



Reallocating multiple inputs and outputs of units to improve overall performance

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ABSTRACT

All decision-making units (DMUs) in the private or public sector are provided with a set of inputs of different values by their governing decision maker (GDM), and are required to generate a set of outputs. The GDM is able to reallocate the inputs/outputs among the DMUs to estimate the maximum absolute decision making efficiency of the sector. Serial models are presented to manage the interaction between two decision-making levels, GDM and DMUs, to provide the reallocated targets of inputs/outputs for DMUs in the next operating period. The 25 branches of a commercial bank in Taiwan are used as an illustration.

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1. Introduction

A set of performance indices is used to measure the efficiency of a group of decision-making units (DMUs) in the private or public sector. These DMUs operate under their governing decision maker (GDM), who has the power to allocate the resources and set targets for the individual DMUs. The relative efficiency of each DMU or the efficiency of the GDM may be evaluated to determine optimal practices with the available data of each DMU in the indices. Available literature measures the ‘relative decision-making efficiency’ of each DMU, for example, by using the data on all of the DMUs in the sector as a reference set. The conventional data envelopment analysis (DEA) would obtain a set of favorable weights from the indices and associate those with a target for improved efficiency to reduce the values of the inputs and increase values of the outputs [1–3]. The set of weights for each DMU represents the best course of measurement, among a collection of possible alternatives, en route to selecting the optimal approach. In this capacity, the set of weights serves to indicate *ex post facto* evaluations of the relative importance among the indices.

Centralized resource allocation models may also be used to obtain the set of weights from the indices for the GDM. Resource allocation problems arise when the GDM, which possesses authority, seeks to reallocate the inputs and outputs among the DMUs to maximize the ‘absolute decision-making efficiency’ of the sector. Our use of the terms ‘DMU’ and ‘GDM’ help emphasize our interest in the decision making by GDM and DMUs on different levels.

Thanassoulis and Dyson [4] combined goal programming (GP) and DEA to obtain the maximal interests of each DMU. Athanassopoulos [5] suggested another goal programming model based on DEA, in which the central decision maker, the GDM, considers the goal of the whole organization when determining global targets and the maximal contribution of each DMU. In a later study, Athanassopoulos [6] proposes another non-linear programming model that includes the restriction of the weights in the model.

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Golany et al. [7] proposed three models based on an additive DEA model [8]. They proposed suggestions concerning the allocation of resources in each DMU after considering the costs and benefits of the input/output. In addition, there are five stages related to the allocation of the resources. This model does not consider output targets, but only maps out the input resources of DMUs. Golany and Tamir [9] suggested an output-oriented model (maximum output) that considers input and output targets and resource allocation simultaneously. However, this model discusses a single output: each output index must be weighed subjectively before analyzing multiple output indices.

Beasley [10] utilizes the method of cross-efficiencies to propose a non-linear programming model that aims to maximize the average efficiency of DMUs, and also discusses the fixed allocation of costs and resource allocation of the inputs. Korhonen and Syrjänen [11] suggest a multi-objective linear programming model (MOLP) to perform the resource allocation. Fang and Zhang [12] propose a bicriteria DEA-based model that the GDM can search to find the preferred resource allocation solution, by exploring trade-offs between the total efficiency of the organization and the equity among the individual DMUs, according to the preference of the GDM. Golany [13] and Golany and Tamir [9] emphasized that resource reallocation is an important approach for improving overall performance.

Similar to the conventional radial-based DEA, the radial-based centralized resource allocation model is considered either input-oriented or output-oriented, depending on whether it is concerned with minimum consumption or maximum total output production, respectively. The model proposed by Lozano and Villa [14] can be considered a special case, with the common weights restrictions under the radial-based model. Lozano and Villa [15] also suggest three models, which discuss resource allocation when the number of DMUs decreases and the output remains unchanged. The first model addresses whether the DMUs should be deleted or retained for maximal efficiency. In this model, only the DMUs with high efficiency are selected. The second model addresses the number of the DMUs that should be reserved and resources that should be reallocated for maximum efficiency. The final model looks for the resource reallocation that minimizes the number of DMUs and maximizes the overall efficiency. Lozano et al. [16] propose a serial model that corresponds to three objectives that are pursued lexicographically to address the problem of emission permits. Asmild et al. [17] reconsider the centralized model proposed by Lozano and Villa [14] and suggest modifying it to consider only adjustments to previously inefficient DMUs, to stabilize the original efficient frontier.

Pachkova [18] considers the restrictions on reallocation. For example, access to resources can be restricted, or the resources can be extremely expensive, especially in the short run, so that moving production between individual DMUs becomes impossible. The organization may thus be unable to achieve full efficiency due to the existing limits on reallocation. The approach is a trade-off between the maximum allowed reallocation cost and the highest level of efficiency that the organization can achieve.

However, the efficiency of the radial-based model is not able to consider the slacks of inputs and outputs. For example, the efficiency score that is estimated by a radial-based model might be achieved with positive slacks. Liu and Tsai [19] propose a slacks-based centralized resource allocation model. By incorporating this model, the problem of a missing slack can be solved. Hosseinzadeh Lotfi et al. [20] proposed an enhanced Russell model that can be expressed as a non-radial centralized resource allocation.

Liu and Tsai [19] proposed [CSBM-CW] model which is used to maximize the aggregate efficiency score of the GDM. The two decision-making levels, the GDM and the DMUs under the GDM, would interpret the primal and dual solutions in different ways. The primal solution provides a set of reallocated values of inputs and outputs to those DMUs as targets to improve the performance of the GDM. Each DMU would then strive to achieve its deadline targets in the indices during the next operation period. By contrast, the dual solution is a set of common weights of inputs and outputs that is applied to all DMUs. The set of common weights indicates the relative importance among the inputs and outputs, regarding the performance of the GDM in the current period. Therefore, during the next period, DMUs are supposed to meet all their targets but may expend more effort on the indices with higher weights. Finally, several indices would have values beyond the targets. At the end of the next period, the set of common weights is used to measure the performance of DMUs in the following period. The GDM would then re-evaluate the aggregate score for the next period, and set new targets for DMUs in the following period.

We consider that certain inputs and outputs are uncontrollable, and their values cannot be altered because they owe their influence to certain congenital or acquired causes. For example, the total square footage of floor space in a bank is one of the performance indices used to assess a bank branch. However, it can be difficult to find another suitable location to achieve the desired square footage of floor space because a change in location directly influences other factors, such as sales. We thus introduce the general resource (re)allocation model [CSBM-G]. Therefore, the [CSBM-CW] model is a special case of the [CSBM-G] model, in which all input and output values can be altered.

The [CSBM-G] model provides a set of common weights for controllable inputs and outputs, and a favorable weight for each uncontrollable input or output. Furthermore, side constraints may be added to the [CSBM-G] model to limit the ranges of alteration in the desired inputs and outputs.

The remainder of the paper is arranged as follows. In the next section, we demonstrate our serial slacks-based centralized resource reallocation model, and discuss ways to reallocate the input resources to achieve optimal performance. We also discuss the restrictions affecting resource reallocation, as decision makers may set restrictions to each index in DMUs, to meet practical needs. In Section 3, the case of a commercial bank is analyzed. Lastly, Section 4 presents a discussion of other resource allocation models, and suggests follow-up studies.

2. Slacks-based centralized resource allocation models

We demonstrate serial slacks-based centralized resource allocation models that employ the idea of a radial-based centralized resource allocation model [14], and a slacks-based measure (SBM) [21]. The models are described in the following subsections.

2.1. [CSBM-CW] model

Liu and Tsai [19] proposed a slacks-based centralized resource allocation model called [CSBM-CW]. An organization could improve its overall performance by adjusting the m resources and s production of n DMUs under its governance. The GDM desires to use the same standard (weights) to adjust the modified targets of the DMUs. For DMU _{j} , the amount of input i consumed and quantity of output r produced are denoted as x_{ij} and y_{rj} , respectively.

The decision variables used in the centralized resource allocation model are listed below.

ρ	the aggregate efficiency score,
$q_{ik} (p_{rk})$	the slack of input i (output r) for projecting DMU _{k} ,
$q_i (p_r)$	total slack of input i (output r),
λ_{jk}	the linear combination weights of DMU _{j} when DMU _{k} changes its inputs and outputs.

(M1) [CSBM-CW]

$$\rho^{CW*} = \min \left[1 - (1/m) \sum_{i=1}^m \left(q_i / \sum_{k=1}^n x_{ik} \right) \right] / \left[1 - (1/s) \sum_{r=1}^s \left(p_r / \sum_{k=1}^n y_{rk} \right) \right], \tag{1.1}$$

$$\text{s.t. } \sum_{k=1}^n \sum_{j=1}^n x_{ij} \lambda_{jk} = \sum_{k=1}^n x_{ik} - q_i, \quad i = 1, \dots, m, \tag{1.2}$$

$$\sum_{k=1}^n \sum_{j=1}^n y_{rj} \lambda_{jk} = \sum_{k=1}^n y_{rk} + p_r, \quad r = 1, \dots, s, \tag{1.3}$$

$$\sum_{j=1}^n \lambda_{jk} = 1, \quad k = 1, \dots, n, \tag{1.4}$$

$$q_i \geq 0, \quad i = 1, \dots, m, \tag{1.5}$$

$$p_r \geq 0, \quad r = 1, \dots, s, \tag{1.6}$$

$$\lambda_{jk} \geq 0, \quad j = 1, \dots, n, \quad k = 1, \dots, n, \tag{1.7}$$

Let $Q_i = tq_i$, $P_r = tp_r$, and $A_{jk} = t\lambda_{jk}$, (M1) is further transferred into a linear programming model for computing.

(M2) [Computing C-SBM-CW]

$$\tau^{CW*} = \min t - (1/m) \sum_{i=1}^m \left(Q_i / \sum_{k=1}^n x_{ik} \right), \tag{2.1}$$

$$\text{s.t. } t + (1/s) \sum_{r=1}^s \left(P_r / \sum_{k=1}^n y_{rk} \right) = 1, \tag{2.2}$$

$$\sum_{k=1}^n \sum_{j=1}^n x_{ij} A_{jk} = t \sum_{k=1}^n x_{ik} - Q_i, \quad i = 1, \dots, m, \tag{2.3}$$

$$\sum_{k=1}^n \sum_{j=1}^n y_{rj} A_{jk} = t \sum_{k=1}^n y_{rk} - P_r, \quad r = 1, \dots, s, \tag{2.4}$$

$$\sum_{j=1}^n A_{jk} = t, \quad k = 1, \dots, n, \tag{2.5}$$

$$Q_i \geq 0, \quad i = 1, \dots, m, \tag{2.6}$$

$$P_r \geq 0, \quad r = 1, \dots, s, \tag{2.7}$$

$$A_{jk} \geq 0, \quad j = 1, \dots, n; \quad k = 1, \dots, n, \tag{2.8}$$

$$t > 0. \tag{2.9}$$

The optimal solutions for (M2), $(\tau^*, \tau^*, A_{jk}^*, Q_i^*, P_r^*)$, could be converted to the optimal solution for (M1) by the following equations:

$$\rho^* = \tau^*, \quad \lambda_{jk}^* = \Lambda_{jk}^*/t^*, \quad q_i^* = Q_i^*/t^*, \quad p_r^* = P_r^*/t^*.$$

The modified targets of the DMU_j in all indices are expressed by the following equations:

$$\tilde{x}_{ik} = \sum_{j=1}^n \lambda_{jk}^* x_{ij}, \quad i = 1, \dots, m, \quad k = 1, \dots, n,$$

$$\tilde{y}_{rk} = \sum_{j=1}^n \lambda_{jk}^* y_{rj}, \quad r = 1, \dots, s, \quad k = 1, \dots, n,$$

x_{ij} and y_{rj} are modified by the amounts q_{ik}^* and p_{rk}^* , respectively. $q_{ik}^* = x_{ik} - \sum_{j=1}^n \lambda_{jk}^* x_{ij}$, $p_{rk}^* = \sum_{j=1}^n \lambda_{jk}^* y_{rj} - y_{rk}$. q_{ik}^* and p_{rk}^* could be positive or negative.

We transferred (M2) into its dual model [22] by the dual model variables: ζ^{CW} , v_{ik} , u_{rk} , ξ_k^{CW} , a_i , and b_r to (2.2)–(2.7), respectively. The dual model of (M2) is shown as (M3).

(M3)

$$\max \zeta^{CW}, \tag{3.1}$$

$$\text{s.t. } \zeta^{CW} + \sum_{i=1}^m v_i \sum_{k=1}^n x_{ik} - \sum_{r=1}^s u_r \sum_{k=1}^n y_{rk} - \sum_{k=1}^n \xi_k^{CW} = 1, \tag{3.2}$$

$$\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} + \xi_k^{CW} \leq 0, \quad j = 1, \dots, n, \quad k = 1, \dots, n, \tag{3.3}$$

$$v_i \geq (1/m) \left(1 / \sum_{k=1}^n x_{ik} \right), \quad i = 1, \dots, m, \tag{3.4}$$

$$u_r \geq (\zeta^{CW}/s) \left(1 / \sum_{k=1}^n y_{rk} \right), \quad r = 1, \dots, s, \tag{3.5}$$

$X_i = \sum_{k=1}^n x_{ik}$, $i = 1, \dots, m$ and $Y_r = \sum_{k=1}^n y_{rk}$, $r = 1, \dots, s$ are seen as the inputs and outputs of a virtual DMU. Eq. (3.2) can be rewritten as

$$\zeta^{CW} = 1 - \sum_{i=1}^m v_i \sum_{k=1}^n x_{ik} + \sum_{r=1}^s u_r \sum_{k=1}^n y_{rk} + \sum_{k=1}^n \xi_k^{CW}.$$

Hence, (M3) is employed to search for the common set of weights that maximize the efficiency of a virtual DMU with functional weight restrictions. The [CSBM-CW] model enables the GDM to reallocate the resources of the sector with the concept of common weights, and to maximize the aggregate efficiency.

2.2. [CSBM-G] model

A general resource (re)allocation model is proposed, which is similar to the conventional SBM but different to the [CSBM-CW] model, in which each DMU, out of its favorable weight, maximizes the efficiency of the organization as a whole. However, the [CSBM-G] model is a special case of the above model. Practically, some performance indices could not be easily modified due to the influence of some congenital or acquired causes. c_x and c_y denote the sets of controllable inputs and outputs that can be modified.

(M4) [CSBM-G]

$$\rho^* = \min \left[1 - (1/m') \sum_{i \in c_x} \left(\sum_{k=1}^n q_{ik} / \sum_{k=1}^n x_{ik} \right) \right] / \left[1 + (1/s') \sum_{r \in c_y} \left(\sum_{k=1}^n p_{rk} / \sum_{k=1}^n y_{rk} \right) \right], \tag{4.1}$$

$$\text{s.t. } \sum_{j=1}^n x_{ij} \lambda_{jk} = x_{ik} - q_{ik}, \quad i \in c_x, \quad k = 1, \dots, n, \tag{4.2a}$$

$$\sum_{j=1}^n x_{ij} \lambda_{jk} = x_{ik}, \quad i \notin c_x, \quad k = 1, \dots, n, \tag{4.2b}$$

$$\sum_{j=1}^n y_{rj} \lambda_{jk} = y_{rk} + p_{rk}, \quad r \in c_y, \quad k = 1, \dots, n, \tag{4.3a}$$

$$\sum_{j=1}^n y_{rj} \lambda_{jk} = y_{rk}, \quad r \notin c_y, \quad k = 1, \dots, n, \tag{4.3b}$$

$$\sum_{j=1}^n \lambda_{jk} = 1, \quad k = 1, \dots, n, \quad (4.4)$$

$$\sum_{k=1}^n q_{ik} \geq 0, \quad i \in c_x, \quad (4.5)$$

$$\sum_{k=1}^n p_{rk} \geq 0, \quad r \in c_y, \quad (4.6)$$

$$\lambda_{jk} \geq 0, \quad j = 1, \dots, n, \quad k = 1, \dots, n, \quad (4.7)$$

$$q_{ik}, p_{rk} \text{ free in sign, } i \in c_x, \quad r \in c_y, \quad k = 1, \dots, n. \quad (4.8)$$

m' and s' are the number of controllable inputs and outputs, denoted by $m' = |c_x|$ and $s' = |c_y|$.

In contrast to (1.1), $\sum_{k=1}^n q_{ik}$ and $\sum_{k=1}^n p_{rk}$ are the sum of i th input reductions and the sum of r th output increases, respectively. For the entire organization, the proportions of i th input reductions and r th output increases are $\sum_{k=1}^n q_{ik} / \sum_{k=1}^n x_{ik}$ and $\sum_{k=1}^n p_{rk} / \sum_{k=1}^n y_{rk}$ and $(1/s') \sum_{r \in c_y} (\sum_{k=1}^n p_{rk} / \sum_{k=1}^n y_{rk})$ are the average proportions of each controllable input reduction and each controllable output increase separately. Hence, the numerator and denominator in (4.1) are the reduced percentage of the total inputs and the increased percentage of the total outputs, respectively. The proportion of the numerator and denominator is the aggregate efficiency score. In other words, the score representing the greatest efficiency is 100%. If the average improvement proportion of inputs and outputs in an organization is 0, then the usage of its inputs and outputs is efficient. Therefore, (4.1) is interpreted as the aggregate preference of the sector, and the efficient score does not exceed 1.

To find the projection for each DMU $_k$, $k = 1, 2, \dots, n$, the weights of n DMUs are $\lambda_{1k}, \lambda_{2k}, \dots, \lambda_{nk}$ as (4.2a) and (4.3b). Considering DMU $_k$, $k = 1, \dots, n$, (4.2a) indicates the modifications for i th controllable input of DMU $_j$. It is equal to the weighted sum of i th controllable input for total DMUs, the same as the linear combination of total DMUs when λ_{jk} are the weights of DMU $_j$, where $j = 1, \dots, n$. Similarly, (4.3a) expresses the modifications of the r th controllable output of DMU $_j$, which is equal to the weighted sum of r th controllable output of total DMUs. Eqs. (4.2b) and (4.3b) are the constraints for the uncontrollable indices. Eq. (4.4) is the constraint for the sum of weights λ_{jk} to one, $j = 1, \dots, n$. This leads to a variable returns-to-scale (VRS) characterization [2].

To reallocate the resources, each controllable input i and controllable output j of each DMU can be increased or reduced arbitrarily. There is no restriction to the improvement of inputs and outputs for each DMU. However, we consider the sector in its entirety, and expect that the aggregate efficiency will improve. The total improvements of the inputs and outputs should be positive, as (4.5) and (4.6).

2.3. Linearization and duality of the [CSBM-G] model

To solve for the [CSBM-G] model, we multiply a scalar variable t (>0) to the numerator and denominator separately, and allow the term of the denominator to equal 1. As with the [CSBM-CW] model, the [CSBM-G] model is further transferred into a linear programming model for computing, as shown in (M5).

(M5) [Computing C-SBM]

$$\tau^* = \min t + (1/s') \sum_{r \in c_y} \left(\sum_{k=1}^n Q_{rk} / \sum_{k=1}^n y_{rk} \right) = 1, \quad (5.1)$$

$$\text{s.t. } t + (1/s') \sum_{r \in c_y} \left(\sum_{k=1}^n P_{rk} / \sum_{k=1}^n y_{rk} \right) = 1, \quad (5.2)$$

$$\sum_{j=1}^n x_{ij} A_{jk} = tx_{ik} - Q_{ik}, \quad i \in c_x, \quad k = 1, \dots, n, \quad (5.3a)$$

$$\sum_{j=1}^n x_{ij} A_{jk} = tx_{ik}, \quad i \notin c_x, \quad k = 1, \dots, n, \quad (5.3b)$$

$$\sum_{j=1}^n y_{rj} A_{jk} = ty_{rk} + P_{rk}, \quad r \in c_y, \quad k = 1, \dots, n, \quad (5.4a)$$

$$\sum_{j=1}^n y_{rj} A_{jk} = ty_{rk}, \quad r \in c_y, \quad k = 1, \dots, n, \quad (5.4b)$$

$$\sum_{j=1}^n A_{jk} = t, \quad k = 1, \dots, n, \tag{5.5}$$

$$\sum_{k=1}^n Q_{ik} \geq 0, \quad i \in c_x, \tag{5.6}$$

$$\sum_{k=1}^n P_{rk} \geq 0, \quad r \in c_y, \tag{5.7}$$

$$A_{jk} \geq 0, \quad j = 1, \dots, n, \quad k = 1, \dots, n, \tag{5.8}$$

$$t > 0, \tag{5.9}$$

$$Q_{ik}, P_{rk} \text{ free in sign, } i \in c_x, \quad r \in c_y, \quad k = 1, \dots, n. \tag{5.10}$$

We can acquire the optimal solutions for the [CSBM-G] model by the optimal solutions of (M5), as in the [CSBM-CW] model.

According to Eqs. (4.2a) and (4.3a), with respect to DMU_k, the modified targets of DMU_j in all indices are expressed by the following equations:

$$\tilde{x}_{ik} = \sum_{j=1}^n \lambda_{jk}^* x_{ij} = x_{ik} - q_{ik}^*, \quad i \in c_x, \quad k = 1, \dots, n,$$

$$\tilde{y}_{rk} = \sum_{j=1}^n \lambda_{jk}^* y_{rj} = y_{rk} + p_{rk}^*, \quad r \in c_y, \quad k = 1, \dots, n,$$

$$\tilde{x}_{ik} = x_{ik}, \quad i \notin c_x, \quad k = 1, \dots, n, \quad \tilde{y}_{rk} = y_{rk}, \quad r \notin c_y, \quad k = 1, \dots, n.$$

In contrast to the [CSBM-CW] model, we can derive each of the DMU_k's improvement of inputs and outputs directly from the model q_{ik}^* and $q_{rk}^* \cdot p_{rk}^*$, and could be positive or negative. The total modifications of the organization in controllable inputs and outputs are computed by the following equations:

$$q_i^* = \sum_{k=1}^n q_{ik}^*, \quad i \in c_x, \quad p_r^* = \sum_{k=1}^n p_{rk}^*, \quad r \in c_y.$$

(M5) is transferred into its dual model [22] by the dual model variables: ζ , v_{ik} , u_{rk} , ζ_k , a_i , and b_r , respectively, to (5.2), (5.3a), (5.3b), (5.5)–(5.7). The dual model of (M5) is shown as (M6).

(M6)

$$\max \zeta, \tag{6.1}$$

$$\text{s.t. } \zeta + \sum_{i=1}^m \sum_{k=1}^n v_{ik} x_{ik} - \sum_{r=1}^s \sum_{k=1}^n u_{rk} y_{rk} - \sum_{k=1}^n \zeta_k = 1, \tag{6.2}$$

$$- \sum_{i=1}^m v_{ik} x_{ij} + \sum_{r=1}^s u_{rk} y_{rj} - \zeta_k \leq 0, \quad j = 1, \dots, n, \quad k = 1, \dots, n, \tag{6.3}$$

$$v_{ik} = (1/m') \left(1 / \sum_{k'=1}^n x_{ik'} \right) + a_i, \quad i \in c_x, \quad k = 1, \dots, n, \tag{6.4}$$

$$u_{rk} = (\zeta/s') \left(1 / \sum_{k'=1}^n y_{rk'} \right) + b_r, \quad r \in c_y, \quad k = 1, \dots, n, \tag{6.5}$$

$$a_i \geq 0, \quad i \in c_x, \tag{6.6}$$

$$b_r \geq 0, \quad r \in c_y. \tag{6.7}$$

The dual variables v_{ik} and u_{rk} can be interpreted as the multiplier (i.e., cost/price) assigned to the i th input and the r th output, respectively. In other words, v_{ik} and u_{rk} can also be seen as the weights of i th input and r th output, for evaluating the efficiency of DMU_k. ζ_k is the scalar associated with (5.5), the VRS auxiliary variable for DMU_k.

(6.3) can be rewritten as $(\sum_{r=1}^s u_{rk} y_{rj} + \zeta_k) / \sum_{i=1}^m v_{ik} x_{ij} \leq 1, j, k = 1, \dots, n$. The numerator is the sum of the virtual price and the scalar of VRS. The denominator is the sum of the virtual cost. The ratio is the efficiency score of DMU_j with respect to DMU_k. The efficiency score for all DMUs does not exceed 1. The sets of constraints for v_{ik} and u_{rk} , (6.4) and (6.5) restrict the feasible v_{ik} and u_{rk} to semi-positive. The conventional radial-based DEA models, CCR [1] and BCC [2], restrict the indices' weights by $v_{ik} \geq \varepsilon > 0$ and $u_{rk} \geq \varepsilon > 0$ as evaluating the object DMU_k (decision-making unit), where ε is a non-Archimedean infinitesimal positive constant.

In (6.1) and (6.2), the value of the aggregate profit, $\sum_{r=1}^s \sum_{k=1}^n u_{rk} y_{rk} - \sum_{i=1}^m \sum_{k=1}^n v_{ik} x_{ik}$, plus the sum of the scalars, $\sum_{k=1}^n \zeta_k$, are maximized. The value of $\sum_{k=1}^n \zeta_k$ could be greater, equal to, or lesser than 0, respectively, indicating that the total return-to-scale is either increasing, constant, or decreasing.

We compare the [CSBM-G] model with the conventional slacks-based DEA models [SBM-V] [3,21]. The [SBM-V] model is used to find the projection of each DMU to improve the individual efficiency; and the average improvement proportion of inputs and outputs is seen as the efficiency score of the evaluated DMU. In our proposed [CSBM-G] model, we can consider n DMUs at the same time, in an aggregated model. The [SBM-V] model is used to analyze the relative performance of each DMU, and to set improved targets for each DMU separately. However, there are situations in which all of the DMUs are under the same organization, and the GDM has an interest in maximizing the efficiency of the individual DMUs at same time. The [CSBM-G] model is concerned with the overall performance of all DMUs by their total inputs and total outputs, instead of by their separate performance.

The [CSBM-CW] model provides outstanding rules for managers to collectively manage the target for improvement of each DMU. Using the [CSBM-CW] model to adjust the resources allocated to each DMU is easier and more appropriate than using the [CSBM-G] model. By linearly combining the n constraints of (4.2a) and (4.3a) separately, the linear combinations of (1.2) and (1.3) are produced. Let $V_i = v_{ik}$ for $\forall k$ and $U_r = u_{rk}$ for $\forall k$ in (M6). (M3) is the special case of (M6).

The [CSBM-CW] model does not consider the set of uncontrollable inputs and outputs. Hence, the proposed [CSBM-G] model for evaluating the performance of DMUs provides the common weights for controllable inputs and outputs, and favorable weights for uncontrollable inputs and outputs. The DMUs not only strive to achieve the targets, but also consider the common weights of each controllable set of inputs and outputs, to improve the specific inputs and outputs that require more weight to achieve greater efficiency. In practice, the model is more suitable in allowing the GDM to manage controllable inputs and outputs univately. In the following section, we evaluate the performance and the (re)allocated resources of 25 branches of a commercial bank in northern Taiwan.

3. Resource allocation problems of a commercial bank

In the case of the commercial bank, the district manager controls resource adjustments and reallocation in the branches. The four input indices are the number of employees, the operating costs (tens of thousands of dollars/year), the rental costs (monthly), and the number of ATMs, denoted as $x_1, x_2, x_3,$ and $x_4,$ respectively. The five output indices are the business transactions in a branch (monthly), the amount of money drawn from ATMs (monthly), the amount of savings, the amount of credit (tens of thousands of dollars/year), and the operating income (tens of thousands of dollars/year), denoted as $y_1, y_2, y_3, y_4,$ and $y_5,$ respectively.

The following information from the commercial bank in Taiwan is the statistical data from the first financial quarter of 2007 (Table 1). Due to issues of confidentiality, details will not be shown. As the model possesses a unit invariance property, and the data of inputs and outputs are in different units, the data of each input and output are divided by its maximal values. Therefore, we can obtain the weights for providing the managing standing.

For these branches, it is difficult to perform resource adjustment and reallocation on rental costs and the number of ATMs in the short term. Rental costs are determined by the locations of the branches, and it is not easy to change the location and size of the branches. Similarly, the number of ATMs cannot be changed easily. Therefore, in this case, $x_1, x_2 \in C_x$ and $y_1, y_2, y_3, y_4, y_5 \in C_y.$

The [CSBM-G] model is suitable if the GDM wants to control the controllable index with common rules for managing DMUs. To obtain the reallocation improvement, we can use unified data in the model and multiply the results by the maximum value of each input and output to restore data. The details of reallocation improvements of the [C-SBM-M] model are shown in (Table 2). The improvements to each DMU are obtained. The operating cost (x_2) can decrease 7203.37 (tens of thousands of dollars a year), and $y_1, y_2, y_4,$ and y_5 can increase to 162,687.30, 8160.23, 336,026.48, and 4,719.65, respectively. The weights for each of the DMUs are listed in (Table 3). There are two columns in the table typed in boldface numbers. That are showing DMUs may possess different weights in the two indices. In the other columns, all the DMUs have same weight. While each DMU is striving to achieve its targets, it also considers the weights of each input and output, and pays more attention to improving its number of employees (x_1), the amount of saving (y_3), and its operating costs (x_2) to obtain greater efficiency in the next period. The favorable weights of each DMU are different for x_3 and $x_4.$

(Table 4) refers to the improved percentage of the inputs and outputs of the DMUs. In regard to the input indices, q_{ik}^*/x_{ij} , the positive percentage means that the input decreases in the DMU. In contrast, the negative percentage denotes the increased input. In regard to the output indices, p_{rk}^*/y_{rj} , the positive percentage refers to increased output, and the negative percentage indicates decreased output. As per (Table 4), the percentages of DMU₈, DMU₉, DMU₁₃, and DMU₁₅ on y_1 are significantly high, particularly DMU₁₃ and DMU₁₅. For these four branches, it may be difficult to increase the operating target twofold. Hence,

Table 1
The statistical data of the inputs and outputs of 25 branches.

	x_1	x_2	x_3	x_4	y_1	y_2	y_3	y_4	y_5
Total	856	275,906	19,103,275	125	247,617	288,865	16,871,595	14,390,840	508,274
Ave.	34	11,036	764,131	5	9905	11,555	67,4864	575,634	20,331
Med.	32	9831	525,000	5	8360	10,465	608,657	507,260	17,770
Std.	11	4501	564,830	2	4966	44,388	258,598	352,252	9577
Min.	21	7018	40,000	2	5500	3874	396,164	236,765	11,420
Max.	69	26,437	2,323,000	8	31,451	23,622	1,499,762	1,712,440	50,452

Table 2
The improvements of [CSBM-G].

DMU _j	q _{1j}	q _{2j}	p _{1j}	p _{2j}	p _{3j}	p _{4j}	p _{5j}
4	10.69	1232.92	112.81	-1805.15	-191,658.16	48,726.93	226.47
5	-1.65	640.26	7840.51	2665.59	26,778.56	26,247.42	1099.41
7	-6.96	-2574.41	4138.21	-25.68	349,351.72	161,063.00	7842.04
8	8.56	3563.89	14,617.41	-997.05	-125,429.07	-306,292.37	-8273.87
9	-3.45	619.11	19,587.12	356.09	-59,726.73	151,856.31	244.04
12	-6.08	-1580.08	5360.58	2592.79	122,769.20	231,543.37	4294.31
13	-7.96	-586.66	22,281.36	-3264.31	89,393.79	190,865.57	4457.20
14	-6.08	-1580.08	5360.58	2592.79	122,769.20	231,543.37	4294.31
15	-4.17	1714.84	23,409.10	4145.92	-79,012.19	704.98	-611.94
17	4.19	2770.04	15,606.31	124.72	-173,681.22	-15206.34	-3037.50
18	-2.03	367.09	8998.94	1599.13	99,493.06	-14,057.28	1983.78
20	0.17	-172.17	9474.88	1006.25	41,629.00	-17,340.71	606.75
21	5.73	1510.80	11,615.16	-810.08	-229,436.37	42,647.92	-1984.28
22	2.38	-1427.13	308.75	2430.50	195,699.25	-192,304.63	-824.38
24	6.69	2704.94	13,975.63	-2451.25	-188,940.04	-203,971.06	-5596.69
Total	0.00	7203.37	162,687.30	8160.23	0.00	336,026.48	4719.65

Note: There are no improvement in DMU₁, DMU₂, DMU₃, DMU₆, DMU₁₀, DMU₁₁, DMU₁₆, DMU₁₉, DMU₂₃, and DMU₂₅.

Table 3
The weights of DMUs in [CSBM-G] (M5).

DMU _j	v [*] _{1j}	v [*] _{2j}	v [*] _{3j}	v [*] _{4j}	u [*] _{1j}	u [*] _{2j}	u [*] _{3j}	u [*] _{4j}	u [*] _{5j}
1	0.1040	0.0479	0.1391	0.0285	0.0219	0.0141	0.0643	0.0205	0.0171
2	0.1040	0.0479	0.0274	0.0210	0.0219	0.0141	0.0643	0.0205	0.0171
3	0.1040	0.0479	0.3452	0.3873	0.0219	0.0141	0.0643	0.0205	0.0171
4	0.1040	0.0479	0.0333	0.0002	0.0219	0.0141	0.0643	0.0205	0.0171
5	0.1040	0.0479	0.0230	0.0230	0.0219	0.0141	0.0643	0.0205	0.0171
6	0.1040	0.0479	0.0333	0.0002	0.0219	0.0141	0.0643	0.0205	0.0171
7	0.1040	0.0479	0.0274	0.0210	0.0219	0.0141	0.0643	0.0205	0.0171
8	0.1040	0.0479	0.0274	0.0210	0.0219	0.0141	0.0643	0.0205	0.0171
9	0.1040	0.0479	0.0333	0.0002	0.0219	0.0141	0.0643	0.0205	0.0171
10	0.1040	0.0479	0.0274	0.0210	0.0219	0.0141	0.0643	0.0205	0.0171
11	0.1040	0.0479	0.0333	0.0002	0.0219	0.0141	0.0643	0.0205	0.0171
12	0.1040	0.0479	0.0333	0.0002	0.0219	0.0141	0.0643	0.0205	0.0171
13	0.1040	0.0479	0.0333	0.0002	0.0219	0.0141	0.0643	0.0205	0.0171
14	0.1040	0.0479	0.0333	0.0002	0.0219	0.0141	0.0643	0.0205	0.0171
15	0.1040	0.0479	0.0333	0.0002	0.0219	0.0141	0.0643	0.0205	0.0171
16	0.1040	0.0479	0.0230	0.0230	0.0219	0.0141	0.0643	0.0205	0.0171
17	0.1040	0.0479	0.1391	0.0285	0.0219	0.0141	0.0643	0.0205	0.0171
18	0.1040	0.0479	0.0230	0.0230	0.0219	0.0141	0.0643	0.0205	0.0171
19	0.1040	0.0479	0.0056	0.0284	0.0219	0.0141	0.0643	0.0205	0.0171
20	0.1040	0.0479	0.1391	0.0285	0.0219	0.0141	0.0643	0.0205	0.0171
21	0.1040	0.0479	0.0056	0.0284	0.0219	0.0141	0.0643	0.0205	0.0171
22	0.1040	0.0479	0.0230	0.0230	0.0219	0.0141	0.0643	0.0205	0.0171
23	0.1040	0.0479	0.0230	0.0230	0.0219	0.0141	0.0643	0.0205	0.0171
24	0.1040	0.0479	0.0230	0.0230	0.0219	0.0141	0.0643	0.0205	0.0171
25	0.1040	0.0479	0.0000	0.0000	0.0219	0.0141	0.0643	0.0205	0.0171

the GDM can add the possible limitations to the linear programming model to obtain an applicable plan for allocating the level of each input and output. For the DMU_k, g₁ and g₂ are the sets of the input $iq_{ik}^* \geq 0$ and $q_{ik}^* < 0$, respectively, and g₃ and g₄ are the sets of the output $rp_{rk}^* \geq 0$ and $p_{rk}^* < 0$, respectively. The regulated model can be rewritten as (M7).

(M7)

$$\tilde{p}^* = \text{Minimize } 1 - (1/m') \sum_{i \in C_x} \left(\frac{\sum_{k=1}^n \tilde{q}_{ik}}{\sum_{k=1}^n x_{ik}} \right) / 1 + (1/s') \sum_{r \in C_y} \left(\frac{\sum_{k=1}^n \tilde{p}_{rk}}{\sum_{k=1}^n y_{rk}} \right), \tag{7.1}$$

$$\text{s.t. } \sum_{j=1}^n x_{ij} \lambda_{jk} = x_{ik} - \tilde{q}_{ik}, \quad i \in C_x, k = 1, \dots, n, \tag{7.2a}$$

$$\sum_{j=1}^n y_{rj} \lambda_{jk} = y_{rk} - \tilde{q}_{rk}, \quad r \in C_y, k = 1, \dots, n, \tag{7.3a}$$

Table 4
The improved percentage of the inputs/outputs in [CSBM-G] (M5).

DMU _j	q _{1j} (%)	q _{2j} (%)	p _{1j} (%)	p _{2j} (%)	p _{3j} (%)	p _{4j} (%)	p _{5j} (%)
4	20.77	9.03	0.88	-10.27	-19.73	6.53	0.93
5	-6.26	6.85	124.66	45.86	4.93	8.38	8.11
7	-16.91	-19.84	36.55	-0.21	44.46	27.77	31.96
8	17.40	22.75	160.78	-6.76	-14.34	-31.13	-26.49
9	-11.18	6.28	216.26	2.38	-9.30	47.04	1.44
12	-17.73	-13.53	68.29	23.52	20.02	36.24	21.13
13	-30.26	-6.73	264.30	-15.99	18.15	70.49	34.13
14	-17.73	-13.53	68.29	23.52	20.02	36.24	21.13
15	-14.57	16.95	269.70	37.82	-12.69	0.18	-3.80
17	11.44	24.09	149.97	0.95	-23.58	-3.72	-16.05
18	-5.92	3.45	62.70	15.32	16.72	-2.93	12.13
20	0.54	-2.02	99.11	10.29	8.27	-4.15	3.98
21	13.91	13.70	102.59	-4.66	-25.75	9.10	-9.35
22	7.16	-17.37	3.27	54.86	43.20	-31.65	-4.64
24	17.20	24.06	107.65	-15.12	-25.09	-33.75	-26.28

Note: There are no improvement in DMU₁, DMU₂, DMU₃, DMU₆, DMU₁₀, DMU₁₁, DMU₁₆, DMU₁₉, DMU₂₃, and DMU₂₅.

Table 5
The weights of DMUs in the regulated [CSBM-G] (M7).

DMU _j	v _{1j}	v _{2j}	v _{3j}	v _{4j}	u _{1j}	u _{2j}	u _{3j}	u _{4j}	u _{5j}
1	0.0884	0.0512	0.1262	0.0332	0.0225	0.0145	0.0552	0.0211	0.0176
2	0.0884	0.0512	0.0264	0.0241	0.0225	0.0145	0.0552	0.0211	0.0176
3	0.0884	0.0512	0.3212	0.3595	0.0225	0.0145	0.0552	0.0211	0.0176
4	0.0884	0.0512	0.0340	0.0027	0.0225	0.0145	0.0552	0.0211	0.0176
5	0.0884	0.0512	0.0172	0.0282	0.0225	0.0145	0.0552	0.0211	0.0176
6	0.0884	0.0512	0.0340	0.0027	0.0225	0.0145	0.0552	0.0211	0.0176
7	0.0884	0.0512	0.0264	0.0241	0.0225	0.0145	0.0552	0.0211	0.0176
8	0.0884	0.0512	0.0227	0.0179	0.0161	0.0145	0.0552	0.0211	0.0176
9	0.0884	0.0512	0.0188	0.0100	0.0111	0.0145	0.0552	0.0211	0.0176
10	0.0884	0.0512	0.0264	0.0241	0.0225	0.0145	0.0552	0.0211	0.0176
11	0.0884	0.0512	0.0340	0.0027	0.0225	0.0145	0.0552	0.0211	0.0176
12	0.0884	0.0512	0.0340	0.0027	0.0225	0.0145	0.0552	0.0211	0.0176
13	0.0884	0.0512	0.0078	0.0058	0.0082	0.0145	0.0552	0.0211	0.0176
14	0.0884	0.0512	0.0340	0.0027	0.0225	0.0145	0.0552	0.0211	0.0176
15	0.0884	0.0512	0.0078	0.0058	0.0082	0.0145	0.0552	0.0211	0.0176
16	0.0884	0.0512	0.0172	0.0282	0.0225	0.0145	0.0552	0.0211	0.0176
17	0.0884	0.0512	0.1262	0.0332	0.0225	0.0145	0.0552	0.0211	0.0176
18	0.0884	0.0512	0.0172	0.0282	0.0225	0.0145	0.0552	0.0211	0.0176
19	0.0884	0.0512	0.0002	0.0380	0.0225	0.0145	0.0552	0.0211	0.0176
20	0.0884	0.0512	0.1262	0.0332	0.0225	0.0145	0.0552	0.0211	0.0176
21	0.0884	0.0512	0.0002	0.0380	0.0225	0.0145	0.0552	0.0211	0.0176
22	0.0884	0.0512	0.0172	0.0282	0.0225	0.0145	0.0552	0.0211	0.0176
23	0.0884	0.0512	0.0172	0.0282	0.0225	0.0145	0.0552	0.0211	0.0176
24	0.0884	0.0512	0.0172	0.0282	0.0225	0.0145	0.0552	0.0211	0.0176
25	0.0884	0.0512	0.0000	0.0000	0.0225	0.0145	0.0552	0.0211	0.0176

(4.2b), (4.3b) and (4.4),

(7.2b), (7.3b), and (7.4)

$$\sum_{k=1}^n \tilde{q}_{ik} \geq 0, \quad i \in c_x, \tag{7.5}$$

$$\sum_{k=1}^n \tilde{p}_{rk} \geq 0, \quad r \in c_y, \tag{7.6}$$

$$\tilde{q}_{ik}/x_{ij} \leq \alpha_{ik}, \quad i \in g_1, \quad \forall j, k, j = k, \tag{7.7}$$

$$-\tilde{q}_{ik}/x_{ij} \leq \alpha_{ik}, \quad i \in g_2, \quad \forall j, k, j = k, \tag{7.8}$$

$$-\tilde{p}_{rk}/y_{ij} \leq \beta_{ik}, \quad r \in g_3, \quad \forall j, k, j = k, \tag{7.9}$$

$$-\tilde{p}_{rk}/y_{ij} \leq \beta_{ik}, \quad r \in g_4, \quad \forall j, k, j = k, \tag{7.10}$$

$$\lambda_{jk} \geq 0, \quad \forall j, k. \tag{7.11}$$

Table 6
The applicable improvements of the regulated [CSBM-G] (M7).

DMU _j	q _{1j} (%)	q _{2j} (%)	p _{1j} (%)	p _{2j} (%)	p _{3j} (%)	p _{4j} (%)	p _{5j} (%)
2	18.61	22.48	34.01	-0.79	-35.56	-11.08	-33.79
4	20.66	8.98	0.88	-10.22	-19.64	6.50	0.93
5	-6.23	6.82	124.05	45.63	4.90	8.34	8.07
6	4.97	7.15	2.68	-7.69	-6.97	-6.72	-8.10
7	2.64	4.90	72.42	-0.87	-4.07	14.51	-10.08
8	15.47	19.83	150.00	-7.21	-8.43	-29.88	-21.68
9	-10.51	0.59	150.00	3.60	-0.86	54.48	12.53
12	-55.18	-77.65	49.25	105.46	85.35	106.19	108.54
13	-16.12	0.01	150.00	-20.61	13.49	41.71	19.06
14	-55.18	-77.65	49.25	105.46	85.35	106.19	108.54
15	-6.65	11.86	150.00	30.71	-6.14	-3.39	-2.81
17	11.38	23.98	149.24	0.94	-23.47	-3.70	-15.97
18	-5.89	3.43	62.40	15.24	16.64	-2.91	12.07
20	0.54	-2.01	98.63	10.24	8.23	-4.13	3.96
21	13.84	13.63	102.10	-4.63	-25.62	9.06	-9.31
22	7.13	-17.28	3.26	54.60	42.99	-31.50	-4.61
24	17.12	23.94	107.13	-15.05	-24.97	-33.59	-26.15

Note: There are no improvement in DMU₁, DMU₃, DMU₁₀, DMU₁₁, DMU₁₆, DMU₁₉, DMU₂₃, and DMU₂₅.

For example, the GDM can set the parameter as β_{1k} = 1.5 for DMUs, k = 8, 9, 13, 15 in model (M7), and then obtain the regulated optimal solution (ρ̃*, q̃_{ij}*, p̃_{rj}*, x̃_{ik}*). One may examine the results of model (M7) to ensure they are realistic improvement targets. Several β_{ik} setting may be needed for testing to select the most feasible one.

The regulated outcomes are shown in (Table 5) and (Table 6). DMU₂ and DMU₆ will change their indices to achieve the maximum aggregate efficiency with the added constraints. It was found that the revised consequences are more applicable. Those bounds may preclude the projection onto the efficient frontier (i.e. the targets computed may not be efficient).

4. Conclusion and discussion

The [CSBM-CW] and the [CSBM-G] models are introduced to solve the resource (re)allocation problems by maximizing the aggregated efficiency score of the GDM. The solutions are associated with reallocated values of inputs and outputs for those DMUs in the next operation period. The general model [CSBM-G] can handle uncontrollable inputs and outputs for the practical problems.

The values for each input and output index can be unified and still retain the same differentiations among the DMUs, as the [CSBM-CW] model and the [CSBM-G] model preserve the property of units invariant. The obtained dual solutions of the two models are the relative weights among the inputs and outputs, and could be interpreted directly, as the original data were unified. A higher weight indicates that the index possesses a higher influence on performance. The DMU, while striving to achieve its targets, also attempts to achieve greater efficiency during the next period by considering the weight of each set of inputs and outputs, to improve the specific inputs and outputs that require more weight.

The objective functions of current [CSBM-CW] and [CSBM-G] models indicate that the influence of both input and output indices are considered for performance measurement. In situations where either input-oriented or output-oriented is considered, the following models, [CSBM-I] and [CSBM-O], should be used.

(M8) [CSBM -I]

$$\rho_I^* = \min 1 - (1/m') \sum_{i=1}^{m'} \left(\frac{\sum_{k=1}^n q_{ik}}{\sum_{k=1}^n x_{ik}} \right), \tag{8.1}$$

s.t. (4.2a) and (4.8).

(M9) [C-SBM-O]

$$\rho_O^* = \max 1 + (1/s') \sum_{r=1}^{s'} \left(\frac{\sum_{k=1}^n p_{rk}}{\sum_{k=1}^n y_{rk}} \right), \tag{9.1}$$

s.t. (4.2a) and (4.8).

Once the preferences among the inputs and outputs indices are considered, the following [CSBM-Preference] model is available. w_i and π_r are the preference parameters for the total of ith input and rth output, respectively.

$$\left[\sum_{i=1}^{m'} w_i \left(\frac{\sum_{k=1}^n x_{ik} - \sum_{k=1}^n q_{ik}}{\sum_{k=1}^n x_{ik}} \right) \right] / m' = \left[\sum_{i=1}^{m'} w_i - \sum_{r=1}^{s'} w_i \left(\frac{\sum_{k=1}^n q_{ik}}{\sum_{k=1}^n x_{ik}} \right) \right] / m'.$$

Let $\sum_{i=1}^{m'} w_i = m'$, the function can be rewritten as the numerator of (10.1), the average weighted improvement ratio of all input indices. Here, $\sum_{k=1}^n x_{ik} - \sum_{k=1}^n q_{ik}$ is the total amount of UOAs in input i after improvement; $(\sum_{k=1}^n x_{ik} - \sum_{k=1}^n q_{ik}) / \sum_{k=1}^n x_{ik}$ is the improvement ratio of input i .

$$\left[\sum_{r=1}^{s'} \pi_r \left(\left(\sum_{k=1}^n y_{rk} + \sum_{k=1}^n p_{rk} \right) / \sum_{k=1}^n y_{rk} \right) \right] / s' = \left[\sum_{r=1}^{s'} \pi_r + \sum_{r=1}^{s'} \pi_r \left(\sum_{k=1}^n p_{rk} / \sum_{k=1}^n y_{rk} \right) \right] / s'.$$

Let $\sum_{r=1}^{s'} \pi_r = s'$, the function can be rewritten as the denominator of (10.1), the average weighted improvement ratio of all output indices. Here, $\sum_{k=1}^n y_{rk} + \sum_{k=1}^n p_{rk}$ is the total amount of UOAs in output r after improvement; $(\sum_{k=1}^n y_{rk} + \sum_{k=1}^n p_{rk}) / \sum_{k=1}^n y_{rk}$ is the improvement ratio of output r .

(M10) [CSBM-Preference]

$$\rho_{pref}^* = \min \frac{1 - (1/m') \sum_{i=1}^{m'} w_i (\sum_{k=1}^n q_{ik} / \sum_{k=1}^n x_{ik})}{1 + (1/s') \sum_{r=1}^{s'} \pi_r (\sum_{k=1}^n p_{rk} / \sum_{k=1}^n y_{rk})}, \quad (10.1)$$

s.t. (4.2a) and (4.8).

The calculation models of (M8), (M9), and (M10) are omitted, as they are similar to the [CSBM-G] model. The upper and lower bounds may be added to the virtual weights on input and output indices in (M6) [23], such as $\alpha_i^l \leq v_i x_{ij} / \sum_r v_r x_{rj} \leq \alpha_i^u$, $\forall i, j$ and $\beta_r^l \leq u_r y_{rj} / \sum_i u_i y_{ij} \leq \beta_r^u$, $\forall r, j$.

This study suggests a number of resource reallocation models that central GDMs may employ, to adjust the resources, and to achieve optimal overall performance. The radial-based model proposed by Lozano and Villa [14] can be taken as a special case with the common weights restrictions. We aim to not only achieve optimal performance theoretically, but to also search for applicable solutions to practical problems. These models can be applied widely in organizations that have subordinate branches, such as banks, government bureaus, educational institutions, and chain markets and convenience stores.

The DMUs are sometimes classified by their properties in practical application. For example, bank branches can be categorized into different regions. Among the different regions, the levels of economy are not equal. In future studies, researchers should consider the classified DMUs in (re)allocation problems, by merging the idea of the [CSBM-G] model.

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