QRD-based Antenna Selection for Maximum-Likelihood MIMO Detection

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***Abstract*—Antenna selection is a simple but effective method to exploit the transmit diversity in multiple-input multiple-output (MIMO) wireless communications. For maximum-likelihood (ML) detectors, the criterion for the selection is to maximize the free distance of the MIMO system. Since the optimum selection is difficult to conduct, a lower bound of the free distance is typically used as the selection criterion instead. The singular-value-decomposition (SVD) based selection criterion is well known in the literature. In this paper, we propose a QR decomposition (QRD) based selection criterion for antenna selection with the ML detector. Using some matrix properties, we theoretically prove that the lower bound achieved with the QRD-based criterion is tighter than that with the SVD-based criterion. We also propose another QRD-based criterion that can further tighten the lower bound. The proposed algorithms can be directly applied to the receive, and joint transmit/receive antenna selection schemes. Simulations show that the performance of the proposed selection criteria can significantly outperform the SVD-based selection criterion.**

I. INTRODUCTION

Multiple-input multiple-output (MIMO) wireless communication systems have become increasingly important for past years. The spatial multiplexing scheme is a well-known technique to achieve high spectral efficiencies in MIMO systems. However, the more bit-streams we transmit, the less diversity gain we can exploit. Transmit antenna selection, equipping extra antenna elements at the transmitter, is a simple approach to increase the transmit diversity.

The problem of transmit antenna selection has been extensively studied, and many selection criteria have been proposed. Antenna selection criteria for linear receivers were proposed in [1], including post signal-to-noise ratio (SNR) maximization and mean-square-error (MSE) minimization. In [2], the selection, aiming to minimize the conditional error probability, was derived for the maximum-likelihood (ML) receiver. It is well known that the error rate performance of ML detection under high SNR strongly depends on the minimum Euclidean distance between the received symbol vectors, which is generally referred to as the free distance. A selection method that maximizes the free distance by exhaustive search was then proposed in [3]. To reduce the computational complexity of the optimum selection with the free distance, a singular-value-decomposition (SVD) based selection criterion was further proposed in [3]. It has been shown that the SVD-based selection criterion will minimize a lower bound of the free distance. Different from the approaches discussed above, a selection criterion based on maximizing the channel capacity was also proposed in [1] and [3].

In this paper, we propose using a QR decomposition (QRD) based criterion for the antenna selection problem. With some matrix properties, we theoretically prove that the lower bound of the QRD-based criterion is tighter than that of the SVD-based one. Moreover, we propose an extended QRD-based criterion to further tighten the lower bound. Except for transmit antenna selection, receive antenna selection [5], [6] and joint transmit/receive antenna selection [7] are also popular in MIMO communication systems. We will show that the proposed schemes can be directly applied to those scenarios.

The remainder of this paper is organized as follows. Section II outlines the system and signal model. Section III gives the existing and proposed selection criteria, and Section IV reports simulation results. Finally, Section V concludes the paper.

II. SYSTEM AND SIGNAL MODEL

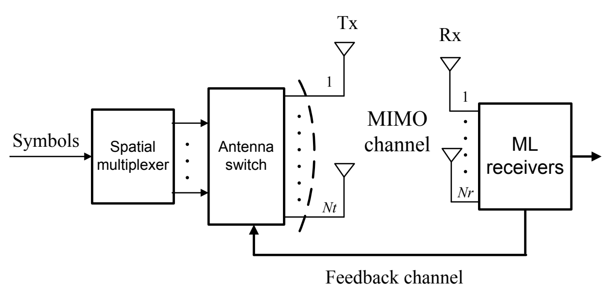

Fig. 1. The system model for transmit antenna selection in spatial multiplexing MIMO systems.

Consider a wireless MIMO system with N_t transmit antennas and N_r receive antennas, as described in Fig. 1. Let $\mathbf{H}$ denote an $N_r \times N_t$ $(N_t > N_r)$ channel matrix, and assume that the channel state information (CSI) is perfectly available to the receiver, but not to the transmitter. In transmit antenna selection, the receiver first determines the index $p \in P$ according to a selection criterion, where p represents the selected subset of the transmit antennas, and P is all possible $\binom{N_t}{M}$ antenna subsets. Then, via a feedback channel, the receiver sends this index p to the transmitter, and finally the transmitter uses the selected antenna for signal transmission. For the application of spatial multiplexing, the input symbols are multiplexed into an $M \times 1$ symbol vector $\mathbf{s}$, and then sent over the $N_r \times M$ $(N_r \geq M)$ MIMO channel subset $\mathbf{H}_p$. The received signal vector can then be expressed as

$$\mathbf{y} = \mathbf{H}_p \mathbf{s} + \mathbf{n} \quad (1)$$

where $\mathbf{n}$ is an $N_r \times 1$ channel noise vector. The ML detector searches all possible symbol vectors $\mathbf{s}_i$ to obtain an estimate such that

$$\hat{\mathbf{s}} = \min_{\mathbf{s}_i \in S} \|\mathbf{y} - \mathbf{H}_p \mathbf{s}_i\|^2 \quad (2)$$

where S is the set of all possible transmitted vectors.

III. Antenna Selection Criterion

The free distance, which dominates the performance of ML detection in high SNR regions, is defined as

$$\hat{d}^2_{min,\mathbf{H}_p} = \min_{\mathbf{s}_i, \mathbf{s}_j \in S, \mathbf{s}_i \neq \mathbf{s}_j} \|\mathbf{H}_p(\mathbf{s}_i - \mathbf{s}_j)\|^2 . \quad (3)$$

where $(\mathbf{s}_i - \mathbf{s}_j)$ is the difference vector. The optimal selection criterion for the ML receiver is equivalent to selecting the antenna subset whose $\mathbf{H}_p$ gives the maximum free distance [3]. Using this criterion, we can describe the selection scheme as: Compute the free distance for the channels corresponding to all antenna subsets, and choose the antenna subset with the largest $\hat{d}^2_{min,\mathbf{H}_p}$. The solution of the optimal selection criterion is conducted by exhaustive search over all possible $\binom{N_t}{M}$ channel matrices and all difference vectors. The exhaustive search, however, requires a high computational complexity that may be prohibitive for larger QAM constellations. As an alternative, one can consider a suboptimum solution in which a lower bound of the free distance, instead of the free distance itself, is maximized.

A. SVD-based selection criterion

Assume that $\mathbf{H}_p$ is of full column rank, and its SVD is given as $\mathbf{H}_p = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^*$($*$ represents Hermitian transpose), where both $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices with the size $N_r \times N_r$ and $M \times M$ respectively, and $\mathbf{\Lambda} = diag(\lambda_1, \lambda_2, \ldots, \lambda_M)$ is an $N_r \times M$ tall matrix. Based on the Rayleigh-Rits theorem, Heath *et al.* [3] proposed the SVD approach to obtain a lower bound for the free distance

$$\hat{d}^2_{min,\mathbf{H}_p} \geq \lambda^2_M \hat{d}^2_{min} \quad (4)$$

where λ_M is the minimum singular value of the channel corresponding to the selected antenna subset $\mathbf{H}_p$, and d^2_{min} is the minimum distance between any two distinct transmit symbol vectors. Note that $\hat{d}^2_{min}$ is a deterministic value for a fixed QAM modulation size. Thus, the free distance hereby only depends on the minimum singular value of the channel matrix, and we then have the SVD-based selection scheme described as follows: Conduct SVD for the channels corresponding to all antenna subsets, and choose the antenna subset with the largest λ_M.

With this criterion, only computing the minimum singular value of each $\mathbf{H}_p$ is required, and the computational complexity can be reduced significantly. In [4], another lower bound via QRD for the free distance was developed. In this paper, we propose to use this lower bound in the antenna selection problem, and refer this approach as the QRD-based selection criterion.

B. QRD-based selection criterion

Performing QRD, we have $\mathbf{H}_p = \mathbf{Q}\mathbf{R}$, where $\mathbf{Q}$ is a tall $N_r \times M$ column-wise orthonormal matrix and $\mathbf{R}$ is an $M \times M$ square upper triangular matrix with positive real-valued diagonal entries

$$\mathbf{R} = \begin{pmatrix} R_{1,1} & R_{1,2} & \ldots & R_{1,M} \\ 0 & R_{2,2} & \ldots & R_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & R_{M,M} \end{pmatrix}.$$

Via this decomposition, the free distance can be bounded as

$$\hat{d}^2_{min,\mathbf{H}_p} \geq [\mathbf{R}]^2_{min} \hat{d}^2_{min} \quad (5)$$

where $[\mathbf{R}]_{min}$ is the minimum diagonal entry of $\mathbf{R}$. Thus, the selection scheme with the criterion can be formulated as follows: Conduct QRD for the channels corresponding to all antenna subsets, and choose the antenna subset with the largest $[\mathbf{R}]_{min}$.

Since both the SVD and QRD methods are suboptimum solutions, we now have a question that which one, (4) or (5), is tighter. From simulations, we can find that (5) is tighter. We now give a theoretical proof for the result. To do this, we have to show that $[\mathbf{R}]^2_{min}$ is larger than λ^2_M for all channels.

Definition 1: Let $\underline{a} = (a_1, a_2, \ldots, a_m)$ and $\underline{b} = (b_1, b_2, \ldots, b_m)$ be two positive, real-valued sequences satisfying

$$a_1 \geq a_2 \geq \ldots \geq a_m$$

and

$$b_1 \geq b_2 \geq \ldots \geq b_m.$$

We say that $\underline{a}$ majorizes $\underline{b}$ in the product sense [8], [9] if

$$\prod_{k=1}^{l} a_k \geq \prod_{k=1}^{l} b_k \quad (6)$$

for all $l = 1, 2, \ldots, m$, and with equality when $l = m$.

Consider an arbitrary $M \times M$ positive-definite matrix $\mathbf{Z} = \mathbf{H}^*_p \mathbf{H}_p$ with the eigenvalue decomposition $\mathbf{Z} = \mathbf{P}\mathbf{\Sigma}\mathbf{P}^*$ and Cholesky fraction $\mathbf{Z} = \mathbf{L}^*\mathbf{D}\mathbf{L}$, where $\mathbf{P}$ is a unitary matrix, $\mathbf{D} = diag(d_1, d_2, \ldots, d_M)$ is a diagonal matrix, and $\mathbf{L}$ is a unit upper-triangular matrix

$$\mathbf{L} = \begin{pmatrix} 1 & L_{1,2} & \ldots & L_{1,M} \\ 0 & 1 & \ldots & L_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1 \end{pmatrix}.$$

Let $\underline{\sigma} = (\sigma_1, \sigma_2, \ldots, \sigma_M)$ and $\underline{d} = (d_1, d_2, \ldots, d_M)$ denote the eigenvalues and Cholesky values [10] of $\mathbf{Z}$ respectively. We arrange the elements of both sequences $\underline{\sigma}$ and $\underline{d}$ in the decreasing order, that is, $(\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_M)$ and $(d_1 \geq d_2 \geq \ldots \geq d_M)$.

Lemma 1: For a positive-definite matrix, the sequence $\underline{\sigma}$ majorizes the sequence $\underline{d}$ in the product sense, i.e.,

$$\prod_{k=1}^{l} \sigma_k \geq \prod_{k=1}^{l} d_k \tag{7}$$

for all $l = 1, 2, \ldots, M$, and with equality when $l = M$. The above lemma and its proof can be found in [8], [10].

Theorem 1: For an $N_r \times M (N_r \geq M)$ full column rank matrix $\mathbf{H}_p$ with QRD $\mathbf{H}_p = \mathbf{QR}$ and SVD $\mathbf{H}_p = \mathbf{U \Lambda V}^*$, the minimum diagonal entry of $\mathbf{R}$ is larger than the minimum singular value of $\mathbf{\Lambda}$, that is,

$$[\mathbf{R}]_{min} \geq \lambda_M. \tag{8}$$

Proof: Since $\mathbf{H}_p$ is of full column rank, we can have a positive-definite matrix $\mathbf{Z}$

$$\mathbf{Z} = \mathbf{H}_p^* \mathbf{H}_p = \mathbf{R}^* \mathbf{R} = \tilde{\mathbf{L}}^* \sqrt{\tilde{\mathbf{D}}}^* \sqrt{\tilde{\mathbf{D}}} \tilde{\mathbf{L}} = \tilde{\mathbf{L}}^* \tilde{\mathbf{D}} \tilde{\mathbf{L}} \tag{9}$$

where $\tilde{\mathbf{L}}$ is a unit upper-triangular matrix, and $\tilde{\mathbf{D}}$ is a diagonal matrix. Let $\underline{r} = (r_1, r_2, \ldots, r_M)$ and $\underline{\tilde{d}} = (\tilde{d}_1, \tilde{d}_2, \ldots, \tilde{d}_M)$ denote the diagonal entries of $\mathbf{R}$ and $\tilde{\mathbf{D}}$ respectively. From (9), we know that $r_1 = \sqrt{\tilde{d}_1}, r_2 = \sqrt{\tilde{d}_2}, \ldots, r_M = \sqrt{\tilde{d}_M}$. Furthermore, a positive-definite matrix exists a unique Cholesky fraction, which implies $\underline{\tilde{d}}$ is exactly the Cholesky values $\underline{d}$ of $\mathbf{Z}$. Assume $\underline{r}$ and $\underline{\tilde{d}}$ are in the decreasing order. Then, by Lemma 1, we can obtain

$$\prod_{k=1}^{l} \sigma_k \geq \prod_{k=1}^{l} d_k = \prod_{k=1}^{l} \tilde{d}_k = \prod_{k=1}^{l} r_k^2 \tag{10}$$

for all $l = 1, 2, \ldots, M$, and with equality when $l = M$. It is well known that $\sigma_k = \lambda_k^2$ for all k, where λ_k is the k-th largest singular value of $\mathbf{H}_p$. We thus have

$$\prod_{k=1}^{l} \lambda_k^2 = \prod_{k=1}^{l} \sigma_k \geq \prod_{k=1}^{l} r_k^2 \tag{11}$$

for all $l = 1, 2, \ldots, M$, and with equality when $l = M$. From (11) we arrive at that r_M^2 is larger than λ_M^2, which completes the proof.

C. Enhanced QRD-based selection criterion

So far we have theoretically shown that the lower bound of the QR-based scheme is tighter than that of the SVD-based scheme. By examining the proof of (5) derived in [4], we observe that the tightness of the QRD-based lower bound will degrade when M is large. To overcome this problem, we propose a simple method to enlarge $[\mathbf{R}]_{min}$.

In QRD, the different permutation orderings of the columns of $\mathbf{H}_p$ will result in different values of $[\mathbf{R}]_{min}$. Note that there are $M!$ permutation patterns for M columns. Assume $\mathbf{H}_{p,1} = \mathbf{H}_p \mathbf{P}_1, \mathbf{H}_{p,2} = \mathbf{H}_p \mathbf{P}_2, \ldots, \mathbf{H}_{p,M!} = \mathbf{H}_p \mathbf{P}_{M!}$ are the matrices whose columns are of the distinct permutation orderings, where $\mathbf{P}_n$ is a permutation matrix. Also assume that their QRD can be expressed as: $\mathbf{H}_{p,1} = \mathbf{Q}_{p,1} \mathbf{R}_{p,1}, \mathbf{H}_{p,2} = \mathbf{Q}_{p,2} \mathbf{R}_{p,2}, \ldots, \mathbf{H}_{p,M!} = \mathbf{Q}_{p,M!} \mathbf{R}_{p,M!}$. Thus we can have the enhanced QRD-based selection as follows. For each antenna subset $p \in P$, compute its related $M!$ matrices $\mathbf{H}_{p,n}$, where $n = 1, \ldots, M!$. Then, conduct QRD on every $\mathbf{H}_{p,n}$ to obtain their corresponding minimum diagonal entries, that is, $[\mathbf{R}_{p,1}]_{min}, [\mathbf{R}_{p,2}]_{min}, \ldots, [\mathbf{R}_{p,M!}]_{min}$. Pick the maximum value $[\mathbf{R}_{p,max}]_{min}$ as the minimum diagonal entry of $\mathbf{H}_p$. Finally, choose the antenna subset with the largest $[\mathbf{R}_{p,max}]_{min}$.

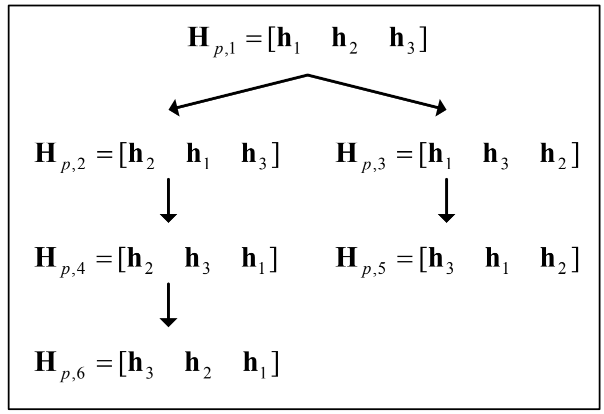

Fig. 2. The ordering of computing each $\mathbf{R}_{p,n}$ of $\mathbf{H}_p$, $M = 3$

The permutation method can improve the tightness of (5), but the computational complexity will increase due to the additional QRD operations. To reduce the complexity, we suggest using Givens rotations [11] when implementing QRD on each $\mathbf{H}_{p,n}$. Assume that $\mathbf{H}_{p,1} = \mathbf{Q}_{p,1} \mathbf{R}_{p,1}$ has been available via the complete QRD, and we seek to obtain $\mathbf{R}_{p,2}$ of $\mathbf{H}_{p,2}$. Denote $\bar{\mathbf{P}}$ as a permutation matrix that exchanges any two neighbor columns of a matrix. We then have

$$\mathbf{H}_{p,2} = \mathbf{Q}_{p,1} \mathbf{R}_{p,1} \bar{\mathbf{P}} = \mathbf{Q}_{p,1} \bar{\mathbf{R}}_{p,1} \tag{12}$$

Note that a QRD-like expression is shown in (12) except that $\bar{\mathbf{R}}_{p,1}$ is a near upper-triangular matrix. Since $\bar{\mathbf{P}}$ only exchanges two neighbor columns of $\mathbf{R}_{p,1}$, we can upper-triangulize $\bar{\mathbf{R}}_{p,1}$ by a simple Givens rotation matrix $\mathbf{G}_1$; that is, $\mathbf{G}_1 \bar{\mathbf{R}}_{p,1} = \mathbf{T}$, where $\mathbf{T}$ is a upper-triangular matrix. Thus we can rewrite (12) as

$$\mathbf{H}_{p,2} = \mathbf{Q}_{p,1} \mathbf{G}_1^* \mathbf{T} = \bar{\mathbf{Q}}_{p,2} \mathbf{T} \tag{13}$$

where $\bar{\mathbf{Q}}_{p,2} = \mathbf{Q}_{p,1} \mathbf{G}_1^*$ is a unitary matrix. According to the fact that the QRD of a full-rank matrix is unique, we know that $\bar{\mathbf{Q}}_{p,2} \mathbf{T}$ in (13) represents the QRD of $\mathbf{H}_{p,2}$, and $\mathbf{T}$ is exactly equal to $\mathbf{R}_{p,2}$. In other words, we obtain $\mathbf{R}_{p,2}$ by left-multiplying a Givens rotation matrix on $\bar{\mathbf{R}}_{p,1}$ rather than by performing the complete QRD on $\mathbf{H}_{p,2}$. Therefore, we can reduce the computational complexity of the enhanced QRD-based scheme. Fig. 2 illustrates an example (for $M = 3$) how each $\mathbf{R}_{p,n}$ can be derived with Givens rotations.

D. Capacity-based selection criterion

Except for the criteria discussed above, there is another criterion, maximizing the capacity $C(\mathbf{H}_p)$ of a MIMO channel $\mathbf{H}_p$. The resultant selection criterion is given by

$$C(\mathbf{H}_p) = \log_2 \det(\mathbf{I}_M + \frac{\rho}{M}\mathbf{H}_p^*\mathbf{H}_p) \tag{14}$$

where ρ is the average SNR per receive antenna, $\det()$ denotes the determinant, and $\mathbf{I}_M$ is an $M \times M$ identity matrix. Thus, the capacity selection criterion can be described as follows: Compute the capacity for the channels corresponding to all antenna subsets, and choose the antenna subset with the largest $C(\mathbf{H}_p)$.

The capacity selection criterion is based on a general mutual information formula, which is independent of the receiver types. Additionally, we remark that the permutation method in our enhanced QRD-based scheme cannot be adopted in the SVD-based and capacity-based schemes since both the capacity and singular values of $\mathbf{H}_p$ do not depend on the columns permutation.

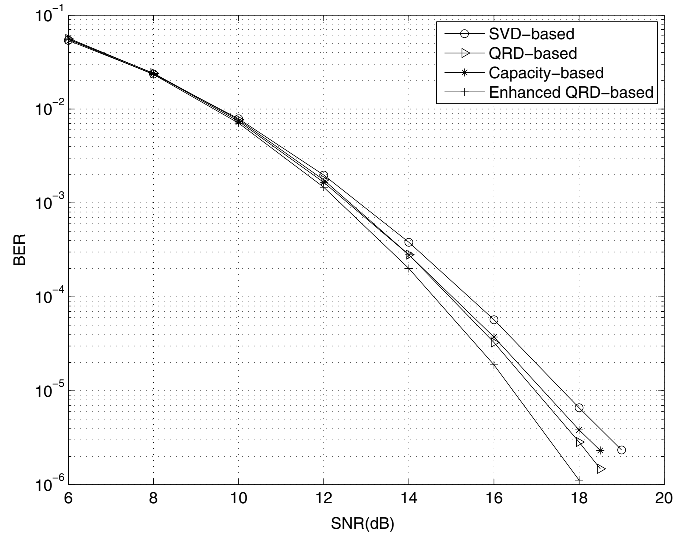

Fig. 3. The BER performance comparison for QPSK when $N_t = 6, N_r = 3$, and $M = 3$

E. Complexity comparisons

One way to quantify the complexity of the matrix computation is to count the number of required floating points, referred to as flops. Several efficient algorithms to perform QRD and SVD are given in [11]. In general, SVD requires more flops than QRD does. Therefore, our QRD-based selection scheme not only have better performance, but also requires lower computational complexity.

We now consider the computational complexity of the capacity-based algorithm. Computing the determinant of $\mathbf{H}_p$ in (14) requires $O(M^3)$ flops, which is roughly the same as computing SVD and QRD. However, the main drawback of the capacity-based method is that the receiver needs to know the variance of the channel noise that is not required for SVD- or QRD-based selection criterion. Moreover, an extra matrix multiplication $\mathbf{H}_p^*\mathbf{H}_p$ in (14) needs to be conducted requiring additional $O(M^3)$ flops. As a result, the computational complexity of the capacity-based scheme is higher.

For the enhanced QRD-based scheme, performing the complete QRD on all $\mathbf{H}_{p,n}$ of $\mathbf{H}_p$ needs $O(M^4)$ flops. As mentioned, we can reduce the complexity via the Givens rotation method, where only once complete QRD and $(M-1)$ Givens rotation matrix computations are required. Note that $(M-1)$ upper-triangulization operations need $O(M^2)$ flops. Thus, the order of the overall computational complexity, which includes one complete QRD and $(M-1)$ Givens rotation matrix computations, is $O(M^3)$. We observe that the enhanced QRD-based scheme only increases the computational complexity slightly.

F. Receive and joint transmit/receive antenna selection

Receive antenna selection has been widely considered as a means of reducing the cost of radio-frequency components in MIMO wireless communications. This scheme is similar to transmit antenna selection except for the assumption that the size of the channel matrix satisfies $N_r > N_t = M$. Assume that the CSI is perfectly known at the receiver, and we aim to select M out of N_r receive antenna elements. We can construct the channel matrix as we did in transmit antenna selection, and select a sub-matrix as the actual channel matrix. The main difference from transmit antenna selection is that instead of columns, we now select rows. Also, the receiver does not have to send the index of the selected antenna subset back to the transmitter.

Using a similar idea, we can conduct antenna selection at both transmit and receive sides. Consider an $N_r \times N_t$ MIMO channel $\mathbf{H}$, and assume that $N_r > M$ and $N_t > M$, meaning that there are $\binom{N_t}{M} \times \binom{N_r}{M}$ possible matrices $\mathbf{H}_p$. Note that only $\lceil \log_2 \binom{N_t}{M} \rceil$ bits are required to send back the index p to the transmitter, where $\lceil t \rceil$ denotes the smallest integer not smaller than t. For the same number of antennas for selection, the joint transmit/receive selection scheme is a compromise between the number of antennas used at the receiver and the number of feedback bits. For example, there are 16 matrix candidates for $N_t = N_r = 4, M = 3$, and only $\lceil \log_2 4 \rceil = 2$ feedback bits are required. If we let $N_t = 5, N_r = M = 3$, we will need at least $\lceil \log_2 \binom{5}{3} \rceil = 4$ bits for feedback. Note that the total number of antenna elements are the same $(N_t + N_r = 8)$. Taking all the antennas into consideration, we can extend the use of the proposed QRD-based selection scheme.

IV. Simulation results

In this section, we provide simulation results evaluating the performance of MIMO systems with different antenna selection criteria. In the simulations, we consider a flat-fading MIMO channel $\mathbf{H}$, of which the entries are assumed to be i.i.d complex Gaussian random variables with zero mean and unit variance. The ML detection is used at the receiver with perfect CSI, and the proposed methods are compared with other existing methods for QPSK modulation.

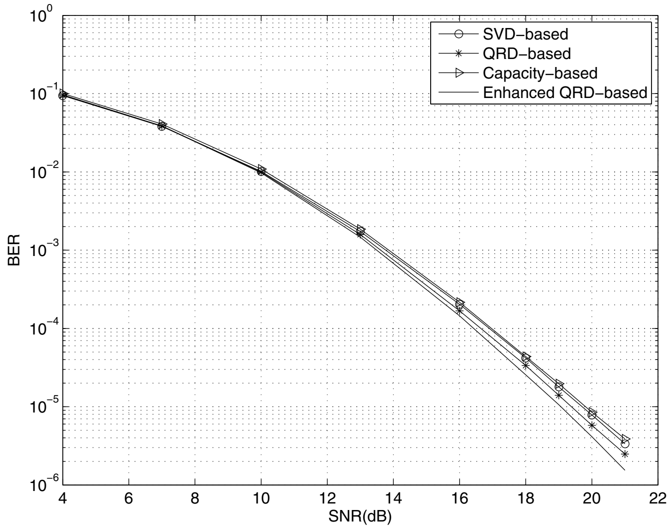

Fig. 4. The BER performance comparison for QPSK when $N_t = 2, N_r = 4$, and $M = 2$

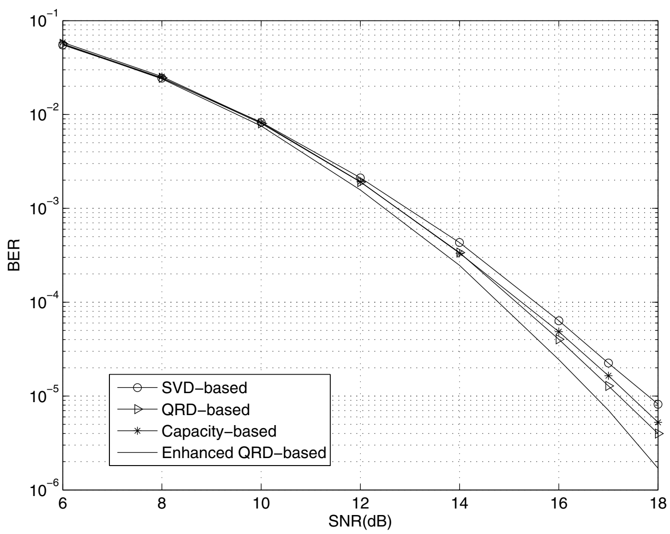

Fig. 5. The BER performance comparison for QPSK when $N_t = 4, N_r = 4$, and $M = 3$

Fig. 3 shows the bit error rate (BER) performance for the MIMO system with $N_t = 6$, $N_r = 3$, and $M = 3$. From the figure, we see that the enhanced QRD-based scheme achieves the best performance, about 1.5 dB better than the SVD-based scheme at BER = 10^{-6}. The proposed QRD-based scheme outperforms the capacity-based scheme slightly; however, the computational complexity of QRD-based criterion is lower.

Fig. 4 shows the BER comparison for the receive antenna selection scheme with $N_t = 2$, $N_r = 4$, and $M = 2$. We observe that the enhanced QRD-based criterion achieves one dB improvement compared to the SVD-based scheme. In this case, the improvements of the proposed schemes are less significant since the number of antenna subsets to be selected is small. Also, we notice that the SVD-based scheme gives the comparable performance to the capacity-based scheme. Note here that the capacity-based selection criterion is designed for the maximization of the total capacity of the channel, not the capacity of individual MIMO sub-channels, which implies that its performance may be seriously affected in some channel conditions. Fig. 5 shows the performance comparison for joint transmit/receive antenna selection with $N_t = 4$, $N_r = 4$, and $M = 3$. The behaviors of all selection criteria are similar to those in Fig. 3.

V. Conclusions

In this paper, we have proposed to use a QRD-based lower bound as the transmit antenna selection criterion. The theoretical analysis and simulation results show that the QRD-based lower bound is tighter than the conventional SVD-based lower bound. Furthermore, we proposed a simple approach to enhance the tightness of the QRD-based lower bound. For receive antennas selection and joint antenna selection, the proposed algorithms can be directly adopted as the selection criteria. As mentioned, in practical implementations, the QRD is of less complexity than the SVD. Furthermore, the QRD-based approach will exhibit a significant advantage when sphere-decoding (SD) [12], an efficient algorithm for the ML detection, is used at the receiver. We know that the QRD is also required in the SD algorithm, which results in the fact that a same QRD unit can be shared by the antenna selection and the SD algorithm. Based on the above reasons, finally, we conclude that the QRD-based selection algorithm will be much more efficient in real-world applications.

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