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# Formation and recurrence of quasicrystalline patterns from quantum dynamics of suddenly released matter waves: Analogous manifestation of optical waves

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**Abstract** – We analytically and numerically verify that two-dimensional (2D) quasicrystalline matter waves can be formed from the free-time evolution of multiple Gaussian waves regularly distributed on a circular ring. We also demonstrate that an interesting recurrence phenomenon can be manifested by adding an extra Gaussian wave on the center of multiple Gaussian waves uniformly distributed on a ring. We finally employ an optical experiment for the coherent light passing through the mask to analogously illustrate the formation of 2D quasicrystalline patterns and the associated recurrence.



**Introduction.** – The hallmark of matter waves in quantum theory is mainly revealed by the phenomena of diffraction and interference. Moshinsky in 1952 [1] first presented the concept of the diffraction-in-time effect by exploring the edge diffraction of the matter plane wave suddenly released from a shutter in one dimension. Moshinsky also verified that the diffraction fringes in the course of time are mathematically similar to the ones obtained in the Fresnel diffraction of light. Subsequently, numerous researchers have investigated various quantum transients and diffraction effects. Felber *et al.* [2] and Szriftgiser *et al.* [3] have first observed the diffraction of matter waves with cold neutrons and cold atoms, respectively. Godoy [4,5] studied the diffraction in time in the context of Fresnel and Fraunhofer for a slit and spherical traps. More recently, the concept of diffraction in time has been extended to explore the topics of matter waves diffracted by linear potential and moving mirrors [6–8] and to investigate the transient dynamics of atom lasers influenced by the optical confinement [9].

With the advent of technological developments, there will be more experiments with subatomic, atomic, and molecular particles to be relevant to the quantum transient phenomena of matter waves. It is believed that theoretical proposals for interesting phenomena of quantum diffraction in time domain could provide valuable features for emergent experiments. The aim of this work is to make a theoretical proposal to generate two-dimensional (2D) quasicrystalline patterns from the free-time evolution of multiple Gaussian waves regularly distributed on a circular ring. The quasicrystalline structures have fascinated artists, mathematicians, and scientists in both ancient and modern cultures [10,11]. It will be of general interest to test experimentally quasicrystalline patterns from the quantum diffraction of matter waves. Furthermore, we find an intriguing recurrence phenomenon from the free-time evolution of 2D quasicrystalline patterns by adding an extra Gaussian wave on the center of multiple Gaussian sources uniformly distributed on a ring. Quantum recurrence phenomena originate from the simultaneous excitation of discrete quantum levels [12] and have been studied in atomic and molecular wave packet evolution [13–16]. We expect that the recurrence phenomenon of matter waves in free-time evolution can lead to some enchanting ideas for future experiments. Finally, we exploit the diffraction of light by spatial slits to analogously demonstrate the formation of 2D quasicrystalline patterns and the related recurrence.

**Formation and recurrence of quasicrystalline patterns from diffraction in time.** – In terms of the 2D free propagator, the free-time evolution of the quantum wave function $\psi_o(x,y)$ suddenly released at time $t=0$ is

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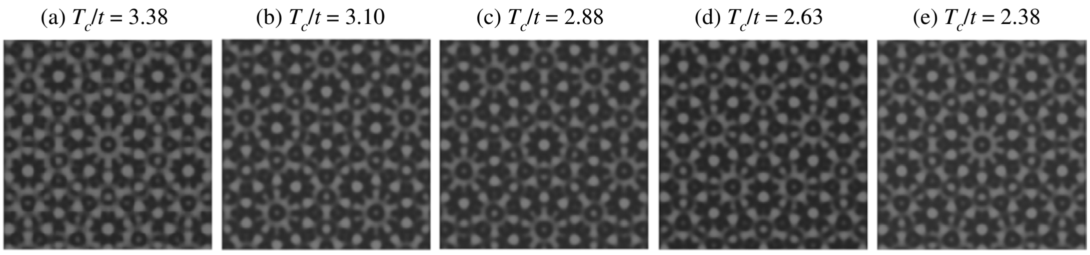


Fig. 1: Numerical patterns calculated with eq. (5) for the case of $q=5$ at different times.

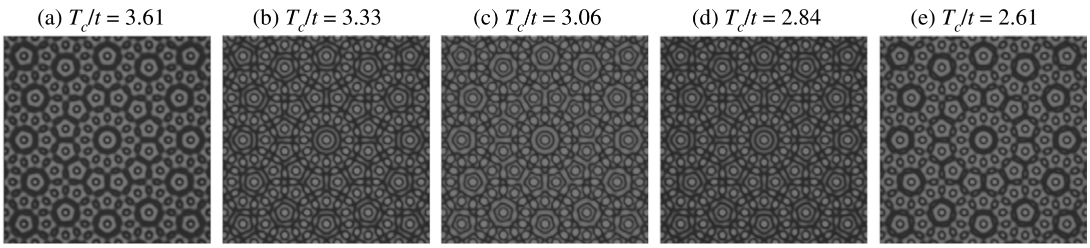


Fig. 2: Numerical patterns calculated with eq. (5) for the case of $q=12$ at different times.

given by [1,2]

$$\psi(x,y,t)=\frac{m}{2\pi i\hbar t}\int \mathrm{d}y'\int \mathrm{d}x' e^{i\frac{m}{2\hbar}\frac{[(x-x')^2+(y-y')^2}{t}}\psi_o(x',y'). \quad (1)$$

Considering the initial field to be formed by $q$ Gaussian waves regularly distributed on a circle, the wave function can be expressed as

$$\psi_o\left(x',y'\right)=\left(\frac{2}{\pi a^2}\right)^{1/2}\sum_{s=0}^{q-1} e^{-\frac{\left[x'-R\cos(2\pi s/q)\right]^2+\left[y'-R\sin(2\pi s/q)\right]^2}{a^2}}, \quad (2)$$

where the constant $a$ is the packet size of Gaussian waves and the constant $R$ is the radius of the circle. Substitution of eq. (2) into eq. (1) leads to

$$\psi(x,y,t) = \left(\frac{2}{\pi a^2}\right)^{1/2}\left(1+\frac{4\hbar^2 t^2}{m^2 a^4}\right)^{-1/2} \times e^{-i\theta} e^{-\left[\frac{r^2+R^2}{a^2+2i\hbar t/m}\right]}\sum_{s=0}^{q-1} e^{\frac{2R}{a^2+2i\hbar t/m} r\cos\left(\phi-\frac{2\pi s}{q}\right)}, \quad (3)$$

where $\theta=\tan^{-1}[2\hbar t/(ma^2)]$ and $(r,\ \phi)$ is the polar coordinate of the point $(x,\ y)$. When the time $t$ is large enough such that $t\gg T_1=ma^2/(2\hbar)$, eq. (3) can be simplified as

$$\psi(x,y,t) = \left(\frac{ma}{\sqrt{2\pi} i\hbar t}\right) \times e^{i\frac{mr^2}{2\hbar t}}\left\{e^{i\frac{mR^2}{2\hbar t}}\left[\sum_{s=0}^{q-1} e^{-i\frac{mR}{\hbar t} r\cos\left(\phi-\frac{2\pi s}{q}\right)}\right]\right\}. \quad (4)$$

The wave structure in eq. (4) is completely fixed by the term in the square brackets. With the new variable $K=mR/(\hbar t)$, the term in the square brackets can be expressed as $\Psi_q(r,\phi;K)=\sum_{s=0}^{q-1} e^{-iKr\cos(\phi-(2\pi s/q))}$ that represents the 2D periodic lattices for $q=2$, 3, 4, 6 and the 2D quasicrystalline structures for all other values of $q$ [11,17]. To be brief, the global structures of the wave functions in eq. (4) reveal periodic or quasicrystalline patterns and are independent of time.

When an extra Gaussian wave is added on the center of original $q$ Gaussian waves, eq. (4) becomes

$$\psi(x,y,t)=\left(\frac{ma}{\sqrt{2\pi} i\hbar t}\right) e^{i\frac{mr^2}{2\hbar t}}\left[e^{i\frac{mR^2}{2\hbar t}}\Psi_q(r,\phi;K)+1\right]. \quad (5)$$

The global structure of the wave pattern in eq. (5) can be seen to be time dependent and is associated with the factor $\exp[i(mR^2)/(2\hbar t)]$. In terms of a new variable $T_c=mR^2/(4\pi\hbar)$, the factor $\exp[i(mR^2)/(2\hbar t)]$ is given by $\exp[i2\pi T_c/t]$. It is clear that $\psi(x,y,t)$ and $\psi(x,y,t+\Delta t)$ can share the same wave structure, when the phase difference satisfies $2\pi[T_c/t-T_c/(t+\Delta t)]=2\pi$. Consequently, the time period for the recurrence of the wave pattern in eq. (5) can be derived to be $T_{rev}=\Delta t=t^2/(T_c-t)$. The time-dependent revival period $T_{rev}$ is close to be a parabolic function of the time $t$, when the time $t$ satisfies $t\ll T_c$. For an instant, the characteristic time $T_c$ for cold neutrons with $R=100\,\mu$m is given by $13.5\,\mu$s. As a result, the revival period $T_{rev}$ for suddenly released cold neutrons is generally on the order of $\mu$s. Figures 1(a)–(e) depict the numerical patterns for the wave functions in eq. (5) for the case of $q=5$ at the times of $T_c/t=3.38$, 3.10, 2.88, 2.63, and 2.38. Figures 2(a)–(e) depict another

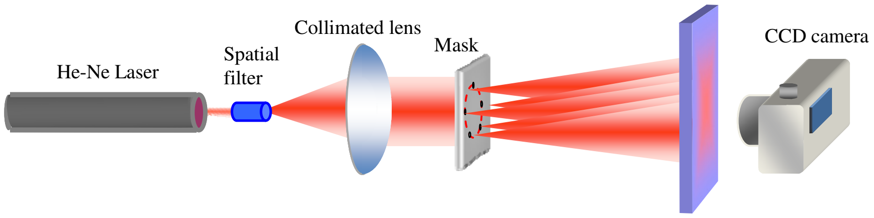


Fig. 3: (Color online) Experimental setup for demonstrating the formation and recurrence of the 2D quasicrystalline patterns.

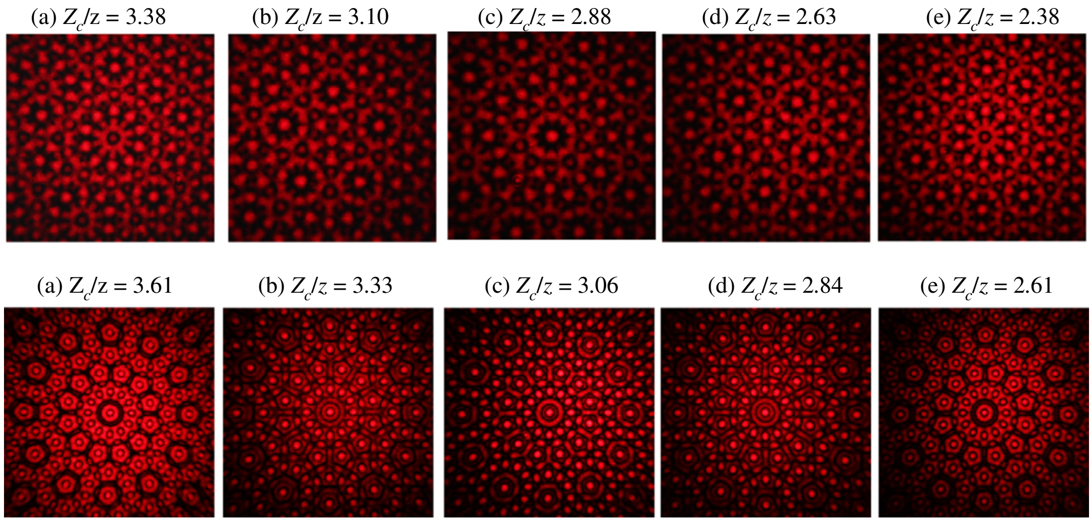


Fig. 4: (Color online) Experimental transverse patterns for the free-space propagation of the collimated laser light passing through the masks with $q = 5$ (upper row) and $q = 12$ (lower row) at the positions corresponding to the results shown in fig. 1 and fig. 2.

case of $q = 12$ at the times of $T_c/t = 3.61$, 3.33, 3.06, 2.84, and 2.61. It can be seen that the transient dynamics of the wave function displays the feature of 2D quasicrystalline structures. More intriguingly, there is a recurrence phenomenon, as shown in the patterns of figs. 1(a) and (e) and the patterns of figs. 2(a) and (e).

**Optical analogy.** – Mathematical similarities between paraxial optics and non-relativistic quantum mechanics have been identified for a long time and have been recently employed to analogously explore the quantum phenomena [18]. More recently, this analogy has been applied to the use of the oxide-confined vertical-cavity surface-emitting lasers (VCSELs) to manifest the spatial morphology of wave functions [19] and energy-level statistics [20] in 2D quantum billiards. In addition to confined eigenstates, the free-time evolution of quantum wave functions has been verified to be mathematically similar to the free-space propagation of the coherent light. Here we use the diffraction of light by spatial slits to analogously demonstrate the time evolution and recurrence of 2D quasicrystalline patterns.

To analogously implement $q + 1$ localized Gaussian waves, we experimentally employ a collimated light to illuminate the stencil mask that is precisely fabricated to have $q$ small apertures regularly distributed on a circle and a small hole on the center. The optical wave emitting from the mask at $z = 0$ to the position in the direction of the $+z$-axis can be expressed as the Fresnel transformation:

$$\psi(x, y, z) = \frac{ie^{-ikz}}{\lambda z} \times \int \mathrm{d}y' \int \mathrm{d}x' e^{-i\frac{k}{2}\frac{\left[(x-x')^2+(y-y')^2\right]}{z}} \psi_o(x', y'). \quad (6)$$

From the comparison of eq. (1) with eq. (6) it is evident that the time evolution of a 2D quantum state is equivalent to the Fresnel transformation of a near-field optical field with the substitution of $t \to z$ and $m/\hbar \to k$, where $k$ is the wave number of the optical wave. Consequently, the recurrence distance for the optical diffraction can be analogously found to be $Z_{rev} = z^2/(Z_c - z)$ with $Z_c = kR^2/(4\pi)$. Figure 3 depicts the experimental optical configuration for manifesting the free-time evolution and recurrence of the 2D quasicrystalline patterns. The light source was a linearly polarized 20 mW He-Ne laser with a wavelength of 632.8 nm. A beam expander

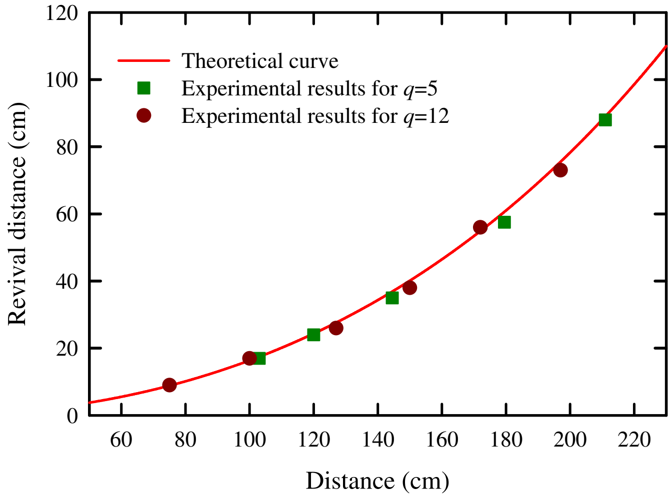


Fig. 5: (Color online) Experimental revival distances for the free-space propagation of the collimated laser light passing through the masks with $q = 5$ and $q = 12$ and numerical results calculated by $Z_{rev} = z^2/(Z_c - z)$.

was employed to reduce the beam divergence less than 0.1 mrad. Metal masks of different forms were fabricated with a laser stencil-cutting machine. The radii of the aperture and the ring are 0.08 mm and 3.0 mm, respectively. Interference patterns formed in the region behind the mask were imaged by a CCD camera. With $Z_c = kR^2/(4\pi)$, $k = 2\pi/\lambda$, $\lambda = 0.632\,\mu$m, and $R = 3$ mm, the parameter $Z_c$ can be found to be approximately 712 cm.

Figure 4 shows the experimental transverse patterns for the free-space propagation of the collimated light passing through the masks corresponding to the cases shown in fig. 1 and fig. 2 with the substitution of $T_c/t \to Z_c/z$. It can be seen that the experimental patterns agree quite well with the numerical results shown in fig. 1 and fig. 2. The good agreement validates that the free-space propagation of coherent optical light diffracted from the specific mask can be employed as an analogous observation of the time evolution of matter waves with $q$ Gaussian sources regularly distributed on a circle and a Gaussian source on the center. Finally, we systematically measure the revival distance as a function of the observed distance from the optical experiment to verify the expression $T_{rev} = t^2/(T_c - t)$ for the revival period. Figure 5 shows the experimental revival distances and the numerical results calculated by $Z_{rev} = z^2/(Z_c - z)$. The excellent agreement confirms the recurrence phenomenon and the expression $T_{rev} = t^2/(T_c - t)$.

**Conclusion.** – In conclusion, we have analytically and numerically explored the generation of 2D quasicrystalline patterns from the diffraction-in-time effect of suddenly released matter waves. We also disclose an intriguing method to manifest the quantum recurrence phenomenon in the free-time evolution of 2D quasicrystalline patterns. Furthermore, we have exploited an optical diffraction experiment to analogously demonstrate the formation and recurrence of 2D quasicrystalline patterns in the quantum dynamics of suddenly released matter waves. It is expected that the present exploration could offer an important insight into quantum and optical physics.

* * *

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