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Microwave absorption in the cores of the Abrikosov vortices pinned by artificial insulator inclusion

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ABSTRACT

The spectrum of core excitations of the Abrikosov vortex pinned by a dielectric inclusion or nanoholes of the size of the coherence length is considered using the Bogoliubov–deGennes equations beyond the semiclassical approximation. While the lowest excitation, mini-gap, in the unpinned (or pinned by a metallic defect) vortex is of the order of Δ^2/ε_F, it becomes of the order of the superconducting gap Δ. The reconstruction of the quasiparticle excitations' spectrum has a significant impact on optical properties and on the tunneling density of states. We calculate the absorption amplitude and point out that, while in STM the energy gap Δ_{DOS} is between a quasiparticles state with angular momentum $\mu^e = \mu_s > 1/2$ and quasi-hole with $\mu^h = -\mu_s$ the microwave absorption gap, Δ_{dir} is between states with $\mu^e = \mu^h + 1$. It is shown that $\Delta_{dir} > \Delta_{DOS}$. The large mini-gap might play a role in magneto-transport phenomena broadly associated with the "superconductor–insulator" transition in quasi 2D systems in which small insulating inclusions are generally present.

A distinctive remarkable feature of a superconducting state, a quantum mechanical macroscopic coherent state, is its rigidity. This robust rigidity (even a certain amount of nonmagnetic impurities does not spoil this feature due to the Anderson theorem) leads to the absence of dissipation. This is a result of the energy gap Δ in the spectrum of excitations of the many body system around the Fermi level observed early on both in measurements of microwave and of tunneling density of states (DOS). The magnetic field always destroys this property. In type II superconductors it creates Abrikosov vortices with cores of radius ξ (coherence length), in center of which the superfluid density vanishes due to a nonzero vorticity. This feature is of topological nature and therefore unavoidable. In the vortex core center the material resembles a normal metal in the sense that the spectrum of the fermions excitations is almost continuous. In fact, the spectrum is discrete since the quasiparticles are confined by Andreev reflection inside the vortex core with typically small inter-level spacing and the minimal. Theoretically the core excitation spectrum of unpinned vortices in clean s-wave superconductors was calculated a long time ago using the Bogoliubov–deGennes equations (BdG) [1]. It is linear, as a function of the half integer angular momentum $\mu = 1/2, 3/2, \ldots$. Therefore, the minimal excitation, of Δ^2/ε_F occurs at the smallest possible angular momentum $\mu_0 = 1/2$ and is much smaller than Δ. The above theories considered unpinned vortices although vortices are seldom free in real superconductors. Usually they are weakly pinned by spatial inhomogeneities. It is expected that spectrum of a vortex pinned on metallic defects is not modified significantly due to the proximity effect. However, in addition to inhomogeneities, superconducting material can also tolerate small insulating inclusions and even holes. In fact, an early method to ensure pinning of vortices was irradiation that creates such an insulating columnar defect. Recently a more efficient method, the ultra fast laser and electron-beam lithography, was realized experimentally in thin films to produce arrays of dielectric inclusions or holes [2]. It has an advantage over the randomly distributed intrinsic or columnar defects since the pinning centers can be periodically arranged into an array. An even more important feature of the artificial pinning is that the diameter of the hole can be made of order of the coherent length. In these cases the effect on the core excitation spectrum might be stronger. Recently, Mel'nikov et al. [3] studied the spectrum of quasiparticles of a vortex pinned by columnar defects of radius smaller than the coherence length using the semiclassical approach. For excitations with a large momentum perpendicular to the magnetic field, $k_\perp$, they found an indication that, even when radius of the columnar defect is as small as $R = 0.1\xi$, the minigap becomes of order Δ rather than Δ^2/ε_F. More specifically, states with a small impact parameter $b = b = \mu/k_\perp$ have energy of order Δ (asymptotically $E\ \ (b \to 0) = \Delta$), then at larger angular momentum the energy of the bound state decreases linearly. When the impact parameter approaches R, the energy abruptly decreases (infinite slope) signaling the breakdown of

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the method at $b = R$. It looks like this decrease is intercepted (just before the breakdown) by the linear behavior characteristic to the unpinned vortex spectrum. The excitation energy at the intercept is still larger than $\Delta_{DOS} = \Delta^2/\varepsilon_F$ however the minimum is beyond the semiclassical approximation applicability region and one might worry about the quantitative result. Qualitatively this allowed them to conclude that one enhances the minigap by placing an insulator, albeit as thin as the columnar defect. If one was to address the dramatic increase of the minigap experimentally, one had to consider all the values of $k_\perp < k_F$ including the small ones. This necessarily requires calculations beyond the semiclassical approximation.

In the present paper we ascertain, by solving the BdG equations numerically, that the artificial pinning by an array of holes changes dramatically the excitation spectrum: while the minigap, that in the spectrum of unpinned vortex (or captured by a metallic defect) is of order Δ^2/ε_F, it becomes of the order Δ. We consider arbitrary values of the in-plane wave vector $k_\perp$ and an arbitrary inclusion radius R for both the quasiparticles branch $E^e(\mu)$ and the quasiholes branch. The results are in agreement with the semiclassical approximation within its range of applicability. It is emphasized that the lowest Andreev bound state appears at elevated angular momentum $\mu_s > 1/2$. The reconstruction of the spectrum of the quasiparticle excitations has a most significant impact on the optical properties and the tunneling density of states. While in STM the energy gap is between a quasiparticle state with angular momentum μ_s and a quasihole with $-\mu_s$, $\Delta_{DOS} = E^e(\mu_s) - E^h(-\mu_s)$ while the microwave absorption gap is between the states with $\mu_s = \mu_s \pm 1$, $\Delta_{dir} = E^e(\mu_s) - E^h(\mu_s \pm 1)$ for circular polarizations.

It is shown that generally $\Delta_{dir} > \Delta_{DOS}$. The absorption intensity increases with $k_\perp$.

We start with the Bogoliubov–deGennes equations in the operator matrix form for the BdG "spinor"

$$\begin{pmatrix} \widehat{H} & \Delta \\ \Delta^* & -\widehat{H} \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = E_v \begin{pmatrix} u \\ v \end{pmatrix};$$

$$\widehat{H} = \tfrac{1}{2}\Pi^2 - E_F; \quad \overrightarrow{\Pi} = -i\hbar\nabla - \tfrac{e}{c}\overrightarrow{A}. \qquad (1)$$
$$\Delta(r) = g \sum_{E_v < \omega_D} u_{E_v} v^*_{E_v} (1 - 2f_F(E/T))$$

where g is the phonon coupling.

Using the well known Insets in the form

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} f_+ e^{i(\mu-1/2)\varphi} \\ f_- e^{i(\mu+1/2)\varphi} \end{pmatrix} e^{ik_z z} \qquad (2)$$

and substituting Eq. (2) into Eq. (1) one obtain in dimensionless form

$$\left[\begin{pmatrix} \widehat{H}_0 & \Delta \\ \Delta^* & -\widehat{H}_0 \end{pmatrix} - \frac{\mu}{r^2}\right] \begin{pmatrix} f_+ \\ F_- \end{pmatrix} = E_v \begin{pmatrix} f_+ \\ F_- \end{pmatrix};$$

$$\widehat{H}_0 = -\tfrac{1}{r}\tfrac{\partial}{\partial r}\left(r\tfrac{\partial}{\partial r}\right) - k_\perp^2 + \tfrac{\mu^2+1/4}{r^2} \qquad (3)$$
$$k_\perp^2 = k_F^2 - k_z^2$$

ξ is the unit of length, the bulk energy gap, Δ will be units of energy. The spatial distribution of the dimensionless order parameter is Θ.

To find the BdG excitations localized near the vortex, one has to solve Esq. (1)–(3) using boundary conditions for BdG localized at the vortex core wave functions,

$$\begin{pmatrix} f_+ \\ f_- \end{pmatrix}_{r=R,\infty} = 0 \qquad (4)$$

In principle, the spatial distribution of the order parameter $\Theta(r)$ should be determined self-consistently, however it was shown that a simple approximation,

$$\begin{array}{ll} \Theta_{free}(r) = \frac{r}{\sqrt{r^2+1}} & \text{at } r > R \\ 0 & \text{at } r < R \end{array} \qquad (5)$$

provides a sufficiently accurate description of the order parameter for free (unpinned) vortex. It should be noted that in a microscopic theory of the SI interface, the order parameter rises abruptly from zero in dielectric (where amplitudes of normal excitations $f_\pm \to 0$) to a finite value inside the superconductor within an atomic distance a from the interface, namely with an "infinite" slope $\propto 1/a$. This means that the boundary condition Eq. (4) on the amplitudes $f_\pm$ is consistent with zero order parameter at the boundary point $r = R - a$ given by third of the set of the Eq. (1).

The spectrum of the quasiparticles of these equations for unpinned vortex can be simply summarized by an approximate universal formula for positive μ found by Clinton [4]. The system possesses the following symmetries. The magnetic field breaks the time reversal symmetry T, however the equations are invariant under conditions $u \to v^*$, $v \to -u^*$.Each quasiparticle state is accompanied by a corresponding quasihole state with energy $E_{k_\perp,\mu} = -E_{k_\perp,-\mu}$.

The BdG equations subject to the insulator inclusion boundary condition were solved numerically for clean superconductors. The radial equation was discretized and the whole spectrum of particle and hole excitation was calculated despite the appearance of oscillations of the order $1/k_F$. This required a very small step (2×10^{-3}) and large size for elevated angular moment a (up to 120, large enough for realistic situations considered below). As a test for the accuracy of the calculation, the threshold at energy Δ was required to be within 0.1% for both quasiparticles and quasiholes. The results for the positive angular momentum (quasiparticle) branch of the BdG spectrum for $k_\perp = 10, 50, 200$ are presented in Fig.1 for values of the hole radius $R = 0.4$ (dots). As estimated below, the maximal value of $k_\perp^{max}$ in metals can reach several hundreds. The results at small positive angular momenta, corresponding to a small impact parameter $b = \mu/k_\perp$ are lower than the semiclassical approximation plotted as solid lines. In this case the wave function is restricted to avoid insulator inclusion. The energy decreases as the angular momentum increases and reaches a minimum at μ_0, given in Table 1.

Unfortunately at μ_0, precisely the region in which the threshold for the excitations of the vortex core is observed, the semiclassical

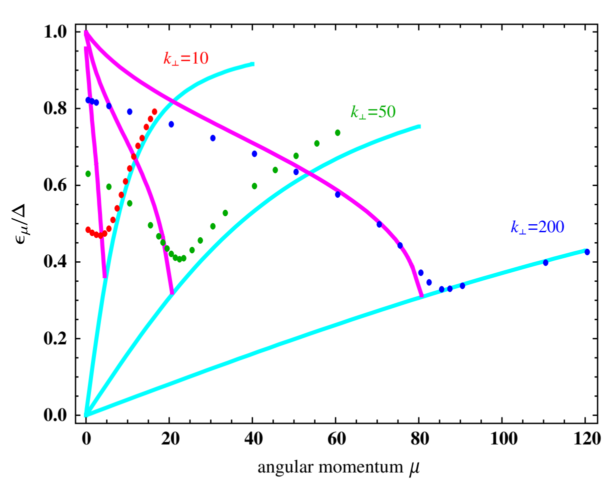

Fig. 1. Energy of quasiparticles for positive angular momentum for three values of $k_\perp$ for the hole radius $R = 0.4\xi$.Our numerical results are given by dots, while the semiclassical approximation and the Clinton formula are given by pink (monotonically decreasing) and blue (monotonically increasing) solid lines correspondingly.

Table 1
The tunneling and microwave minigaps.

$k_\perp \xi$	R/ξ	μ_0	I_{μ_0}	Δ_{dir}/Δ	Δ_{DOS}/Δ
10	0.1	1.5	8.27	0.51	0.5
10	0.4	3.5	7.58	0.96	0.93
10	0.8	6.5	7.71	1.42	1.37
50	0.1	7.5	40.79	0.76	0.39
50	0.4	22.5	45.22	1.1	0.88
50	0.8	42.5	47.44	1.47	1.25
200	0.1	24.5	–	1.04	0.2
200	0.4	73.5	–	1.46	0.71

approximation breaks down. For large $k_\perp$ the semiclassical spectrum becomes accurate before the approximation breaks down near $b = R$. Of course the spectrum at small μ is different, even for R as small as 0.1ξ, compared to the spectrum of the unpinned vortex that is close to the $b > R$ semiclassical.

According to Ref. [4], the results of quasiclassical theory are valid under conditions, $k_\perp \xi \gg 1$ while perturbation theory hinges on an additional condition $b/R > 1$.Clearly our results agree with the quasiclassical approach better for large $k_\perp$ (see Fig.1). To calculate the microwave absorption across the minigap, one needs also the negative energy states as well as negative μ. According to the symmetry, it is sufficient to calculate the $\mu > 0$ quasihole states. The whole spectrum is shown in Fig. 2 for $R = 0.4$. It demonstrates that due to the lack of $\mu \to -\mu$ symmetry the minimal gap occurs between a particle with μ_0 and a hole with $-\mu_0$.It is important to note that μ_0 grows monotonically as $k_\perp$ increases, see the 5th column in Table 1.

The minimal gap $\Delta_{DOS} = 2E_{\mu_0}(k_\perp^{\max})$, see arrows, can be measured by STM from the expression for tunneling differential conductance:

$$\frac{dI}{dV} = \left(\frac{dI}{dV}\right)_N \int_{-\infty}^{\infty} \frac{N(r,E)}{N_N} \frac{\partial f_F(E-eV)}{\partial V} dE$$
$$N(r,E) = \sum_{\mu,k_\perp} \left| f^e_{+\mu,k_\perp} \right|^2 \Delta(E - E_{\mu,k_\perp}) + \left| f^e_{-\mu,k_\perp} \right|^2 \Delta(E + E_{\mu,k_\perp}) \tag{6}$$

While the minigap seen by STM, Δ_{DOS} is between a quasiparticle state with angular momentum $\mu^e = \mu_s$ and quasihole with $\mu^h = -\mu_s$, the microwave absorption gap, Δ_{dir} is between the states with $\mu^e = \mu^h - 1$ and shown on the bottom plot. It is clear that $\Delta_{dir} > \Delta_{DOS}$. In the presence of the magnetic field the direct energy minigap Δ_{dir} should be observable in microwave absorption. The calculation of the energy absorption in one vortex is very similar to that of the inter-band transition in a cylindrically symmetric quantum dot [5] in the present case at zero temperature the hole branch is occupied, while the electron branch is empty. Coupling of time dependent external field A_{mw} (assumed to be a plane wave propagating parallel to the z axis in the gauge $\nabla \bullet A_{mw} = 0$) in linear response is described by the operator (returning to physical units

$$V = -\frac{e}{mc}\begin{pmatrix} A_{mw} \bullet \Pi & 0 \\ 0 & -A_{mw} \bullet \Pi \end{pmatrix} \tag{7}$$

In the dipole approximation, the real part of the AC conductivity for the left-handed circular polarization $E_{mw} = E_0(\cos \omega t, \sin \omega t)$ is

$$\sigma(\omega) = \frac{2\pi e^2 B}{\Phi_0 m^2 \omega} \sum_{\mu\mu' k_z} \left| M_{\mu\mu' k_z} \right|^2 \Delta(E^e_{\mu k_z} - E^h_{\mu' k_z} - \omega)$$

$$M_{\mu\mu' k_z} = \int d^2 r \begin{pmatrix} u^{e*}_{\mu' k_z} & v^{e*}_{\mu' k_z} \end{pmatrix} \mathrm{K} \begin{pmatrix} u^h_{\mu k_z} \\ v^h_{\mu k_z} \end{pmatrix}$$
$$\mathrm{K} = \begin{pmatrix} \Pi_x + i\Pi_y & 0 \\ 0 & -\Pi_x - i\Pi_y \end{pmatrix} \tag{8}$$
$$M_{\mu\mu' k_z} = 2\pi^2 i I_\mu \Delta_{\mu-\mu'=1}$$
$$I_\mu = \int r\, dr \left\{ f^{e-}_{\mu-1}\left(\frac{\mu+1}{r} + \partial_r\right) f^{h-}_\mu - f^{e+}_{\mu-1}\left(\frac{\mu}{r} + \partial_r\right) f^{h+}_\mu \right\}$$

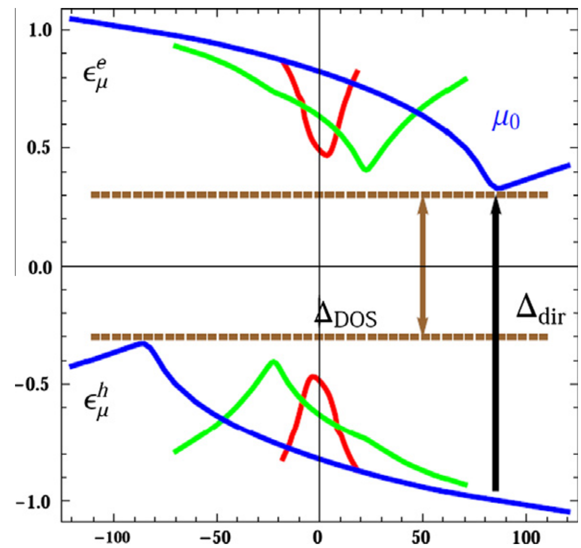

Fig. 2. Full spectrum of Andreev bound states (in the units of superconducting gap) versus angular momentum, for various $k_\perp \xi = 10, 50, 200$ (see Table 1) $R = 0.4$.

Both components of the wave functions at μ_0, are given in Fig. 3 for $R = 0.4$ at $k_\perp = 50$. At yet higher angular momenta the wave function is concentrated far from the inclusion and energy approaches the universal envelop.

The matrix element at $\mu_0(k_\perp)$ was calculated numerically and the results are given in Table 1 for various parameters.

To exemplify possible signatures of the "large" mini-gaps Δ_{DOS} and Δ_{dir}, we consider two extreme realizations of the "clean" superconducting thin film. The first is $L_z = 10$ nm thick Pb film for which $T_c = 7.15$ K, $\xi = 50$ nm, $E_F = 9.5$ eV. The states on the Fermi surface (considered isotropic) obey

$$E_F = \frac{\pi^2 \hbar^2 n_z^2}{2m_c} + \frac{\hbar^2 k_\perp^2}{2m_\perp}$$

Here the integer n_z can take values from 1 to $L_z k_F/\pi = 50$.

Correspondingly, the in-plane component of the momentum varies from zero to $\xi k_\perp = 790$.The second, is the optimally doped YBCO with $T_c = 93$ K, $\xi = 2.5$ nm, $m_c = 7.5 m_e$. The d-wave nature of pairing in cuprites has a certain effect on Andreev bound states that has been studied intensively for unpinned vortices and vortices pinned on metallic inclusions. The absorption threshold, in principle sharp, is smeared due to the d-wave nature of the energy gap. The effect of nodal quasiparticles is strongly suppressed by the limited phase space (measure zero) of the nodes [6]. The vortex structure prevents quasiparticles from approaching the nodes. When insulator inclusions are present the situation does not change in this respect. One therefore can use the s-wave results with a small value of $E_F = 0.3$ eV. For a film of thickness $L_z = 0.5$ nm only 10 levels are effective. The highest values of $k_\perp$ are consequently very small $\xi k_\perp < 8$. The tunneling and microwave (or direct) minigap are given in Table 1.

To summarize, the spectrum of the Andreev bound states, localized by the Abrikosov vortex pinned by the insulator inclusion (or nanoholes) of the size of the coherence length, was studied theoretically. We show that in this case the BdG spectrum dramatically changes in comparison with that inside an unpinned Abrikosov vortex.

While for the unpinned vortex (or the one pinned by a slight inhomogeneity or a normal metal inclusion) the spectrum is monotonic in the angular momentum μ, with lowest excitation at $\mu = 1/2$ of order Δ^2/E_F, in the case of the dielectric inclusion of a nanosize R, the low angular momenta excitations are pushed up towards the

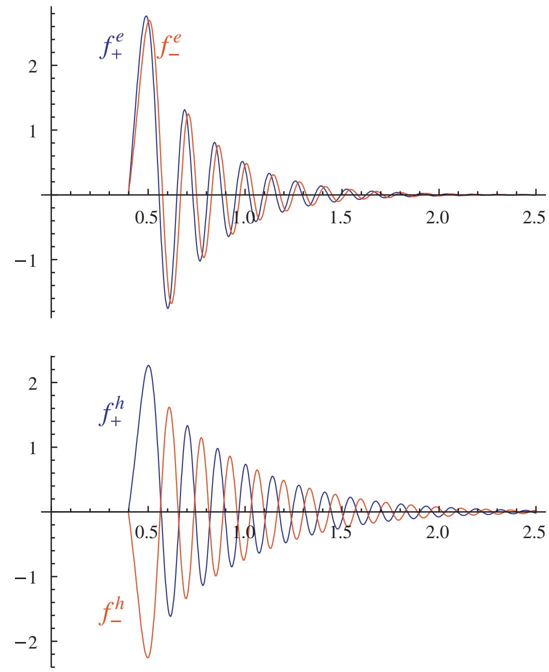

Fig. 3. Components of the eigenvectors of the BdG equations for $R = 0.4\xi,\ k_\perp\xi = 50$ at angular momentum μ as functions of the distance from the center of inclusion (in units of ξ). Quasiparticles (upper plot) and quasiholes (lower plot) differ by the sign of the lower component of the spinor.

threshold Δ (gap of the superconductor) since the dielectric prevents electron orbits of radius R. As the angular momentum increases, the excitation energy decreases to a minimum see Figs. 1 and 2, and then rises approaching the threshold Δ at large angular momenta, very much like in an unpinned vortex. Due to the approximate electron-hole symmetry, the lowest energy gap exists for the largest possible $k_\perp$.

The reconstruction of the quasiparticle excitations' spectrum has a significant impact on optical, transport and thermodynamic properties of the superconductor under magnetic field. The excitation with minimal energy manifests itself in optical response. Generally, unpinned or pinned by metallic inclusion vortices lead to excitations in the microwave range of the spectrum. On the other hand, on the basis of the above estimate, when all the vortices are pinned by dielectric inclusions or holes, the absorption spectrum should move to the infrared range. One therefore is lead to conclude that the sample with nanoholes becomes "transparent" to the microwave pulse similarly, dissipation in the vortex system at sufficiently high frequencies due to "vortex viscosity" closely associate with the quasiparticle excitations, is greatly reduced.

We speculate that the spectrum reconstruction also plays a role in transport phenomena broadly associated with the "superconductor–insulator" transition in quasi 2D systems. In these materials small insulating inclusions (islands) are generally present and, since the width is small, can be considered as "columnar". In that case our results are applicable. Experimentally magnetoresistance first rises dramatically as the magnetic field is increased and subsequently decreases. It has been established that the long-range superconducting correlations are suppressed so there is no supercurrent contribution to conductivity. As a result transport is dominated by the normal component. Why does this normal component of the conductivity become so small? It might be that the normal excitations are suppressed by the spectrum reconstruction due to pinning on insulator inclusions. At very high magnetic fields not all the vortices are pinned and the normal component is no longer suppressed.

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References

[1] J.B. Ketterson, S.N. Song, Superconductivity, Cambridge University Press, New York, 1999.
[2] J.C. Keay et al., Phys. Rev. B 80 (2009) 165421;
I. Sochnikov et al., Nat. Nanotechnol. 5 (2010) 516;
V. Metlushko et al., Phys. Rev. B 59 (1999) 603;
Yu. Aladyshkin et al., Supercond. Sci. Technol. 22 (2009) 053001. and references therein;
V.R. Misko et al., Phys. Rev. B 82 (2010) 184512;
R. Wördenweber, P. Dymashevski, V.R. Misko, Phys. Rev. B 69 (2004) 184504;
M. Kemmler et al., Phys. Rev. Lett. 97 (2006) 147003.
[3] A.S. Mel'nikov, A.V. Samokhvalov, M.N. Zubarev, Phys. Rev. B 79 (2009) 134529.
[4] W.L. Clinton, Phys. Rev. B 46 (1992) 5742.
[5] J.H. Davies, The Physics of Low Temperature Superconductors, Cambridge University Press, New York, 1998.
[6] G.E. Volovik, JETP Lett. 57 (1993) 244.