

Theoretical study of two-frequency coherence of MST radar returns

Yen-Hsyang Chu

Institute of Space Science, National Central University, Chung-Li, Taiwan

Tsai-Yuan Chen

Institute of Electronics, National Chiao Tung University, Hsin-Chu, Taiwan

Abstract. The theoretical expressions of two-frequency coherence of the MST radar returns from an atmospheric thin layer with sharp boundaries, which embeds within the radar volume and cannot be resolved by conventional radar techniques, are derived in this article. It shows that the derived frequency domain interferometry (FDI) coherence is not only the function of layer thickness and the components of radar wave vector, but also related to the wavenumber power spectrum of refractivity irregularities. A quantitative examination of the derived FDI coherence indicates that the effect of the irregularity power spectrum on FDI coherence is negligible for power law spectral model, while it cannot be ignored for other spectral forms, for example, a Gaussian model. The zenith angle dependence of FDI coherence is also investigated in this article. According to the observations made with Chung-Li VHF radar, it shows that the ratio of the observed FDI coherence at the vertical to that at 17° off-zenith angle is between 1.3 and 2.4. This feature can be illustrated satisfactorily by using the theory developed in this article. The effect of echoing mechanism on FDI coherence is also studied, showing that the expression of FDI coherence derived from the turbulent scattering theory can be treated to be identical to that from the Fresnel reflection theory as long as the condition $\Delta k/k \ll 1$ is met. This result implies that it seems to be impossible to identify the echoing mechanism of MST radar by using FDI technique. The problem of estimating the thickness of a thin layer having sharp boundaries by using a Gaussian function with the expression of $\exp(-\Delta k^2 \sigma_r^2)$ is also discussed. It suggests that the pertinent formula used for layer thickness estimate is $L = 2.364\sigma_r$. A comparison of this work with other results is also made in the text.

1. Introduction

It is well known that operating at one carrier frequency with a monostatic pulse radar makes it impossible to resolve an isolated, thin atmospheric turbulent layer which embeds within the scattering volume defined by the pulse length. However, the advent of the frequency domain interferometry (FDI) technique, first developed by *Kudeki and Stitt* [1987], provides us with the ability to detect such a thin layer by sending two closely spaced frequencies. With this technique, the thickness and the position of the thin layer can be estimated, respectively, from the magnitude (i.e., coherence) and phase of the normalized complex cross-corre-

lation function calculated from the two radar returns with slightly different operating frequencies. In view of its high potential in the observation of the atmospheric thin layer, the FDI technique has been employed successfully by many investigators on various MST radar to study the mesospheric, stratospheric, and tropospheric turbulent layer [*Kudeki and Stitt*, 1987, 1990; *Franke*, 1990; *Franke et al.*, 1992; *Palmer et al.*, 1990; *Chu and Franke*, 1991].

In deriving FDI equations for the estimate of the thickness and position of an atmospheric thin layer, the proper assumptions have to be made to obtain the analytical expression. Obviously, the assumption changes will result in different expressions of the FDI equation, implying that the suitability of a theoretical FDI equation is restricted by the conditions imposed in the derivation of theoretical equation. An examination of the existing FDI theories

Copyright 1995 by the American Geophysical Union.

Paper number 95RS02056.
0048-6604/95/95RS-02056\$08.00

[Franke, 1990; Kudeki and Stitt, 1987, 1990] shows that a modeled turbulent layer weighted by a Gaussian function of the form $\exp [-(r - r_o)^2/2\sigma^2]$, where r_o is the height to the center of scattering layer and 2σ is defined as the layer thickness, is employed in deriving the normalized two-frequency cross-correlation function of the radar scattering fields. Strictly speaking, the FDI formula derived on the basis of the assumption of Gaussian model layer can be only used to estimate the thickness and position of a Gaussian layer. The error will be induced as such FDI formula is used to estimate a non-Gaussian turbulent layer, for example, uniform turbulent layer with sharp boundaries. In fact, abundant evidences of a turbulent layer with sharp boundaries existing both in atmosphere and in deep ocean have already been shown by many investigators [Crain, 1955; Ottersten, 1969; Stewart, 1969; Barat, 1982; Gossard and Strauch, 1983; Gossard, 1990]. If the estimation error is significantly large, the development of a more proper FDI equation for a sharp-edged layer is necessary. Furthermore, in developing the existing FDI theory, the contribution due to the wavenumber power spectrum of the refractive index inhomogeneities to the normalized two-frequency cross-correlation function with closed frequency spacing is thought to be negligible, without providing any quantitative evidence. Although this speculation may be true for some forms of irregularity power spectrum, it may be incorrect for other specific spectral model. In order to access the role of irregularity power spectrum in the two-frequency coherence, the quantitative examination for the cases of various irregularity wavenumber power spectrum models is needed. Recently, the result of an oblique FDI experiment conducted by Palmer *et al.* [1992] provides an observational evidence showing a close connection between coherence and aspect sensitivity due to the anisotropy of refractivity irregularities. This feature can not be explained satisfactorily by using current FDI theory. In addition, in view of the fact that the echoing mechanisms involved in the MST radar returns are basically turbulent scattering and Fresnel reflection, the question arises as to whether the existing FDI formula based on the turbulent scattering theory is valid in the estimate of the thickness and position of a partially reflecting layer with transverse coherent structure characterized by a sharp vertical refractive index gradient. Because the gov-

erning equations are different, it seems intuitively that the FDI formula suitable for turbulent layer will be different from that for the partially reflecting layer. To clarify this question, a more extensive investigation on the theoretical FDI analysis is necessary.

This paper is an attempt to answer the questions addressed above from observational and theoretical points of view. Because the absolute position of the atmospheric layer can not be determined by using the observed phase difference of two operating frequencies unless the radar phase has been calibrated precisely, only the magnitude of normalized two-frequency cross-correlation function (i.e., FDI coherence) will be emphasized in this article. In section 2, by including the effect of irregularity power spectrum, an analytical expression for the normalized two-frequency cross-correlation function between radar returns generated from a turbulent layer having sharp boundaries is derived. Additionally, on the basis of theoretical expression of the Fresnel reflection coefficient, the FDI coherence for a partially reflecting layer is also evaluated in this section. The contribution of the irregularity power spectrum to FDI coherence is examined in section 3 by substituting various forms of irregularity power spectrum into the derived FDI equations. It shows that for the power law spectral model the effect of irregularity power spectrum can be ignored, while for other spectral forms, for example, the Gaussian spectral model, it can not be neglected. The discussions on the zenith angle dependence of FDI coherence is made in section 4. It shows that the observed vertical FDI coherence is systematically larger than the oblique one, and this feature can be accounted for by using the theory developed in this article. In section 5, the effect of echoing mechanism on FDI coherence is investigated. Comparing the analytical expression of the FDI coherence for a turbulent layer with that for a spectral layer shows that the difference between these two FDI coherences is negligibly small, implying that it seems to be unlikely to discriminate the echoing mechanism from FDI experiment. In section 6, the estimation of thickness of a thin layer with sharp edges is studied. Finally, the comparison of this work with other results is made in section 7. The conclusion is given in section 8.

2. Normalized Two-Frequency Cross-Correlation Function

In this section, the analytical expressions of the normalized two-frequency cross-correlation function of the MST radar returns from a thin atmospheric turbulent layer and a partially reflecting layer are derived. For mathematical simplicity, the following assumptions are made in the mathematical manipulation: (1) the range weighting function is taken to be rectangular, (2) a narrow antenna beam with uniform distribution is considered, (3) the refractivity irregularities responsible for the radar returns are distributed uniformly in the illuminating volume, and (4) the center of the layer, if it exists in the radar volume, is located at the center of the radar volume at where the origin point is set. The mathematical derivations of the cross-correlation function are given as below.

Case for Turbulent Scattering

It is well known that if a pencillike radar beam operating at frequency f is used such that the horizontal dimension of the radar volume is smaller than the first Fresnel radius, the strength of the radar echo backscattering from the atmospheric refractive index random fluctuations, Δn , can be expressed as [Tatarskii, 1971; Doviak and Zrnic, 1984]

$$E(k, \mathbf{r}) = \frac{Gk^2 A_o \exp(-i2kR)}{2\pi R^2} \cdot \int_V \Delta n(\mathbf{r}) \exp(-i2\mathbf{k}\mathbf{a}_s \cdot \mathbf{r}) dv \quad (1)$$

where G is the antenna gain, k is the wavenumber ($=2\pi f/c$), A_o is the amplitude of incident wave, \mathbf{a}_s is the unit vector in the direction of radar beam, R is the range, and V is the scattering volume dependent on the layer thickness and radar beam width. Because a sharp-edged layer with a thickness smaller than the vertical extent of radar volume is considered and the effect of irregularity power spectrum is also taken into account in deriving FDI coherence, the integration limits of (1) should be finite, instead of infinite, as was taken by the earlier investigators. If the origin point is set at the center of scattering volume V , the upper and lower limits of the volume integration for (1) will be $-L_x/2$ and $L_x/2$, $-L_y/2$ and $L_y/2$, and $-L_z/2$ and $L_z/2$, where L_x , L_y , and L_z are the dimensions of the scattering volume in the x ,

y , and z directions, respectively. We note that in carrying out the FDI experiment, a radar pulse with frequency f_2 is usually transmitted a short time after a previous pulse with a slightly different carrier frequency f_1 . If the time interval between these two successive radar pulses is much shorter than the correlation time of refractivity fluctuations Δn , it is reasonable to assume that the refractivity irregularities responsible for the radar returns of these two transmitted pulses are identical. Accordingly, the cross-correlation function of the radar returns for these two pulses can then be expressed as follows:

$$\langle E_1 E_2^* \rangle = \frac{G^2 k_1^2 k_2^2 A_o^2 \exp(-i2\Delta k R)}{4\pi^2 R^4} \cdot \int_{V_1} \int_{V_2} \langle \Delta n(\mathbf{r}_1) \Delta n(\mathbf{r}_2) \rangle \exp(-i2\mathbf{k}\mathbf{a}_s \cdot \mathbf{U}) dv_1 dv_2 \quad (2)$$

where $\langle \rangle$ means ensemble average, the asterisk represents the complex conjugate, $\Delta k = k_2 - k_1$, and $\mathbf{U} = k_1 \mathbf{r}_1 + k_2 \mathbf{r}_2$. If the atmospheric refractivity fluctuations are assumed to be stationary and homogeneous, the autocorrelation function of Δn in (2), namely, $\langle \Delta n(\mathbf{r}_1) \Delta n(\mathbf{r}_2) \rangle$, can be represented by $B_n(\mathbf{r}_1 - \mathbf{r}_2)$. To proceed, the appropriate coordinate transformations are introduced in (2) such that the analytical form of the integration can be obtained. For that purpose, let the new variables of integration $\boldsymbol{\sigma}$ and $\boldsymbol{\delta}$ be related to the old ones \mathbf{r}_1 and \mathbf{r}_2 by the transformations $\boldsymbol{\sigma} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ and $\boldsymbol{\delta} = \mathbf{r}_1 - \mathbf{r}_2$. Substituting these relationships into (2) and rearranging it, we have

$$\langle E_1 E_2^* \rangle = \frac{G^2 k_1^2 k_2^2 A_o^2 \exp(-i2\Delta k R)}{4\pi^2 R^4} \int_{V_\delta} \int_{V_\sigma} B_n(\boldsymbol{\delta}) \cdot \exp[-i(2k_1 + \Delta k)\mathbf{a}_s \cdot \boldsymbol{\delta} + 2\Delta k\mathbf{a}_s \cdot \boldsymbol{\sigma}] dv_\delta dv_\sigma \quad (3)$$

Note that the integration limits for the new variables $\boldsymbol{\sigma}$ and $\boldsymbol{\delta}$ in (2) are significantly different from those for the old ones \mathbf{r}_1 and \mathbf{r}_2 in (1). For example, the upper and lower limits of the integration for the variable of integration σ_x , which is the component of $\boldsymbol{\sigma}$ in the x direction, are $L_x/2 - |\delta_x|/2$ and $-L_x/2 + |\delta_x|/2$, respectively, while the upper and lower integration limits for variable of integration δ in the x component are L_x and $-L_x$, respectively. The integration limits for other components are similar to those for the x component, except that the subscripts of the corresponding variables are differ-

ent. After performing the mathematical manipulation, (3) reduces to

$$\langle E_1 E_2^* \rangle = Q \int_{V_s} B_n(\delta) P \exp[-i(k_1 + k_2)\mathbf{a}_s \cdot \delta] dV_\delta \quad (4)$$

where

$$P = \frac{\sin[\Delta k_x(L_x - |\delta_x|)]}{\Delta k_x} \frac{\sin[\Delta k_y(L_y - |\delta_y|)]}{\Delta k_y} \cdot \frac{\sin[\Delta k_z(L_z - |\delta_z|)]}{\Delta k_z}$$

$$Q = \frac{A_0^2 k_1^2 k_2^2 e^{i2\Delta k R}}{4\pi^2 R^4}$$

If we assume that the correlation length of $B_n(\delta)$ is fairly smaller than the dimension of the scattering volume, and if $\Phi(\mathbf{K})$ is the Fourier transform of $B_n(\delta)$, then (4) becomes

$$\langle E_1 E_2^* \rangle = Q V_s \Phi[(2k_1 + \Delta k)\mathbf{a}_s] \cdot \text{sinc}(\Delta k_x L_x) \text{sinc}(\Delta k_y L_y) \text{sinc}(\Delta k_z L_z) \quad (5)$$

where $\Phi[(2k_1 + \Delta k)\mathbf{a}_s]$ is the average spatial power spectrum of refractive index irregularities over the scattering volume $V_s (= L_x L_y L_z)$, and Δk_x , Δk_y , and Δk_z are the wavenumber differences projecting on the x , y , and z axes, respectively. Because the normalized cross-correlation function S_{12} of the signals E_1 and E_2 is defined as

$$S_{12} = \frac{\langle E_1 E_2^* \rangle}{\sqrt{\langle |E_1|^2 \rangle \langle |E_2|^2 \rangle}} \quad (6)$$

from (4) and (5) we then have

$$S_{12} = \frac{\Phi[(k_1 + k_2)\mathbf{a}_s]}{\sqrt{\Phi(2k_1\mathbf{a}_s)\Phi(2k_2\mathbf{a}_s)}} \cdot \text{sinc}(\Delta k_x L_x) \text{sinc}(\Delta k_y L_y) \text{sinc}(\Delta k_z L_z) e^{i2\Delta k R} \quad (7)$$

Equation (7) shows that by considering the finite integration in performing mathematical manipulation, the normalized two-frequency cross-correlation for the radar returns from a uniform turbulent layer with sharp edges is not only the function of the dimension of scattering volume defined by radar beam width and layer thickness but also related to the components of wavenumber spacing in accor-

dance with the sinc function. These theoretical results predict that FDI coherence as a function of layer thickness will vary with the zenith angle of radar beam, and the phase of the normalized cross-correlation function is not affected by the form of the refractive index spectrum. In addition, from (7) it is also indicated that the magnitude of S_{12} is associated with a three-dimensional wavenumber power spectrum of the refractive index inhomogeneities, contradicting the predictions achieved by earlier investigators [Kudeki and Stitt, 1987; Franke, 1990]. This result indicates that the effect of refractivity irregularities plays a role on FDI coherence, implying that the layer thickness estimated by using existing FDI equations may be inaccurate. Detailed discussions on the characteristics of (7) will be made in section 3.

Case for Fresnel Reflection

It is obvious that an EM wave will be partially reflected if it is incident normally to an atmospheric stable layer with a substantial gradient of refractive index cross the layer. If this layer is horizontally stratified, it can be shown that the theoretical partial reflection coefficient ρ of a vertically incident EM wave can be formulated as follows:

$$\rho = \frac{1}{2} \int_{-L/2}^{L/2} \frac{dn}{dz} e^{-i2kz} dz \quad (8)$$

where the origin point is set at the center of the layer, L is the thickness of the stable layer, $k (= 2\pi/\lambda)$ is the wavenumber of the incident EM wave, and dn/dz is the gradient of refractive index of the layer. We note from (8) that the magnitude of the partial reflection coefficient as the function of the incident wave frequency is determined by the Fourier component of dn/dz at the spatial scale of $\lambda/2$. Because the Fresnel reflection echo strength is proportional to ρ , it is apparent that the FDI coherence of the Fresnel reflection echoes will be proportional to that of reflection coefficient. Accordingly, for two EM waves with slightly different frequencies reflecting from a stable layer, the cross-correlation function of the reflection coefficient $\langle \rho_1 \rho_2^* \rangle$ can thus be calculated from (8) analytically. Assume that the variation of the refractive index is statistically random across the stable layer. Then from (8), $\langle \rho_1 \rho_2^* \rangle$ can be expressed as

$$\langle \rho_1 \rho_2^* \rangle = \frac{1}{4} \int_{-L/2}^{L/2} \int_{-L/2}^{L/2} \left(\frac{dn(z_1)}{dz_1} \frac{dn(z_2)}{dz_2} \right) \cdot \exp [-i2k_1(z_1 - z_2) + i2\Delta kz_2] dz_1 dz_2 \quad (9)$$

where ρ_1 and ρ_2 represent the reflection coefficients of the EM waves with wavenumber k_1 and k_2 , respectively. If the variation of refractive index in the z direction is thought to be a stationary process and the new variables of integration σ and δ are introduced in (9) in such a way that the old ones z_1 and z_2 are superseded in accordance with the rules $\delta = z_1 - z_2$ and $\sigma = (z_1 + z_2)/2$, rearranging (9) reduces the equation to [Papoulis, 1965]

$$\langle \rho_1 \rho_2^* \rangle = \frac{1}{4} \int_{-L}^L \int_{-L/2+\delta/2}^{-L/2-\delta/2} \left(-\frac{d^2 R_n(\delta)}{d\delta^2} \right) \cdot \exp \{-i2k_1 \delta + i2\Delta k[\sigma - (\delta/2)]\} d\sigma d\delta \quad (10)$$

Where $R_n(\delta)$ is the autocorrelation function of random process $n(z)$. We further assume that the correlation length of $R_n(\delta)$ is considerably smaller than the vertical extent of the illuminating region (or layer thickness). By employing the differential theorem of the Fourier transform, namely,

$$-k^2 \Phi(k) = \int_{-\infty}^{\infty} \frac{d^2 R_n(\delta)}{d\delta^2} e^{-ik\delta} d\delta \quad (11)$$

where $\Phi(k)$ is the Fourier transform of $R_n(\delta)$, (10) thus becomes

$$\langle \rho_1 \rho_2^* \rangle = L(k_1 + k_2)^2 \hat{\Phi}(k_1 + k_2) \text{sinc}(\Delta kL) \quad (12)$$

Similarly, $\hat{\Phi}(k_1 + k_2)$ represents the average wavenumber power spectrum of the refractivity irregularities in the height coverage of the stable layer L . Following the definition of coherence shown in (6), the coherence of ρ_1 and ρ_2 can be expressed as

$$\rho_{12} = \frac{\hat{\Phi}(k_1 + k_2) (k_1 + k_2)^2}{\sqrt{\hat{\Phi}(2k_1)\hat{\Phi}(2k_2)} 4k_1 k_2} \text{sinc}(\Delta kL) \quad (13)$$

Equation (13) indicates that if the finite limits are taken into account in performing the mathematical integration, the derived two-frequency coherence of the partial reflection coefficient is not only the function of power spectrum of the refractive index but also the function of layer thickness following the Sinc function form, which is quite similar to the turbulent layer case. A detailed comparison of the

derived FDI coherence for the turbulent layer with that for the specular layer will be made in section 4 in order to examine the echoing mechanism effect on the FDI coherence to what extent. In the following section, the effect of refractivity inhomogeneities on FDI coherence will be investigated quantitatively.

3. Effect of Wavenumber Power Spectrum of Irregularities

In the preceding section, by carrying out the finite integration, we have derived the analytical expressions of the FDI coherence for the radar returns from a turbulent layer and partially reflecting layer, showing a close connection between the FDI coherence and wavenumber power spectrum of refractive index fluctuations. In order to investigate the effect of refractivity irregularities on the FDI coherence, the relevant wavenumber power spectral model has to be given in (7). Assuming that the theoretical wavenumber power spectrum of anisotropic irregularities is followed, the power law form, that is,

$$\Phi(\mathbf{k}) = \frac{C}{(1 + \zeta_x^2 k_x^2 + \zeta_y^2 k_y^2 + \zeta_z^2 k_z^2)^p} \quad (14)$$

where

$$k_x = k \sin(\theta) \cos(\phi)$$

$$k_y = k \sin(\theta) \sin(\phi)$$

$$k_z = k \cos(\theta)$$

and θ and ϕ are the zenith angle and azimuth angle of the radar beam, respectively; C is a constant independent of the wavenumber; and ζ_x , ζ_y , and ζ_z are the correlation lengths in the x , y , and z directions, respectively. For the special case in which $\zeta_z < \zeta_y = \zeta_x = \zeta_r$, (14) can be expressed as

$$\Phi(\mathbf{k}) = \frac{C}{(1 + \zeta_r^2 k_r^2 + \zeta_z^2 k_z^2)^p} \quad (15)$$

where $k_r^2 = k_x^2 + k_y^2$. Substituting (15) into (7), we have

$$S_{12} = E \text{sinc}(\Delta k_x L_x) \text{sinc}(\Delta k_y L_y) \text{sinc}(\Delta k_z L_z) e^{i2\Delta kR} \quad (16)$$

where

$$E = \left\{ \frac{(U + k_{i1}^2 + A^2 k_{z1}^2)(L + k_{i2}^2 + A^2 k_{z2}^2)}{U + \left(\frac{k_{i1} + k_{i2}}{2}\right)^2 + A^2 \left(\frac{k_{z1} + k_{z2}}{2}\right)^2} \right\}^{p/2} \quad (17)$$

and $U = (1/2\zeta_t)^2$, A is the anisotropy of irregularities and is defined as the ratio of ζ_z to ζ_t , and the subscripts 1 and 2 correspond to the operational frequencies f_1 and f_2 , respectively. Note that the expression of E is determined by the given wavenumber spectrum of irregularities. For other theoretical wavenumber spectral forms, such as the Gaussian spectral model, that is,

$$\Phi(\mathbf{k}) = C \exp [-(\zeta_x^2 k_x^2 + \zeta_y^2 k_y^2 + \zeta_z^2 k_z^2)] \quad (18)$$

where C is a constant, E can be expressed as

$$E = \exp (\zeta_t^2 \Delta k_t^2 + \zeta_z^2 \Delta k_z^2) \quad (19)$$

where the condition that $\zeta_x = \zeta_y = \zeta_t$ has been assumed. Calculations show that the magnitude of E for the power law spectral form is almost equal to 1 and insensitive to the values of ζ_t , ζ_z , θ , and ϕ as long as $\Delta k/k \ll 1$, while the magnitude of E for Gaussian spectral form increase exponentially with ζ_t , ζ_z , and θ . Figure 1 shows the variations of magnitude of E for Gaussian and power law wavenumber spectra with frequency spacing, where solid and dashed lines represent magnitude of E for Gaussian and power law spectral models, respectively, and the spectral index of 11/6, ζ_t of 50 m, ζ_z of 3 m, f_1 of 50 MHz, and θ of 17° are given in the calculation of E . It is clear from Figure 1 that for the power law spectral form the effect of refractivity irregularities on the FDI coherence is negligible. However, the irregularity effect may not be ignored for other spectral models, for example, the Gaussian type. It is conclusive from these results that because the effect of refractivity irregularities on the magnitude of S_{12} cannot be ignored for a certain spectral form, the interpretation of the observed FDI coherence must be treated with some caution.

4. Zenith Angle Dependence of FDI Coherence

It is obvious from (16) and (19) that the theoretical FDI coherence is the function of the components of the EM wave vector and the anisotropy of irregularities, implying the possible dependence of FDI coherence on zenith angle. By including the contribution due to irregularity power spectrum,

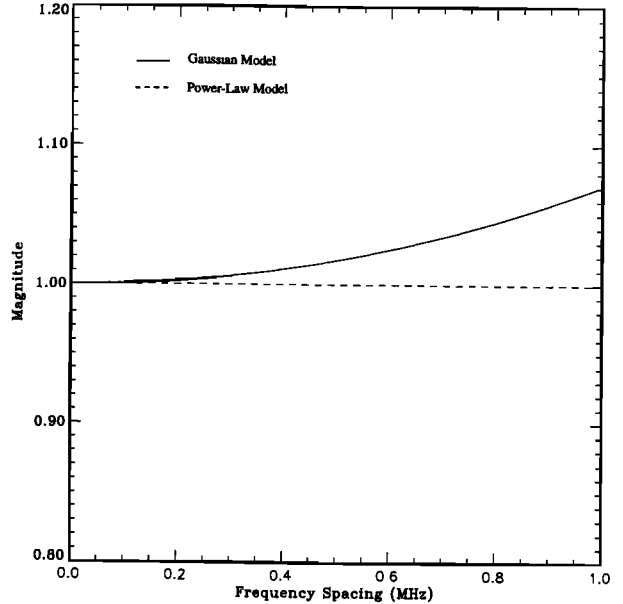


Figure 1. The comparison of the magnitude of E for the Gaussian model (solid line) with that for power law model (dashed line) (for explanation of E , see text). The parameters used in the calculation of E are given as the spectral index of 11/6, ζ_t of 50 m, ζ_z of 3 m, f_1 of 50 MHz, and θ of 17° .

the variations of FDI coherence versus zenith angle are plotted in Figure 2, where solid and dashed lines are the same as in Figure 1. The values of parameters used in Figure 2 are given as follows: f_1 of 50 MHz, f_2 of 50.5 MHz, layer thickness of 100 m, radar beam width of 3° , altitude of 6 km, ζ_z of 40 m, and ζ_t of 5 m. Figure 2 shows that the FDI coherence for the Gaussian spectral model is greater than that for the power law spectral form, owing to the contribution of refractivity irregularities. The larger the correlation length of the irregularities, the greater will be the difference. Generally speaking, it is believed that the anisotropy of the refractivity inhomogeneities will lead to the aspect sensitivity of MST radar echo power [Gage and Balsley, 1980; Doviak and Zrnica, 1984; Woodman and Chu, 1989]. Because FDI coherence is related to ζ_t and ζ_z , a possible connection between FDI coherence and aspect sensitivity can be inferred. A FDI experiment was conducted by using Chung-Li VHF radar on June 3, 1990, to investigate the zenith angle dependence of FDI coherence. The key radar parameters were as follows: peak transmitted power of 35 kW, radar pulse length of 4 μ s, and coherent integration time of 0.15 s. The radar beam was

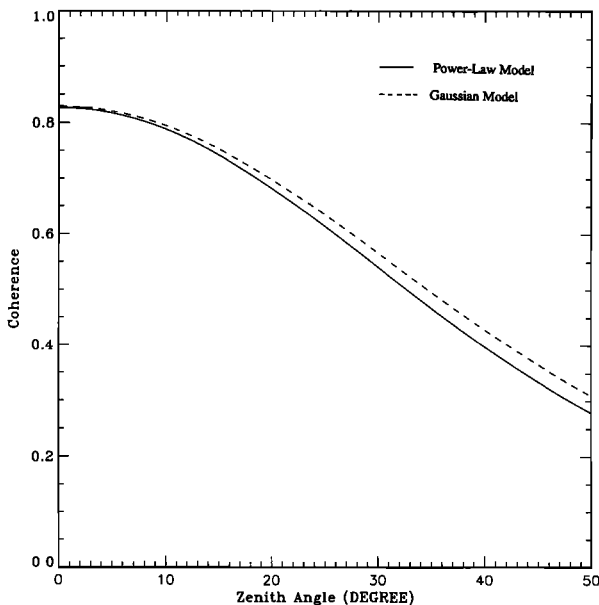


Figure 2. The comparison of the FDI coherence for Gaussian spectral model (solid curve) with that for the power law spectral model (dashed curve). The parameters used for the calculation are given as follows: f_1 of 50 MHz, f_2 of 50.5 MHz, layer thickness of 100 m, radar beam width of 3° , altitude of 6 km, ζ_z of 40 m, and ζ_r of 5 m.

steered first toward the zenith and then toward the east at 17° off-vertical angle with the duration of 20 min each. Two operating frequencies, 52 and 52.35 MHz (corresponding to frequency spacing of 0.35 MHz), were employed in this FDI experiment. Figure 3 shows the profiles of the observed vertical (dotted line with asterisks) and 17° off-zenith (solid line with open circles) echo power at the operational frequency of 52.3 MHz, while Figure 4 presents the height variations of the vertical and oblique FDI coherence. Note that the absence of the oblique coherence above 12 km is due to the poor data quality. Each profile shown in Figures 3 and 4 is a 20-min average. Figure 3 shows that significant aspect sensitivity is observed below 10 km, in which extremely high aspect sensitivity can be seen at heights of 4, 7, and 9 km. Note that the enhancement of vertical echo power at around 18 km is due to high atmospheric stability of tropopause. Examining the height variations of FDI coherence shown in Figure 4 indicates that the vertical coherence is systematically greater than the oblique one through the entire height range. Corresponding to the enhancement of echo power at the height of tropopause, a salient peak can also be seen in the

vertical profile of the coherence. In addition, it is noteworthy from Figure 4 that the vertical FDI coherences at the heights of 7 and 9 km, where the distinct layers exist, are significantly larger than the oblique ones in the ratios of 2.4 and 2.1, respectively. In order to account for this feature, the theoretical variations of FDI coherence with zenith angle for the cases of five different layer thicknesses are plotted in Figure 5 in accordance with (16). As indicated, with increasing zenith angle of the radar beam, the FDI coherence as the function of layer thickness decreases rapidly. Inspecting the behavior of the FDI coherence varied with the zenith angle shows that for the case of the 120-m layer thickness, the theoretical ratio of the FDI coherence at the zenith to that at 17° off-vertical angle is about 1.6, in good agreement with the observations. This result seems to suggest that the layer thickness at the heights of 7 and 9 km are about 120 m. In an analogous way, the layer thickness at the height of 10 km can be estimated to be about 260 m. From Figures 3 and 4, it is also demonstrated that vertical and oblique FDI coherence are both large at the place where aspect sensitivity is significant. *Palmer et al.* [1992] also provides a distinct observational evidence showing a close connection between FDI coherence and aspect sensitivity. Irrespective of somewhat unrealistic property of Gaussian model, the relation between coherence and anisotropy of irregularities (i.e., aspect sensitivity) can be illustrated qualitatively by (19).

5. Effect of Echoing Mechanism

In section 2 the theoretical FDI coherences for the echoing mechanisms of turbulent scattering and Fresnel reflection are derived. In order to examine quantitatively the extent of the effect of the echoing mechanism on the magnitude of the FDI coherence has, we assume that the propagation of the EM wave is in the z direction for the comparison of (7) with (13). In this special case the components of the wavenumbers in the x and y directions will be both zero, that is, $k_x = k_y = 0$, and (7) reduces to

$$S_{12} = \frac{\hat{\Phi}[(k_1 + k_2)]}{\sqrt{\hat{\Phi}(2k_1)\hat{\Phi}(2k_2)}} \text{sinc}(\Delta kL) e^{i2\Delta kR} \quad (20)$$

where $\Delta k (= k_2 - k_1)$ is the wavenumber difference, and L is the layer thickness. If the power law

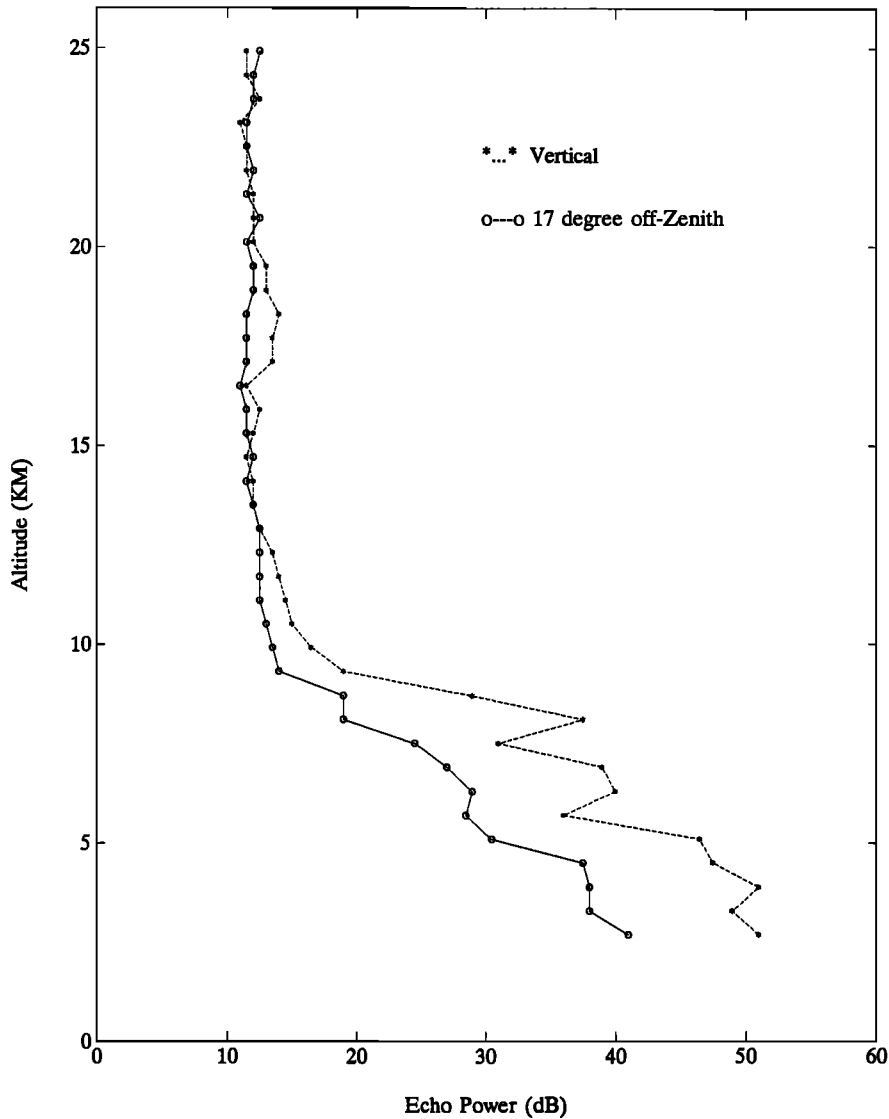


Figure 3. The profiles of vertical (dotted line with asterisks) and 17° off-zenith echo power (solid line with open circles) at 20-min averages, each observed with the Chung-Li radar.

spectral model of irregularities is chosen, from (20) the corresponding FDI coherence becomes

$$|S_{12}| = \frac{\sin(\Delta kL)}{\Delta kL} \tag{21}$$

Comparing (20) with (13) shows that the FDI coherence for a turbulent layer is different slightly from that for a partially reflecting layer only by the factor $(k_1 + k_2)^2/4k_1k_2$. If the wavenumber spacing is so small that $\Delta k/k_1 \ll 1$, by using binomial expansion, $(k_1 + k_2)^2/4k_1k_2$ can be expressed as

$$\frac{(k_1 + k_2)^2}{4k_1k_2} \cong 1 + \frac{1}{4} \left(\frac{\Delta k}{k_1}\right)^2 \left(1 + \frac{1\Delta k}{2k_1}\right) \tag{22}$$

For a VHF radar, if the frequency spacing Δf and operating frequency f_1 are set as 0.5 and 52 MHz respectively, we then have $(\Delta k/k_1)^2 \approx 0.0000925$. In view of $(k_1 + k_2)^2/4k_1k_2 \approx 1$, (20) can be treated to be identical to (13), indicating that FDI coherence is insensitive to the echoing mechanism. This result also suggests that it seems to be impossible to identify the echoing mechanism by using the FDI technique.

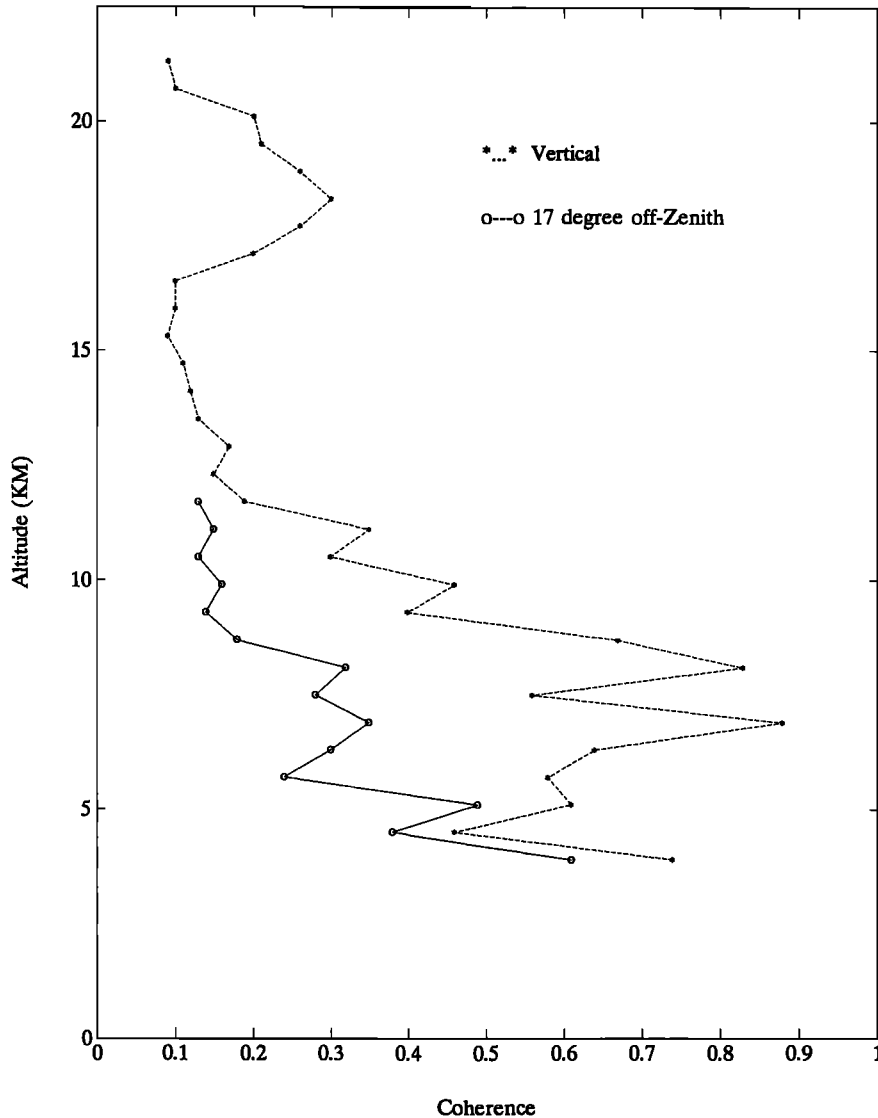


Figure 4. The height variations of the FDI coherence at the vertical (dotted line with asterisks) and at 17° off-zenith angle (solid line with open circles) at 20-min averages, each observed with the Chung-Li radar.

6. Estimation of Layer Thickness

In section 4, with the help of theoretical curves, the layer thickness with large aspect sensitivity can be determined from the observed FDI coherence at different zenith angle. In this section the thickness of a layer with sharp boundaries estimated by using a Gaussian function is discussed. According to the earlier investigations, an exponential decrease of coherence with the square of wavenumber spacing Δk and layer thickness L is predicted under the

conditions in which the infinite integration is carried out and a Gaussian layer structure is assumed. In this article, by considering an atmospheric layer with distinct edges, the theoretical FDI coherence is obtained. It shows that a factor $\text{sinc}(\Delta kL)$ can always be found in the theoretical expressions of FDI coherence, no matter what the echo mechanism is. Apparently, this factor results from the consideration of an isolated atmospheric layer with sharp boundaries in performing the mathematical

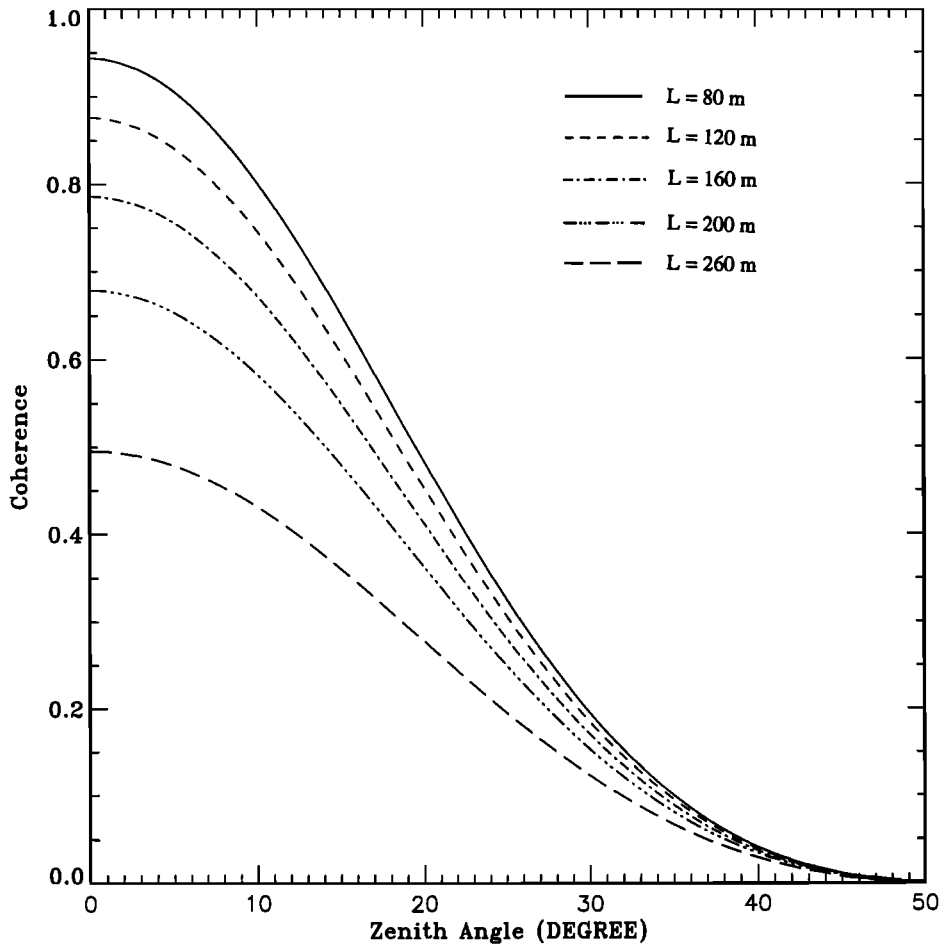


Figure 5. The variations of theoretical coherence with zenith angle for the cases of $L = 80, 120, 150, 180,$ and 260 m in accordance with (16), where the power law spectral model is assumed. The other parameters used for calculation are given as follows: radar beam width of 7.4° , altitude of 6 km, and frequency spacing of 0.35 MHz.

integration. It has been shown in the preceding section that for a more realistic irregularity spectral model, for example, a power law form, the contribution due to irregularity power spectrum to the FDI coherence can be ignored. In this case the magnitude of FDI coherence varies with Δk and L in accordance with the sinc function. Figure 6 shows the behaviors of such FDI coherence varied with frequency spacing Δf and layer thickness, where four cases of FDI coherence for the layer thickness of 50 m (solid line), 80 m (dotted line), 120 m (dashed line), and 150 m (dash-dotted line) given in the calculation of the coherence are presented. The other parameters used in Figure 6 are the same as those in Figure 5. As indicated, by increasing the

frequency spacing and layer thickness, the magnitude of the coherence becomes small.

Although the analytical expression of FDI coherence for the atmospheric layer with distinct boundaries has been derived, the problem remains as to how to estimate the layer thickness from $|S_{12}| = \sin(\Delta kL)/(\Delta kL)$, where $|S_{12}|$ is the observed coherence with a vertical radar beam at a given Δk . In view of its special property, it is not easy to evaluate directly the value of L from sinc function. However, making use of the quasi-bell-shaped behavior of the sinc function, this problem can be resolved as follows. It is easy to show that the -3 -dB width B_s of the sinc function $\sin(\Delta kL)/(\Delta kL)$ can be expressed in terms of L as

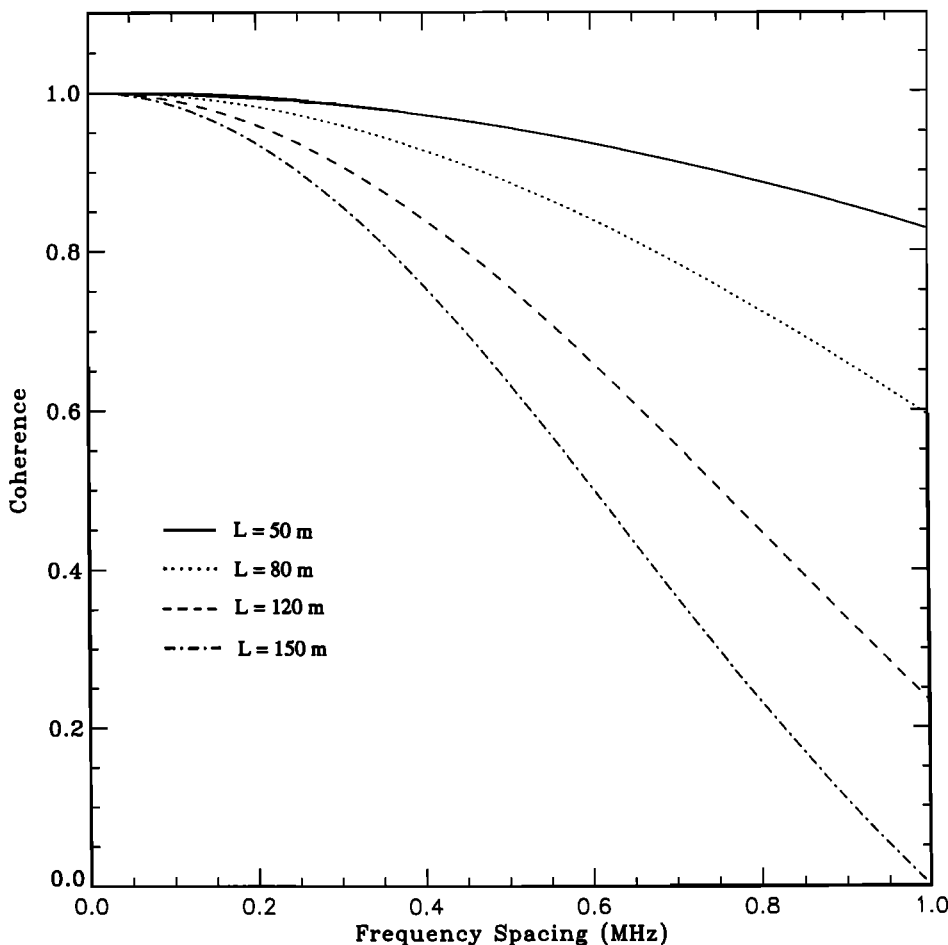


Figure 6. The variation of FDI coherence with frequency spacing in accordance with (16) for the cases of $L = 50, 80, 120,$ and 150 m. The parameters used for calculation are the same as those in Figure 5.

$$B_s = \frac{2.784}{L} \tag{23}$$

Similarly, the -3 -dB width B_g of the Gaussian function $\exp(-\sigma^2 \Delta k^2)$ can be written as

$$B_g = \frac{1.178}{\sigma} \tag{24}$$

The sinc function will approximate to Gaussian function within the region of -3 -dB width if the condition that $B_s = B_g$ is required. The comparison of the sinc function (solid line) with Gaussian function (dashed line) for the case of $L = 120$ m is shown in Figure 7, where the value of σ in the Gaussian function is given as $0.423L$. As indicated, these two

curves coincide very well in the frequency spacing region of smaller than 0.6 MHz. Therefore, with employing Gaussian function, the thickness of the sharp edged layer can be estimated unambiguously.

7. Comparison of Other Results

By considering the Gaussian probability distribution of the scatterers in an isolated layer, *Kudeki and Stitt* [1987], through their pioneering work on FDI theory, proposed a set of simple equations to estimate position and thickness of the layer. If an isolated layer with distinct edges is dealt with, and the probability distribution function $p(r)$ of the scatterers in this layer is assumed to be uniform, namely,

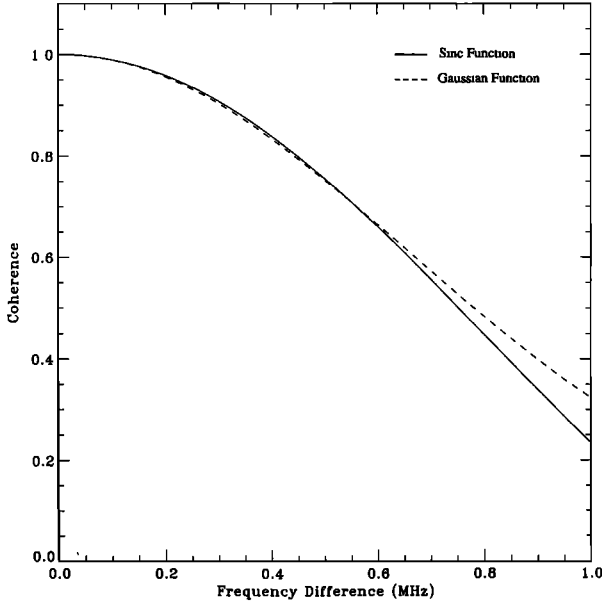


Figure 7. The comparison of the Gaussian function with the form of $\exp(-\Delta k^2 \sigma^2)$ and the sinc function with the expression of $\sin(\Delta kL)/(\Delta kL)$, where $\sigma = 0.423L$ and $L = 120$ m are given.

$$p(r) = \frac{1}{2L} \quad -L < r < L$$

$$p(r) = 0 \quad \text{elsewhere} \quad (25)$$

where r is the range of scatterer and $2L$ is the layer thickness, it is obvious that the FDI coherence will be different from that obtained by *Kudeki and Stitt* [1987]. Following *Kudeki and Stitt's* derivation, the FDI coherence can be expressed as

$$|S_{12}| = \frac{\sin(\Delta kL)}{\Delta kL} \quad (26)$$

This equation is exactly identical to (21), indicating that the same results can be achieved, although the approaches are different. In a recent paper, by including the range weighting and antenna beam pattern effects, *Franke* [1990] obtained a more complicated equation of the FDI coherence, that is,

$$|S_{12}| = \frac{\exp(-2\Delta k^2 \sigma_r^2)}{\sqrt{(1 + N_1/S_1)(1 + N_2/S_2)} \sqrt{1 + \Delta k^2 \sigma_r^4/z^2}} \quad (27)$$

where Δk is the wavenumber spacing ($= 2\pi\Delta f/c$, $\Delta f = f_1 - f_2$, where f_1 and f_2 are two operating frequencies with close spacing, and c is the speed of

light, z is the height; N_i and S_i are the noise and signal power, respectively, where the subscripts correspond to the different operating frequencies; $1/\sigma_r^2 = 1/\sigma_v^2 + 2/\sigma_l^2$, where $2\sigma_l$ is defined as the layer thickness, while σ_v is the second central moment of the range weighting function for backscatter power; and $\sigma_l = 0.425z\theta_6$, where θ_6 is the 6-dB width of the antenna beam. Note that if the transmitted pulse is rectangular with width τ and the receiver filter is well approximated by a Gaussian transfer function with a 6-dB bandwidth equal to $1/\tau$, then $\sigma_v = 0.35c\tau/2$. For an FDI experiment conducted by an MST radar with a relatively broad antenna beam width of 3° , if the frequency spacing of 0.5 MHz is set and the thickness of a layer located in the troposphere is assumed to be much smaller than the vertical extent of the radar volume, in this case the magnitude of $\Delta k^2 \sigma_r^4/z^2$ will be less than 1, and (27) can be written approximately as

$$|S_{12}| = \exp(-\sigma_l^2 \Delta k^2) \quad (28)$$

where the signal power has been assumed to be much greater than the noise power and the value of N/S is negligible. Once σ_l is calculated from given Δk and observed $|S_{12}|$ in accordance with (28), the layer thickness can then be determined in terms of σ_l . By doing so, the thickness of a layer weighted by the Gaussian function is $2\sigma_l$, however, the more pertinent formula for the estimation of the thickness of a layer having sharp edges will be $2.364\sigma_l$, as mentioned in the section 6.

8. Concluding Remarks

The analytical expressions of the normalized two-frequency cross correlation for the thin layer with sharp boundaries are derived in this article. It shows that the theoretical FDI coherences, as the function of the components of EM wave vector and the layer thickness following the sinc function, are related to the wavenumber power spectrum of the refractivity irregularities. In order to examine the extent of the effect of the irregularity power spectrum on the coherence, two spectral models, namely, Gaussian and power law, are employed. As a result of calculation, it is indicated that under the condition of $\Delta k/k_1 \ll 1$ the contribution of the wavenumber power spectrum of irregularities with the power law form can be reasonably ignored. However, for the other kind of spectral form, such as the Gaussian type, the effect of irregularity

power spectrum on the coherence may not be neglected. In addition, in view of the similar expressions of FDI coherence, identifying the echoing mechanism of MST radar, that is, turbulent scattering and Fresnel reflection, by using FDI technique seems unlikely. On the basis of derived FDI equations, the zenith angle dependence of FDI coherence is also investigated. It shows, in agreement with the observations, that FDI coherence decreases with the zenith angle of the radar beam. The problem of estimating the thickness of a sharp-edged layer from observed coherence and a given Δk by using a Gaussian function $\exp(-\sigma_1^2 \Delta k^2)$ is also discussed. It suggests that the pertinent formula for the thickness estimate of this kind of the layer is $2.364\sigma_1$, not $2\sigma_1$. Finally, by taking range weighting and antenna beam pattern effects into account, the generalization of the FDI theory developed in this article is in progress. Because it is impossible to obtain the analytical solution, the numerical computation and analysis are carried out. We hope that the analyzed results can be reported in the near future.

Acknowledgment. This work is supported partially by the National Science Council of Republic of China (ROC) under grant NSC82-0202-M008-056.

References

- Barat, J., Some characteristics of clear-air turbulence in the middle stratosphere, *J. Atmos. Sci.*, *39*, 2553–2564, 1982.
- Chu, Y. H., and S. J. Franke, A study of frequency coherence of stratospheric and tropospheric radar echoes made with Chung-Li VHF radar, *Geophys. Res. Lett.*, *18*, 1849–1852, 1991.
- Crain, C. M., Survey of airborne microwave refractometer measurements, *Proc. IRE*, *43*, 1405–1411, 1955.
- Doviak, R. J., and D. S. Zrnic, Reflection and scattering formula for anisotropically turbulent air, *Radio Sci.*, *19*, 325–336, 1984.
- Franke, S. J., Pulse compression and frequency domain interferometry with a frequency-hopped MST radar, *Radio Sci.*, *25*, 565–574, 1990.
- Franke, S. J., J. Rottger, C. LaHoz, and C. H. Liu, Frequency domain interferometry of polar mesosphere summer echoes with the EISCAT VHF radar, *Radio Sci.*, *27*, 417–428, 1992.
- Gage, K. S., and B. B. Balsley, On the scattering and reflection mechanisms contributing to clear air radar echoes from the troposphere, stratosphere, and mesosphere, *Radio Sci.*, *15*, 243–257, 1980.
- Gossard, E. E., Radar research on the atmospheric boundary layer, in *Radar in Meteorology*, edited by D. Atlas, 806 pp., Am. Meteorol. Soc., Boston, 1990.
- Gossard, E. E., and R. G. Strauch, *Radar Observation of Clear Air and Clouds*, 280 pp., Elsevier, New York, 1983.
- Kudeki, E., and G. R. Stitt, Frequency domain interferometry: A high-resolution radar technique for studies of atmospheric turbulence, *Geophys. Res. Lett.*, *14*, 198–201, 1987.
- Kudeki, E., and G. R. Stitt, Frequency domain interferometry studies of mesospheric layers at Jicamarca, *Radio Sci.*, *25*, 575–590, 1990.
- Ottersten, H., Atmospheric structure and radar backscattering in clear air, *Radio Sci.*, *4*, 1179–1193, 1969.
- Palmer, R. D., R. F. Woodman, S. Fukao, M. F. Larsen, M. Yamamoto, T. Tsuda, and S. Kato, Frequency domain interferometry observations of tropo/stratospheric scattering layers using the MU radar: Description and first results, *Geophys. Res. Lett.*, *17*, 2189–2192, 1990.
- Palmer, R. D., S. Fukao, M. F. Larsen, M. Yamamoto, T. Tsuda, and S. Kato, Oblique frequency domain interferometry measurements using the middle and upper radar, *Radio Sci.*, *27*, 713–720, 1992.
- Papoulis, A., *Probability, Random Variable, and Stochastic Process*, McGraw-Hill, New York, 1965.
- Stewart, R. W., Turbulence and waves in a stratified atmosphere, *Radio Sci.*, *4*, 1269–1278, 1969.
- Tatarskii, V. I., The effect of the turbulent atmosphere on wave propagation, *TT-68-50464*, 472 pp., U.S. Dep. of Commer., Washington, D.C., 1971.
- Woodman, R. F., and Y. H. Chu, Aspect sensitivity measurements of VHF backscatter made with the Chung-Li radar: Plausible mechanism, *Radio Sci.*, *24*, 113–125, 1989.

T.-Y. Chen, Institute of Electronics, National Chiao Tung University, Hsin-Chu, Taiwan.

Y.-H. Chu, Institute of Space Science, National Central University, Chung-Li, Taiwan.

(Received November 16, 1994; revised June 27, 1995; accepted July 3, 1995.)