# A Low-complexity Precoder Searching Algorithm for MIMO-OFDM Systems

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***Abstract*—Precoding is an effective technique enhancing the performance of MIMO-OFDM systems. In practical systems, the precoding matrix is computed at the receiver, and then fed back to the transmitter. To reduce the amount of the feedback data, only the index representing a quantized precoding matrix is fed back. The quantized matrix is selected from a set of predetermined matrices called a codebook. Since the number of matrices in the codebook may be large, the search for the optimum precoder requires a high computational complexity. In this paper, we propose a low-complexity precoder searching algorithm to solve the problem. The basic idea is to construct a tree-like search strategy such that the complexity can be reduced from $O(L)$is $O(\log_2(L))$ where $L$ is the number of the codewords. Compared to the exhaustive search, the proposed searching method can reduce the searching complexity significantly while the performance loss is small.**

## I. Introduction

Multiple-input multiple output (MIMO) transceivers, created by placing multiple antennas at both the transmitter and the receiver, have been extensively discussed in recent years. Space-time coding (STC) and spatial multiplexing (SM) are two popular schemes chosen to exploit the benefits offered by MIMO systems. Spatial multiplexing (SM) is known to be a simple and effective scheme enhancing the spectral efficiency of wireless channels.

Unfortunately, SM is sensitive to the condition number of the MIMO channel. When the channel matrix becomes ill-conditioned, the performance of SM becomes poor. Linear precoding, a technique that pre-multiplying the transmitted data vectors by a precoding matrix, is an effective way to solve the problem. Optimum precoder designs under different performance criteria have been studied in [1][2]. However, a problem associated with precoding is that the channel state information (CSI) must be known at transmitter. This may be difficult since the bandwidth of the feedback channel is usually limited. Thus, a codebook-based limited feedback precoding scheme is generally used. The main idea is to quantize the precoding matrix and feedback the index of the optimum precoder chosen from a finite set of precoding matrices, called a codebook known to both the receiver and the transmitter. The codeword selection and codebook design criteria are discussed in [1]. A practical codebook construction method used in [1] is described in [5], known as the Fourier-based design. In general, the exhaustive search is used in the determination of the optimum precoder. If the codebook size is large, which is the usual case, the required computational complexity will be high.

The precoding technique originally proposed for the narrowband channel can be easily extended to the broadband channel by using the technique of orthogonal frequency division multiplexing (OFDM). The combination of MIMO and OFDM, known as MIMO-OFDM, converts a broadband MIMO channel into a series of parallel narrowband MIMO channels, one for each OFDM subcarrier. The codebook-based feedback precoding scheme can then be performed independently at each subcarrier. Since the number of the subcarrier in OFDM systems is typically large, the searching complexity for optimum precoders can be very high. To solve this problem, [6] proposes a tree-structured codebook construction method using the Linde, Buzo and Gray (LBG) approach [7]. With the specially designed codebook, codeword searching can be conducted with a tree search algorithm, reducing the search complexity significantly. However, for codebooks not constructed with the method in [6], this method cannot be applied. In this paper, we propose a new low-complexity codeword searching algorithm. The distinct advantage of our method is that it can be applied for any given codebooks. The main idea is to partition codewords successively such that the tree search algorithm can be leveraged in the searching process. Simulation results show that the proposed method can significantly reduce the required complexity while the precoding performance is affected slightly.

## II. System Model

Fig. 1 and 2 show the block diagram of a precoded MIMO-OFDM system.

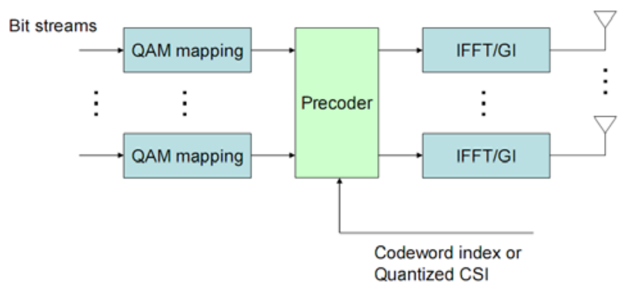


Fig. 1 Transmitter of precoded MIMO-OFDM system

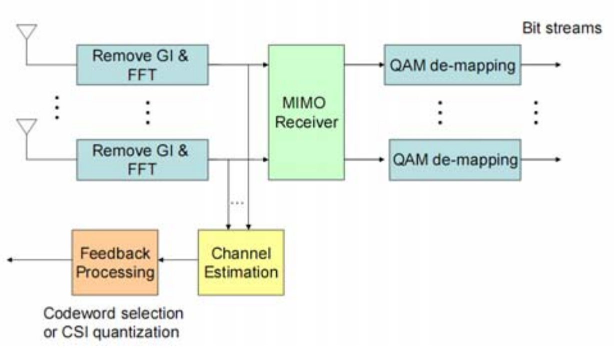


Fig. 2 Receiver of precoded MIMO-OFDM system

Information bits for subcarrier $k$ are first divided into $M$ different bit streams and sent into QAM mappers. Each of the $M$ bit streams is then mapped to QAM symbols. Let $\mathbf{s}_k$ be the symbol vector collecting of the symbols. Then,

$$\mathbf{s}_k = [s_{k,1}\ s_{k,2} \cdots s_{k,M}]^T \tag{1}$$

For convenience, we assume that the $M$ data streams are equally-powered and independent to each other. That is,

$$E[\mathbf{s}_k \mathbf{s}_k^*] = \mathbf{I}_M , \tag{2}$$

where $\mathbf{A}^*$ denotes complex-conjugate of matrix **A.** The symbol vector $\mathbf{s}_k$ is then multiplied by an $N_t \times M$ precoding matrix **F** (which is chosen as a function of the channel using criteria to be described below) producing a length $N_t$ vector $\mathbf{x}_k$,

$$\mathbf{x}_k = \sqrt{\frac{E_s}{M}} \cdot \mathbf{F} \cdot \mathbf{s}_k , \tag{3}$$

where $N_t$ is the number of transmit antennas and $E_s$ is the total transmit energy for subcarrier $k$. Here, we assume that $N_t > M$. Let the number of receive antennas be $N_r$. Then, the received signal vector at subcarrier $k$ can be represented as:

$$\begin{aligned} \mathbf{r}_k &= \mathbf{H} \cdot \mathbf{x}_k + \mathbf{n}_k \\ &= \sqrt{\frac{E_s}{M}} \cdot \mathbf{H} \cdot \mathbf{F} \cdot \mathbf{s}_k + \mathbf{n}_k \end{aligned} \tag{4}$$

where **H** is the $N_r \times N_t$ channel matrix and $\mathbf{n}_k$ is the $N_r \times 1$ noise vector. We assume that the entries of **H** are independent and identically distributed (i.i.d.) and their distributions are complex normal with zero mean and unit variance, denoted by $CN(0,1)$. Similarly, the entries of $\mathbf{n}_k$ are also i.i.d. and the distribution is $CN(0,N_0)$.

Here, we assume that the channel matrix **H** can be perfectly estimated, and a linear zero-forcing (ZF) receiver is applied. Let **G** be the $M \times N_r$ equalization matrix, and the estimated symbol vector be $\tilde{\mathbf{s}}$. Then,

$$\tilde{\mathbf{s}} = \mathbf{G} \cdot \mathbf{r}_k \tag{5}$$

It can be shown that the optimum **G** for the ZF receiver is

$$\mathbf{G} = (\mathbf{HF})^+ \tag{6}$$

where "+" denotes the matrix pseudo-inverse. It was also proved in [1] that for the ZF receiver, the optimum precoding matrix **F** selected from the codebook ***C*** maximizes the minimum singular value of **HF**. We call this the minimum singular value selection criterion (MSV-SC), i.e.,

$$\mathbf{F} = \arg\max_{\mathbf{F}_i \in \mathbf{C}} \lambda_{\min}\{\mathbf{HF}_i\} \tag{7}$$

The optimum un-quantized precoder for MSV-SC has also been shown in [1] [2]. Let the singular value decomposition (SVD) of the channel matrix **H** be represented by

$$\mathbf{H} = \mathbf{V_L \Sigma V_R}^* \tag{8}$$

Then, the optimum un-quantized precoding matrix for MSV-SC is

$$\mathbf{F}_{opt} = \bar{\mathbf{V}}_\mathbf{R} \tag{9}$$

which is the first $M$ columns of the right singular matrix $\mathbf{V_R}$.

In practice, the optimum precoder shown in (9) is difficult to feed back. Using the criterion in (7), we can select a precoding matrix from a codebook, and feed back the index to the transmitter. This is referred to as the codebook-based precoding scheme. In general, the exhaustive search is used to find the optimum precoding matrix in the codebook. If the codebook size is $L$ and the number of the subcarriers is $N$, we then have to conduct the operation in (7) for $LN$ times. The required computational complexity can be very high.

## III. Proposed Codeword Search Method

To facilitate the development of the proposed search method, we use a sub-optimum codeword selection criterion which minimizes the chordal distance between the chosen codeword and the ideal (un-quantized) optimum precoder, i.e.,

$$\mathbf{F} = \arg\min_{\mathbf{F}_i \in \mathbf{C}} d_{Chordal}\{\mathbf{F}_i, \mathbf{F}_{opt}\} \tag{10}$$

The chordal distance can be defined as follows [3][4]:

$$\begin{aligned} d_{chord}(\mathbf{F}_1, \mathbf{F}_2) &= \frac{1}{\sqrt{2}} \left\| \mathbf{F}_1\mathbf{F}_1^* - \mathbf{F}_2\mathbf{F}_2^* \right\|_F \\ &= \sqrt{M - \sum_{i=1}^{M} \lambda_i^2 \{\mathbf{F}_1^*\mathbf{F}_2\}} \end{aligned} \tag{11}$$

where $\| . \|_F$ denotes the matrix Frobenius norm. Simulation results show that this criterion can perform comparably to the MSV-SC criterion. With this distance-based codeword selection criterion, we propose a low complexity codeword search method as follows.

The proposed codeword search method is composed of two parts: the codebook partition and codeword searching parts.

The codebook partition can be done off-line, while the codeword searching is performed on-line

### A. Codebook partition

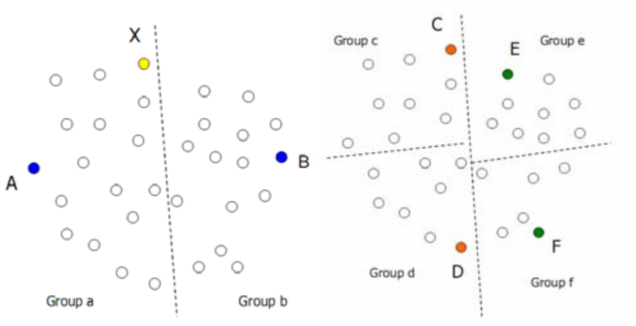


Fig. 3 Partition the codebook into groups

With a given codebook, we first find two codewords **A** and **B** which have the maximum chordal distance in the codebook. This can be done with an exhaustive search manner. With the two farthest codewords, all the codewords can then be partitioned into two groups with a simple distance comparison algorithm. Assume **X** is a codeword,

If $d(\mathbf{A}, \mathbf{X}) < d(\mathbf{B}, \mathbf{X})$
  **X** will be referred to group ***a***.
else
  **X** will be referred to group ***b***.

$d(\mathbf{A}, \mathbf{X})$ denotes the chordal distance between **A** and **X**. We refer **A** and **B** as reference codewords. Then, we can further find two reference codewords **C** and **D** in group ***a***, and two reference codewords **E** and **F** in group ***b***. Following the distance comparison algorithm, group ***a*** can be further partitioned into group ***c*** and group ***d***, and group ***b*** can be partitioned into group ***e*** and group ***f***. Continuing this process, we can finally partition the codebook into 2, 4, 8, … ,$2^m$ groups, where the integer $m$ is defined as the searching depth. The reference codewords within each group must be recorded during the partition process. Note that groups at depth $m$ are all contained in groups at depth $m$-1. In other words, the partitioned codebook has a nested structure.

### B. Codeword searching

After the codebook is partitioned, the tree structure is determined and stored. To find a codeword for a channel matrix, we can first conduct the SVD and obtain the optimum precoder $\mathbf{F}_{\text{opt}}$ . Then, we can conduct a tree searching in the partitioned codebook to locate the optimum codeword having a minimum distance with $\mathbf{F}_{\text{opt}}$.

The first step of the search is to compare $d(\mathbf{A}, \mathbf{F}_{\text{opt}})$ and $d(\mathbf{B}, \mathbf{F}_{\text{opt}})$, where **A** and **B** are two reference codewords in search depth one. If $\mathbf{F}_{\text{opt}}$ is nearer to the codeword **A**, the optimum codeword is assumed to be within group ***a***. Then, we can further compare $d(\mathbf{C}, \mathbf{F}_{\text{opt}})$ and $d(\mathbf{D}, \mathbf{F}_{\text{opt}})$, where **C** and **D** are two reference codewords within the group ***a*** (depth two). If $\mathbf{F}_{\text{opt}}$ is nearer to the codeword **C**, the optimum codeword is assumed to be within group ***c***. Repeat this process, and we can finally find the optimum codeword for a designated depth.

## IV. Simulation Results

We consider uncoded and precoded spatial multiplexing (SM) MIMO-OFDM systems, sending two independent data streams ($M$ = 2) simultaneously. For the uncoded case, a 2×2 system is used while for the precoded case, a 4 × 2 system is used ($N_t$ = 4, $N_r$ = 2). The QAM size is set as 16, the FFT size as 512, and the cyclic prefix (CP) size as 64. For simplicity, the precoded system uses a ZF receiver.

Fig. 4 shows the bit-error-rate (BER) comparison for a 2×2 uncoded system, a 4×2 ideally precoded system, and a 4×2 codebook-based precoded system. For codebook-based precoding, the codebook size $L$ is 64 and the codeword selection criterion is MSV-SC. We use the Fourier-based designed codebook [5] which can be downloaded from [8]. Here, the exhaustive search is conducted to find the optimum codeword for the codebook-based system. As we can see, precoding can significantly improve the system performance.

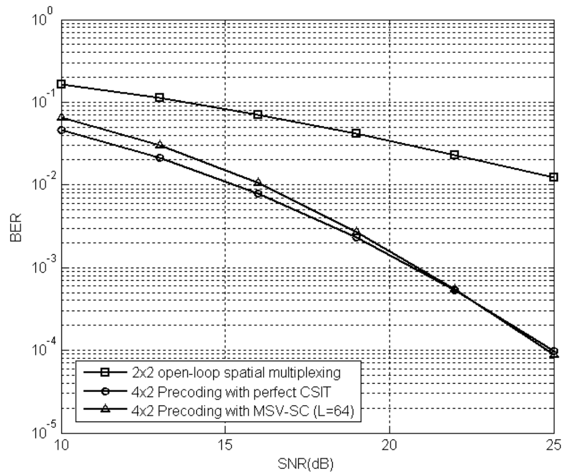


Fig. 4 BER comparison for 2×2 open-loop SM and 4×2 precoding

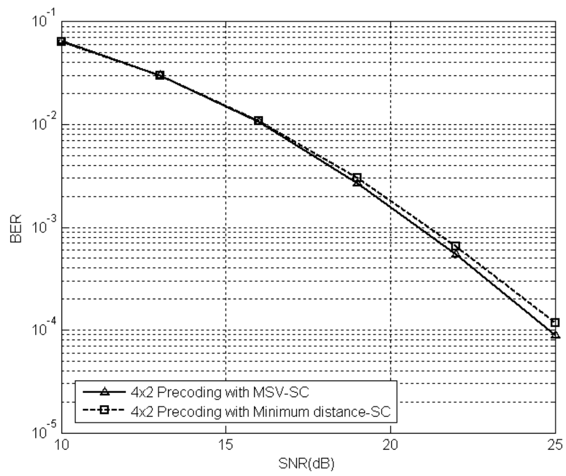


Fig. 5 BER comparison for precoding schemes with different codeword selection criteria

Fig. 5 shows the performance comparison between codebook-based precoding with different codeword selection criteria. It is apparent that minimum chordal distance selection criterion has comparable performance to MSV-SC.

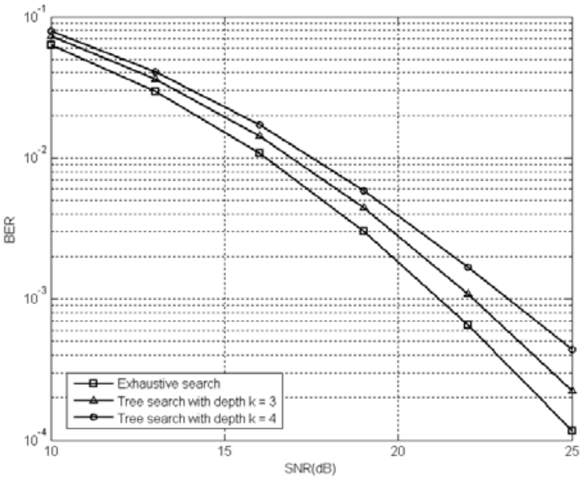


Fig. 6 BER comparison between exhaustive search and tree search (L=64)

Fig. 6 shows the BER performance for the exhaustive and the proposed codeword search methods. The codebook size used here is 64. From the figure, we can that the proposed codeword searching method with $m = 3$ has about 1dB performance loss compared to the exhaustive search. If we increase the searching depth $m$ to 4, the performance will further degrade about 1 dB. The performance degradation is caused by the codeword searching error. Increasing the searching depth $m$ will decrease the searching complexity (as discussed below), but the probability of codeword searching error will also increase.

Fig. 7 shows the BER performance for the proposed codeword search method with the codebook size of 128. Comparing Fig. 6 and Fig. 7, we can find that increasing the codebook size under the same searching depth will improve the performance. This is because increasing the codebook size will lower the probability of codeword searching error.

Table 1 shows the average complexity ratio, defined as the computational complexity of the proposed method divided by that of the exhaustive method.

TABLE I
AVERAGE COMPLEXITY RATIO

| Searching depth **m** | **Average complexity ratio** | |
|---|---|---|
| | **L** = 64 | **L** = 128 |
| 3 | 0.2177 | 0.1725 |
| 4 | 0.1866 | 0.1257 |
| 5 | | 0.1099 |

From Table 1, we can see that increasing the searching depth $m$ will decrease the complexity ratio. For example, if $L = 64$ and $m = 3$, the computational complexity for the proposed algorithm only requires 21.77% of that for the exhaustive search. If we increase $m$ to 4, the complexity ratio can be further reduced to 18.66%. The computational complexity of the proposed method is significantly lower than that of the exhaustive search method.

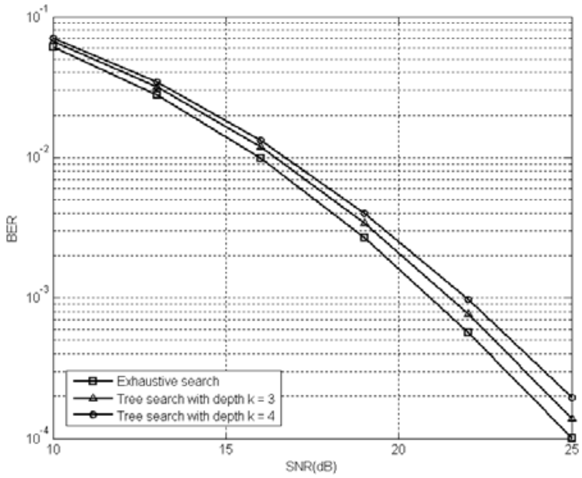


Fig. 7 BER comparison between exhaustive search and tree search (L=128)

## V. Conclusions

In this paper, we propose a low-complexity precoder searching algorithm, which consists of an off-line codebook partition part, and an on-line codeword searching part. Compared to the exhaustive search, the proposed searching method can reduce about 80% searching complexity with acceptable performance loss. The distinct advantage of the proposed method is that it can be used in any existing codebooks, and its application to real-world systems is highly feasible.

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