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*Abstract*—Simple cooperative automatic retransmission request (ARQ) protocols are presented based on the selective amplified-and-forward (SAF) relaying method. Analysis shows that the temporal diversity of ARQs can be exploited with AF relaying only if the channel quality to the relay exceeds a threshold that depends on the source data rate requirement. Based on this analysis, an effective ARQ protocol is first developed from the concept of selective relaying to attain the full temporal diversity. Moreover, the notion of SAF relaying is further extended to systems with multiple relays to exploit the spatial and temporal diversities, incorporating the mechanism of opportunistic relaying. Two types of opportunistic-selective AF (OSAF) relaying methods are thus proposed for cooperative ARQs. Analysis shows that both OSAF protocols can offer much higher diversities than ARQ schemes with the typical AF relaying method. And the throughput of ARQs with OSAF is more robust to the variations of channel qualities and is close to their decode-and-forward counterparts.

*Keywords*— Selective amplified-and-forward, cooperative ARQ and opportunistic relaying.

## I. INTRODUCTION

Since the pioneering work of [1], a host of relaying protocols have been presented to exploit the spatial diversity offered by cooperative relaying. They can be roughly categorized into the amplified-and-forward (AF) and decoded-and-forward (DF) methods. In view of the simplicity of AF relaying and its corresponding effect of noise enhancement, selective AF (SAF) relaying method has been considered in [2] to improve the power efficiency of typical AF relaying, or in [3] for multi-hop relaying and in [4] with phase feedbacks,

On the other hand, to avoid the difficulty of synchronization among all participating relays and to prevent from the complexity of using distributed space-time coding or beamforming, opportunistic relaying (OR) has been introduced in [5] to exploit the spatial diversity offered by distributed relays. Inspired by the above results, we study herein effective opportunistic and selective AF methods for cooperative automatic retransmission request (ARQ) to exploit the spatial and temporal diversities via cooperative relaying.

According to our analysis, it shows that the temporal diversity of ARQs can be exploited with AF relaying only if the channel quality to the relay exceeds a threshold that depends on the source data rate requirement. Based on this result, an effective ARQ protocol is first proposed to employ the SAF relaying to attain the full temporal diversity. Moreover, by incorporating the OR mechanism, this SAF relaying scheme is further extended to exploit the spatial and temporal diversities in systems with multiple relays. Different from the opportunistic AF (OAF) in [5], the opportunistic selection methods studied herein only rely on the channel qualities to the destination, which prevents from the need

of extra channel state information at the destination. Based on this opportunistic mechanism, two types of opportunistic-selective AF (OSAF) relaying methods are developed for cooperative ARQs. Analysis shows that both OSAF protocols can offer much higher diversities than ARQ schemes with the typical AF relaying method if proper thresholds are set for each hop along the relaying. Besides, numerical studies also demonstrate that the throughput of ARQs with OSAF is more robust to the variations of channel qualities and is close to their decode-and-forward counterparts.

## II. COOPERATIVE ARQ WITH SELECTIVE AF RELAYING

We introduce in this section the ARQ schemes with the assistance of SAF relaying evolved from the original AF in [1] and its variation in [3]. For the clearness of presentation, we first consider a system that consists of only one source, one destination and a single relay. The result of this system will then be extended to systems with multiple relays. Throughout the paper, the channel between any transmit and receive pair is considered flat Rayleigh and, for simplicity of analysis, the channel coefficients remain unchanged within a period of time and change randomly from period to period. This assumption, though rather optimistic in practice, allows us to proceed with the analysis based on the outage probability [6].

### A. ARQ with selective AF relaying (ARQ-SAF)

Different from the typical AF relaying, the relay in this ARQ scheme first compares the instantaneous source-to-relay channel quality  $\rho|h_{sr}|^2$  against a predetermined threshold,  $\Delta$ , before retransmission. If  $\rho|h_{sr}|^2$  is less than or equal to  $\Delta$ , then the source will be asked to do the retransmission by itself, while, in the mean time, the relay keeps overhearing the signal during retransmissions. Once  $\rho|h_{sr}|^2 > \Delta$ , the relay proceeds with the retransmission using the AF relaying and will continue to use the same quantity for retransmission until it is decoded successfully at the destination or when the maximal number of ARQs is reached, namely no ARQ is further needed. The corresponding received SNR at the destination is given by

$$\text{SNR}_d = \frac{\rho^2|h_{sr}|^2|h_{rd}|^2}{\rho|h_{sr}|^2 + \rho|h_{rd}|^2 + 1} \quad (1)$$

where  $h_{sr}$ ,  $h_{sd}$ , and  $h_{rd}$  stand for the channel coefficients of source-to-relay (S-R), source-to-destination (S-D), and relay-to-destination (R-D), respectively, and are all complex Gaussian random variables with zero mean and variances equal to  $\beta_1$ ,  $\beta_0$  and  $\beta_2$ , respectively. Besides, without the loss of generality, the transmit SNR is assumed to be the same at the source and the relay, and is denoted by  $\rho \triangleq \frac{E_s}{N_0}$ .

### III. THE OUTAGE PROBABILITY OF ARQ-SAF

In this section, we will show that the threshold  $\Delta$  for the selective AF relaying is crucial for ARQ schemes to attain their full diversities. In other words, the ARQ scheme with the simple AF relaying ( $\Delta = 0$ ) is not able to benefit from diversity enhancement via retransmissions. The analysis is mainly based on the outage probability of the form

$$\Pr\{\log_2(\text{SNR} + 1) < R\} = \Pr\{\text{SNR} < 2^R - 1\} \quad (2)$$

where  $R$  is the information rate in bits/sec per channel use. For convenience of expression, we define a number of notations for variables to be used frequently in the analysis. Specifically, we have  $a \triangleq \rho|h_{sr}|^2$ ,  $b \triangleq \rho|h_{rd}|^2$  and  $w \triangleq \rho|h_{sd}|^2$ , and denote the outage probability after  $n$  rounds of ARQs with scheme A by  $P_n^A$ , and the maximal number of ARQ rounds by  $\mathcal{N}$ . Besides, we also have  $\delta_1 \triangleq 2^R - 1$ , and redefine  $\Delta \triangleq k\delta_1$  with  $k \geq 0$ . Finally, the diversity order  $d$  is defined as [6]

$$d = - \lim_{\rho \rightarrow \infty} \frac{\log P_n^A(\rho)}{\log \rho}. \quad (3)$$

Given  $\Delta$ , the outage probability after  $n$  rounds of ARQs with the selective AF relaying can be expressed as

$$P_n^{\text{SAF}} = \Pr\{w < \delta_1\} \times \sum_{l=0}^n \left[ (\Pr\{a \leq \Delta\} \Pr\{w < \delta_1\})^{n-l} F(\Delta, l) \right] \quad (4)$$

where  $(\Pr\{a \leq \Delta\} \Pr\{w < \delta_1\})^{n-l}$  is the outage probability after  $n-l$  consecutive retransmissions by the source, and  $F(\Delta, l)$  stands for the outage probability of the subsequent  $l$  consecutive retransmissions by the relay, which are characterized by the joint probability of the outage events of

$$F(\Delta, l) = \Pr\left\{ a > \Delta, \frac{ab_1}{a+b_1+1} < \delta_1, \dots, \frac{ab_l}{a+b_l+1} < \delta_1 \right\}, \text{ for } l > 0 \quad (5)$$

with  $F(\Delta, 0) \triangleq 1$ . Since the R-D channel fades independently in each ARQ round,  $b_l$  is used to distinguish the corresponding channel quality in each ARQ round.

Apparently, the outage events in  $F(\Delta, l)$  are correlated as the S-R channel quality “ $a$ ” remains unchanged throughout the ARQs even if  $b_l$  are statistically independent. With some mathematical manipulations, it can be shown that the form of  $F(\Delta, l)$  also depends on  $\delta_1$ . For the conciseness of presentation, the result is summarized in the following lemma.

*Lemma 1:* Given  $\Delta$  and  $R$  and, hence,  $\delta_1$ , we have

$$F(\Delta, l) = e^{-\frac{\Delta}{\rho\beta_1}} + \begin{cases} \sum_{i=1}^l C_i^l (-1)^i e^{-\left(\frac{1}{\rho\beta_1} + \frac{i}{\rho\beta_2}\right)\delta_1} \Gamma\left(1, 0; \frac{i(\delta_1^2 + \delta_1)}{\rho^2\beta_1\beta_2}\right) & , \Delta < \delta_1 \\ \sum_{i=1}^l C_i^l (-1)^i e^{-\left(\frac{1}{\rho\beta_1} + \frac{i}{\rho\beta_2}\right)\delta_1} \Gamma\left(1, \frac{\Delta - \delta_1}{\rho\beta_1}; \frac{i(\delta_1^2 + \delta_1)}{\rho^2\beta_1\beta_2}\right) & , \Delta \geq \delta_1 \end{cases} \quad (6)$$

where  $\Gamma(\alpha, x; b) = \int_x^\infty t^{\alpha-1} e^{-t} dt$  is the *generalized incomplete gamma function* [7], and  $C_i^l$  is the total number of combinations of picking  $i$  out of  $l$  distinct objects.

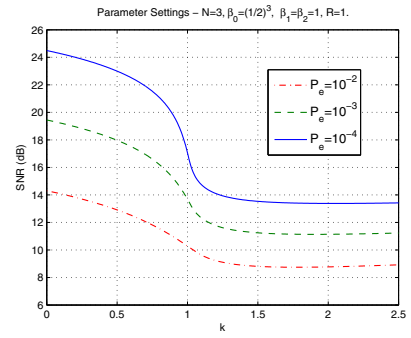


Fig. 1. The effect of  $k = \frac{\Delta}{\delta_1}$  v.s. the SNR at a target  $P_3^{\text{SAF}} = P_e$ .

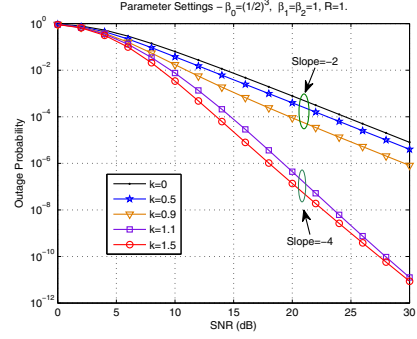


Fig. 2. Outage probabilities after three rounds of ARQ-SAFs ( $P_3^{\text{SAF}}$ ), with thresholds  $\Delta = k\delta_1$  in different  $k$ 's.

Substituting (6) into (4) gives the closed form expression of (4). The relation between  $\Delta \triangleq k\delta_1$  and the outage probability(4) is illustrated in Fig. 1. As can be seen in the figure, the required SNR for  $P_3^{\text{SAF}} = P_e$  dramatically reduces around  $k = 1$ . This in fact results from the diversity loss for  $k < 1$  shown in Fig. 2. The dependence of the diversity order on  $\Delta$  is characterized in the following lemma. Due to the space limitation, the proof is not presented in the paper.

*Lemma 2:* If  $\Delta > \delta_1$ , then the diversity order of  $P_n^{\text{SAF}}$  in (4) is equal to  $(n+1)$ ; whereas, if  $\Delta < \delta_1$ , it is equal to 2.

Lemma 2 shows that the full temporal diversity of ARQs can be achieved if a basic channel quality of  $\Delta$  is met before the AF relaying. This gives an interesting reminiscence of the selective DF relaying in [1], even if the source signal is not decoded here before the AF retransmission.

The result of Lemma 2 also shows that the diversity order of ARQ with direct AF relaying (ARQ-AF) is equal to 2 as it is simply a special case of ARQ-SAF with  $\Delta = 0$ , which is always less than  $\delta_1$  for  $R \geq 0$ . According to Lemma 1, the corresponding outage probability for ARQ-AF is given by

$$P_n^{\text{AF}} = (1 - e^{-\frac{\delta_1}{\rho\beta_0}}) \times F(0, n). \quad (7)$$

### IV. COOPERATIVE ARQ WITH OPPORTUNISTIC-SELECTIVE AF RELAYING

In this section, we extend the notion of selective AF relaying to systems with multiple relays. Motivated by the opportunistic relay selection method in [5], we will discuss and analyze three protocols herein based on their outage

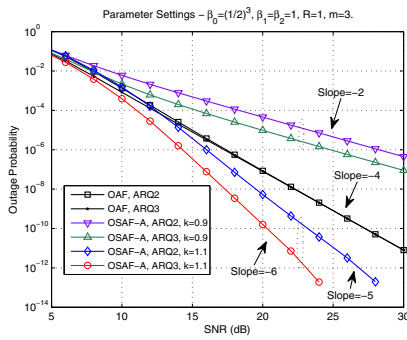


Fig. 3. Outage probabilities for ARQs with OAF and OSAF-A relaying. For OSAF-A with  $\Delta = k\delta_1 \geq \delta_1$ , the diversity orders increase by 1 with each round of ARQ. Otherwise, they are limited to 2.

probabilities. We assume there are  $m$  relays available in the system and denote the channel coefficient between the source and the relay  $j$  by  $h_{j,sr}$  and the channel between the relay and the destination by  $h_{j,rd}$ . In addition, the channel coefficient between relay  $i$  and relay  $j$  is denoted by  $h_{i,j}$ . For simplicity of analysis, all channel coefficients are still assumed complex Gaussian random variables with zero mean. The variances of  $h_{j,sr}$  for different  $j$  are the same and equal to  $\beta_1$ . Similarly, the variance of  $h_{j,rd}$  is  $\beta_2$ ,  $\forall j = 1, \dots, m$ . Furthermore, the variance of  $h_{i,j}$  is  $\beta_3$ ,  $\forall i, j = 1, \dots, m$ .

#### A. ARQ with opportunistic AF relaying

We first investigate the outage probability of ARQ with the opportunistic AF (ARQ-OAF) relaying method presented in [5]. The ARQ-OAF basically chooses the relay  $i$  in each round of ARQ that satisfies

$$i = \arg \max_{j \in \{1, \dots, m\}} \left\{ \frac{\rho^2 |h_{j,sr}|^2 |h_{j,rd}|^2}{\rho |h_{j,sr}|^2 + \rho |h_{j,rd}|^2 + 1} \right\} \quad (8)$$

to directly amplify and forward the signal. Based on the previous results, the outage probability after  $n$  rounds of this OAF-based ARQs is provided in the following proposition.

*Proposition 1:* Given  $R$  and  $m$ , the outage probability after  $n$  rounds of ARQs with OAF is given by

$$P_n^{\text{OAF}} = (1 - e^{-\frac{\delta_1}{\rho\beta_0}}) \times (F(0, n))^m \quad (9)$$

and its diversity order is limited to  $(m+1)$ ,  $\forall n = 1, \dots, \mathcal{N}$ .

*Proof:* The detailed derivation for the outage probability is omitted here for space limitation. By Lemma 2, if  $\Delta < \delta_1$ , then  $F(\Delta, n)$  is of the order of  $\rho^{-1}$  at high SNR. Thus, the diversity order of  $P_n^{\text{OAF}}$  is equal to  $(m+1)$ . ■

In fact, the ARQ-OAF scheme offers the full cooperative diversity only for the first ARQ round. In the subsequent ARQs however,  $a_j \triangleq \rho |h_{j,sr}|^2$  remain unchanged in the AF signal. Similar to the ARQ-AF scheme, this results in the loss of the temporal diversity as shown in Fig. 3. This motivates us to develop another two protocols based on the SAF relaying method in Section II to recover the diversity.

#### B. ARQ with opportunistic-selective AF relaying

The essence of ARQ-SAF lies in setting a sufficiently high threshold that the relay is allowed for forwarding only if the

S-R link quality,  $\rho |h_{sr}|^2$ , exceeds the threshold. By ensuring the quality of the received signal, the channel diversity in the subsequent ARQs can be exploited to reduce the outage probability. Inspired by this result, we define a qualified set  $\mathcal{Q}$  of the relays whose  $\rho |h_{j,sr}|^2 > \Delta$  for opportunistic AF relaying. In each ARQ, the relay in  $\mathcal{Q}$  with the highest  $\rho |h_{j,rd}|^2$  gets selected for AF relaying. In case of  $\mathcal{Q} = \emptyset$ , then the source will do the retransmission until  $\mathcal{Q} \neq \emptyset$  or when no ARQ is further needed. We note that the relay selection method here is *unrelated* to the S-R channel quality any more, *i.e.*  $h_{j,sr}$  is not needed at the destination.

Based on this opportunistic-selective AF (OSAF) relaying method, we discuss in the next section two types of ARQ schemes, referred to as the type A and B of ARQ-OSAF, respectively. The type-A scheme forms  $\mathcal{Q}$  by overhearing the signals from the source only, while type-B continues to enlarge the cardinality of  $\mathcal{Q}$  by overhearing the transmitted signals from relays in  $\mathcal{Q}$  as well. Their performance are analyzed in the following two subsections.

#### B.1 ARQ with the type A of OSAF relaying (OSAF-A)

Under the assumption that all R-D channels have the same statistical property, every relay in  $\mathcal{Q}$  has equal possibility to be chosen as the active relay for AF. With this simplified setting, the outage probability after  $n$  rounds of ARQs with OSAF-A can be expressed in the following compact form.

*Proposition 2:*

$$P_n^{\text{OSAF-A}} = \Pr\{w < \delta_1\} \times \sum_{l=0}^n \left\{ (\Pr\{a \leq \Delta\})^m \times \Pr\{w < \delta_1\}^{n-l} \times G(m, l) \right\} \quad (10)$$

where  $G(m, l) \triangleq 1$  for  $l = 0$ ; in addition, for  $l > 0$ , it follows

$$G(m, l) = \sum_{i=1}^m C_{m-i}^m (\Pr\{a \leq \Delta\})^{m-i} \left(\frac{1}{i}\right)^l F^{(i)}(\Delta, l, i) \quad (11)$$

in which for  $i = 1, 2$ , we have

$$F^{(1)}(\Delta, l, r) \triangleq F(\Delta, l) \quad (12)$$

$$F^{(2)}(\Delta, l, r) \triangleq \sum_{\zeta=0}^l C_{\zeta}^l (e^{-\frac{\Delta}{\rho\beta_1}})^{q(l, \zeta)} \times F(\Delta, r \cdot \zeta) \times F(\Delta, r \cdot (l - \zeta)) \quad (13)$$

with  $q(l, \zeta) = \delta[l - \zeta] + \delta[\zeta] - \delta[l + \zeta]$ . While for  $i > 2$ ,  $F^{(i)}(\Delta, l, r)$  can be expressed as a recursive form of

$$F^{(i)}(\Delta, l, r) = \sum_{\zeta=0}^l C_{\zeta}^l (e^{-\frac{\Delta}{\rho\beta_1}})^{q(l, \zeta)} \times F^{(i-1)}(\Delta, \zeta, r) \times F(\Delta, r \cdot (l - \zeta)) \quad (14)$$

*Proof:* Due to the space limitation, the proof is not included in the paper. Basically we make use of the Binomial Theorem and Lemma 1 to complete the proof. ■

As we have known from Lemma 2, different thresholds for ARQ-SAF will result in different outage probabilities or even diversity losses. For ARQ with OSAF-A, we also have similar results which are summarized in the next proposition.

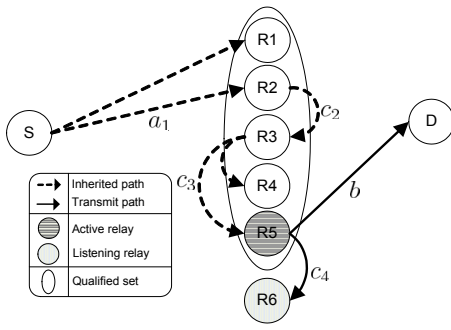


Fig. 4. An illustration for ARQs with the OSAF-B relaying.

*Proposition 3:* If the threshold  $\Delta$  for ARQ-OSAF-A is large than  $\delta_1$ , then the diversity order of  $P_n^{\text{OSAF-A}}$  increases with  $n$  and is equal to  $(m+n)$ . However, if  $\Delta < \delta_1$ , then the diversity order of  $P_n^{\text{OSAF-A}}$  is just equal to 2 regardless of the number of relays and the ARQ rounds.

*Proof:* : By Proposition 2 and Lemma 2, the diversity order of  $P_n^{\text{OSAF-A}}$  could be easily verified. ■

The diversity orders of ARQ-OSAF-A can be verified with the outage probabilities presented in Fig. 3. Although only  $\rho|h_{j,rd}|^2$  is considered for relay selection in OSAF-A, the OSAF relaying scheme is able to exploit the temporal diversity through ARQs if  $\Delta > \delta_1$ . Nevertheless, the diversity order only increases by 1 in each round after the first round of ARQ. This limitation is due to the worse case of  $\mathcal{Q}$  in which only one relay is inside the set.

On the other hand for  $\Delta < \delta_1$ , the diversity order is limited to 2 due to the poor S-R channel qualities and the selection rule of OSAF. In comparison, the diversity order of ARQ-OAF is equal to  $m+1$  as both  $\rho|h_{j,sr}|^2$  and  $\rho|h_{j,rd}|^2$  are required for the destination to choose the best relay according to (8).

## B.2 ARQ with the type B of OSAF relaying (OSAF-B)

Based on the previous discussions on OSAF-A relaying, the key to further improve the diversity via ARQs is to increase the cardinality of  $\mathcal{Q}$ , denoted by  $|\mathcal{Q}|$ , through ARQs as well. This can be made possible only if the unqualified relays continue to overhear the signals transmitted by relays in  $\mathcal{Q}$  during the process of ARQs. If conditions can be set on the link qualities,  $\rho|h_{i,j}|^2$ , between the transmitting and receiving relays to qualify and bring new relays into  $\mathcal{Q}$ , then the diversity may no longer be limited to the case of  $|\mathcal{Q}| = 1$ . This type of OSAF scheme is referred to as the ARQ with the OSAF-B relaying. The functioning of the protocol is illustrated in Fig. 4.

As shown in the figure, the active relay,  $R_5$ , of  $\mathcal{Q}$  received the signal from  $R_3$  in the previous ARQ and is currently forwarding the signal to the destination. The relay  $R_6$  in the forbidden set, denoted by  $\overline{\mathcal{Q}}$ , overhears the signal from  $R_5$ . If  $c_4 \triangleq \rho|h_{5,6}|^2$  exceeds a threshold, say  $\Delta_4$  with 4 being the number of hops before reaching the destination, then  $R_6$  will be taken out of the forbidden set  $\overline{\mathcal{Q}}$  and join the qualified set  $\mathcal{Q}$ . In the next round of ARQ, if any, the destination still chooses the relay in  $\mathcal{Q}$  with the highest  $\rho|h_{j,d}|^2$  for forward-

ing, even if the signal from  $R_6$  has accumulated more noise through the hops from the source to  $R_2$ ,  $R_3$  and then  $R_5$ .

In general, for an active relay that forwards a signal already gone through  $p$  hops, the received SNR at the destination can be expressed as

$$\text{SNR}_d = \frac{1}{\frac{1}{a_1} + \frac{1}{c_2} + \cdots + \frac{1}{c_p} + \frac{1}{b_*^{[q]}} + (\text{higher order terms})} \quad (15)$$

where  $b_*^{[q]}$  represents the highest  $\rho|h_{j,d}|^2$  in  $\mathcal{Q}$  with  $q \triangleq |\mathcal{Q}|$ . Since the higher order terms can be ignored in the high SNR regime, the denominator becomes simplified into the reciprocal sum of all the passed channel qualities. As a result, the corresponding outage probability at high SNR becomes

$$\lim_{\rho \rightarrow \infty} P_{out} = \lim_{\rho \rightarrow \infty} \Pr \left\{ \frac{1}{b_*^{[q]}} > \frac{1}{\delta_1} - \left( \frac{1}{a_1} + \cdots + \frac{1}{c_p} \right) \right\}. \quad (16)$$

We next characterize the thresholds for (16) to achieve its full diversity. To this end, we first define a requirement on the reciprocal sum of  $\frac{1}{a_1} + \cdots + \frac{1}{c_p}$ .

*Requirement 1:* Given a fixed and arbitrarily small positive number  $\epsilon$ , for an active relay that forwards a signal already gone through  $p$  hops, the corresponding reciprocal sum satisfies  $\frac{1}{a_1} + \frac{1}{c_2} + \cdots + \frac{1}{c_p} \leq \xi \triangleq \frac{1}{\delta_1} + \epsilon$ .

For ARQ with OSAF-B relaying that satisfies Requirement 1, an upper bound can be obtained on the corresponding outage probability of (16), given by

$$\lim_{\rho \rightarrow \infty} P_{out} \leq \lim_{\rho \rightarrow \infty} \Pr \left\{ b_*^{[q]} < \frac{1}{\epsilon} \right\}. \quad (17)$$

Based on this upper bound, we have the next two lemmas.

*Lemma 3:* If there exists a qualified set  $\mathcal{Q}$  with  $q = |\mathcal{Q}|$  and in  $\mathcal{Q}$ , every relay chosen by the destination according to the OSAF method satisfies Requirement 1, then the diversity order contributed by the relaying is  $q$ .

*Lemma 4:* Following Lemma 3, if the listening relays to be added into  $\mathcal{Q}$  also satisfy Requirement 1, then the maximum diversity order that can be achieved for each ARQ is  $m$ .

Based on the above results, we arrive at a theorem for the diversity order of the ARQ-OSAF-B scheme.

*Theorem 1:* If all relays chosen according to OSAF-B out of  $\mathcal{Q}$  satisfy Requirement 1, then the diversity order of the outage probability after  $n$  rounds of ARQ-OSAF-B is given by  $(m \times n + 1)$ .

*Proof:* By Proposition 3, at the first ARQ round, the OSAF-B scheme achieves a diversity order of  $(m+1)$  as the first ARQ of OSAF-B is exactly the same to that of OSAF-A. For the subsequent rounds of ARQs, by Lemma 4, the diversity order increases by  $m$  with every extra ARQ round. As a result, the overall diversity of the outage probability is equal to  $(m \times n + 1)$  after  $n$  rounds of ARQ-OSAF-B. ■

## B.3 Threshold assignment for ARQ-OSAF-B

Apparently, for ARQ-SAF and ARQ-OSAF-A, the threshold  $\Delta$  should be set greater than  $\delta_1$  according to Lemma 2 and Proposition 3, respectively. For ARQ-OSAF-B however, the source signal may go through multiple hops before arriving at the destination. Thus we need to define a threshold for each hop to control the channel quality of the entire

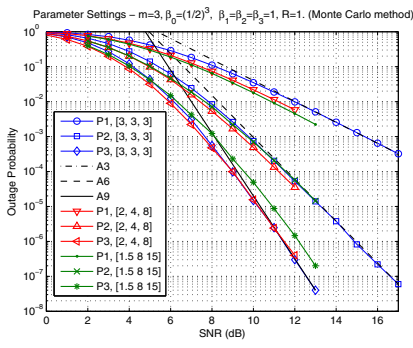


Fig. 5. Outage probabilities of ARQ-OSAF-B. The diversity contributed by  $\Pr\{w < \delta_1\}$  is ignored here. Therefore, the full diversity becomes  $(m \times i)$ .  $P_i$  denotes the outage probability after  $i$  rounds of ARQs, and  $[a, b, c]$  stands for the thresholds of  $\delta_1 \times [a, b, c]$ . Besides,  $A_j$  corresponds to the asymptotic line of diversity order  $j$  at high SNR.

relaying path. Since the maximal number of hops is limited to  $\min[m, \mathcal{N}]$ , we therefore define an array of thresholds as  $[\Delta_1, \dots, \Delta_i, \dots, \Delta_{\min[m, \mathcal{N}]}]$  with  $\Delta_i$  corresponding to the threshold for the  $i$ -th hop.

Fig. 5 demonstrates the outage probabilities for three different assignments of  $\Delta_i$  with  $m = 3$  and  $\mathcal{N} = 3$ . The thresholds are  $[3\delta_1, 3\delta_1, 3\delta_1] + \epsilon/3$ ,  $[2\delta_1, 4\delta_1, 8\delta_1]$  and  $[1.5\delta_1, 8\delta_1, 15\delta_1]$ , respectively, and all satisfy Requirement 1. As characterized by Theorem 1, all lead to the full diversity order at high SNR, while with small offsets among them.

## V. NUMERICAL STUDIES

We study herein the performance of the proposed OSAF schemes from the perspective of delay-limited (DL) throughput [8]. The DL throughput is defined as

$$\eta = R - \sum_{l=1}^{\mathcal{N}} \frac{R}{l \times (l+1)} P_{l-1} - \frac{R}{\mathcal{N}+1} P_{\mathcal{N}}. \quad (18)$$

According to this metric, we study the performance with the rate assignment obtained with

$$\max_{R \geq 0} \eta \quad \text{subject to } P_{\mathcal{N}} \leq P_e. \quad (19)$$

Two scenarios are considered in the study to characterize the effects of good and bad S-R channel qualities. The variances of the channel coefficients in Fig. 6 are assigned as  $\beta_0 = (\frac{1}{2})^v$ ,  $\beta_1 = 2^v$ , and  $\beta_2 = 1$ , and the variances for Fig. 7 are  $\beta_0 = (\frac{1}{2})^v$ ,  $\beta_1 = (\frac{1}{2})^v$  and  $\beta_2 = 1$ . Besides, for opportunistic relaying, the variance  $\beta_3$  for the channels between relays is set to  $2^v$ . The path loss exponent  $v$  and the target outage probability  $P_e$  are set to 3 and  $10^{-3}$ , respectively.

For comparisons, the results of DF relaying are also presented in the figures, which include those of ARQ-DF and ARQ-OSDF-B of the DF counterparts of ARQ-AF and ARQ-OSAF-B, respectively. It can be seen from Fig. 6 that the throughput of ARQ-OAF can approach that of the OSAF schemes since the S-R channel qualities are in good condition. However, in poor S-R channel conditions as illustrated in Fig. 7, the performance of ARQ-OAF deteriorates significantly. Although the throughput of OSAF-A is slightly worse than that of OSAF-B, its performance is

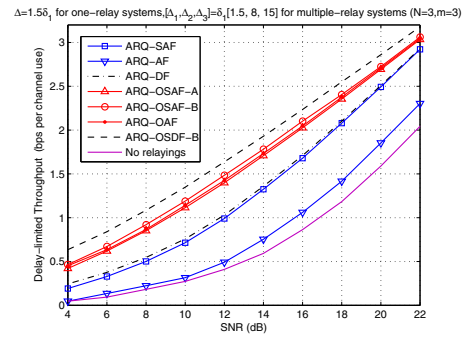


Fig. 6. The throughput of various ARQ schemes with  $\beta_0 = (\frac{1}{2})^3$ ,  $\beta_1 = 2^3$ , and  $\beta_2 = 1$  to simulate a good S-R channel condition.

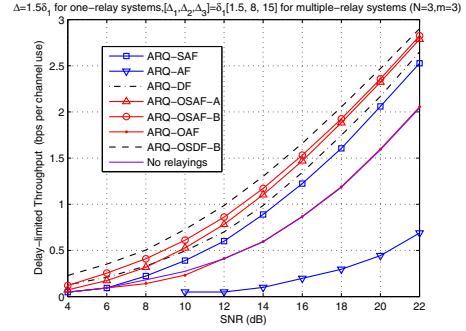


Fig. 7. The throughput of various ARQ schemes with  $\beta_0 = (\frac{1}{2})^3$ ,  $\beta_1 = (\frac{1}{2})^3$  and  $\beta_2 = 1$  to simulate a poor S-R channel condition.

in fact pronounced considering its much simpler mechanism for relaying. In general, the ARQ schemes with OSAF relaying are more robust than the typical AF relaying, and their performance are very close to their DF counterparts.

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