Trace Aloha for Random Multiple Access in Wireless MIMO Networks

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***Abstract*—In this paper, we propose and evaluate the Trace Aloha protocol for distributed medium access control in wireless MIMO networks. We study the case in which a node only knows the MIMO channel matrix from the node to the access point and the total number of nodes in the network. Given a MIMO channel matrix, a node could calculate the corresponding singular values. According to the Trace Aloha protocol, a node transmits data if and only if the sum of the squared singular values exceeds a predetermined threshold. We use both analytical results and simulation results to justify the usage of the proposed protocol.**

***Index Terms*—Medium access control, wireless networks, MIMO, random multiple access.**

I. INTRODUCTION

IN this paper, we propose a novel opportunistic medium access control scheme based on the matrix traces of MIMO (multiple input multiple output) channels [1] in a wireless network. In a wireless MIMO network, each node has multiple transmitting antennas and the access point has multiple receiving antennas. MIMO technologies have been applied to one-to-one communications in order to increase the data transmission rate. In addition, multi-user MIMO technologies such as successive interference cancellation [1] could reach the information-theoretic network capacity. Many previous works on medium access control with MIMO channels concentrate on the centralized schemes. In contrast, we focus on distributed medium access control with MIMO capabilities in this paper.

The proposed Trace Aloha algorithm exploits the singular values [2] of a MIMO channel matrix and is based on the well-known slotted Aloha protocol [3]. Early works on Aloha used the $(0, 1, e)$ collision channel model [3] in which the data transmission rate is a constant. Ghez, Verdu, and Schwartz [4] derived stability properties of slotted ALOHA with multipacket reception capability. In the last decade, many works on medium access control with multipacket reception emerged. Naware, Mergen, and Tong [5] studied the impact of multiple packet reception on the stability and delay of slotted Aloha when the buffer size is infinity. Gau [6] analytically derived the non-saturation throughput for slotted Aloha in wireless networks with multipacket reception. Dua [7] proposed a user-centric approach for evaluating the performance of slotted Aloha with multipacket reception in a wireless network in which the total number of nodes is finite but the buffer size at each node is infinity. Lotfinezhad, Liang, and Sousa [8] derived the optimal retransmission probabilities for slotted Aloha in wireless sensor networks with multipacket reception. Recently, Guo, Hu, Zhang, and Chen [9] proposed the adaptive space-time diversity slotted Aloha protocol for random access in wireless MIMO networks. They focused on collision resolution/interference cancellation at the signal processing level. In contrast, we propose a novel transmission policy based on the squared singular values of a MIMO channel matrix. Gong, Perahia, Stacey, Want, and Mao [10] investigated the problem of medium access control in wireless LANs with downlink multi-user MIMO capabilities. In contrast, we focus on uplink medium access control with multi-user MIMO capabilities. Qian, Zheng, Zhang, and Shroff [11] studied the problem of distributed scheduling in multi-hop MIMO networks. We focus on single-hop wireless MIMO networks.

II. SYSTEM MODELS

In the wireless network, there are an access point and $n \geq 2$ nodes. The access point has n_r antennas, while a node has n_t antennas. We focus on uplink transmissions from the nodes to the access point. Time is partitioned into time slots. Let T be the length of a time slot. Typically, the length of a time slot is smaller than the coherence time of the wireless channel [1]. Denote the set of complex numbers by $\mathcal{C}$. Let $\mathbf{x}_k[t] \in \mathcal{C}^{n_t}$ be the transmitted signal/vector from node k to the access point in time slot t, $\forall t$. Let $\mathbf{H}_k[t] \in \mathcal{C}^{n_r \times n_t}$ be the MIMO channel matrix from node k to the access point in time slot t, $\forall t$. Let $\mathbf{y}[t] \in \mathcal{C}^{n_r}$ be the received signal at the access point in time slot t. Let $\mathbf{w}[t] \in \mathcal{C}^{n_r}$ be the additive white Gaussian noise in time slot t, $\forall t$. Note that for each fixed t, $\mathbf{x}_k[t]$, $\mathbf{y}[t]$, and $\mathbf{w}[t]$ are complex-valued column vectors. Denote the expected value of a random variable X by $\mathbb{E}[X]$. Let $\mathbf{A}^*$ be the Hermitian transpose (the conjugate transpose) [2] of the matrix $\mathbf{A}$. It is assumed that the background noise $\mathbf{w}[t]$ is circularly symmetric complex Gaussian [1], $\forall t$. In particular, $\mathbb{E}[\mathbf{w}[t]] = 0$ and $\mathbb{E}[\mathbf{w}[t] \times (\mathbf{w}[t])^*] = N_0 \mathbf{I}_{n_r}$, $\forall t$. According to [1], for each fixed t,

$$\mathbf{y}[t] = \sum_{k=1}^{n} \mathbf{H}_k[t] \times \mathbf{x}_k[t] + \mathbf{w}[t]. \quad (1)$$

It is assumed that $\mathbf{H}_\alpha[t_1]$ and $\mathbf{H}_\beta[t_2]$ are statistically independent, $\forall \alpha \neq \beta, t_1, t_2$. In addition, it is assumed that for each fixed k, $\mathbf{H}_k[1], \mathbf{H}_k[2], \mathbf{H}_k[3], ...$ are IID (independent and identically distributed) random variables. We adopt the IID Rayleigh fading model [1]. In particular, for each fixed pair (k, t), the entries of the channel matrix $\mathbf{H}_k[t]$ are IID circular symmetric complex Gaussian random variables. Namely, $[\mathbf{H}_k[t]]_{i,j} \sim \mathcal{CN}(0, \frac{1}{\mu})$ [1], $\forall k, t, i, j$. Let P be the total transmission power of a node. Let W be the bandwidth used for information transmission from a node to the access point.

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We focus on the saturation case in which a node always has data to send. Let $C(\mathbf{H})$ be the capacity of a MIMO channel when the MIMO channel matrix is $\mathbf{H}$. Given the values of $\mathbf{H}$, P, N_0, and W, the capacity of the MIMO channel can be obtained by singular value decomposition and the well-known water-leveling algorithm [1]. It is assumed that at the beginning of time slot t, node k knows the value of $\mathbf{H}_k[t]$ (through channel estimation) and the value of μ. However, at the beginning of time slot t, node k does not know the value of $\mathbf{H}_i[t], \forall i \neq k$. In time slot t, if node k transmits, it transmits with rate equals $C(\mathbf{H}_k[t])$. Let $A[t]$ be the set composed of the indexes of the nodes that transmit in time slot t. Based on network information theory [1], if $|A[t]| = 1$, the access point successfully receives/decodes data for sure in time slot t. On the other hand, if $|A[t]| \geq 2$, the access point could not receive/decode any data in time slot t. Recall that when $|A[t]| \geq 2$, $\sum_{i:i \in A[t]} C(\mathbf{H}_i[t])$ is larger than the maximum achievable sum rate of the multiple access channel in time slot t.

For a continuous random variable X, denote the probability density function by $f_X(x)$ and the cumulative distribution function by $F_X(x)$. Denote the set of real numbers by $\mathcal{R}$. Recall that $\left|\left|a + b\sqrt{-1}\right|\right|^2 = a^2 + b^2$, $\forall a, b \in \mathcal{R}$. Let $\mathbf{1}\{condition\}$ be the indicator function with value one if the condition is true or with value zero if the condition is not true.

III. The Trace Aloha Protocol

We propose and analyze the Trace Aloha protocol in this section. In order to optimize the network throughput, it is desired that a node transmits only when the instantaneous MIMO channel capacity is relatively large. Since it is quite difficult to compute the cumulative distribution function of $C(\mathbf{H}_k[t])$, the proposed Trace Aloha protocol is based on the cumulative distribution function of the trace [2] of the matrix $\mathbf{H}_k[t] \times (\mathbf{H}_k[t])^*$. Define $n_{min} = \min(n_t, n_r)$. Let $\lambda_{k,1}[t], \lambda_{k,2}[t], .., \lambda_{k,n_{min}}[t]$ be the singular values of the matrix $\mathbf{H}_k[t]$. Recall that the singular value decomposition of $\mathbf{H}_k[t]$ can be interpreted as two coordinate transformations and $(\lambda_{k,i}[t])^2$'s correspond to the magnitude responses of the corresponding Gaussian parallel channels [1]. Thus, for each pair (k, t), we define $Z_k[t]$ as follows.

$$Z_k[t] \quad = \quad \sum_{i=1}^{n_{min}} (\lambda_{k,i}[t])^2. \tag{2}$$

According to [1], $(\lambda_{k,1}[t])^2, (\lambda_{k,2}[t])^2, .., (\lambda_{k,n_{min}}[t])^2$ are eigenvalues of the matrix $\mathbf{H}_k[t] \times (\mathbf{H}_k[t])^*$. Based on [2], the trace of a matrix equals the sum of all eigenvalues of the matrix. Denote the trace of the matrix $\mathbf{A}$ by $tr(\mathbf{A})$. Then,

$$Z_k[t] \quad = \quad tr(\mathbf{H}_k[t] \times (\mathbf{H}_k[t])^*). \tag{3}$$

A. Symmetric MIMO networks

When the Trace Aloha protocol is used, in time slot t, node k transmits data to the access point if and only if $Z_k[t] \geq \theta$, where θ is a real number to be determined. Abbreviate $Z_k[t]$ by Z_k. In order to improve the network throughput, we propose setting θ to be the unique root of the following equation in $[0, \infty)$.

$$\mathbb{E}[\sum_{k=1}^{n} \mathbf{1}\{Z_k \geq \theta\}] = 1. \tag{4}$$

Since Z_k's are IID random variables and $\mathbb{E}[\mathbf{1}\{Z_k \geq \theta\}] = P\{Z_1 \geq \theta\}$, based on Equation (4),

$$P\{Z_1 \geq \theta\} \quad = \quad \frac{1}{n}. \tag{5}$$

Before deriving the value of θ, we elaborate on the difference between the Trace Aloha algorithm and the basic Aloha algorithm. Let $V_k^1[t] \in \{0, 1\}$ be a binary random variable such that $V_k^1[t] = 1$ if and only if node k transmits in time slot t, when the basic Aloha algorithm is used. Let $V_k^2[t] \in \{0, 1\}$ be a binary random variable such that $V_k^2[t] = 1$ if and only if node k transmits in time slot t, when the Trace Aloha algorithm is used. When the basic Aloha algorithm is used, in time slot t, node k transmits with probability $\frac{1}{n}$ regardless of the value of $\mathbf{H}_k[t]$. Thus, $P\{V_k^1[t] = 1 | tr(\mathbf{H}_k[t] \times (\mathbf{H}_k[t])^*) < \theta\} = \frac{1}{n}$. When the Trace Aloha algorithm is used, in time slot t, node k transmits if and only if $tr(\mathbf{H}_k[t] \times (\mathbf{H}_k[t])^*) \geq \theta$. Thus, $P\{V_k^2[t] = 1 | tr(\mathbf{H}_k[t] \times (\mathbf{H}_k[t])^*) < \theta\} = 0$. Although $P\{V_k^1[t] = 1\} = P\{V_k^2[t] = 1\} = \frac{1}{n}$, the random vector $(V_k^1[t], \mathbf{H}_k[t])$ and the random vector $(V_k^2[t], \mathbf{H}_k[t])$ have different probability distribution functions. Thus, as shown later in the paper, the throughput of the Trace Aloha algorithm is larger than the throughput of the basic Aloha algorithm. Based on the principle of symmetry, $P\{\arg\max_{i:1 \leq i \leq n} V_i^2[t] = k | |A[t]| = 1\} = \frac{1}{n}$, $\forall 1 \leq k \leq n$. Namely, when the Trace Aloha algorithm is used, given that a unique node successfully transmits in time slot t, the probability that the unique node is node k equals $\frac{1}{n}$, $\forall 1 \leq k \leq n$.

Define $g(x) = P\{Z_1 \geq x\} - \frac{1}{n}$. Since Z_1 is a continuous random variable and $f_{Z_1}(x) > 0, \forall x > 0$, $g(x)$ is a continuous and decreasing function of x. Furthermore, $g(0) = 1 - \frac{1}{n} > 0$ and $\lim_{x \to \infty} g(x) = 0 - \frac{1}{n} < 0$. Thus, based on the theorem of intermediate value, $g(x)$ has a unique root in $(0, \infty)$. Thus, θ is well-defined. Since Z_1 is a non-negative random variable, based on Markov inequality [12], $P\{Z_1 \geq x\} \leq \frac{\mathbb{E}[Z_1]}{x}$. Thus, $g(n\,\mathbb{E}[Z_1] + 1) \leq \frac{\mathbb{E}[Z_1]}{n\,\mathbb{E}[Z_1]+1} - \frac{1}{n} < 0$. Therefore, based on the theorem of intermediate value, $\theta \in (0, n\,\mathbb{E}[Z_1] + 1)$.

We now derive the value of θ. Note that $P\{Z_1 \geq \theta\} = 1 - P\{Z_1 \leq \theta\}$. Since the proposed protocol is memoryless, $\mathbf{H}_k[t]$ is abbreviated by $\mathbf{H}_k$ whenever appropriate. Since $[\mathbf{H}_k]_{i,j}$'s are IID circular symmetric complex Gaussian random variables, $||[\mathbf{H}_k]_{i,j}||^2$'s are IID exponential random variables [1]. In addition, $\mathbb{E}[||[\mathbf{H}_k]_{i,j}||^2] = \frac{1}{\mu}$ and $F_{||[\mathbf{H}_k]_{i,j}||^2}(x) = 1 - e^{-\mu x}$, $\forall x \geq 0$, $\forall k, i, j$. Furthermore, $\forall 1 \leq k \leq n, 1 \leq i \leq n_r$,

$$\begin{aligned} [\mathbf{H}_k \times \mathbf{H}_k^*]_{i,i} \quad &= \quad \sum_{j=1}^{n_t} [\mathbf{H}_k]_{i,j} \times ([\mathbf{H}_k]_{i,j})^* \\ &= \quad \sum_{j=1}^{n_t} ||[\mathbf{H}_k]_{i,j}||^2. \end{aligned} \tag{6}$$

Thus, for each fixed pair (k, i), $[\mathbf{H}_k \times \mathbf{H}_k^*]_{i,i}$ is a n_t-th order Gamma distributed random variable. In particular, $\mathbb{E}[[\mathbf{H}_k \times \mathbf{H}_k^*]_{i,i}] = \frac{n_t}{\mu}$, $f_{[\mathbf{H}_k \cdot \mathbf{H}_k^*]_{i,i}}(x) = \frac{\mu^{n_t} \cdot x^{n_t - 1}}{(n_t - 1)!} e^{-\mu x}$, $\forall x \geq 0$, and

$F_{[\mathbf{H}_k \cdot \mathbf{H}_k^*]_{i,i}}(x) = 1 - \sum_{k=0}^{n_t-1} \frac{(\mu \cdot x)^k}{k!} e^{-\mu x}$, $\forall x \geq 0$ [12]. Since $[\mathbf{H}_k \times \mathbf{H}_k^*]_{i,i}$ is a n_t-th order Gamma distributed random variable, $Z_k = \sum_{i=1}^{n_r} [\mathbf{H}_k \times \mathbf{H}_k^*]_{i,i}$ is a $(n_t \cdot n_r)$-th order Gamma distributed random variable. In addition, $\mathbb{E}[Z_k] = \frac{n_t \cdot n_r}{\mu}$, $\forall k$. Since Z_1 is a $(n_t \cdot n_r)$-th order Gamma distributed random variable, based on Equation (5),

$$\sum_{m=0}^{n_t n_r - 1} \frac{(\mu \cdot \theta)^m}{m!} e^{-\mu \theta} - \frac{1}{n} \quad = \quad 0. \tag{7}$$

The above equation can be solved by numerical methods such as binary search.

Let λ_D be the network throughput, when the Trace Aloha algorithm is used. In particular, the network throughput is defined to be the average number of bits that are successfully received by the access point per time unit per Hertz. Then,

$$\begin{aligned}
& \lambda_D \\
= \; & \mathbb{E}[\sum_{k=1}^{n} \mathbf{1}\{Z_k \geq \theta, Z_i < \theta, \forall i \in \{1, 2, .., n\} - \{k\}\} \\
& \times C(\mathbf{H}_k)] \\
= \; & n \times \mathbb{E}[\mathbf{1}\{Z_1 \geq \theta, Z_i < \theta, \forall i \geq 2\} \times C(\mathbf{H}_1)] \\
= \; & n \times P\{Z_1 \geq \theta, Z_i < \theta, \forall i \geq 2\} \times \\
& \mathbb{E}[C(\mathbf{H}_1) | Z_1 \geq \theta, Z_i < \theta, \forall i \geq 2] \\
= \; & (1 - \frac{1}{n})^{n-1} \times \mathbb{E}[C(\mathbf{H}_1) | Z_1 \geq \theta].
\end{aligned} \tag{8}$$

Recall that node k transmits with rate $C(\mathbf{H}_k[t])$ in time slot t if and only if $Z_k[t] \geq \theta$. Thus, we have the first equality. The second equality is based on that $\mathbb{E}[\sum_{i=1}^{n} X_i] = \sum_{i=1}^{n} \mathbb{E}[X_i]$ and symmetry. The third equality is based on that $\mathbb{E}[X] = \mathbb{E}[\mathbb{E}[X|Y]]$. Note that Z_i's are IID random variables and $P\{Z_i < \theta\} = 1 - \frac{1}{n}$, $\forall i$. In addition, $\mathbf{H}_1$ and Z_k are statistically independent, $\forall k \neq 1$. Thus, we have the last equality.

Let $\lambda_D^{'}$ be the network throughput when the basic Aloha algorithm is used. Recall that when the basic Aloha algorithm is used, node k transmits with probability $\frac{1}{n}$ in time slot t regardless of the value of $\mathbf{H}_k[t]$. In addition, if node k transmits in time slot t, it transmits with rate equals $C(\mathbf{H}_k[t])$. Then,

$$\lambda_D^{'} \quad = \quad (1 - \frac{1}{n})^{n-1} \times \mathbb{E}[C(\mathbf{H}_1)]. \tag{9}$$

B. Asymmetric MIMO networks

We now modify the Trace Aloha algorithm for the asymmetric case in which $[\mathbf{H}_k[t]]_{i,j} \sim \mathcal{CN}(0, \frac{1}{\mu_k})$, $\forall k, t, i, j$. Typically, μ_k depends on the location of node k. Let $(\theta_1, \theta_2, .., \theta_n)$ be a real vector such that

$$\sum_{k=1}^{n} \sum_{m=0}^{n_t n_r - 1} \frac{(\mu_k \cdot \theta_k)^m}{m!} e^{-\mu_k \cdot \theta_k} \quad = \quad 1. \tag{10}$$

Similar to the symmetric case, when the Trace Aloha algorithm is used, in time slot t, node k transmits if and only if $Z_k[t] \geq \theta_k$. Let $\theta^\dagger$ be a positive real number such that $\sum_{k=1}^{n} \sum_{m=0}^{n_t n_r - 1} \frac{(\mu_k \cdot \theta^\dagger)^m}{m!} e^{-\mu_k \cdot \theta^\dagger} = 1$. When network throughput rather than fairness is the primary concern, we can simply set $\theta_k = \theta^\dagger$, $\forall k$. In this case, the Trace Aloha algorithm is called the throughput Trace Aloha algorithm. When fairness is a major concern, we can choose θ_k such that $\sum_{m=0}^{n_t n_r - 1} \frac{(\mu_k \cdot \theta_k)^m}{m!} e^{-\mu_k \cdot \theta_k} = \frac{1}{n}$, $\forall k$. In this case, the Trace Aloha algorithm is called the fairness Trace Aloha algorithm.

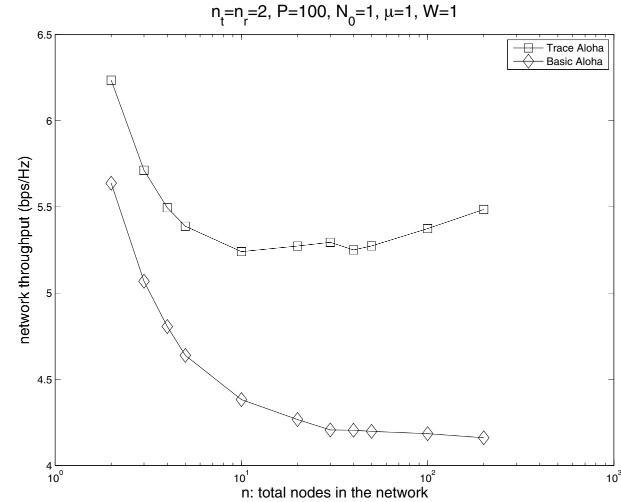

Fig. 1. The network throughput for the Trace Aloha algorithm in symmetric networks.

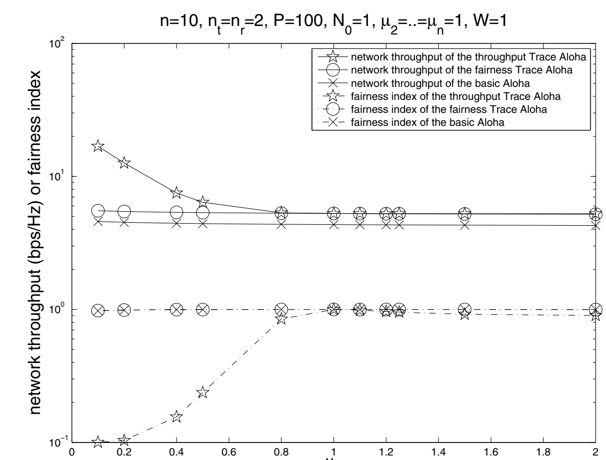

Fig. 2. The network throughput and Jain's fairness index for two variants of the Trace Aloha algorithm in asymmetric networks.

IV. Simulation Results

We wrote a C program to obtain event-based simulation results. Each simulation instance contains $100,000$ time slots. We first studied the case in which $2 \leq n \leq 200$, $n_t = n_r = 2$, $P = 100$, $T = 1$, $W = 1$, $N_0 = 1$, and $\mu = 1$. In Figure 1, we show the values of λ_D and $\lambda_D^{'}$ for symmetric MIMO networks. Regardless of the total number of nodes in the network, the proposed Trace Aloha algorithm outperforms the basic Aloha algorithm. When $n = 200$, the throughput improvement is more than $= \frac{5.48}{4.16} - 1 = 31\%$. Note that in both algorithms, whenever a node transmits, the data transmission rate adapts to the MIMO channel matrix. As the total number of nodes in the network increases, the value of $\lambda_D^{'}$ decreases.

In contrast, when the Trace Aloha algorithm is used, the network throughput when $n = 200$ is larger than the network throughput when $n = 10$. We elaborate on the above result as follows. When the value of n is large, given that a unique node transmits in a time slot, the corresponding data rate tends to be significantly larger than the value of $\mathbb{E}[C(\mathbf{H})]$. Thus, λ_D is not a strictly decreasing function of n. We have also used Jain's fairness index to evaluate the fairness of the Trace Aloha algorithm. In Figure 2, for asymmetric networks in which $n = 10$ and $\mu_1 \neq \mu_2 = \mu_3 = .. = \mu_n$, we show the network throughput and the fairness index for the throughput Trace Aloha algorithm, the fairness Trace Aloha algorithm, and the basic Aloha algorithm. Regardless of the value of μ_1, the throughput of the Trace Aloha algorithm is larger than the throughput of the basic Aloha algorithm. When $\mu_1 \leq 0.5$, in terms of the network throughput, the throughput Trace Aloha algorithm is superior to the fairness Trace Aloha algorithm. However, in terms of fairness, the throughput Aloha algorithm is inferior to the fairness Trace Aloha algorithm. The fairness Trace Aloha algorithm and the basic Aloha algorithm have almost identical fairness indexes. Nevertheless, in terms of the network throughput, the former outperforms the latter.

V. Conclusion

In this paper, we have proposed and evaluated the Trace Aloha protocol for distributed medium access control in wireless MIMO networks. In a wireless MIMO network, each node has multiple transmitting antennas and the access point has multiple receiving antennas. We have focused on the case in which a node only knows the MIMO channel matrix from the node to the access point and the total number of nodes in the network. Given a MIMO channel matrix, a node could calculate the corresponding singular values. According to the Trace Aloha protocol, a node transmits data if and only if the sum of the squared singular values exceeds a predetermined threshold. In order to optimize the network throughput, we have proposed an approach to select the threshold. We have used both analytical results and simulation results to justify the usage of the proposed protocol. Future work includes designing novel random multiple access schemes based on the joint probability density function of the singular values of random matrices of MIMO channels. Another direction of future research is minimizing the average packet delay of random multiple access when packet arrival times are random.

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