# A Cost-Effective Strategy for Road-Side Unit Placement in Vehicular Networks 

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#### Abstract

In this paper, we study the Roadside Unit (RSU) placement problem in vehicular networks. We focus on the highway-like scenario in which there may be multiple lanes with exits or intersections along the road. In our model, each vehicle can access RSUs in two ways: 1) direct delivery, which occurs when the vehicle is in the transmission range of the RSUs, and 2) multi-hop relaying, which takes place when the vehicle is out of RSU transmission range. We account for both access patterns in our placement strategy and formulate this placement problem via an integer linear programming model such that the aggregate throughput in the network can be maximized. We also take into account the impact of wireless interference, vehicle population distribution, and vehicle speeds in the formulation. The performance of the proposed placement strategy is evaluated via ns2 simulations together with VanetMobisim to generate vehicle mobility patterns. The results show that our strategy leads to the best performance as compared with the uniformly distributed placement and the hot spot placement. More importantly, our solution needs the least number of RSUs to achieve the maximal aggregate throughput in the network, indicating that our scheme is indeed a cost effective yet highly efficient placement strategy for vehicular networks.


Index Terms-Vehicular networks, RSU placement.

## I. Introduction

RESEARCH in vehicular networks has attracted much attention in recent years [1]-[6]. The main components of a vehicular network include Road Side Units (RSU) and vehicles. Each vehicle can access the wired network whenever it enters the transmission range of an RSU. Alternatively, it can take advantage of multi-hop relaying via other vehicles to reach the coverage of an RSU ahead. Nevertheless, the link condition to the RSU determines the achievable data rate and the connection lifetime. Therefore, the RSU placement strategy is the key to maximize the network capacity.

Vehicles move at a speed within a certain range, based on the speed limit imposed on the road. According to the empirical data as shown in [7] and the analytical study in

Paper approved by Y. Fang, the Editor for Wireless Networks of the IEEE Communications Society. Manuscript received September 16; March 18, October 9, and November 11, 2011.
This work was supported in part by the Excellent Research Projects of National Taiwan University, under Grant Number AE00-00-04, and in part by the National Science Council (NSC), Taiwan, under Grant Number NSC99-2221-E-002-029-MY3 and Grant Number NSC 97-2221-E-009-098-MY3.
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Digital Object Identifier 10.1109/TCOMM.2012.062512.100550
[8], the vehicle population on the highway is non-uniformly distributed and may vary with time (i.e., time of day, day of week, etc.) and with location (e.g., the car density near highway intersection is higher than that in between, and the density in urban areas is higher than that in suburban areas). Typically, the relative speed of vehicles is small in a dense area, but may vary in a sparse area. The link is more robust with smaller relative speed among vehicles due to longer link lifetime, and vice versa. However, the problem with interference and wireless resource contention is more severe in a denser area. This causes negative impact on achievable data rate for vehicles.

Compared with a cell in cellular systems, The coverage of each RSU is relatively small. As a result, it is hard to provide seamless roaming for vehicles. Determining an efficient yet cost-effective RSU placement is a key issue for vehicular networks. The simplest RSU placement strategy is uniform distribution, namely, RSUs are spaced apart at a fixed distance. While simple, this placement strategy leads to intermittent disconnection. Alternatively, RSUs can be placed in hot spot areas such that vehicles passing by the areas can transmit packets to the RSUs. Placing multiple RSUs in hotspot regions can achieve load balancing among the RSUs and partially alleviate the contention problem. In [9], Pan et al. propose a placement strategy for a set of Access Points to access a single gateway in an open space, similar to the base station placement in cellular systems. However, that scheme does not take into account the interference problem and the road topology. Therefore, it is unsuitable for real systems.

In this paper, we tackle the RSU placement problem on a highway-like roadway. In this problem, each vehicle can access RSUs in two ways: $i$ ) direct delivery, which occurs when the vehicle enters the transmission of each RSU, and ii) multi-hop relaying, which takes place when the vehicle is out of RSU's transmission range. We account for both access patterns in our placement strategy and formulate this placement problem via an Integer Linear Programming (ILP) model such that the aggregate throughput in the network can be maximized. We refer to such a placement strategy as Capacity Maximization Placement (CMP). We also take into account the impact of wireless interference, vehicle population distribution, and vehicle speeds in the formulation. The performance of the proposed placement strategy is evaluated via ns-2 simulations, together with VanetMobisim [10] to generate vehicle mobility patterns, under different scenarios. The results show that our strategy leads to the best performance as compared with the uniformly distributed placement and hot spot placement. Our scheme is adaptive to different settings:


Fig. 1. The network model for the vehicular network.
it behaves like the uniformly distributed placement and the hot spot placement in certain cases, and chooses the optimal placement in all cases. More importantly, our solution needs the least number of RSUs to achieve the saturated throughput as compared with existing solutions, indicating that our CMP scheme is indeed a cost effective yet highly efficient placement strategy for vehicular networks.

The rest of the paper is organized as follows. In Section II, the system model is described. The RSU placement problem is formulated in Section III. The simulation results and discussions are shown in Section IV. Finally, this paper is concluded in Section V.

## II. System Model

The network we consider is a vehicular network over a highway-like roadway. The road consists of multiple lanes and is partitioned into segments. The distance of each segment is set to the transmission range of each RSU. Without loss of generality, the segments are numbered from 1 to $K$, increasing in the car moving direction, as shown in Fig. 1. There is at most one RSU placed in each segment, and the RSU is placed at the center of the segment.

Vehicles on the road move at a speed over a pre-defined certain range. Vehicle population on the road follows a certain distribution, which can be obtained from historical data. RSUs are the only gateways with backhaul access to the Internet. Each vehicle can connect to an RSU in two ways: $i$ ) direct access to an RSU and $i i$ ) multi-hop relaying between two RSUs. With multi-hop relaying, each vehicle can either deliver packets forward to the RSU ahead, or backward to the RSU it just passed, depending on through which RSU can the packets be delivered at a higher data rate due to smaller hop counts. We refer to the car moving direction as the forward direction, and the opposite direction as the backward direction in this paper.

We consider IEEE 802.11p [11] and CSMA/CA as our medium access control protocol, and the free space model as the channel propagation model, i.e., the receiving power is proportional to

$$
P_{r} \propto \frac{P_{t}}{(l)^{2}}
$$

where $P_{t}$ is the transmission power and $l$ is the distance between the transmitter and the receiver .

We assume that all RSUs and vehicles have a common transmission range (denoted by $d_{T}$ ), and a common inter-
ference range (denoted by $d_{I}$ ); $d_{I} \geq d_{T}{ }^{1}$. The common transmission power of each vehicle and RSU is denoted by $P_{t}$. Recall that the size of each segment is set equal to the transmission range of each RSU. As a result, vehicles in the same segment are neighboring vehicles and can hear the transmission from another. This fact also implies that at most one vehicle can be active in one segment at any given time.

Typically, the transmission range of an RSU is much larger than the road width and the empirical data for the distance between two vehicles in the $x$-axis are more accessible than their real distance. Accounting for this fact, in this paper we can approximate the distance between two vehicles on the road by the difference in their x -axis distance, i.e., for two vehicles on the road located at $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}, y_{2}\right)$, the distance between the two vehicles is approximated by $\left|x_{1}-x_{2}\right|$.

## III. Problem Formulation and Solution Approach

In this section, we formulate the RSU placement problem as an ILP model. We define the following decision variables: $i$ ) $X_{i}$ indicates the number of RSUs in segment $i$, and $\left.i i\right) Y_{i}^{j}=1$ means that there are vehicles in segment $j$ being served by the RSU in segment $i$; otherwise $Y_{i}^{j}=0$.

The objective of this problem is to determine the placement scheme of RSUs in terms of how many RSUs to deploy and where to place them such that the achievable aggregate throughput in the network can be maximized. We consider both cases of direct delivery and multi-hop relaying to RSUs. Mathematically, this problem can be formulated as follows.

Given:

- $R^{d}(i)$ : the achievable data rate over the link via direct access to an RSU located in segment $i$;
- $R^{m+}(i)$ : the achievable data rate for a link from segment $i$ to segment $(i+1)$ via multi-hop relaying (i.e., toward an RSU in the forward direction);
- $R^{m-}(i)$ : the achievable data rate for a link from segment $i$ to segment $(i-1)$ via multi-hop relaying (i.e., toward an RSU in the backward direction);
- $T_{f}^{d}(i)$ : the link lifetime for direct access to an RSU located in segment $i$;
- $T_{f}^{m+}(i)$ : the link lifetime from segment $i$ to segment $(i+$ 1) via multi-hop relaying (i.e., toward an RSU in the forward direction);

[^0]- $T_{f}^{m-}(i)$ : the link lifetime from segment $i$ to segment $(i-$ 1) via multi-hop relaying (i.e., toward an RSU in the backward direction);
- $\xi$ : a positive integer indicating the hop count limit for multi-hop relaying;
- $C_{x}$ : the cost of one RSU;
- $C_{B}$ : the total deployment budget for this placement;

The problem can be formulated by:

$$
\begin{align*}
& \max \sum_{i=1}^{K} X_{i}\left\{R^{d}(i) \cdot T_{f}^{d}(i)\right. \\
& +\sum_{j=1}^{i-1} Y_{i}^{j} \cdot \frac{1}{\sum_{k=j}^{i-1} \frac{1}{R^{m+}(k) \cdot T_{f}^{m+}(k)}}  \tag{1}\\
& \left.+\sum_{j=i+1}^{K} Y_{i}^{j} \cdot \frac{1}{\sum_{j=i+1}^{K} \frac{1}{R^{m-}(k) \cdot T_{f}^{m-}(k)}}\right\}
\end{align*}
$$

subject to:

$$
\begin{gather*}
X_{i} \in\{0,1\}, \forall i \in\{1,2, \cdots, K\}  \tag{2}\\
\sum_{i=1}^{K} Y_{i}^{j} \leq 1, \forall j \in\{1,2, \cdots, K\}  \tag{3}\\
X_{i}=1-\prod_{j}\left(1-Y_{i}^{j}\right), \forall i  \tag{4}\\
Y_{i}^{j}=0 \quad i f|i-j| \geq \xi, \forall i, j  \tag{5}\\
\sum_{i} X_{i} \cdot C_{x}<C_{B} \tag{6}
\end{gather*}
$$

Constraint (2) indicates that there is at most one RSU in each segment. Constraint (3) says that each multi-hop relaying transmission can be served by at most one RSU. Constraint (4) states that there must be an RSU placed in segment $i$ if there are vehicles in any other segments being served by the RSU in segment $i$. Constraint (5) places a hop count limit (i.e., $\xi$ ) in each multi-hop relaying path for performance reason. Constraint (6) indicates the deployment budget for this RSU placement. Note that for the multi-hop relaying scenario, the achievable path data rate may decrease when the hop count increases [9]. Therefore, we cannot simply add all the successful bits for each hop (i.e., $R^{m+}(k) \cdot T_{f}^{m+}(k)$ ) in the path, as this would infer the following: "a longer path leads to better throughput due to more successful bits." Instead, we take the inverse of the successful bits per hop first, and add them together. We then take the inverse of this sum to obtain the achievable path throughput in (1) (i.e., the last two terms in (1)). Intuitively, given packet size for a flow, per-hop throughput will depend on the transmission time of each hop. Here, the transmission time of each hop is determined by the SNR value of the link. Thus, the end-to-end throughput will be highly affected by the total transmission time of the path. The larger hop count indicates longer transmission time, thus leading to lower throughput. This approach is reasonable as it preserves the characteristics of a multi-hop path, namely, $i$ ) a longer multi-hop path may lead to lower throughput, $i i$ )
increasing link rate will alleviate the impact of hop count, and $i i i$ ) the link with the minimum rate dominates the path throughput. Note that our objective function (1) by itself is not equivalent to the expression of "aggregate throughput" of the networks. By maximizing this objective function, it could lead to the similar effect of throughput maximization.

The achievable link data rates (i.e., $R^{d}(i), R^{m+}(k)$, and $\left.R^{m-}(k)\right)$ are affected by the vehicle population distribution on the road and the way of transmission (i.e., direct transmission vs. multi-hop relaying, forward or backward delivery); whereas the link lifetime is determined by the relative speed and the distance between two vehicles. In the following, we will determine the achievable link data rate and the link lifetime by accounting for several factors that include interference, vehicle population, and vehicle speed on a segmented, multilaned, highway-like roadway. Due to space limitations, we will only show the derivation for the multi-hop scenario in the forward direction, i.e., $R^{m+}(k)$ and $T_{f}^{m+}(k)$. The other quantities can be obtained similarly.

## A. Cross-Segment Link

In our model, each link is formed across two segments except when the vehicles are within the same segment of an RSU. The successful transmission over a cross-segment link is then significantly affected by the wireless interference (particularly for multi-hop relaying), vehicle population distribution, and vehicle density in the network.

Consider a specific cross-segment link $\left(t_{i}, r_{i+1}\right)$, i.e., the transmitting vehicle in segment $i$ is located at $t_{i}$ and the receiving vehicle in segment $i+1$ is at $r_{i+1}$ (i.e., it is toward an RSU in the forward direction). We would like to determine the successful transmission probability for link $\left(t_{i}, r_{i+1}\right)$. For a successful transmission on a link, the signal to interference and noise ratio (SINR) at the receiver must exceed a certain threshold $\delta$ in order for the packets over the link to be successfully decoded.

We first consider the case that there is a single active vehicle within the interference range of the receiver $r_{i+1}$. A vehicle within the interference range of the link's receiver is called an interfering vehicle in this paper. According to the free space model, the receiving power is only affected by the distance between the transmitter and the receiver. Therefore, our approach is to determine, within the interference range of $r_{i+1}$, the maximum coverage area in which a single active interfering vehicle will generate enough interference to fail the link transmission. We refer to this area as the Strong Interference Area (sIFA) for $\left(t_{i}, r_{i+1}\right)$, denoted by $\theta\left(t_{i}, r_{i+1}\right)$, as shown in Fig. 2.

The sIFA for $\left(t_{i}, r_{i+1}\right)$ can be determined as follows. Suppose that there is only one active vehicle within the transmission range of $r_{i+1}$. Based on the free space model and the capture effect of the channel, we have

$$
\begin{gather*}
P_{r_{i+1}}=\eta \cdot \frac{P_{t}}{\left(r_{i+1}-t_{i}\right)^{2}}  \tag{7}\\
P_{I_{i+1}}=\eta \cdot \frac{P_{t}}{\left(l_{I_{i+1}}\right)^{2}} \tag{8}
\end{gather*}
$$



Fig. 2. The strong interference area for a transmission link.

$$
\begin{equation*}
\operatorname{SINR}_{r_{i+1}}=\frac{P_{r_{i+1}}}{P_{I_{i+1}}+\sigma}>\delta \tag{9}
\end{equation*}
$$

where $P_{t}$ is the transmission power from $t_{i}, P_{r_{i+1}}$ is the power received at $r_{i+1}, P_{I_{i+1}}$ is the interference experienced at $r_{i+1}, l_{I_{i+1}}$ is the distance between the receiver and the active interfering vehicle, $\operatorname{SINR}_{r_{i+1}}$ is the SINR value at the receiver $r_{i+1}$, and $\sigma$ is the environmental noise which is assumed to be negligible.

From (7), (8), and (9), we can obtain the radius of $\theta\left(t_{i}, r_{i+1}\right)$ by equating $\operatorname{SINR}_{r_{i+1}}$ to $\delta$, i.e.,

$$
l_{I_{i+1}}=\sqrt{\delta} \cdot\left(r_{i+1}-t_{i}\right)
$$

Accordingly, $\theta\left(t_{i}, r_{i+1}\right)$ can be expressed by

$$
\begin{equation*}
\theta\left(t_{i}, r_{i+1}\right)=\left\{x \mid r_{i+1}-l_{I_{i+1}}<x<r_{i+1}+l_{I_{i+1}}\right\} . \tag{10}
\end{equation*}
$$

Let $P_{\theta}$ denote the probability that there is one vehicle in region $\theta\left(t_{i}, r_{i+1}\right)$, which is

$$
P_{\theta}=\int_{\theta\left(t_{i}, r_{i+1}\right)} b_{X}(x) d x
$$

where $b_{X}(x)$ is the probability density function (p.d.f.) of the location distribution.

The total number of vehicles within $\theta\left(t_{i}, r_{i+1}\right)$, denoted by $N_{\theta\left(t_{i}, r_{i+1}\right)}$, is a binomial random variable, as each vehicle on the road is located independently according to the location distribution $b_{X}(x)$. Taking expectation of $N_{\theta\left(t_{i}, r_{i+1}\right)}$, we obtain the average number vehicles within $\theta\left(t_{i}, r_{i+1}\right)$ as follows.

$$
N_{\theta}=E\left[N_{\theta\left(t_{i}, r_{i+1}\right)}\right]=N \cdot P_{\theta} \approx\left\lceil N \cdot p_{\theta}\right\rceil,
$$

where $N$ is the total number of vehicles in the network.
The probability that no other vehicle in $\theta\left(t_{i}, r_{i+1}\right)$ is active except the transmitter of link $\left(t_{i}, r_{i+1}\right)$ is then expressed by

$$
\begin{equation*}
P_{S}^{m+}\left(\theta\left(t_{i}, r_{i+1}\right)\right)=\tau \cdot(1-\tau)^{N_{\theta}-1} \tag{11}
\end{equation*}
$$

where $\tau$ is the successful channel access probability in a multihop wireless network as obtained in [12], i.e.,

$$
\tau \approx \frac{2 W_{\min }}{\left(W_{\min }+1\right)^{2}} q
$$

where $W_{\min }$ is the minimum contention window size specified for the back-off operation of CSMA/CA, and $q$ is the probability of successful handshake which can be assumed to be a constant. Note that with this equation, the RTS/CTS mechanism adopted in the IEEE 802.11p standard can also be supported in our model.

Next we consider the case with multiple active interfering vehicles which affect link $\left(t_{i}, r_{i+1}\right)$. By the definition of sIFA, a single active vehicle outside $\theta\left(t_{i}, r_{i+1}\right)$ cannot by generating enough interference fail the transmission on link $\left(t_{i}, r_{i+1}\right)$. However, two or more active interfering vehicles outside $\theta\left(t_{i}, r_{i+1}\right)$ may or may not fail the link transmission, depending on the accumulated interference at the receiver. For this, we consider the following area that is outside $\theta\left(t_{i}, r_{i+1}\right)$ but still within the interference range (i.e., the shaded areas of the interference range of $r_{i+1}$ in Fig. 2):

$$
\Lambda\left(r_{i+1}\right)=\left\{x \mid r_{i+1} \pm d_{I}\right\} \backslash\left\{x \mid r_{i+1} \pm l_{I+i+1}\right\} .
$$

Vehicles which are within $\Lambda\left(r_{i+1}\right)$ and which will affect the link transmission when they are active are called affecting vehicles. Due to space limitations, we will just show how to calculate the case for two affecting vehicles to fail the transmission on link $\left(t_{i}, r_{i+1}\right)$. The case of more affecting vehicles can be obtained in a similar manner. We will show shortly in the simulations that even with $k=2$, we can obtain good performance. Note that the maximum number of active affecting vehicles is expressed by $\left\lceil\frac{d_{I}}{d_{T}}\right\rceil$, which should be a small integer in practice.

Consider the case that there are only two affecting vehicles in $\Lambda\left(r_{i+1}\right)$, say, $c_{1}$ and $c_{2}$. Suppose that $c_{1}$ is located at $x_{1}$, and which alone will not fail the transmission on link $\left(t_{i}, r_{i+1}\right)$. We further suppose that $c_{2}$, the other vehicle in $\Lambda\left(r_{i+1}\right)$, is at $x_{2}$, and together with $c_{1}$ will fail the transmission on link $\left(t_{i}, r_{i+1}\right)$. With these two affecting vehicles whose locations are at $x_{1}$ and $x_{2}$, the condition that link transmission will fail (due to the capture effect) is expressed by

$$
\begin{equation*}
\operatorname{SINR}_{\Lambda\left(r_{i+1} ; 2\right)}=\frac{P_{r_{i+1}}}{\frac{P_{t}}{\left|r_{i+1}-x_{2}\right|^{2}}+\frac{P_{t}}{\left|r_{i+1}-x_{2}\right|^{2}}}<\delta \tag{12}
\end{equation*}
$$

By definition, the possible value of $x_{1}$ is within the following region:

$$
\begin{aligned}
A_{1}=\{ & x_{1} \mid r_{i+1}+l_{I_{i+1}}<x_{1}<r_{i+1} \\
& \text { or } \left.r_{i+1}-d_{I}<x_{1}<r_{i+1}-l_{I_{i+1}}\right\} .
\end{aligned}
$$

Given that the location of $c_{1}$ is $x_{1}$, the location of $c_{2}$ can only be within the region expressed as follows:

$$
\begin{aligned}
A_{2}=\{ & x_{2} \left\lvert\, r_{i+1}+l_{I_{i+1}}<x_{2}<r_{i+1}+\left(\frac{d_{I}}{\left(l_{I_{i+1}}\right)^{2}}-\frac{1}{\left(x_{1}\right)^{2}}\right)\right. \\
& \text { or } \left.r_{i+1}-\left(\frac{d_{I}}{\left(l_{I_{i+1}}\right)^{2}}-\frac{1}{\left(x_{1}\right)^{2}}\right)<x_{2}<r_{i+1}-l_{I_{i+1}}\right\},
\end{aligned}
$$

which can be obtained together with (12) and region $A_{1}$.
Denote by $P_{X_{1}}$ the probability that there is a vehicle in region $A_{1}$ and it is active, and by $P_{X_{2} \mid X_{1}=x_{1}}$, the conditional probability that there is one active vehicle within region $A_{2}$ given that the location of the other vehicle who is active in $A_{1}$ is at $x_{1}$. Let $P_{1}=\int_{A_{1}} b_{X}\left(x_{1}\right) d x_{1}$ and $P_{2}=\int_{A_{2}} b_{X}\left(x_{2}\right) d x_{2}$. Therefore,

$$
\begin{align*}
P_{X_{1}}= & P_{1}\left(1-P_{1}\right)^{N-1} \tau+P_{1}^{2}\left(1-P_{1}\right)^{N-2} \tau(1-\tau) \cdot 2+\cdots \\
& +P_{1}^{N} \tau(1-\tau)^{N-1} \cdot N  \tag{13}\\
P_{X_{2} \mid X_{1}=x_{1}}= & P_{2}\left(1-P_{2}\right)^{N-2} \tau+P_{2}^{2} \cdot\left(1-P_{2}\right)^{N-3} \tau(1-\tau) \cdot 2 \\
& \quad+\cdots+\left(1-P_{2}\right)^{N-1} \tau(1-\tau)^{N-2}(N-1) \tag{14}
\end{align*}
$$

Let $P_{K_{i+1}}^{m+}(k)$ be the probability that the transmission of $\left(t_{i}, r_{i+1}\right)$ fails due to $k$ affecting vehicles in $\Lambda\left(r_{i+1}\right), k \geq 2$ (i.e., $k$ concurrently active interfering vehicles in $\Lambda\left(r_{i+1}\right)$ ). Based on the derivation above, the probability of two active affecting vehicles in $\Lambda\left(r_{i+1}\right)$ is

$$
\begin{equation*}
P_{K_{i+1}}^{m+}(2)=P_{X_{1}} \cdot \int_{A_{1}} P_{X_{2} \mid X_{1}=x_{1}} \cdot b_{X_{1}}\left(x_{1}\right) d x_{1}, \tag{15}
\end{equation*}
$$

where $b_{X_{1}}\left(x_{1}\right)$ is the truncated p.d.f. of the location for the affecting vehicle at $x_{1}$ in $\Lambda\left(r_{i+1}\right)$, which is

$$
b_{X_{1}}\left(x_{1}\right)=\frac{b_{X}\left(x_{1}\right)}{\int_{A_{1}} b_{X}\left(x_{1}\right) d x} .
$$

Similarly, $P_{K_{i+1}}^{m+}(k)$ can be obtained, for $k=3, \ldots,\left\lceil\frac{d_{I}}{d_{T}}\right\rceil$.
Let $P_{S}^{m+}\left(t_{i}, r_{i+1}\right)$ denote the probability for a successful transmission on link $\left(t_{i}, r_{i+1}\right)$, which is equivalent to the probability that there are no active vehicles in $\theta\left(t_{i}, r_{i+1}\right)$ and there are no more than $k$ affecting vehicles within $\Lambda\left(r_{i+1}\right)$. Accordingly, we have

$$
\begin{equation*}
P_{S}^{m+}\left(t_{i}, r_{i+1}\right)=P_{S}^{m+}\left(\theta\left(t_{i}, r_{i+1}\right)\right) \cdot\left(1-\sum_{k=2}^{\left\lceil\frac{d_{I}}{d_{T}}\right\rceil} P_{K_{i+1}}^{m+}(k)\right) \tag{16}
\end{equation*}
$$

## B. Achievable Link Data Rate

Having derived the successful probability for link $\left(t_{i}, r_{i+1}\right)$, i.e., $P_{S}^{m+}\left(t_{i}, r_{i+1}\right)$, from the transmitting vehicle at $t_{i}$ to the receiving vehicle at $r_{i+1}$, we are now ready to calculate the achievable data rate for this link. With the Shannon Capacity theorem, the maximum achievable rate for this link can be approximated by

$$
\begin{equation*}
R_{\left(t_{i}, r_{i+1}\right)}^{m+}(i) \approx P_{S}^{m+}\left(t_{i}, r_{i+1}\right) \cdot W \log _{2}\left(1+S N R_{r_{i+1}}^{t_{i}}\right), \tag{17}
\end{equation*}
$$

where $W$ is the bandwidth of the transmission channel, and $S N R_{r_{i+1}}^{t_{i}}$ is the signal to noise ratio at receiver $r_{i+1}$ with transmission power $P_{t}$.

Suppose that the location of the receiver in segment $i+1$ is at $r_{i+1}$. Since at most one vehicle can be active in each segment at any given time, the possible location for a transmitter in segment $i$ is within the following range:

$$
\begin{equation*}
A_{t_{i}}=\left\{x \mid\left(r_{i+1}-d_{T}\right)<x<i \cdot d_{T}\right\}, \tag{18}
\end{equation*}
$$

where $d_{T}$ is the transmission range of each vehicle.
Thus, the occurrence probability of a link across two segments $i$ and $i+1$ (i.e., a forward link from segment $i$ to segment $i+1$ ) can be expressed by

$$
\begin{equation*}
P_{L}^{m+}(i)=P_{R_{x}} \cdot \int_{i \cdot d_{T}}^{(i+1) d_{T}} P_{T_{x} \mid R_{x}=r_{i+1}}\left(t_{i}\right) d B_{x_{i+1}}\left(r_{i+1}\right) \tag{19}
\end{equation*}
$$

where $P_{R_{x}}$ is the probability that there is an active vehicle in segment $i+1, P_{T_{x} \mid R_{x}=r_{i_{1}}}$ is the conditional probability that the transmitter is at $t_{i}$ given that the receiver is at $r_{i+1}$, and $B_{x_{i+1}}\left(r_{i+1}\right)$ is the cumulative distribution function (c.d.f.) of the truncated p.d.f. $b_{x_{i+1}}\left(r_{i+1}\right)$.
$P_{R_{x}}, P_{T_{x} \mid R_{x}=r_{i_{1}}}$, and $b_{x_{i+1}}\left(r_{i+1}\right)$ can be expressed by:

$$
\begin{gathered}
P_{R_{x}}=\left(1-\left(1-\int_{i \cdot d_{T}}^{(i+1) \cdot d_{T}} b_{X}\left(r_{i+1}\right) d r_{r+1}\right)^{N}\right) . \\
P_{T_{x} \mid R_{x}=r_{i+1}}\left(t_{i}\right)=\left(1-\left(1-\int_{r_{i+1}-d_{T}}^{i \cdot d_{T}} b_{X}\left(t_{i}\right) d t_{i}\right)^{N}\right) . \\
b_{X_{i+1}}\left(r_{i+1}\right)=\frac{\int_{i \cdot d_{T}}^{(i+1) \cdot d_{T}} b_{X}\left(r_{i+1}\right)}{} .
\end{gathered}
$$

By un-conditioning on the locations of the transmitter and the receiver in segments $i$ and $i+1$, and together with (16), (17) and (19), the maximum achievable rate for a link across segments $i$ and $i+1$ is then expressed as follows.

$$
\begin{array}{r}
R^{m+}(i)=P_{L}^{m+}(i) \int_{i \cdot d_{T}}^{(i+1) \cdot d_{T}} \int_{r_{i+1}-d_{T}}^{i \cdot d_{T}} R_{\left(t_{i}, r_{i+1}\right)}^{m+}(i)  \tag{20}\\
d B_{X_{i} \mid X_{i+1}=r_{i+1}}\left(t_{i}\right) d B_{X_{i+1}}\left(r_{i+1}\right),
\end{array}
$$

where $B_{X_{i} \mid X_{i+1}=r_{i+1}}\left(t_{i}\right)$ and $B_{X_{i+1}}\left(r_{i+1}\right)$ are the c.d.f. of $b_{X_{i} \mid X_{i+1}=r_{i+1}}\left(t_{i}\right)$ and $b_{X_{i+1}}\left(r_{i+1}\right)$, the truncated p.d.f. of the location for $t_{i}$ and $r_{i+1}$, respectively, given that the receiver is located at $r_{i+1}$, and

$$
b_{X_{i} \mid X_{i+1}=r_{i+1}}\left(t_{i}\right)=\frac{b_{X}\left(t_{i}\right)}{\int_{r_{i+1}-d_{T}}^{i \cdot d_{T}} b_{X}\left(t_{i}\right) d t_{i}} .
$$

Similarly, $R^{m-}(i)$ can be expressed as follows.

$$
\begin{array}{r}
R^{m-}(i)=P_{L}^{m-}(i) \int_{(i-2) \cdot d_{T}}^{(i-1) \cdot d_{T}} \int_{(i-1) \cdot d_{T}}^{r_{i-1}+d_{T}} R_{\left(t_{i}, r_{i-1}\right)}^{m-}(i)  \tag{21}\\
d B_{X_{i} \mid X_{i-1}=r_{i-1}}\left(t_{i}\right) d B_{X_{i-1}}\left(r_{i-1}\right),
\end{array}
$$

$R^{d}(i)$ can also be obtained in a similar manner, except that the receiver is replaced by an RSU located in segment $i$, whereas the transmitter is in the same segment of the RSU, i.e., in segment $i$. Thus,

$$
\begin{equation*}
R^{d}(i)=P_{T_{x}}\left(t_{i}\right) \int_{(i-1) \cdot d_{T}}^{i \cdot d_{T}} R_{t_{i}}^{d}(i) d B_{X_{i}}\left(t_{i}\right) \tag{22}
\end{equation*}
$$

where $P_{T_{x}}\left(t_{i}\right)$ is the probability that we can find one vehicle in the RSU's transmission range in segment $i, R_{\left(t_{i}\right)}^{d}(i)$ is the average successful receiving rate for the RSU located in segment $i, B_{X_{i}}\left(t_{i}\right)$ is the c.d.f. of the location distribution $b_{X_{i}}\left(t_{i}\right)$ for the transmitter at $t_{i}$ within the RSU's segment. $R_{\left(t_{i}\right)}^{d}(i)$ can be derived in a similar way as we did in (20), except that the location of the RSU (i.e., the receiver) in segment $i$ is placed at the middle of the segment, i.e., at $\left(i-\frac{1}{2}\right) \cdot d_{T}$.

## C. Link lifetime

First, consider the link lifetime for $\left(t_{i}, r_{i+1}\right)$, which is denoted by $T_{f}^{m+}\left(t_{i}, r_{i+1}\right)$. Let $V_{i}$ and $V_{i+1}$ be the average speeds in segment $i$ and segment $i+1$, respectively. The link is broken whenever a vehicle moves out of the transmission range of each other. Accordingly, we obtain

$$
\begin{align*}
T_{f}^{m+}\left(t_{i}, r_{i+1}\right) \approx \min \left\{\frac{\left|i \cdot d_{T}-t_{i}\right|}{V_{i}}\right. & , \frac{\left|(i+1) \cdot d_{T}-r_{i+1}\right|}{V_{i+1}} \\
& \left.\frac{d_{T}+s \cdot\left|r_{i+1}-t_{i}\right|}{\left|V_{i+1}-V_{i}\right|}\right\} \tag{23}
\end{align*}
$$

where $s=-1$ if $V_{i}<V_{i+1}$ (i.e., they are moving away from each other), and $s=1$ otherwise. The first two terms in (23) are to avoid the case that either vehicle moves out of the segment they are staying, thereby causing the link broken, and the last term corresponds to the link lifetime calculated by the relative speed between the two vehicles.

Again, by un-conditioning on the locations of the transmitter and the receiver, we obtain

$$
\begin{array}{r}
T_{f}^{m+}(i)=P_{L}^{m+}(i) \int_{i \cdot d_{T}}^{(i+1) \cdot d_{T}} \int_{r_{i+1}-d_{T}}^{i \cdot d_{T}} T_{f}^{m+}\left(t_{i}, r_{i+1}\right)  \tag{24}\\
d B_{X_{i+1}}\left(t_{i}\right) d B_{X_{i+1}}\left(r_{i+1}\right)
\end{array}
$$

Similarly, the link life time between segments $i$ and $i-1$ and the link lifetime for a vehicle in the same segment of an RSU can be derived, respectively, as follows.

$$
\begin{gather*}
T_{f}^{m-}(i)=P_{L}^{m-}(i) \int_{(i-2) \cdot d_{T}}^{(i-1) \cdot d_{T}} \int_{(i-1) \cdot d_{T}}^{r_{i-1}+d_{T}} T_{f}^{m-}\left(t_{i}, r_{i+1}\right)  \tag{25}\\
d B_{X_{i} \mid X_{i-1}=r_{i-1}}\left(t_{i}\right) d B_{X_{i-1}}\left(r_{i-1}\right) . \\
T_{f}^{d}(i)=P_{T_{x}}\left(t_{i}\right) \int_{(i-1) \cdot d_{T}}^{i \cdot d_{T}} T_{f}^{d}\left(t_{i}\right) d B_{X_{i}}\left(t_{i}\right) \tag{26}
\end{gather*}
$$

## IV. Computational Experiments and Discussion

In this section, we evaluate the performance of our proposed placement strategy CMP via simulations. The road topology is shown in Fig. 3(a), which consists of two roads intersecting at three points. The region for each road measures 1250 m by 150 m , and the three intersections are located at $0 \mathrm{~m}, 750$ m , and 1250 m of the horizontal axis, respectively. There are 30 vehicles moving at a speed over a range of $60-80 \mathrm{~km}$ per hour on the roads. Each marked point on the roads indicates one car. Vehicles on the upper road move from left to right, whereas those on the bottom road drive from right to left. The mobility model is generated by VanetMobisim to emulate realistic vehicular environments and Fig. 3(a) is a snapshot taken in the simulation. It shows that the vehicle population on the roads is distributed non-uniformly, and more vehicles are clustered in the intersection areas. Fig. 3(b) shows the vehicle population distribution of Fig. 3(a), which is the input to the ns-2 simulations. We adopt IEEE 802.11 b for channel access, and the free space model for channel propagation. The common transmission range of each RSU and vehicle is 250 m

(a) A snapshot of the road topology with car distribution

(b) Vehicle population distribution of (a)

Fig. 3. The network topology and vehicle population distribution for the simulations.
and the common interference range is 550 m , resulting in five segments in our simulations. RSUs are placed between the two roads, so they can serve vehicles on both roads. Each vehicle can transmit to the RSU in its forward or backward driving direction, using the shortest path strategy to choose which RSU to access.

We compare our proposed CMP with two placement strategies, namely, uniformly distribution and hot-spot placement. Specifically, we evaluate the aggregate throughput in the network achievable by each placement scheme under the budget constraint. We also investigate the number of RSUs required to achieve the maximum throughput for each placement strategy without the budget constraint.

The first scenario is described as follows. Given that there is only one RSU to deploy in the network shown in Fig. 3, we would like to determine the best location for the RSU. In our ns-2 simulations, each vehicle generates bulk UDP traffic at a constant rate of 1 Mbps . The routing protocol for multi-hop relaying is AODV [13]. We plot in Fig. 4 the aggregate network throughput when the RSU is placed in different locations as indicated in the x -axis of the figure. It shows that placing the RSU in different location indeed leads to different performance. We also mark the location of the RSU determined by our CMP scheme and the hotspot scheme. As can be seen in Fig. 4, the performance of our CMP is very close to the optimal performance, while there is still a gap for the hot-spot scheme. Fig. 5 shows the aggregate throughputs for the three placements schemes with two RSUs in the network model shown in Fig. 3. Again, our CMP outperforms the other two schemes and the improvement is up to $60 \%$ for the uniform placement and up to $80 \%$ for the hot spot placement.

To further explore the characteristic of our CMP scheme, we generate several strategic vehicle population distributions in the following simulations. Unlike in Fig. 3(b), in the strategic population distributions, there are more fluctuations in the distributions. For each strategic distribution, 500 vehicles moving at speeds of 60 to 100 km per hour on a road of 3000 m long are tested. The transmission range and interference range


Fig. 4. Aggregate throughputs with one RSU at different locations.


Fig. 5. Aggregate throughputs with one RSU at different locations.
of vehicles are set to 100 m and 225 m , respectively, resulting in 30 segments with this setting. Each vehicle is always backlogged. Fig. 6(a) shows one such distributions and Fig. 6(b) is its placement result for CMP which leads to the maximal network capacity (i.e., there are 7 RSUs for Fig. 6 ). We can observe that with CMP, RSUs tend to be placed near the hot spot areas (i.e., high population density), instead of being directly placed in the hotspots so as to avoid severe interference and contentions. Specifically, RSUs are usually placed in medium density areas, at a certain distance to the hot spots so as to avoid generating large hop count paths to access the RSUs. We can also observe that CMP enjoys the merits of both uniform distribution and hot spot distribution under different population distributions. When there are more fluctuations in the population distribution, it spreads out more as in uniform distribution, whereas with a few hot spots, it behave more like a hot spot placement, i.e., clustering in several spots.

Fig. 7 further plots the aggregate throughputs for the three schemes under the vehicle population distribution shown in Fig. 6. As can be seen, our scheme outperforms the other placement schemes in all cases. More importantly, with our CMP scheme, the aggregate throughput in the network is saturated with about seven RSUs, while the uniformly distributed scheme takes 15 and the hot spot needs 20 . The


Fig. 6. Different vehicle population distributions and their resulting RSU placements with CMP.


Fig. 7. Aggregate throughputs for the three schemes.
deployment budget for our scheme is the minimal among the three. Therefore, CMP is indeed a cost-effective solution.

Fig. 8 shows the placement results of CMP with different


Fig. 8. RSU placements with high and low relative speeds.
relative vehicle speeds given the population distribution shown in Fig. 6. With higher relative speeds (i.e., speeds varying from 60 to 100 km per hour), the placement for the same number of RSUs tends to more spread out. This is because with high relative speed, the link is more error-prone (due to shorter lifetime) so that vehicles benefit from a placement in which they can encounter an RSU more often and transmit packets via direct delivery. On the other hand, with low relative speed (i.e., the speeds of the vehicles on the road are similar), the link is more robust, the transmission can go through multi-hop relaying to RSUs to better utilize the wireless resource [14]. Fig. 9 plots the aggregate throughputs for the three schemes under higher relative speed (as compared to Fig. 7). Again, our CMP outperforms the other two schemes in all cases, and the required number of RSUs to achieve the saturated throughput of the network is even smaller than that with lower relative speed.

In summary, our proposed CMP scheme is a much better placement strategy than the existing solutions for high-way like vehicular networks in terms of the aggregate throughput and the deployment budget.


Fig. 9. Aggregate throughputs for the three schemes with high relative speed.

## V. Conclusion

In this paper, we have studied the RSU placement strategy for vehicular networks under a highway-like scenario. We propose a Capacity Maximization Placement (CMP) scheme which adapts to different vehicle population distribution and different vehicle speeds on the road. Specifically, when the vehicle population distribution exhibits more fluctuations, the set of RSUs is spaced apart more uniformly on the road; when there are only a few dense areas on the road, RSUs tend to be placed near these hotspots. Moreover, in a dense area, the relative speed among vehicles is smaller so that the link is more robust due to longer link lifetime. Therefore, our scheme prefers multi-hop relaying for vehicles so as to better utilize wireless resource. On the other hand, in a sparse area, the relative speed is more variable, thereby the link may be more error-prone and unpredictable. Therefore, direct delivery via a RSU is preferable for higher achievable data rates. More importantly, the proposed CMP strategy needs the least number of RSUs to achieve the saturated network throughput as compared with the uniform distribution and the hot spot placement, making it a cost-effective yet highly efficient RSU placement solution in vehicular networks.

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[^0]:    ${ }^{1}$ The focus of this paper is on the uplink aggregate throughput. For simplicity but without loss of generality, we assume a common transmission range and a common interference range for all nodes in this paper.

