A Linear Programming Approach for PAPR Reduction in OFDM Systems

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***Abstract*—Peak-to-average-power ratio (PAPR) is a key design concern of an OFDM system as it determines the system's power efficiency. Clipping is a simple and efficient way of reducing PAPR. A suboptimal recursive clipping and filtering (RCF) method was proposed in [5] to reduce PAPR. This paper presents an alternate RCF approach which selects an error (clipping) vector that minimizes the peaks in both I- and Q-channels while satisfying some time and frequency domain distortion bounds. We formulate the design problem as a constrained linear programming (CLP) problem and extend it to include the tone-reservation option. Like the RCF method, our approach does not need side information and incurs no data rate loss. Since the error vector used in RCF has not been optimized, our approach offers improved PAPR reduction. It also provide tradeoff between PAPR reduction and bit error rate (BER) degradation.**

***Index Terms*—Peak-to-average-power ratio (PAPR), constellation error, clipping threshold, linear programming (LP).**

I. Introduction

Due to its high bandwidth efficiency, robustness against multipath fading induced inter-symbol interference (ISI), and flexibility in allocating multiuser radio resources, orthogonal frequency division modulation (OFDM) has become a very popular transmission technique, having been adopted by many industrial standards. However, a well-known disadvantage of the OFDM technique is its high peak-to-average-power ratio (PAPR) time domain waveform and thus low power efficiency.

Many approaches of PAPR reduction have been proposed. The selective mapping (SLM) [1] and partial transmit sequence (PTS) [1] methods generate many symbol sequences for the same OFDM frame with different phase scrambling sequences and transmits the one with the lowest PAPR. Interleaving [3] produces various sequences by permuting the original data sequence. These techniques belong to the class of signal scrambling schemes which reduce PAPR in a stochastic sense. As the scrambling sequence or permutation used depends on the original data, it must be sent along with the transmitted data. An alternate technique is the coding scheme [2] which maps the original symbol sequence set to a low PAPR symbol sequence subset of a larger codeword set. The code rate of this scheme is usually very low.

The recursive (repeated) clipping and filtering (RCF) approach [7] is a simple clipping technique. We clip the peaks of a time domain OFDM waveform and then filter out the out-of band components of the clipped signal. This time domain clipping and frequency domain filtering process is repeated until a certain criterion is satisfied. Although this method is simple and easy to implement, it introduces additional constellation distortions. To maintain desired BER performance, we can either enforce bounded distortion (BD) in frequency domain [5] for the clipped signal or use the so-called active constellation extension (ACE) [4] instead. Adopting the RCF scheme with BD or ACE, we can achieve low PAPR without changing the receiver structure and the need to transmit side information. The ACE method often requires much higher average transmit power. On the other hand, the BD approach forces out-of-bound frequency domain samples to contract to the prescribed distortion boundaries, effectively performing nonlinear complex frequency domain clipping. The contraction satisfies the BD constraint but does not guarantee that minimum PAPR can be achieved. Tone Reservation (TR) [6], which inserts dummy finite-magnitude complex numbers in some pre-selected subcarriers (tones), is another PAPR-reduction method. Tellado and Cioffi [8] proposed a TR-based PAPR reduction method for real-valued time-domain signal. But data rate is decreased by a half to satisfy the real value constraint.

In this paper, we combine and generalize both RCF and TR approaches by reserving some tones and allowing nonlinear clipping in both time and frequency domain with bounded frequency domain distortion constraints. The nonlinear time domain clipping in effect introduces a time domain vector error signal. This error signal is chosen to minimize the peaks of the time domain sequence while its real and imaginary frequency (error) magnitudes stay in the bounded and extended regions. The time domain clipping is done in real and imaginary parts separately so that we not only have one more degree of freedom but also are able to employ established linear programming (LP) techniques. In contrast to the approach of [8], we consider complex signals by adding complex peak-mitigating signal to the original OFDM signal that uses all in-band subcarriers and guarantee zero loss of data rate.

The rest of the paper is organized as follows: In Section II, we introduce basic definitions of a typical OFDM system. Section III presents the proposed approach formulated as an optimization problem and the following section provides some simulation results. Some concluding remarks are given in Section V.

Notation: j denotes $\sqrt{-1}$. $(\cdot)^T$ and $(\cdot)^H$ represent transpose and Hermitian operations. $Re(\cdot)$ and $Im(\cdot)$ stand for the real and imaginary part of a complex signal.

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II. PAPR and Available Subcarriers in OFDM Systems

An N-channel OFDM symbol consists of N sub-carriers with frequency f_k, $k = 0, 1, 2, ..., N-1$ and frequency spacing $\Delta f = \frac{1}{T}$, where T is the symbol duration without the cyclic prefix (CP). Each subcarrier is modulated by an M-QAM data and a frequency domain OFDM symbol can be expressed as $\mathbf{X} = [X(0), X(1), ..., X(N-1)]^T$.

An over-sampling factor $L \geq 4$ is needed to ensure a negligible approximation error if the discrete PAPR analysis is to be used to approximate analog waveforms. Denote the zero-padded frequency domain symbol by $\mathbf{X} = [X(0), X(1), \cdots, X(N-1), 0, \cdots, 0]^T$, which is an $LN \times 1$ vector. Then the over-sampled time domain signal is given by

$$x[n] = \frac{1}{\sqrt{NL}} \sum_{k=0}^{NL-1} X_k e^{\frac{j2\pi kn}{NL}} \ , \ 0 \leq n \leq NL-1 \tag{1}$$

The PAPR of an OFDM symbol is defined as

$$PAPR(\mathbf{x}) = \frac{\max\limits_{0 \leq n \leq LN-1} |x[n]|^2}{E[|x[n]|^2]} \tag{2}$$

where $\mathbf{x} = \mathbf{QX}$ and $\mathbf{Q}$ is an $(NL \times NL)$ inverse discrete Fourier transform (IDFT) matrix.

Let the set of subcarriers that serves encoded data be denoted by Ω_d, where $|\Omega_d| = N_d$ and $|\cdot|$ denotes the cardinality. The subcarrier set in which free pilots are placed is Ω_{TR}, where $|\Omega_{TR}| = N_{TR}$. The subcarriers $f_k \in \Omega_d \bigcup \Omega_{TR}$ and $|\Omega_d \bigcup \Omega_{TR}| = N_t$ are called available subcarriers. The remaining subcarriers are used as guardband. The time domain representation of the transmitted frequency vector $\mathbf{X} = [X(k)]^T$, $k \in \Omega_d \bigcup \Omega_{TR}$, is given by $\mathbf{x} = \tilde{\mathbf{Q}}\mathbf{X}$, where $\tilde{\mathbf{Q}} =$

$$\frac{1}{\sqrt{NL}} \begin{pmatrix} 1 & 1 & \cdots & 1 \\ e^{\frac{j2\pi f_1}{NL}} & e^{\frac{j2\pi f_2}{NL}} & \cdots & e^{\frac{j2\pi f_{N_t}}{NL}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{\frac{j2\pi f_1 (NL-1)}{NL}} & e^{\frac{j2\pi f_2 (NL-1)}{NL}} & \cdots & e^{\frac{j2\pi f_{N_t} (NL-1)}{NL}} \end{pmatrix} \tag{3}$$

$f_1, \cdots, f_{N_t} \in \Omega_d \bigcup \Omega_{TR}$.

The real and imaginary parts of the over-sampling IDFT matrix are denoted by $\mathbf{Q}_C = Re(\tilde{\mathbf{Q}})$, an $(NL \times N_t)$ matrix, and $\mathbf{Q}_S = Im(\tilde{\mathbf{Q}})$ which is an $(NL \times N_t)$ matrix.

III. Nonlinear Clipping as an Optimization Problem

The idea of our nonlinear clipping technique is to find an optimal clipping threshold and error vector such that all real and imaginary parts of the clipped time-domain signal samples be smaller than the threshold. It is also required that this threshold be minimized while both the real and imaginary parts of the frequency domain error vector be less than the BD constraint δ.

We add to the original vector $\mathbf{x}$ a time-domain error vector $\mathbf{e} = [e_1, e_2, ..., e_{NL}]$ to minimize the peaks in both I- and Q-channels. $\mathbf{e}$ is to be determined at the transmit site. There will be no need to send any side information or modify the receiver structure. The DFT of $\mathbf{e}$, i.e., the frequency domain error vector is defined as $\mathbf{E} = [\underbrace{0, ..., 0}_{\frac{(N-N_t)}{2}}, E_1, E_2, .., E_{N_t}, \underbrace{0, ..., 0}_{N(L-\frac{1}{2})-\frac{N_t}{2}}\ ] =DFT[\mathbf{e}]$. $\mathbf{e}$ and $\mathbf{E}$ are introduced to reduce PAPR. Moreover, there are free pilots (reserved tones) whose locations form a set which is disjoint to data subcarrier set.

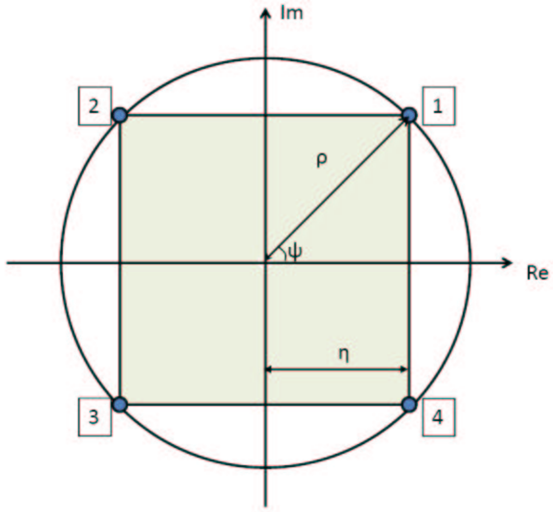

Fig. 1. The time domain clipping rule.

A. Clipping as Multiple Constraints

Conventional clipping techniques [5], [7] clip only those peaks which exceed the clipping threshold while keeping the corresponding phase intact. In contrast, the proposed algorithm clips the real and imaginary parts of the samples separately and thus brings in one more degree of freedom. This extra dimension also manifests in the polar coordinate in which, unlike the conventional approach, the phases $\tan^{-1}\left[\frac{Im(x_i)}{Re(x_i)}\right]$ of the clipped samples are allowed and likely to be different after clipping. Fig. 1 shows that our clipping rule confines all complex samples to stay inside the square with side length 2η, i.e., the magnitudes of their real and imaginary parts cannot be greater than η. Given an error vector $\mathbf{e}$ and the original time-domain vector $\mathbf{x}$, the clipped samples becomes $\tilde{\mathbf{x}} = \mathbf{x} + \mathbf{e}$ which should satisfy the constraints:

$$\tilde{x} = \begin{cases} ||Re(\mathbf{x}) + Re(\mathbf{e})||_\infty \leq \eta \\ ||Im(\mathbf{x}) + Im(\mathbf{e})||_\infty \leq \eta \end{cases} \tag{4}$$

where $||\cdot||_\infty$ stands for infinite norm. The above discussion implies that our approach can be formulated as the following constrained linear programming (CLP) problem

$$\begin{aligned} &\min \ \eta \\ &\textit{subject to}: \quad Re(\mathbf{x}+\mathbf{e}) \preceq \eta\mathbf{1} \\ &\qquad\qquad\quad Im(\mathbf{x}+\mathbf{e}) \preceq \eta\mathbf{1} \\ &\qquad\qquad\quad -Re(\mathbf{x}+\mathbf{e}) \preceq \eta\mathbf{1} \\ &\qquad\qquad\quad -Im(\mathbf{x}+\mathbf{e}) \preceq \eta\mathbf{1} \\ &\textit{in variables}: \quad \mathbf{e} \in \mathbb{C}^{NL} \end{aligned} \tag{5}$$

where $\mathbf{1}$ stands for an $LN \times 1$ vector and $\preceq$ denotes componentwise inequality in $\mathbb{R}^m$: $u \preceq v$ means $u_i \leq v_i$ for $i = 1, ..., m$.

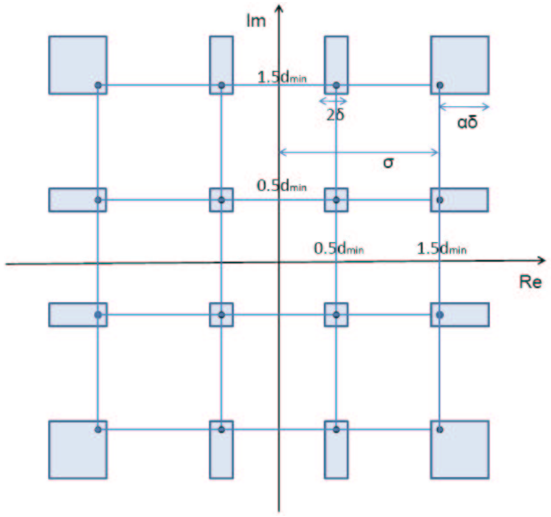

Fig. 2. Constellation error constraints for 16-QAM. Those colored area of internal points are bounded region and those for external points are extended region.

When the signal is clipped in time domain, distortions (sidelobe re-growth) are also generated in frequency domain. The resulting distortion is proportional to the magnitude of error vector $\mathbf{e}$. Although time domain clipping can achieve good PAPR performance, the frequency domain distortion might be very high, leading to BER degradation. To maintain acceptable BER performance, we need also to enforce constraints on the frequency domain error vector.

The difference between error vector magnitude (EVM) and constellation error is that EVM is based on 2-norm metric and there might be large errors in some coordinates without violating the error magnitude constraint but the constellation error measure ensures all coordinates of the error vector be within the constraint.

We use the constellation error to place constraints on the frequency-domain error vector $\mathbf{E}$ to further reduce the feasible set of optimal clipping threshold η. Here, we limit the real and imaginary parts of E_i, the ith component of $\mathbf{E}$, to stay in the bounded region and extended region (see Fig. 2). The constellation error constraint is further elaborated and a new CLP formulation in the next subsection.

B. Bounded Constellation Errors

Recall that $\mathbf{X}_d = [X_1, X_2, ..., X_{N_d}]^T \in \mathbb{C}^{N_d}$ is the original M-QAM data vector and the distorted frequency domain data symbol is given by $\tilde{\mathbf{X}}_d = \mathbf{X_d} + \mathbf{E_d}$, where $\mathbf{E}_d = [E_i]^T \in \mathbb{C}^{N_d}$, $i \in \Omega_d$. Since the decision regions for QAM are squares or half-squares, we allow square bounded (distortion) regions as well. We further divide a QAM constellation into internal points and external points. The external points can not only go inward in the bounded region but extend outward in the extended region. We require that the frequency domain error vector $\mathbf{E}_d$ satisfy the following constraints:

$$
\begin{aligned}
&1)\ if\ Re(X_i) > \gamma : -\delta \le Re(E_i) \le \alpha\delta\ ,\ i \in \Omega_d\\
&2)\ if\ Re(X_i) < -\gamma : -\alpha\delta \le Re(E_i) \le \delta\ ,\ i \in \Omega_d\\
&3)\ if\ Im(X_i) > \gamma : -\delta \le Im(E_i) \le \alpha\delta\ ,\ i \in \Omega_d\\
&4)\ if\ Im(X_i) < -\gamma : -\alpha\delta \le Im(E_i) \le \delta\ ,\ i \in \Omega_d\\
&5)\ if\ -\gamma \le Re(X_i) \le \gamma : -\delta \le Re(E_i) \le \delta\ ,\ i \in \Omega_d\\
&6)\ if\ -\gamma \le Im(X_i) \le \gamma : -\delta \le Im(E_i) \le \delta\ ,\ i \in \Omega_d
\end{aligned}
\tag{6}
$$

and

$$\gamma = (\sqrt{M} - 2) \times \frac{d_{min}}{2}$$

where $M = 2^{2l}, l \in \mathbb{N}$, d_{min} is the minimum distance of the constellation, δ defines the allowable constellation error and $\alpha\delta$ decides the extended region. Note that γ is used to distinguish between internal and external points. Constraints 1)–4) are for external points while constraints 5)–6) are for internal points. We choose appropriate δ's to satisfy different PAPR and BER requirements. Given $\delta = \epsilon$, our algorithm is designed to provide a modified OFDM constellation with the minimum achievable η.

In order to avoid increasing too much average transmit power, we set constraints on the TR pilots $\mathbf{E}_{TR} = [E_i]^T \in \mathbb{C}^{N_{TR}}$, where $i \in \Omega_{TR}$. We allow TR pilots to have exactly the same maximum power as that allowed in data subcarriers which carry signal points lie within the original constellation plus extended region. The TR constraints are thus given by

$$
\begin{aligned}
&-(\sigma + \alpha\delta) \le Re(E_i) \le \sigma + \alpha\delta\ ,\ i \in \Omega_{TR}\\
&-(\sigma + \alpha\delta) \le Im(E_i) \le \sigma + \alpha\delta\ ,\ i \in \Omega_{TR}
\end{aligned}
\tag{7}
$$

where $M = 2^{2l}, l \in \mathbb{N}$, and $\sigma = (\sqrt{M} - 1)d_{min}/2$ stands for the coordinate of external points.

Incorporating the above constraints, the proposed clipping scheme (inserted error vector) must satisfy the combined time and frequency domain constraints:

$$\tilde{\mathbf{x}} = \begin{cases} ||Re(\mathbf{x}) + Re(\mathbf{A}\hat{\mathbf{E}})||_\infty \le \eta \\ ||Im(\mathbf{x}) + Im(\mathbf{A}\hat{\mathbf{E}})||_\infty \le \eta \end{cases} \tag{8}$$

where

$$\mathbf{A} = \begin{pmatrix} \mathbf{Q}_C & -j\mathbf{Q}_S \\ j\mathbf{Q}_S & \mathbf{Q}_C \end{pmatrix}, \quad \hat{\mathbf{E}} = \begin{pmatrix} Re(\mathbf{E}) \\ Im(\mathbf{E}) \end{pmatrix}$$

are $(2NL \times 2N_t)$ and $2N_t \times 1$ matrices, respectively.

By introducing the $2N_t \times 1$ stacked vector $\hat{\mathbf{x}}$

$$\hat{\mathbf{x}} = \begin{pmatrix} Re(\mathbf{x}) \\ Im(\mathbf{x}) \end{pmatrix}$$

we restate the optimal clipper design problem as: Given $\mathbf{X}$, find the frequency domain error vector $\mathbf{E}$ that satisfies the BD and TR constraints such that the resulting peaks η in both I- and Q-channels are minimized.

$$
\begin{aligned}
&\min\ \eta\\
&\text{subject to}: \begin{pmatrix} \mathbf{A} & -\mathbf{1} \\ -\mathbf{A} & -\mathbf{1} \end{pmatrix} \begin{pmatrix} \hat{\mathbf{E}} \\ \eta \end{pmatrix} \preceq \begin{pmatrix} -\hat{\mathbf{x}} \\ \hat{\mathbf{x}} \end{pmatrix}
\end{aligned}
\tag{9}
$$

and

$$
\begin{aligned}
&(a)\ \ -\delta \le Re(E_i) \le \alpha\delta\ ,\ i \in \Omega_d,\ \text{if}\ Re(X_i) > \gamma\\
&(b)\ \ -\alpha\delta \le Re(E_i) \le \delta\ ,\ i \in \Omega_d,\ \text{if}\ Re(X_i) < -\gamma\\
&(c)\ \ -\delta \le Im(E_i) \le \alpha\delta\ ,\ i \in \Omega_d,\ \text{if}\ Im(X_i) > \gamma\\
&(d)\ \ -\alpha\delta \le Im(E_i) \le \delta\ ,\ i \in \Omega_d,\ \text{if}\ Im(X_i) < -\gamma\\
&(e)\ \ -\delta \le Re(E_i) \le \delta\ ,\ i \in \Omega_d,\ \text{if}\ -\gamma \le Re(X_i) \le \gamma\\
&(f)\ \ -\delta \le Im(E_i) \le \delta\ ,\ i \in \Omega_d,\ \text{if}\ -\gamma \le Im(X_i) \le \gamma\\
&(g)\ \ -(\sigma + \alpha\delta) \le Re(E_i) \le \sigma + \alpha\delta\ ,\ i \in \Omega_{TR}\\
&(h)\ \ -(\sigma + \alpha\delta) \le Im(E_i) \le \sigma + \alpha\delta\ ,\ i \in \Omega_{TR}\\
&\qquad \text{in variables: } \hat{\mathbf{E}} \in \mathbb{R}^{2N_t},\ \hat{\mathbf{x}} \in \mathbb{R}^{2NL}
\end{aligned}
$$

where $\mathbf{1}$ denotes a $2NL \times 1$ vectors, and η is real-valued. Note that $(a) \sim (f)$ are the BD constraints while $(g) \sim (h)$ are the TR constraints.

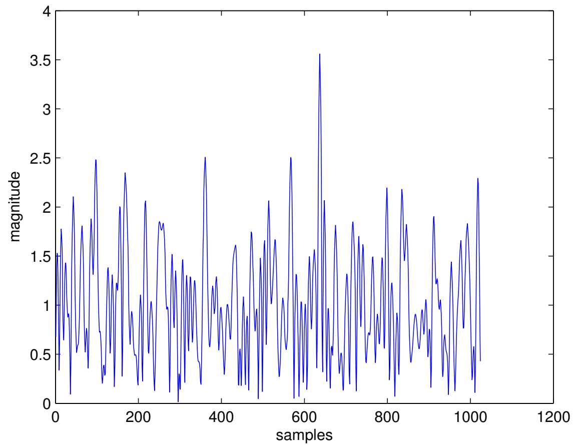

Fig. 3. Original time-domain signal with PAPR=9.6448 dB.

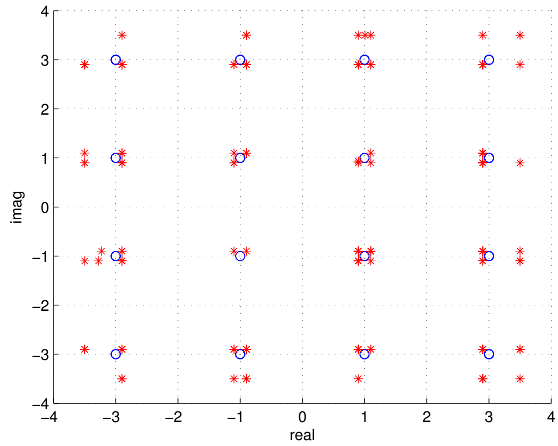

Fig. 5. Constellation of distorted frequency-domain signal achieving 3.7433 dB PAPR reduction.

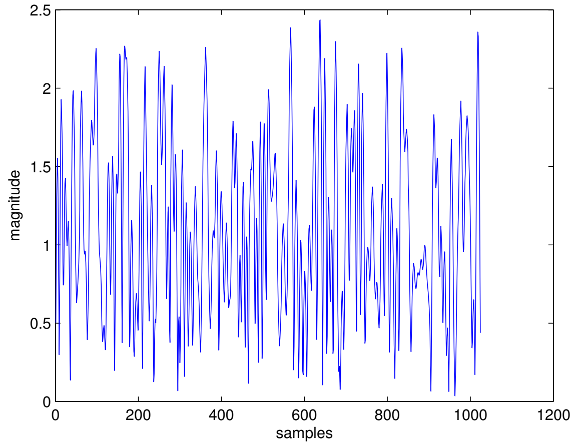

Fig. 4. Clipped time-domain signal with PAPR=5.9015 dB.

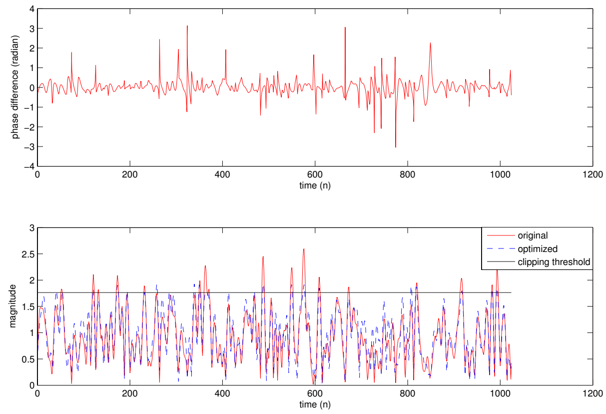

Fig. 6. Phase difference and amplitude trajectories of a typical time-domain sequence before and after nonlinear clipping.

IV. Simulation Results

In this section we provide some numerical performance of our algorithm and an example of time- and frequency-domain signal of original and optimized signal. We consider an OFDM system with 256 subcarriers using 16-QAM modulation for the data carried on each available subcarrier. Only 128 tones are used to carry data, and there are six reserved tones at subcarriers 193, 194, 195, 196, 197, 198, respectively. 0 denotes dc tone and guard-bands are distributed in subcarriers 0 to 64 and 199 to 256. The reserved tones' locations do not affect the PAPR value too much but the number of the reserved tones does. The more tones are reserved, the higher the PAPR reduction gain becomes. For fair comparison, the number of reserved tones is the same for all schemes whose performance is presented in this section.

First, we would like to show the clipping effects in time and frequency domains. Fig. 3 shows the original time-domain signal of an OFDM symbol with a PAPR of 9.6448 dB. After applying the proposed nonlinear clipping algorithm (9), the resulting time-domain signal is plotted in Fig. 4 where the PAPR is reduced to 5.9015 dB. However, we achieve such PAPR reduction at the cost of a smaller minimum distance in the 16-QAM constellation. The constellation setup for our case is 16-QAM having $\{-\frac{3d_{min}}{2}, -\frac{d_{min}}{2}, \frac{d_{min}}{2}, \frac{3d_{min}}{2}\}$ as signal points on real and imaginary axes (I- and Q-channel components) with a maximum constellation error $\delta = 0.05d_{min}$. Red stars in Fig. 5 are clipped frequency domain samples which has deviated from the original constellation points and the resulting minimum distance for this case is reduced to $0.9d_{min}$. A shorter minimum distance means worse BER performance, but larger distortion bound and extended region give better PAPR reduction capability. There is an obvious tradeoff between the PAPR reduction and BER performance degradation.

Fig. 6 shows the phase difference (original phase - optimized phase) and amplitude trajectories of a typical time-domain sequence before and after performing the proposed nonlinear clipping. Obviously, our clipping method does result in (or allow) phase rotations of the time domain samples. Separate constraints on I- and Q-channel magnitudes result in envelop clipping and phase rotation (in this example as

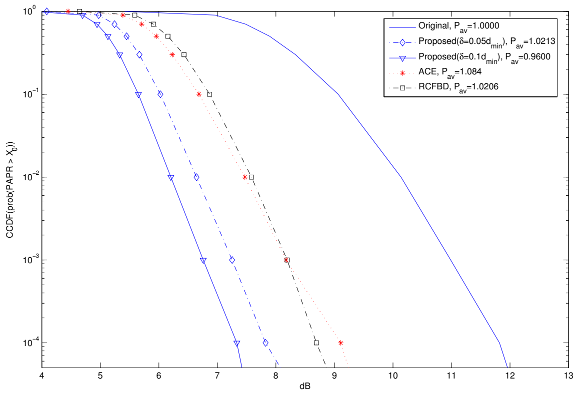

Fig. 7. Effective PAPR gain for 16-QAM 256-carrier OFDM symbol with L=4 over-sampling for $\delta = 0.05d_{min}$, $\delta = 0.1d_{min}$, ACE, RCFBD, and separated coordinate PAPR.

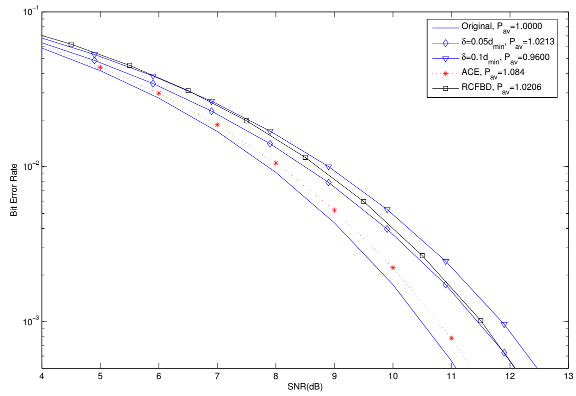

Fig. 8. BER of a 16-QAM and 256-carrier OFDM system.

much as 0.2180 radian). As mentioned before, phase rotation gives us an extra degree of freedom for PAPR reduction. The optimal rotated phase is obtained by solving the corresponding CLP problem (9).

Simulated BER performance of the conventional OFDM and error vector optimized OFDM schemes in AWGN channels is given in Fig. 8. As expected, the larger the allowed constellation error is, the greater the PAPR reduction becomes. As mentioned before, since the constellation (frequency domain) error reduces the minimum-distance of the signal set, it also degrades the BER performance. In this figure we find that a constellation error bound of $\delta = 0.05d_{min}$ gives BER performance better than that with a bounded error of $\delta = 0.1d_{min}$. It is clear that the amount of PAPR reduction is an increasing function of the allowed frequency domain distortion bound.

Figs. 7 and 8 indicate that our clipping scheme yields a better PAPR performance than that achieved by RCF-BD while maintaining the same BER performance. This PAPR reducing gain is due to the fact that the error vector used in RCF-BD has not been optimized.

Although both ours and the ACE schemes use an optimization procedure to reduce PAPR, and our algorithm outperforms the ACE scheme by imposing BD's. A BD region allows optimal modifications for distorted data points, it enlarges the feasible set of linear programming problem whence leads to a greater likelihood to find a solution with lower PAPR.

V. Conclusion

This paper presents a novel nonlinear clipping technique that extends the RCF-BD and TR concepts to reduce the PAPR of OFDM signals. We formulates the proposed algorithm as one for solving a CLP problem. The solution can be easily found by following the established procedure. The proposed approach does not have to modify the receiver structure and needs not to send side information. Moreover, our algorithm guarantees that the optimal error vector is obtained and the resulting PAPR value is minimized under certain BD constraints. The complexity of our algorithm can be greatly reduced by if a fast algorithm to solve the corresponding CLP problem can be found.

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