Bandwidth-Constrained Multistage Bayesian Game for Spectrum Trading in Cognitive Radio Networks

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***Abstract*—In this paper, we study the problem of spectrum trading in cognitive radio (CR) networks from a game theoretical perspective. Particularly, we consider the CR network with multiple primary services (PS's) and a single secondary service (SS), where all PS's are sellers competing in the prices and the SS is the buyer deciding how much spectrum is demanded from each PS in the trading game. Aiming at dealing with the trading behaviors, we propose using a multistage Bayesian game based trading model to account for possible unknown private information in each player, and obtain the perfect Bayesian equilibrium (PBE) sequentially under a bandwidth constraint. By the backward induction principle, we translate the Karush-Kuhn-Tucker (KKT) condition of the SS to form a joint KKT conditions with PS in order to solve the optimization problem. Finally, in the simulations, we compare the proposed approach with that in [1], and numerically study the convergence behaviors of the proposed multistage game.**

***Index Terms*—Spectrum Sharing, Cognitive Radio, Game Theory, Bayesian Game, Joint KKT Condition**

I. Introduction

The promise of providing anytime and anywhere multimedia services demands a large spectrum for broadband wireless communications. On one hand, this drives the advance of radio technology to realize faster, convenient and reliable communications. On the other hand, the enormous demand also unveils the problem of insufficiency and under-utilized inefficiency of current radio spectrum. To tackle the problem, instead of introducing new portions of the radio spectrum, the idea of exploiting under-utilized spectrum, owned by legacy operators, for more flexible and efficient transmissions is receiving significant attentions lately. In particular, the concept of cognitive radios is considered as a promising technique to improve the efficiency of current radio spectrum [2].

A cognitive radio (CR) is a software-defined radio capable of intelligently sensing, adapting and responding to constantly varying environments, particularly the available spectrum temporarily not used by licensed users. However, there still exist many technical challenges before cognitive radios can be practically deployed. One of the critical challenges is how to encourage the licensed service operators so that they are willing to share their unused spectrum to unlicensed cognitive (secondary) services. Leasing available spectrum to unlicensed services is an attractive solution that provides an incentive for legacy operators to support deploying cognitive radios [3]. This gains monetary profits for licensed operators, while fulfilling unlicensed services' satisfaction requirements by renting. Since both licensed and unlicensed services are interactive, rational, and selfish decision makers, it is natural to resort to game theory to analyze the micro-economics and design the best decision strategies in the CR networks.

There have been several studies focusing on the issues of spectrum trading from the game theory perspective [1], [4], [5]. An overview of the general idea and recent developments can be found in [4]. The auction mechanism in CR networks with multiple primary and multiple secondary users is considered in [6], where the authors discuss cheating behaviors in the game and propose using reserve prices and beliefs to prevent collusion. The work in [5] and [1] consider a game model which incorporates both monetary gain and quality-of-service (QoS) satisfaction of wireless services in utility functions. The authors in [5] explicitly model the price of available bandwidth as a function of demand, and obtain the Nash equilibrium (NE) for the spectrum sharing strategy in a network consisting of a single primary service (PS) and multiple secondary services (SS). The work in [1] considers the spectrum trading game in a CR network with multiple PS and a single SS, and models the utilities of the PS and SS separately, wherein the demand of SS implicitly affects the price. However, under certain circumstances, the optimal bandwidth demand for the SS would be negative, and the corresponding NE turns out to be practically infeasible, though theoretically solvable.

In this paper, we address the problem of spectrum trading in a cognitive radio network consisting of multiple PS's and a single SS, as in [1]. Particularly, we assume each player (PS or SS) in the game has its own private information, such as the number of local connections within each service or the channel conditions, that is unknown to other players. With the assumption, we formulate a multistage trading model based on the Bayesian game to statistically account for the unknown private information (incomplete information), and sequentially obtain the perfect Bayesian equilibrium (PBE) in the trading processes. Furthermore, we consider a bandwidth constraint for the demand of the SS such that the demand has to be within feasible supply regions provided by each PS. By the backward induction principle, we translate the SS's Karush-Kuhn-Tucker (KKT) conditions into a constraint of the PS, and form a joint KKT conditions in order to solve the involved sequential optimization problem in finding the PBE. Finally, in the simulations, we compare our work with [1], and justify the convergence of the sequence of strategies in the proposed multistage Bayesian game.

II. System Model

We consider a cognitive radio network with N primary services (*e.g.* the existing cellular services) and 1 secondary

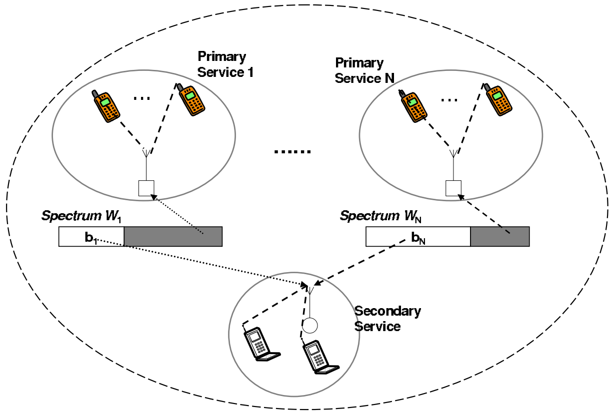

Fig. 1. The cognitive radio network with multiple primary services and a single secondary service .

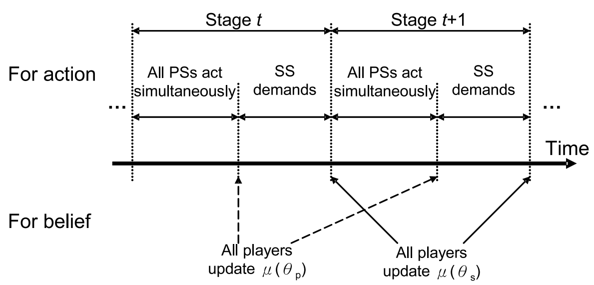

Fig. 2. The evolution of the multistage game.

service (*e.g.* a CR base station with several CR users) as shown in Fig. 1. The ith PS operates on its own exclusive spectrum W_i, within which the ith PS can lease available unused bandwidth to the SS, the player who doesn't have the private spectrum. In the trading process, all PS's compete with each other in the prices they offer to the SS, and the SS decides from whom and how much of the available spectrum to rent. Specifically, we model the spectrum trading process as a multistage game in a manner that all PS's set their own prices, p_i per unit bandwidth for the ith PS, simultaneously at first, and then in a subsequent stage the SS requests bandwidth b_i from the ith PS, for $i = 1, 2, \cdots, N$, according to a net profit the SS can acquire. Practically, each player (PS and SS) may possess its own private information that is unknown to other players. Therefore, each player cannot predict the overall trading behaviors correctly, which makes the decisions of optimal strategies a challenging task. In this incomplete information game, we propose using the theory of Baysian game to deal with the problem. The Bayesian approach allows each player to update its posterior beliefs, *i.e.* probabilities, about the others' private information by observing their actions in prior stages. And, each player can act in the current stage based on the updated beliefs. In this work, our objective is to find the perfect Bayesian equilibrium of the multistage Bayesian spectrum trading game that maximizes the net profit of the PS and the SS alternately as the game evolves sequentially. The evolution of the multistage game is illustrated in Fig. 2.

III. Spectrum Trading Game

A. Utility Model

In this work, we adopt and modify the utility model in [1]. The profit function of the ith PS is given by

$$\mathcal{P}_i(p_i, b_i|\theta_{pi}) = p_i b_i + c_1\theta_{pi} - c_2\theta_{pi}\left(\mathrm{B_i^{req}} - k_i^{(p)}\frac{W_i - b_i}{\theta_{pi}}\right)^2, \tag{1}$$

where c_1 and c_2 are constant weights, $\mathrm{B_i^{req}}$ is the bandwidth requirement for a primary connection, $k_i^{(p)} = \log_2\left(1 + \frac{1.5\gamma_i}{\ln(0.2/\mathrm{BER^{tar}})}\right)$ denotes the spectral efficiency of wireless transmission for the ith PS with γ_i being the signal-to-noise ratio (SNR) at the ith PS's receivers and $\mathrm{BER_i^{tar}}$ being the target bit-error-rate (BER) for the ith PS's local connection [7]. The private informationθ_{pi}, taking values in the set Θ_p, represents the number of connections in the ith PS. The first term in (1) is the monetary gain of selling bandwidths. The second term is the revenue of maintaining primary connections that is proportional to θ_{pi}. The third term is the cost of sharing the spectrum with SS.

The profit function of SS is given by

$$\mathcal{P}_s(\boldsymbol{p}, \boldsymbol{b}|\theta_s) = \frac{1}{\theta_s}\left[\sum_i^N b_i k_i^{(s)} - \frac{1}{2}\left(\boldsymbol{b}^T\boldsymbol{b} + 2\xi\sum_{j\neq i} b_j b_i\right)\right] - \boldsymbol{p}^T\boldsymbol{b}, \tag{2}$$

where $\boldsymbol{p} = (p_1, p_2, ..., p_N)^T$ is the price profile, $\boldsymbol{b} = (b_1, b_2, ..., b_N)^T$ is the demand profile, ξ is spectrum substitutability as defined in [1], and $k_i^{(s)}$ denotes the spectral efficiency of the bandwidth b_i that is acquired by SS.

Compared with the utility in [1], we introduce the private information θ_s of SS in this paper to represent the factor leveraging the weighting between QoS and the spectrum trading expense. This weighting factor is implicitly related to the number of active connections within SS. When there is no connections requested by the cognitive users, the SS must have zero profit in terms of QoS.

Other notations we will use in the paper are defined as follows. We denote the type profile of the private information as $\boldsymbol{\theta} = (\boldsymbol{\theta}_p, \theta_s)$, where $\boldsymbol{\theta}_p = (\theta_{p1}, \theta_{p2}, ...\theta_{pN})$. The actual type value for the ith PS and the SS is denoted by $\widehat{\theta}_{pi}$ and $\widehat{\theta}_s$, respectively. The corresponding type profile is represented by $\widehat{\boldsymbol{\theta}} = (\widehat{\boldsymbol{\theta}}_p, \widehat{\theta}_s)$. Furthermore, $\boldsymbol{\theta}_{-i}$ represents all elements of $\boldsymbol{\theta}$ except θ_{pi}. Similarly, $\boldsymbol{\theta}_{-s}$ is $\boldsymbol{\theta}$ except θ_s, and $\boldsymbol{\theta}_{-pi}$ is $\boldsymbol{\theta}_p$ without θ_{pi}. Likewise for $\boldsymbol{b}_{-i}$ and $\boldsymbol{p}_{-i}$. The sets Θ_p and Θ_s of private information are assumed discrete in the work. Nevertheless, they can be in general either discrete or continuous, accordingly depending on the model considered.

B. Belief System

We assume that each player has no knowledge of other players' private information. Nevertheless, each player can form posterior beliefs about this incomplete information. In this paper, the posterior belief of the ith PS's and the SS about other players' private information is respectively denoted by $\mu_{pi}(\boldsymbol{\theta}_{-pi}|\theta_{pi}, \boldsymbol{h}^t)$ and $\mu_s(\boldsymbol{\theta}_{-s}|\theta_s, \boldsymbol{h}^t)$, where $\boldsymbol{h}^t = (\mathbf{a}^0, \mathbf{a}^1, ...\mathbf{a}^{t-1})^T$ is the action history at the beginning of stage

t that contains all the performed actions prior to stage t with $\mathbf{a}^\tau = (\boldsymbol{p}^\tau, \boldsymbol{b}^\tau)$ being the action profile at stage τ and $\boldsymbol{h}^0 = \varnothing$. We assume that each player has independent and identical initial beliefs to all other users' possible types of their private information. With the assumption, the belief can be updated stage by stage using Bayes' rule, and the players' optimal strategies will change according to the the beliefs updates at each stage. In the game theory literature, the *solution concept*, *i.e.* the rule for predicting how the game will be played, in a dynamic game with incomplete information is called the perfect Bayesian equilibrium (PBE). The belief update rule and conditions for the existence of PBE can be found in [8].

C. Self-Optimization and KKT Translation

Since the SS is a follower of the game, it can observe the sellers' actions $\boldsymbol{p}(\hat{\boldsymbol{\theta}}_p) = (p_1(\hat{\theta}_{p1}), p_2(\hat{\theta}_{p2}), ..., p_N(\hat{\theta}_{pN}))^T$, which gives the profit $\mathcal{P}_s(\boldsymbol{p}(\hat{\boldsymbol{\theta}}_p), \boldsymbol{b}|\theta_s)$ of the SS. Note that $\boldsymbol{p}(\hat{\boldsymbol{\theta}}_p)$ is the optimal price profile corresponding to $\hat{\boldsymbol{\theta}}_p$, and SS can only observe the prices but not $\hat{\boldsymbol{\theta}}_p$. Because of the incomplete information, the secondary service aims at finding the best demand function that maximizes the *expected* profit

$$\boldsymbol{b}^*(\hat{\theta}_s, \hat{\boldsymbol{\theta}}_p) = arg \max_{\boldsymbol{b}} E_{\boldsymbol{\theta}_p}[\mathcal{P}_s(\boldsymbol{p}(\hat{\boldsymbol{\theta}}_p), \boldsymbol{b}|\widehat{\theta}_s)] \quad (3)$$

$$= arg \max_{\boldsymbol{b}} \mathcal{P}_s(\boldsymbol{p}(\hat{\boldsymbol{\theta}}_p), \boldsymbol{b}|\widehat{\theta}_s). \quad (4)$$

The KKT condition of SS with type $\hat{\theta}_s$ for solving the above optimization problem is $\nabla_{\boldsymbol{b}}\mathcal{P}_s(\boldsymbol{p}(\hat{\boldsymbol{\theta}}_p), \boldsymbol{b}^*(\hat{\theta}_s, \hat{\boldsymbol{\theta}}_p)|\hat{\theta}_s) = \mathbf{0}$.

According to the backward induction principle [8], all PS's assume that the SS will ask for the best demand $\boldsymbol{b}^*(\hat{\theta}_s, \hat{\boldsymbol{\theta}}_p)$. This is equivalent to saying that all primary services know the KKT condition of SS. However, since PS i doesn't know the exact type $\hat{\theta}_s$ and $\hat{\theta}_{-pi}$ exactly, PS i views $\boldsymbol{b}^*(\hat{\theta}_{pi}, \boldsymbol{\theta}_{-i})$ as a random variable with uncertain $\boldsymbol{\theta}_{-i}$. Here, the objective of PS i is to maximize its expected profit based on the beliefs $\mu(\theta_s, \boldsymbol{\theta}_{-pi}|\boldsymbol{h}^t)$ about other players' private information, considering the KKT condition of SS. The optimization for PS i of type $\widehat{\theta}_{pi}$ is therefore given by

$$p_i^*(\hat{\theta}_{pi}) = arg \max_{p_i} E_{\boldsymbol{\theta}_{-i}}[\mathcal{P}_i(p_i, b_i^*(\hat{\theta}_{pi}, \boldsymbol{\theta}_{-i})|\widehat{\theta}_{pi})], \quad (5)$$

$$\text{s.t. } 0 \le b_i^*(\hat{\theta}_{pi}, \boldsymbol{\theta}_{-i}), \quad \forall \boldsymbol{\theta}_{-i} \in \Pi_{-i}, \quad (6)$$

$$W_i \ge b_i^*(\hat{\theta}_{pi}, \boldsymbol{\theta}_{-i}), \quad \forall \boldsymbol{\theta}_{-i} \in \Pi_{-i}, \quad (7)$$

$$\nabla_{\boldsymbol{b}}\mathcal{P}_s(\boldsymbol{p}(\hat{\theta}_{pi}, \boldsymbol{\theta}_{-pi}), \boldsymbol{b}^*(\hat{\theta}_{pi}, \boldsymbol{\theta}_{-i})|\hat{\theta}_s) = \mathbf{0}, \quad \forall \boldsymbol{\theta}_{-i} \in \Pi_{-i}, \quad (8)$$

where $\Pi_{-i} = \{\boldsymbol{\theta}_{-i}|\mu(\boldsymbol{\theta}_{-i}|\boldsymbol{h}^t) > 0\}$ is the set of all possible $\boldsymbol{\theta}_{-i}$'s that satisfy $\mu(\boldsymbol{\theta}_{-i}|\boldsymbol{h}^t) > 0$. The constraints in (6) and (7) limit the demand to be within the physically realizable spectrum region afforded by PS i under all possible type profiles of the other players. The constraint in (8) is the KKT condition of SS, which is imposed here following the backward induction principle. In this work, we call this approach the *KKT translation*. With the utility model considered in the paper, the strategy $b^*(\boldsymbol{\theta})$ has a closed-form solution. Consequently, we can replace SS's KKT condition in (8) by $b^*(\boldsymbol{\theta})$. It follows that the best demand $b_i^*(\boldsymbol{\theta})$ of SS to the ith PS is given by

$$\begin{aligned} b_i^*(\boldsymbol{\theta}) &\triangleq \mathcal{D}_i(\boldsymbol{p}(\boldsymbol{\theta}_p), \theta_s) \\ &= D_1(\boldsymbol{p}_{-i}(\boldsymbol{\theta}_{-pi}), \theta_s) - \theta_s p_i(\theta_{pi}) D_2 \end{aligned}$$

where $D_1(\boldsymbol{p}_{-i}(\boldsymbol{\theta}_{-pi}), \theta_s) = \frac{C_i}{A} + \frac{\xi\theta_s \sum_{j\neq i} p_j(\theta_{pj})}{A}$, $D_2 = \frac{\xi(N-2)+1}{A}$ with $A = (1-\xi)(\xi(N-1)+1)$, $C_i = k_i^{(s)}(\xi(N-2)+1) - \xi\sum_{j\neq i} k_j^{(s)}$.

Notably, we observe that as $p_j(\theta_{pj})$ increases, $\mathcal{D}_i(\boldsymbol{p}(\boldsymbol{\theta}_p), \theta_s)$ also increases; while as $p_i(\theta_{pi})$ increases, $\mathcal{D}_i(\boldsymbol{p}(\boldsymbol{\theta}_p), \theta_s)$ decreases. With the observations, we can greatly reduce the number of inequalities in (6) and (7) by finding the minimal sets $\Theta_{m,i}$ and $\Theta_{M,i}$ to represent these two inequalities (*i.e.* the minimal sets that include the minimum and maximum of $\boldsymbol{b}_i^*(\hat{\theta}_{pi}, \boldsymbol{\theta}_{-i})$. The minimum of $\mathcal{D}_i(\boldsymbol{p}(\boldsymbol{\theta}_p), \theta_s)$ occurs when $p_i(\theta_{pi})$ reaches its highest value and $p_j(\theta_{p_j})$ is at the lowest value. A similar reasoning can be applied to explain the maximum of $\mathcal{D}_i(\boldsymbol{p}(\boldsymbol{\theta}_p), \theta_s)$. Therefore, the minimal sets for the constraints in (6) and (7) are given by

$$\Theta_{m,i} = \{\boldsymbol{\theta}_{m,1} \triangleq (\boldsymbol{\theta}_{-pi}^m, \theta_s^m), \boldsymbol{\theta}_{m,2} \triangleq (\boldsymbol{\theta}_{-pi}^m, \theta_s^M)\}, \quad (9)$$

$$\Theta_{M,i} = \{\boldsymbol{\theta}_{M,1} \triangleq (\boldsymbol{\theta}_{-pi}^M, \theta_s^m), \boldsymbol{\theta}_{M,2} \triangleq (\boldsymbol{\theta}_{-pi}^M, \theta_s^M)\}, \quad (10)$$

where $\boldsymbol{\theta}_{-pi}^m$ and $\boldsymbol{\theta}_{-pi}^M$ are the elementwise minimum and elementwise maximum of $\boldsymbol{\theta}_{-pi}$ with $\mu(\boldsymbol{\theta}_{-pi}^m|\boldsymbol{h}^t) > 0$, and $\mu(\boldsymbol{\theta}_{-pi}^M|\boldsymbol{h}^t) > 0$, respectively. And, θ_s^m and θ_s^M are the minimum and the maximum of θ_s with $\mu(\theta_s^m|\boldsymbol{h}^t) > 0$ and $\mu(\theta_s^M|\boldsymbol{h}^t) > 0$, respectively.

D. Perfect Bayesian Equilibrium and Joint KKT Conditions

We are now ready to find the PBE at stage t of the multistage Bayesian game modeled in the considered cognitive radio network. To obtain PBE, we have to solve

$$\begin{aligned} p_i^*(\theta_{pi}) = arg \max_{p_i} & E_{\boldsymbol{\theta}_{-i}}[\mathcal{P}_i(p_i, b_i^*(\theta_{pi}, \boldsymbol{\theta}_{-i})|\theta_{pi})] \\ \text{s.t. } & 0 \le b_i^*(\theta_{pi}, \boldsymbol{\theta}_{-i}), \forall \boldsymbol{\theta}_{-i} \in \Theta_{m,i}, \\ & W_i \ge b_i^*(\theta_{pi}, \boldsymbol{\theta}_{-i}), \forall \boldsymbol{\theta}_{-i} \in \Theta_{M,i}, \\ & b_i^*(\theta) = \mathcal{D}_i(p_i(\theta_{pi}), \boldsymbol{p}_{-i}^*(\boldsymbol{\theta}_{-pi}), \theta_s), \forall \boldsymbol{\theta}_{-i} \in \Pi_{-i}, \\ & \forall \theta_{pi} \in \mu(\theta_{pi}|\boldsymbol{h}^t) > 0, \forall i = 1, 2, ..., N. \end{aligned}$$

It is clear that if the price profile $\boldsymbol{p}_{-i}^*(\boldsymbol{\theta}_{-pi})$ for all type profiles is known, then the KKT condition is sufficient and necessary for solving the convex optimization problem. However, finding the optimal strategy profile $\boldsymbol{p}_{-i}^*(\boldsymbol{\theta}_{-pi})$ for all possible $\boldsymbol{\theta}_{-pi}$ needs the information of $p_i^*(\theta_{pi})$ for all possible θ_{pi}. It follows that each player has to jointly solve all PS's KKT conditions simultaneously. The joint KKT conditions are given by

$$\left\{ \begin{aligned} -\mathcal{D}_i(\boldsymbol{\theta}) &\le 0, \forall\, \boldsymbol{\theta}_{-i} \in \Theta_{m,i}, \\ \mathcal{D}_i(\boldsymbol{\theta}) - W_i &\le 0, \forall\, \boldsymbol{\theta}_{-i} \in \Theta_{M,i}, \\ \lambda_{i,\boldsymbol{\theta}_{-i}} &\ge 0, \forall\, \boldsymbol{\theta}_{-i} \in \Theta_{m,i}, \\ \nu_{i,\boldsymbol{\theta}_{-i}} &\ge 0, \forall\, \boldsymbol{\theta}_{-i} \in \Theta_{M,i}, \\ \lambda_{i,\boldsymbol{\theta}_{-i}} \mathcal{D}_i(\boldsymbol{\theta}) &= 0, \forall\, \boldsymbol{\theta}_{-i} \in \Theta_{m,i}, \\ \nu_{i,\boldsymbol{\theta}_{-i}}(\mathcal{D}_i(\boldsymbol{\theta}) - W_i) &= 0, \forall\, \boldsymbol{\theta}_{-i} \in \Theta_{M,i}, \\ \nabla_{p_i}\mathcal{L}_i(\theta_{pi}) &= 0, \end{aligned} \right. \quad (11)$$

$$\forall \theta_{pi} \in \mu(\theta_{pi}|\boldsymbol{h}^t) > 0, \forall\, i = 1, 2, ..., N.$$

where $\lambda_{i,\boldsymbol{\theta}_{-i}}$ and $\nu_{i,\boldsymbol{\theta}_{-i}}$ are Lagrange multipliers, $\mathcal{L}_i(\theta_{pi})$ is the Lagrangian function of the ith PS with type θ_{pi}, and we use $\mathcal{D}_i(\boldsymbol{\theta})$ to represent $\mathcal{D}_i(p(\theta_p), \theta_s)$ for notational clarity. Since $\nabla_{p_i} E_{\boldsymbol{\theta}_{-i}}[\mathcal{P}_i(p_i, \mathcal{D}_i(\boldsymbol{\theta})|\theta_{pi})] = E_{\boldsymbol{\theta}_{-i}}[\nabla_{p_i}\mathcal{P}_i(p_i, \mathcal{D}_i(\boldsymbol{\theta})|\theta_{pi})]$, we have

$$\begin{aligned}\nabla_{p_i}\mathcal{L}_i(\boldsymbol{\theta}_{pi}) &= E_{\boldsymbol{\theta}_{-i}}[\nabla_{p_i}\mathcal{P}_i(p_i, \mathcal{D}_i(\boldsymbol{\theta})|\theta_{pi})] \\ &= E_{\boldsymbol{\theta}_{-i}}\Big[H(\theta_s,\theta_{pi}) + F(\theta_s,\theta_{pi})\sum_{j\neq i} p_j(\theta_{pj})\Big] \\ &\quad - E_{\boldsymbol{\theta}_{-i}}\left[G(\theta_s,\theta_{pi})p_i(\theta_{pi})\right] \\ &\quad + \theta_s^m D_2(\nu_{i,\theta_{M,1}} - \lambda_{i,\theta_{m,1}}) \\ &\quad + \theta_s^M D_2(\nu_{i,\theta_{M,2}} - \lambda_{i,\theta_{m,2}})\end{aligned}$$

with $H(\theta_s,\theta_{pi}) = \frac{C_i}{A} + 2c_2 k_i^{(p)}\theta_s D_2\left(\mathrm{B}_{\mathrm{i}}^{\mathrm{req}} - k_i^{(p)}\frac{W_i - \frac{C_i}{A}}{\theta_{pi}}\right)$, $F(\theta_s,\theta_{pi}) = \left(\frac{\xi\theta_s}{A} + 2c_2 k_i^{(p)2}\theta_s^2 D_2\frac{\xi}{A\theta_{pi}}\right)$, and $G(\theta_s,\theta_{pi}) = \left(2\theta_s D_2 + 2c_2 k_i^{(p)2}\theta_s^2\frac{D_2^2}{\theta_{pi}}\right)$. The above joint KKT conditions can be solved by active-set method [9], which is summarized in Algorithm 1.

Algorithm 1 Active-set method for solving joint KKT condition

0: **Define**: $S \triangleq \bigsqcup_i \Theta_{m,i} \cup \Theta_{M,i}$, and W is the working set.
1: **Initialize**: Set $W = \emptyset$.
2: **Repeat**: Solve the joint KKT conditions (N linear equations) with that $\lambda_{i,\theta_{-i}} = 0$ and $\nu_{i,\theta_{-i}} = 0$ for those constraints $\notin W$.
3: Condition 1: Check whether $\mathcal{D}_i(p_i(\theta_{pi}^M), p_{-i}(\theta_{-pi}), \theta_s)$ satisfies (6) $\forall\theta_{-i} \in \Theta_{m,i}, \forall i$.
4: Condition 2: Check whether $\mathcal{D}_i(p_i(\theta_{pi}^m), p_{-i}(\theta_{-pi}), \theta_s)$ satisfies (7) $\forall\theta_{-i} \in \Theta_{M,i}, \forall i$.
5: Condition 3: Check whether $\lambda_{i,\theta_{-i}} \geq 0$ and $\nu_{i,\theta_{-i}} \geq 0$, $\forall\theta_{-i} \in W, \forall i$.
6: **If** conditions 1,2, and 3 all are satisfied, **then** we obtain the optimal $\boldsymbol{p}_i^*(\theta_{pi})\ \forall\theta_{pi} \in \mu(\theta_{pi}|h^t) > 0, \forall i$ and finish.
7: **Else** choose another $W \subset S$.
8: **End repeat**

IV. Simulations

A. Simulation Setup

In this section, we consider an example with two PS's and one SS for numerical simulations. The type space of PS is set to be $\Theta_P = \{10, 11, 12\}$. The actual number of connections for PS 1 and PS 2 is $\hat{\theta}_{p1} = 10$ and $\hat{\theta}_{p2} = 10$, respectively. We set the type space of SS as $\Theta_S = \{1, 2, 3, 4\}$, with the actual weighting of SS being $\hat{\theta}_s = 1$. The initial beliefs are uniformly distributed $\mu(\theta_{p1}|\boldsymbol{h}^0) = \mu(\theta_{p2}|\boldsymbol{h}^0) = \frac{1}{3}$, and $\mu(\theta_s|\boldsymbol{h}^0) = \frac{1}{4}$. The received SNR's are $\gamma_{p1} = 15$ dB, $\gamma_{p2} = 15$ dB, $\gamma_{s1} = 22$ dB, and $\gamma_{s2} = 22$ dB. The constants in the PS's utility are chosen as $c_1 = 2$ and $c_2 = 2$, and the spectrum substitutability ξ is 0.4.

B. Numerical Results

1) The Joint KKT Conditions: First, we simulate the multi-stage game with complete information and compare the results of the proposed joint KKT conditions with those in [1] that correspond to the game without constraints (WOC). Note that the constraint for b_i in (6) is denoted here as $f_{i,1}$, and that in (7) is denoted here as $f_{i,2}$. We show the best responses and the corresponding feasible regions in both Fig. 3 and Fig. 4, respectively with $W_1 = 15$ MHz, $W_2 = 15$ MHz, and $W_1 = 5$ MHz, $W_2 = 5$ MHz. Furthermore, we assume the bandwidth requirements are $\mathrm{B}_1^{\mathrm{req}} = 2$ Mbps for PS 1, and $\mathrm{B}_2^{\mathrm{req}} = 2$ Mbps for PS 2 in plotting this figure. The intersection of the best responses is the Nash equilibrium (NE). In Fig. 3, with $W_1 = 15$ MHz and $W_2 = 15$ MHz, the WOC solution satisfies the bandwidth constraints, so it agrees with the solution of the proposed joint KKT conditions. In Fig. 4, with $W_1 = 5$ MHz, $W_2 = 5$ MHz and $\gamma_{s1} = 22$ dB, $\gamma_{s2} = 10$ dB, the WOC solution lies outside the bandwidth constraints, while the optimal strategies $b_1^* = 0$ and $b_2^* = 0$ of the joint KKT conditions satisfy the constraint.

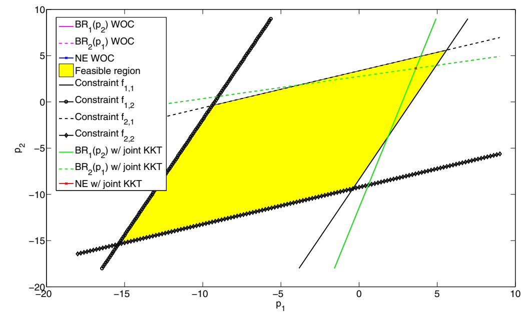

Fig. 3. The best response, Nash equilirium and feasible region for PS's with $W_1 = 15$ MHz and $W_2 = 15$ MHz.

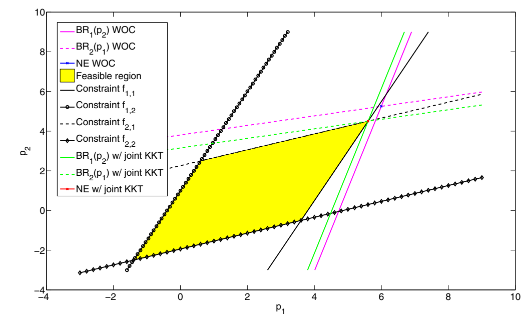

Fig. 4. The best response, Nash equilirium and feasible region for PS's with with $W_1 = 5$ MHz and $W_2 = 5$ MHz.

2) Evolutions of the Actions: Next, we study the behavior of the sequence of optimal strategies as stage evolves under $W_1 = 15$MHz and $W_2 = 15$MHz. The Bayesian game model allows the optimal strategies to update according to the beliefs of all players' private information. Fig. 5 shows the effects of different type profiles on the best demands b_1^* and b_2^*, and the optimal pricing p_1^* and p_2^*. We see that the SS with $\hat{\theta}_s = 2$ demands less than that with $\hat{\theta}_s = 1$. This is because the SS with $\hat{\theta}_s = 2$ puts less emphasis on the QoS satisfaction, or equivalently, is more concerned with the monetary expense. Since the SS with $\hat{\theta}_s = 2$ demands less, both PS's would offer lower prices to the SS with $\hat{\theta}_s = 2$ in order to promote the trading process. The penalty, or the cost, of sharing spectrum for PS 1 with $\hat{\theta}_{p1} = 12$ is higher than that with $\hat{\theta}_{p1} = 10$, since the PS with a higher volume of local

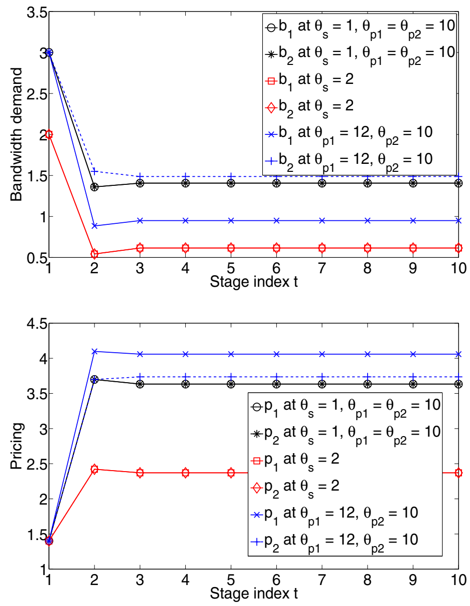

Fig. 5. (Upper) The optimal strategies of the SS at different stages with different types of private information. (Lower) The optimal strategies of the PS at different stages with different types of private information.

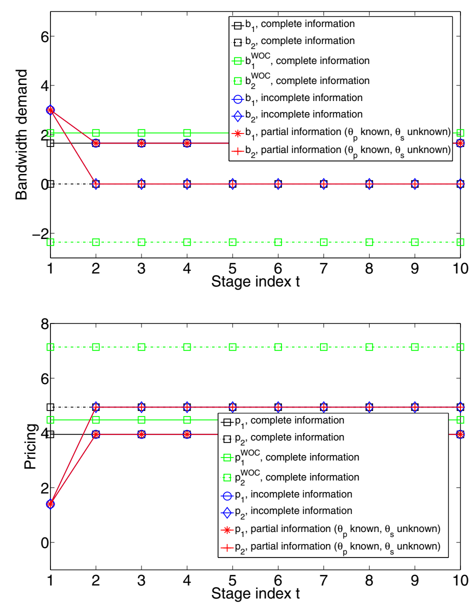

Fig. 6. (Upper) The optimal strategies of the SS at different stage with $\mathrm{B}_1^{\mathrm{req}} = 2$, and $\mathrm{B}_2^{\mathrm{req}} = 4$. (Lower) The optimal strategies of the PS at different stage with $\mathrm{B}_1^{\mathrm{req}} = 2$, and $\mathrm{B}_2^{\mathrm{req}} = 4$.

connections is more reluctant to share the spectrum, in order to fulfill its primary users' QoS satisfaction. Consequently, PS 1 of $\hat{\theta}_{p1} = 12$ would set a higher price, that yields a lower b_1^*. Under this circumstance, the SS demands more b_2^* from PS 2 to compensate the insufficiency of b_1^*. As a result, PS 2 increases the price p_2^* to maximize its profit.

3) Convergence of Beliefs and Actions: Next, we study in Fig. 6 the convergence behaviors of the sequence of action profiles with the settings of $\mathrm{B}_1^{\mathrm{req}} = 2$ Mbps, $\mathrm{B}_2^{\mathrm{req}} = 4$ Mbps, $\hat{\theta}_{p1} = 10$, $\hat{\theta}_{p2} = 10$, and $\hat{\theta}_s = 1$, $W_1 = 15$MHz and $W_2 = 15$MHz. Comparisons are provided among the scenarios of complete information with bandwidth constraint, complete information WOC, partial information (known θ_p and unknown θ_s), and incomplete information with bandwidth constraint. We see that in Fig. 6, under the proposed multistage Bayesian game model, the sequence of actions (strategies), both for the prices and bandwidth demands, with incomplete or partial private information converges to the actions with complete information. In Fig. 6, it can be shown that the beliefs updates about PS 2 cannot lead to a convergence to the actual type. Nevertheless, the optimal sequence of actions can still converges to the strategy profile with complete information. We can analytically show that the sequence of strategies with incomplete private information always converges to the actions with complete information, even if the eventual updated beliefs cannot tell each player the true types of other players' private information [10]. This convergence property demonstrates the robustness of the proposed Bayesian game model.

V. Conclusion

We've studied the spectrum trading game with incomplete information in a sequential manner for a cognitive radio network. To solve the optimization problem with bandwidth constraints in the multistage game, we've proposed using the KKT translation and joint KKT conditions to yield the perfect Bayesian equilibrium at each stage. We've demonstrated that the KKT translation technique provides a general rule that can be applied to optimization problems of multistage game theory. In addition, we've studied the convergence behaviors of action profiles in the simulations. Finally, we've concluded that the proposed multistage Baysian game model with bandwidth constraints is robust and capable of providing more reasonable strategy profiles for players.

References

[1] D. Niyato and E. Hossain, "Competitive pricing for spectrum sharing in cognitive radio networks: dynamic game, inefficiency of nash equilibrium, and collusion," *IEEE J. Select. Areas Commun.*, vol. 26, no. 1, pp. 192–202, Jan. 2008.

[2] J. Mitola, "Cognitive Radio: An Integrated Agent Architecture for Software Defined Radio," *Doctor of Technology, Royal Inst. Technol. (KTH), Stockholm, Sweden*, 2000.

[3] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Select. Areas Commun.*, vol. 23, no. 2, pp. 201–220, Feb. 2005.

[4] Z. Ji and K. J. Ray Liu, "Dynamic spectrum sharing: a game theoretic overview," *IEEE Communications Magazine*, pp. 88–94, May 2007.

[5] D. Niyato and E. Hossain, "Competitive spectrum sharing in cognitive radio networks: a dynamic game approach," *IEEE Trans. Wireless Commun.*, vol. 7, no. 7, pp. 2651–2660, July 2008.

[6] Z. Ji and K. J. Ray Liu, "Multi-stage pricing game for collusion-resistant dynamic spectrum allocation," *IEEE J. Select. Areas Commun.*, vol. 26, no. 1, pp. 182–191, Jan. 2008.

[7] A. J. Goldsmith and S. G. Chua, "Variable rate variable power MQAM for fading channels," vol. 45, no. 10, pp. 1218–1230, Oct. 1997.

[8] D. Fudenberg and J. Tirole, *Game Theory*, The MIT Press, 1991.

[9] J. Nocedal; S. J. Wright, *Numerical Optimization*, Springer, 2006.

[10] C. Y. Lee, *Bayesian Game based Spectrum Sharing for Cognitive Radio Networks*, Master Thesis, NCTU, Taiwan, 2009.