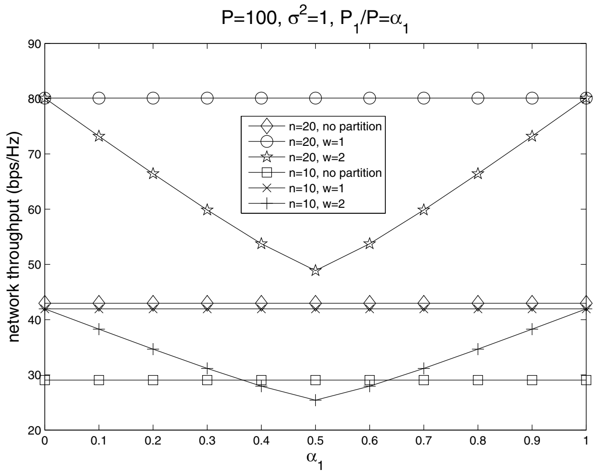

Fig. 1. The network throughput of the optimal group partition algorithm.

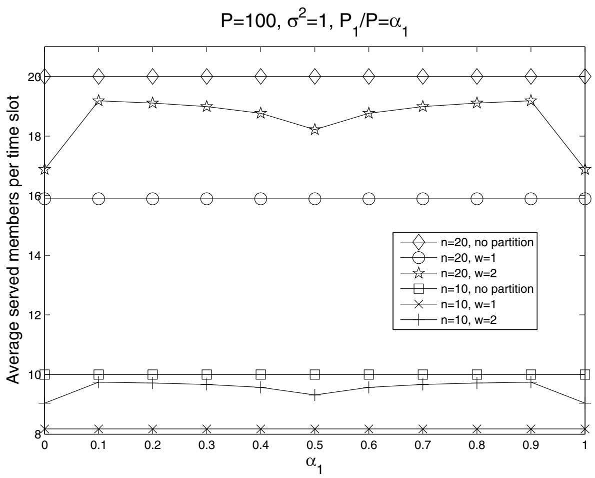

Fig. 2. The average served members of the optimal group partition algorithm.

split $P = P_1 + P_2$ and split in degrees of freedom [12] α_1 are not jointly optimized when $w = 2$. If $\alpha_1 = 0.9$, the network throughput when $w = 2$ is about 90% of the network throughput when $w = 1$. A node is said to be served in a time slot if the node successfully receives some information in the time slot. In Figure 2, we show the average number of group members that are served in a time slot. When $\alpha_1 = 0.9$ and $w = 2$, on average, more than 95% of the group members are served in a time slot. In contrast, when $\alpha_1 = 0.9$ and $w = 1$, on average, about 80% of the group members are served in a time slot. It should be noted that a node might belong to different subgroups in different time slots, since channel gains at different time slots are IID random variables. When the proposed scheme is used in a symmetric network, according to our simulation results, the value of Jain's fairness index is always greater than 0.99. All group members will receive the desired data in the long term.

V. Conclusion

In this paper, we have proposed a novel cross-layer approach of group partition for single-hop wireless multicast flow control. In order to maximize the network throughput, we have formulated and efficiently solved a discrete optimization problem. In addition, we have used simulation results to show group partition is a promising approach for wireless multicast flow control. In particular, we have found that the proposed approach of group partition strikes a good balance between the network throughput and the average number of served group members in a time slot. A direction of future research is predicting some channel gains based on temporal correlations for a large-scale network. Future work also includes taking limits of medium access control protocols into consideration. For asymmetric networks, a promising direction of future research is to partition users into subsets according to their average channel gains and apply the proposed algorithm to each subset. Based on the proposed algorithm, in principle, we can find the optimal order of group partition by solving n optimal group partition problems with orders ranging from 1 to n. However, the computational complexity of the above approach is very high. Thus, we suggest selecting the value of w to be smaller than or equal to five in practice. Designing an efficient algorithm to find the optimal order of group partition is an important future work.

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