

# Joint Estimation and Compensation of Transmitter and Receiver Radio Impairments in MIMO-OFDM Receivers

Chen-Jiu Hsu\*, and Wern-Ho Sheen\*\*

\* Department of Communication Engineering, National Chiao Tung University, Hsinchu, 300 Taiwan

\*\* Department of Information and Communication Engineering, Chaoyang University of Technology, 413 Taiwan  
E-mail : linch.ee89@nctu.edu.tw, whsheen@cyut.edu.tw

**Abstract**—This paper aims to improve the performance of estimation and compensation for both the transmitter and receiver radio impairments in the MIMO-OFDM (multiple-input, multiple-output orthogonal frequency-division multiplexing) systems. First, a joint least-squares estimation of channel and radio impairments is developed with a complete set of radio impairments being taken into consideration, including frequency-dependent and independent I-Q imbalances, dc-offsets and frequency-offset. Previously, only parts of the radio impairments were included and/or treated separately from the radio channel. Second, a novel two-stage compensation scheme is proposed which is applicable to a general form of MIMO operations with any number of transmit and receive antennas. Numerical results show that the new design significantly outperforms the existing ones in error-rate performance and/or the number of training symbols required.

## I. INTRODUCTION

Direct-conversion radio architecture has been widely used in today's wireless devices because it is more amenable to monolithic integration and thus offers a low cost, small form factor design [1]. Nevertheless, it introduces radio impairments such as I-Q imbalance and dc-offset that, along with frequency offset, incur severe degradation in communication performance if not compensated accurately. Estimation and compensation of the radio impairments in the direct-conversion architecture has been a topic of extensive research [2]-[10]; the transmitter and receiver I-Q imbalances were investigated in [5][7][8] for the spatial-multiplexing MIMO-OFDM systems, and in [9] for the space-time block coded (STBC) systems, where a post-FFT compensation was proposed jointly with symbol detection on the extended channel that has a larger system dimension than the original system, which may largely increase the detector's complexity. In [10], the authors proposed per-tone equalization (PTEQ) to the spatial-multiplexing systems in the presence of the transmitter and receiver I-Q imbalances and frequency offset. However, the proposed method is only applicable to a linear MIMO detection and suffers from slow convergence.

This paper aims to improve the performance of estimation and compensation for both the transmitter and receiver radio impairments in the MIMO-OFDM receivers. Taking into consideration both the transmitter and receiver radio impairments is crucial in a wireless peer-to-peer communication, where a less precise analog front-end is likely to be implemented at both sides of communication. This paper is unique in twofold: First, the channel and the radio impairments are

estimated jointly in an optimal way under the least-squares (LS) criterion. Second, a novel two-stage compensation is proposed which is applicable to a general form of MIMO operations with any number of transmit and receive antennas. Simulation results show that the new design significantly outperforms the existing ones in error-rate performance and/or the number of training symbols required.

## II. SYSTEM MODEL

Figure 1 is the considered MIMO-OFDM system with a direct-conversion radio transceiver. Coming out of the OFDM base-band processing, the signal for transmit antenna  $i$  is

$$s_i(t) = \sum_{k=-P}^{K-1} \sum_{n=-N_g}^{N-1} s_{i,k}(n) \delta(t - (k(N + N_g) + n)T_s) \quad (1)$$

where

$$s_{i,k}(n) = \frac{1}{N} \sum_{l=0}^{N-1} S_{i,k}(l) \exp\left[\frac{j2\pi nl}{N}\right] \quad (2)$$

is the inverse fast Fourier transform (IFFT) of the transmitted data  $\{S_{i,k}(l)\}_{l=0}^{N-1}$  in OFDM symbol  $k$ ,  $j = \sqrt{-1}$ ,  $N_g$  is the length of cyclic prefix,  $N$  is FFT size,  $K + P$  is the packet length (in OFDM symbols),  $T_s$  is the data (symbol) duration, and  $\delta(t)$  is the Dirac delta function. The length of cyclic prefix is assumed larger than the maximum delay spread of the overall channel, and therefore there is no inter-block and inter-carrier interference. The data transmission is on a packet-by-packet basis with the first  $P$  OFDM symbols, indexed from  $K = -P$  to  $K = -1$ , as the training sequence for data-aided estimation.

Figure 1 (a) shows the direct-conversion radio transmitter for transmit antenna  $i$ , where  $f_{0,i} = f_{0,i}^I + j f_{0,i}^Q$  is the dc-offset, and  $h_{t,i}^I(t) + j h_{t,i}^Q(t)$  is the (unit-energy) base-band transmit filter. If  $h_{t,i}^I(t) \neq h_{t,i}^Q(t)$ , it is said that there is a frequency-dependent I-Q imbalance. The frequency-independent I-Q imbalance, on the other hand, is characterized by the parameters  $\alpha_{t,i}$  and  $\theta_{t,i}$ , which are the gain and phase imbalance respectively due to imperfect analog circuits of the mixer.  $f_c$  is the carrier frequency which is same for all transmit antennas; in other words, only one oscillator is used for all antennas at the transmitter for implementation simplicity. Figure 1(b) is the MIMO channel, where  $\tilde{h}_{j,i}(t) = \text{Re}\{h_{j,i}(t)e^{j2\pi f_c t}\}$  is the channel response from transmit antenna  $i$  to receive

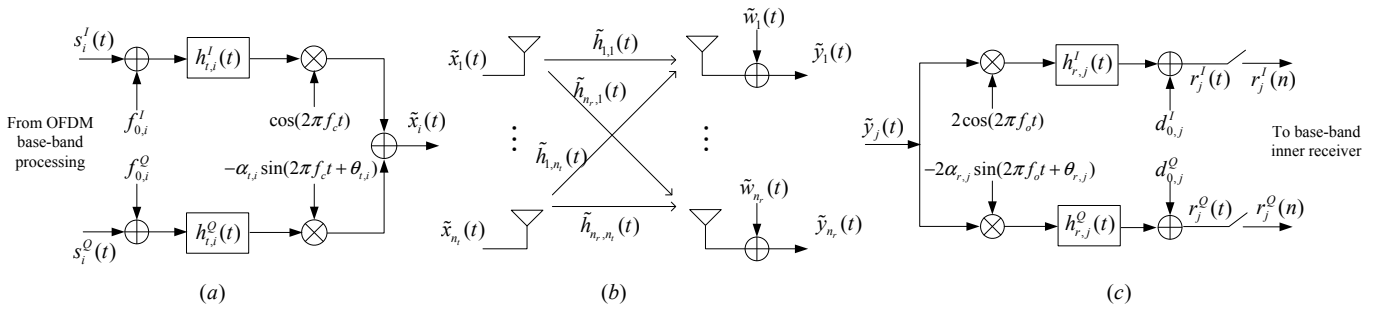


Fig. 1. Direct-conversion radio transceiver with I-Q imbalances, dc-offsets and frequency-offset (a) transmitter, (b) MIMO channel, and (c) receiver.

antenna  $j$ , and  $h_{j,i}(t)$  is the equivalent base-band.  $\tilde{w}_j(t) = \text{Re}\{w_{0,j}(t)e^{j2\pi f_0 t}\}$  is the pass-band additive white Gaussian noise, and  $w_{0,j}(t)$  is its base-band equivalent.

Figure 1(c) is the direct-conversion radio receiver for antenna  $j$ . Let  $\alpha_{r,j}$  and  $\theta_{r,j}$  be the frequency-independent gain and phase imbalance, respectively,  $h_{r,j}^I(t) + jh_{r,j}^Q(t)$  be the (unit-energy) base-band filter, and  $d_{0,j} = d_{0,j}^I + jd_{0,j}^Q$  be the dc-offset. Again, it is said to have frequency-dependent I-Q imbalance if  $h_{r,j}^I(t) \neq h_{r,j}^Q(t)$ .  $f_0 = f_c - \Delta f$  is the local oscillator frequency of the receiver, where  $\Delta f$  is the frequency-offset between transmitter and receiver which is same for all receiver branches. After some manipulations, the received signal after sampling can be expressed as [6]

$$\begin{aligned} r_j(n) &= r_j^I(n) + jr_j^Q(n) \\ &= h_{r+,j}(n) \otimes [y_j(n)e^{j2\pi\nu n} + w_{0,j}(n)] \\ &\quad + h_{r-,j}(n) \otimes [y_j(n)e^{j2\pi\nu n} + w_{0,j}(n)]^* + d_{0,j} \end{aligned} \quad (3)$$

where

$$\begin{aligned} y_j(n) &= \sum_{i=1}^{n_t} [s_i(n) \otimes h_{t+,i}(n) \\ &\quad + s_i^*(n) \otimes h_{t-,i}(n) + f_{1,i}] \otimes h_{j,i}(n), \end{aligned} \quad (4)$$

$h_{r\pm,j}(n) = 1/2 \cdot [h_{r,j}^I(n) \pm \alpha_{r,j} e^{\mp j\theta_{r,j}} h_{r,j}^Q(n)]$ ,  $h_{t\pm,i}(n) = 1/2 \cdot [h_{t,i}^I(n) \pm \alpha_{t,i} e^{j\theta_{t,i}} h_{t,i}^Q(n)]$ ,  $f_{1,i} = f_{0,i} \otimes h_{t+,i}(n) + f_{0,i}^* \otimes h_{t-,i}(n)$ ,  $\nu = \Delta f T_S$  is the normalized frequency-offset, and  $w_{0,j}(n)$  is a zero mean additive white Gaussian noise.  $[x]^*$  and  $\otimes$  denote the complex conjugate of  $x$  and the operation of linear convolution, respectively. In addition,  $h_{r-,j}(n) \otimes [y_j(n)e^{j2\pi\nu n} + w_{0,j}(n)]^*$  in (3), and  $s_i^*(n) \otimes h_{t-,i}(n)$  in (4) are the mirror interferences induced by the receiver and transmitter I-Q imbalances, respectively.

### III. JOINT ESTIMATION OF RADIO IMPAIRMENTS AND CHANNEL

From (3) and (4), it can be seen that the mirror interferences due to the transmitter and receiver I-Q imbalances appear at different mirror frequencies in the presence of frequency offset, and thus they cannot be combined as one and compensated by applying the technique developed in [6], where only the receiver radio impairments were considered. (The same argument applies to the dc-offset). This motivates us to propose a two-stage compensation scheme (detailed in

this and next sections). With the two-stage compensation in mind, a time-domain filter is introduced as follows for each receive branch, say branch  $j$ , aiming to cancel out the mirror interference due to the receiver I-Q imbalance

$$\begin{aligned} r_j(n) - \rho_j(n) \otimes r_j^*(n) \\ = \tilde{h}_{r+,j}(n) \otimes (y_j(n)e^{j2\pi\nu n} + w_{0,j}(n)) \\ + \tilde{h}_{r-,j}(n) \otimes (y_j(n)e^{j2\pi\nu n} + w_{0,j}(n))^* + d_j \end{aligned} \quad (5)$$

where  $\tilde{h}_{r\pm,j}(n) = h_{r\pm,j}(n) - \rho_j(n) \otimes h_{r\mp,j}^*(n)$ , and  $d_j = d_{0,j} - \rho_j(n) \otimes d_{0,j}^*$ . For a perfect cancellation,  $\tilde{h}_{r-,j}(n) = 0$ , that is,  $\rho_j(n) = (h_{r+,j}^*(n))^{-1} \otimes h_{r-,j}(n)$ , where  $(h_{r+,j}^*(n))^{-1}$  is the inverse filter of  $h_{r+,j}^*(n)$ . In practice, however, the cancellation may not be perfect. In this case,

$$\begin{aligned} r_j(n) - \rho_j(n) \otimes r_j^*(n) = e^{j2\pi\nu n} \left[ \left( \sum_{i=1}^{n_t} s_i(n) \otimes h_{+,j,i}(n) \right. \right. \\ \left. \left. + s_i^*(n) \otimes h_{-,j,i}(n) \right) + f_j \right] + d_j + w_j(n) \end{aligned} \quad (6)$$

where

$$\begin{aligned} w_j(n) &= \tilde{h}_{r+,j}(n) \otimes w_{0,j}(n) \\ &\quad + \tilde{h}_{r-,j}(n) \otimes [y_j(n)e^{j2\pi\nu n} + w_{0,j}(n)]^*, \end{aligned} \quad (7)$$

$h_{\pm,j,i}(n) \doteq h_{t\pm,i}(n) \otimes h_{j,i}(n) \otimes (\tilde{h}_{r\pm,j}(n)e^{-j2\pi\nu n})$ , and  $f_j = \sum_{i=1}^{n_t} f_{1,i} \otimes h_{j,i}(n) \otimes (\tilde{h}_{r+,j}(n)e^{-j2\pi\nu n})$ . Here,  $h_{+,j,i}(n)$  is the overall impulse response from transmit branch  $i$  to receive branch  $j$  involving transmit filter, channel, and receive filter after canceling out the receiver mirror interference while  $h_{-,j,i}(n)$  is the overall impulse response due to the transmitter I-Q imbalance.  $f_j$  and  $d_j$  are the equivalent transmitter and receiver dc-offset to be compensated, and  $w_j(n)$  is the composite effects of additive white Gaussian noise and residual error after receiver I-Q cancellation. In (6),  $\{\rho_j(n)\}$ ,  $\{h_{\pm,j,i}(n)\}$ ,  $\{f_j\}$ ,  $\{d_j\}$ , and  $\nu$  are the parameters needed to be estimated and compensated from the received signal before it can be passed on to later signal processing. The estimation will be based on the least-squares principle; that is, the optimum parameters are sought to minimize the square error of  $\{w_j(n)\}$ .

Similar to [6],  $\rho_j(n)$  and  $h_{\pm,j,i}(n)$  are approximated by the FIR filters  $\rho_j$ ,  $\mathbf{h}_{\pm,j,i}$ , with the length  $L_\rho$  and  $L_{\mathbf{h}_{\pm}}$ , respectively.<sup>1</sup> Recall that the first  $P$  OFDM symbols, indexed from  $k = -P, \dots, -1$ , serve as the training sequence for the estimation of radio impairments and channel. Let  $\mathbf{r}_j(k) = [r_{j,k}(0), \dots, r_{j,k}(N-1)]^T$  with  $r_{j,k}(n) \doteq r_j(k(N_g + N) + n)$  be the useful part of OFDM symbol  $k$ ,  $\mathbf{w}_j(k) = [w_{j,k}(0), w_{j,k}(1), \dots, w_{j,k}(N-1)]^T$  with  $w_{j,k}(n) = w_j(k(N_g + N) + n)$ ,  $\mathbf{R}_j(k)$  be the  $N \times L_\rho$  received signal matrix with the  $(p, q)$ -th entry  $[\mathbf{R}_j(k)]_{p,q} = r_{j,k}(p - q)$ ,  $0 \leq p \leq N - 1$ ,  $0 \leq q \leq L_\rho - 1$ , and  $\mathbf{T}_{\pm,i}(k)$  be the  $N \times L_{\mathbf{h}_{\pm}}$  signal matrix with  $[\mathbf{T}_{\pm,i}(k)]_{p,q} = s_{i,k}(p - q)$ ,  $0 \leq p \leq N - 1$ ,  $0 \leq q \leq L_{\mathbf{h}_{\pm}} - 1$ . Furthermore, define  $\mathbf{r}_j = [\mathbf{r}_j^T(-P), \dots, \mathbf{r}_j^T(-1)]^T$ ,  $\mathbf{R}_j = [\mathbf{R}_j^T(-P), \dots, \mathbf{R}_j^T(-1)]^T$ ,  $\mathbf{T}(k) = [\mathbf{T}_{+,1}(k), \dots, \mathbf{T}_{+,n_t}(k), \mathbf{T}_{-1}^*(k), \dots, \mathbf{T}_{-,n_t}^*(k), \mathbf{1}_N]$ ,  $\mathbf{h}_j = [\mathbf{h}_{+,j,1}^T, \dots, \mathbf{h}_{+,j,n_t}^T, \mathbf{h}_{-,j,1}^T, \dots, \mathbf{h}_{-,j,n_t}^T, f_j]^T$ , and  $\mathbf{w}_j = [\mathbf{w}_j^T(-P), \dots, \mathbf{w}_j^T(-1)]^T$ . From (6) and using these notations, we have

$$\mathbf{r}_j - \mathbf{R}_j^* \boldsymbol{\rho}_j = \boldsymbol{\Gamma}(\nu) \mathbf{T} \mathbf{h}_j + d_j \mathbf{1} + \mathbf{w}_j \quad (8)$$

where  $\mathbf{T} = [\mathbf{T}^T(-P), \dots, \mathbf{T}^T(-1)]^T$ ,  $\mathbf{1}$  is the all 1 vector with dimension  $NP$ , and  $\boldsymbol{\Gamma}(\nu) = \text{diag}\{\boldsymbol{\Gamma}_{-P}(\nu), \dots, \boldsymbol{\Gamma}_{-1}(\nu)\}$  with  $\boldsymbol{\Gamma}_k(\nu) = e^{j2\pi k(N_g + N)\nu} \cdot \text{diag}\{1, e^{j2\pi\nu}, \dots, e^{j2\pi\nu(N-1)}\}$ . Since the optimization problem formulation in (8) carries the same form as that in [6], the recursive optimization procedure developed there can be used here to obtain the joint estimates.

#### IV. TWO-STAGE COMPENSATION OF RADIO IMPAIRMENTS

We consider the general MIMO structure of linear-dispersion (LD) codes, which subsumes spatial multiplexing and STBC as special cases and is applicable to any number of transmit and receive antennas [11]. Without loss of generality, the first code block that starts from the zero-th OFDM symbol is considered for notation simplicity. From [11], a set of  $n_s$  data symbols  $\{D_m(l)\}_{m=1}^{n_s}$ , which is to be transmitted on sub-carrier  $l$  of  $\kappa$  consecutive OFDM symbols, is encoded as a  $\kappa \times n_t$  LD code matrix  $\mathbf{S}(l)$  as follows

$$\begin{aligned} \mathbf{S}(l) &= \begin{bmatrix} S_{1,0}(l) & \cdots & S_{n_t,0}(l) \\ \vdots & \ddots & \vdots \\ S_{1,\kappa-1}(l) & \cdots & S_{n_t,\kappa-1}(l) \end{bmatrix} \\ &= \sum_{m=1}^{n_s} (\text{Re}\{D_m(l)\} \mathbf{A}_m + j \text{Im}\{D_m(l)\} \mathbf{B}_m) \quad (9) \end{aligned}$$

where  $\mathbf{A}_m$  and  $\mathbf{B}_m$  are  $\kappa \times n_t$  complex-valued dispersion matrices that are designed to reap diversity and/or degree of freedom gains of the MIMO channel. As in [11], with a proper selection of the dispersion matrices, spatial-multiplexing and STBC can be viewed as special cases of (9). Recall that  $\{S_{i,k}(l)\}_{i=0}^{N-1}$  is the data symbols input to the IFFT in (2).

<sup>1</sup>Throughout this paper, bold uppercase letters denote matrices and bold lowercase letters denote vectors.  $(\cdot)^T$  and  $(\cdot)^H$  represent the operations of transpose and conjugate transpose of a matrix or vector, respectively.

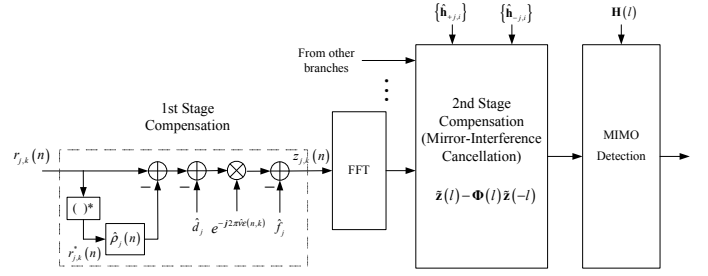


Fig. 2. The two-stage compensation scheme.

The proposed two-stage compensation is shown in Figure 2. From (6), the time-domain compensation is straightforward, where the receiver I-Q imbalance is compensated first, followed by the receiver dc-offset, frequency-offset and the transmitter dc-offset. With perfect compensation, the received signal becomes

$$\begin{aligned} z_{j,k}(n) &= e^{-j2\pi\hat{\nu}\varepsilon(n,k)} [r_{j,k}(n) - \hat{\rho}_j(n) \otimes r_{j,k}^*(n) - \hat{d}_j] - \hat{f}_j \\ &= \sum_{i=1}^{n_t} [s_{i,k}(n) \otimes h_{+,j,i}(n) + s_{i,k}^*(n) \otimes h_{-,j,i}(n)] + \omega_{j,k}(n) \end{aligned} \quad (10)$$

$n = 0, \dots, N - 1$ , where  $\varepsilon(n, k) = k(N + N_g) + n$ , and  $\omega_{j,k}(n) = e^{-j2\pi\hat{\nu}\varepsilon(n,k)} w_{j,k}(n)$ . (10) says that at this point the mirror interference due to the transmitter I-Q imbalance is the only impairment left to be compensated. Here, we propose a new method, called frequency-domain mirror-interference cancellation, for the compensation. Starting from (10), taking FFT on  $z_{j,k}(n)$ ,

$$Z_{j,k}(l) = \sum_{i=1}^{n_t} [H_{+,j,i}(l) S_{i,k}(l) + H_{-,j,i}(l) S_{i,k}^*(-l)] + \Omega_{j,k}(l) \quad (11)$$

where  $\{X(l)\}_{l=0}^{N-1} \doteq \text{FFT}[\{x(n)\}_{n=0}^{N-1}]$ , and  $\{\Omega(l)\}_{l=0}^{N-1} \doteq \text{FFT}[\{\omega(n)\}_{n=0}^{N-1}]$ . In addition, using a matrix form for those  $Z_{j,k}(l)$  corresponding to the LD code matrix  $\mathbf{S}(l)$ , one has

$$\mathbf{Z}(l) = \mathbf{S}(l) \mathbf{G}_+(l) + \mathbf{S}^*(-l) \mathbf{G}_-(l) + \boldsymbol{\Omega}(l) \quad (12)$$

where

$$\mathbf{Z}(l) = \begin{bmatrix} Z_{1,0}(l) & \cdots & Z_{n_r,0}(l) \\ \vdots & \ddots & \vdots \\ Z_{1,\kappa-1}(l) & \cdots & Z_{n_r,\kappa-1}(l) \end{bmatrix} \quad (13)$$

$$\boldsymbol{\Omega}(l) = \begin{bmatrix} \Omega_{1,0}(l) & \cdots & \Omega_{n_r,0}(l) \\ \vdots & \ddots & \vdots \\ \Omega_{1,\kappa-1}(l) & \cdots & \Omega_{n_r,\kappa-1}(l) \end{bmatrix} \quad (14)$$

and

$$\mathbf{G}_{\pm}(l) = \begin{bmatrix} H_{\pm,1,1}(l) & \cdots & H_{\pm,n_r,1}(l) \\ \vdots & \cdots & \vdots \\ H_{\pm,1,n_t}(l) & \cdots & H_{\pm,n_r,n_t}(l) \end{bmatrix} \quad (15)$$

Furthermore, let  $\mathbf{z}_j(l)$ ,  $\boldsymbol{\omega}_j(l)$ , and  $\mathbf{g}_{\pm,j}(l)$  denote the  $j$ -th column of  $\mathbf{Z}(l)$ ,  $\boldsymbol{\Omega}(l)$ , and  $\mathbf{G}_{\pm}(l)$ , respectively. Define  $\tilde{\mathbf{d}}(l) =$

$$\begin{aligned} & [\text{Re}\{D_1(l)\}\text{Im}\{D_1(l)\}, \dots, \text{Re}\{D_{n_s}(l)\}\text{Im}\{D_{n_s}(l)\}]^T, \\ \tilde{\mathbf{z}}(l) &= [\text{Re}\{\mathbf{z}_1^T(l)\}\text{Im}\{\mathbf{z}_1^T(l)\}, \dots, \text{Re}\{\mathbf{z}_{n_r}^T(l)\}\text{Im}\{\mathbf{z}_{n_r}^T(l)\}]^T, \\ \tilde{\omega}(l) &= [\text{Re}\{\omega_1^T(l)\}\text{Im}\{\omega_1^T(l)\}, \dots, \text{Re}\{\omega_{n_r}^T(l)\}\text{Im}\{\omega_{n_r}^T(l)\}]^T, \end{aligned}$$

$$\tilde{\mathbf{A}}_{\pm, m} = \begin{bmatrix} \text{Re}\{\mathbf{A}_m\} & \mp \text{Im}\{\mathbf{A}_m\} \\ \pm \text{Im}\{\mathbf{A}_m\} & \text{Re}\{\mathbf{A}_m\} \end{bmatrix},$$

and

$$\tilde{\mathbf{B}}_{\pm, m} = \begin{bmatrix} -\text{Im}\{\mathbf{B}_m\} & \mp \text{Re}\{\mathbf{B}_m\} \\ \pm \text{Re}\{\mathbf{B}_m\} & -\text{Im}\{\mathbf{B}_m\} \end{bmatrix}.$$

As in [11], it is convenient to rewrite (12) in the form of real matrices and vectors as follows

$$\tilde{\mathbf{z}}(l) = \tilde{\mathbf{G}}_+(l) \tilde{\mathbf{d}}(l) + \tilde{\mathbf{G}}_-(l) \tilde{\mathbf{d}}(-l) + \tilde{\omega}(l) \quad (16)$$

where  $\tilde{\mathbf{G}}_{\pm}(l)$  shown at the bottom of the page is a tall or square matrix with dimension  $2\kappa n_r \times 2n_s$ . From (16), the mirror-interference cancellation is proposed, by introducing the filter  $\Phi(l)$ , as follows

$$\begin{aligned} \tilde{\mathbf{z}}(l) - \Phi(l) \tilde{\mathbf{z}}(-l) &= [\tilde{\mathbf{G}}_+(l) - \Phi(l) \tilde{\mathbf{G}}_-(l)] \tilde{\mathbf{d}}(l) \\ &+ \underbrace{[\tilde{\mathbf{G}}_-(l) - \Phi(l) \tilde{\mathbf{G}}_+(-l)]}_{=0} \tilde{\mathbf{d}}(-l) + [\tilde{\omega}(l) - \Phi(l) \tilde{\omega}(-l)] \\ &= \mathbf{H}(l) \tilde{\mathbf{d}}(l) + \omega(l) \end{aligned} \quad (17)$$

to cancel mirror interference

where  $\mathbf{H}(l) = \tilde{\mathbf{G}}_+(l) - \Phi(l) \tilde{\mathbf{G}}_-(l)$  and  $\omega(l) = \tilde{\omega}(l) - \Phi(l) \tilde{\omega}(-l)$  is the effective channel and noise after compensation. Clearly, to cancel out the mirror interference completely, we need to have

$$\tilde{\mathbf{G}}_-(l) - \Phi(l) \tilde{\mathbf{G}}_+(-l) = \mathbf{0} \quad (18)$$

For the case  $\kappa n_r = n_s$ , (18) has a unique solution of

$$\hat{\Phi}(l) = \tilde{\mathbf{G}}_- \tilde{\mathbf{G}}_+^{-1}(-l) \quad (19)$$

for the case  $\kappa n_r > n_s$ , however, there are infinite solutions. Naturally, the one with minimum noise power  $E\{\|\omega(l)\|^2\}$  is sought in this case. That is, the optimal  $\hat{\Phi}(l)$  is obtained by solving the following constrained optimization problem.

$$\hat{\Phi}(l) = \arg \min_{\tilde{\Phi}(l)} E\{\|\omega(l)\|^2\}, \text{ s.t. } \tilde{\mathbf{G}}_-(l) - \tilde{\Phi}(l) \tilde{\mathbf{G}}_+(-l) = \mathbf{0} \quad (20)$$

Moreover, it can be shown that

$$E\{\|\omega(l)\|^2\} \approx 2\kappa n_r \sigma^2 + \sigma^2 \text{tr}\{\tilde{\Phi}^T(l) \tilde{\Phi}(l)\} \quad (21)$$

Here we have used the approximations  $E\{\tilde{\omega}(l) \tilde{\omega}^T(l)\} \approx \sigma^2 \mathbf{I}_{2\kappa n_r}$  and  $E\{\tilde{\omega}(l) \tilde{\omega}^T(-l)\} \approx \mathbf{0}$ . Therefore, (20) becomes

$$\hat{\Phi}(l) = \arg \min_{\tilde{\Phi}(l)} \text{tr}\{\tilde{\Phi}^T(l) \tilde{\Phi}(l)\}, \text{ s.t. } \tilde{\mathbf{G}}_-(l) - \tilde{\Phi}(l) \tilde{\mathbf{G}}_+(-l) = \mathbf{0}$$

TABLE I  
RF PARAMETER VALUE

$(\alpha_{t,i}, \theta_{t,i})$	$(1.05, 5^\circ), (0.94, -6^\circ)$
$(\alpha_{r,j}, \theta_{r,j})$	$(1.08, 5^\circ), (0.91, 6^\circ), (0.92, -5^\circ), (1.09, -6^\circ)$
$\{h_{t,i}^I(n), h_{t,i}^Q(n)\}$ ,	I part : [1 0.3 0.2 0.1]
$\{h_{r,j}^I(n), h_{r,j}^Q(n)\}$	Q part : [0.9 0.4 0.15 0.15]
$\Delta f$	uniform over -0.5 and 0.5 subcarrier spacing
$ f_{0,i} ,  d_{0,j} $	0.15, -0.1, 0.08, -0.12

The problem amounts to solve the minimum norm solution of the linear equations of  $\tilde{\mathbf{G}}_+^T(-l) \tilde{\Phi}^T(l) = \tilde{\mathbf{G}}_-^T(l)$  for  $\tilde{\Phi}^T(l)$  which is given by

$$\hat{\Phi}(l) = \left[ \tilde{\mathbf{G}}_+(-l) \left( \tilde{\mathbf{G}}_+^T(-l) \tilde{\mathbf{G}}_+(-l) \right)^{-1} \tilde{\mathbf{G}}_-^T(l) \right]^T \quad (22)$$

After the mirror-interference cancellation, any type of MIMO detectors can be used for detecting  $\tilde{\mathbf{d}}(l)$  from (17). Note that the second-stage compensation deals the same problem as that tackled in [5] and [7]-[9], where only the impairment of I-Q imbalance was considered with the detection done on extended channel. This will largely increase the detector's complexity especially if MAP (Maximum a posteriori) or ML (Maximum Likelihood) type of detection is employed. In our approach, on the contrary, the system dimension is kept the same as the one with no I-Q imbalance, which may result in a lower detection complexity.

## V. NUMERICAL RESULTS

The performance of the proposed receiver is evaluated for an uncoded MIMO-OFDM system with 64-QAM modulation order and MMSE detection. The system parameters are set as FFT length  $N = 64$ , cyclic prefix length  $N_g = 16$ , and symbol time  $T_s = 50n_s$ . The dc-offset is given by  $d_{0,j} = |d_{0,j}| \cdot (1+j)/\sqrt{2}$  and  $f_{0,i} = |f_{0,i}| \cdot (1+j)/\sqrt{2}$  with signal power normalized to 1. Table 1 gives the impairments parameters. The transmission is done on a packet-by-packet basis beginning with the training sequence similar to 802.11a spec [12]. An exponential decay multipath channel is considered with root-mean square delay spread  $T_{RMS} = 50n_s$ . The length of channel is 10 taps, and each tap is zero mean independently complex Gaussian random variable. The parameters are set as  $L_{h_+} = 13$  and  $L_{h_-} = 9$ .

Figure 3 shows the BER performance over fading channels with different  $L_\rho$ 's that characterize the effect of FIR approximation of the filters  $\{\rho_j(n)\}_{j=1}^{n_r}$ .  $L_\rho \geq 5$  is usually enough to obtain good performance in all our numerical results. The BER performance with receiver radio impairments compensation only proposed in [6] is also shown for comparison purpose. Clearly, the transmitter radio impairments incurs error floor if left not compensated, as one can expect. Figure 4 compares

$$\tilde{\mathbf{G}}_{\pm}(l) = \begin{bmatrix} \tilde{\mathbf{A}}_{\pm,1} \tilde{\mathbf{g}}_{\pm,1}(l) & \tilde{\mathbf{B}}_{\pm,1} \tilde{\mathbf{g}}_{\pm,1}(l) & \cdots & \tilde{\mathbf{A}}_{\pm,n_s} \tilde{\mathbf{g}}_{\pm,1}(l) & \tilde{\mathbf{B}}_{\pm,n_s} \tilde{\mathbf{g}}_{\pm,1}(l) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \tilde{\mathbf{A}}_{\pm,1} \tilde{\mathbf{g}}_{\pm,n_r}(l) & \tilde{\mathbf{B}}_{\pm,1} \tilde{\mathbf{g}}_{\pm,n_r}(l) & \cdots & \tilde{\mathbf{A}}_{\pm,n_s} \tilde{\mathbf{g}}_{\pm,n_r}(l) & \tilde{\mathbf{B}}_{\pm,n_s} \tilde{\mathbf{g}}_{\pm,n_r}(l) \end{bmatrix}, \quad \tilde{\mathbf{g}}_{\pm,j}(l) = \begin{bmatrix} \text{Re}\{\mathbf{g}_{\pm,j}(l)\} \\ \text{Im}\{\mathbf{g}_{\pm,j}(l)\} \end{bmatrix}$$

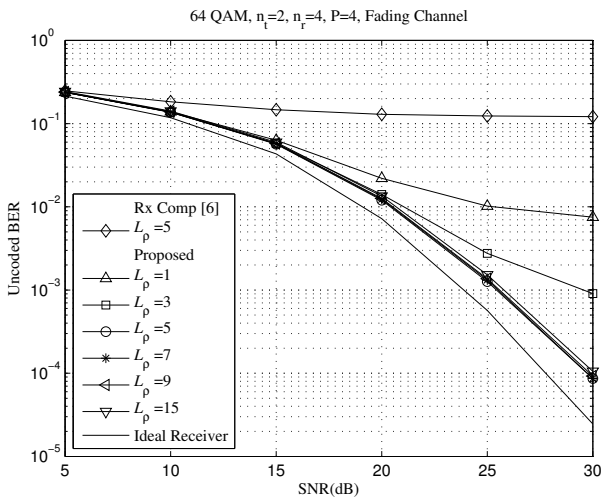


Fig. 3. The effect of FIR approximation on the time-domain filter  $\rho_j(n)$ .

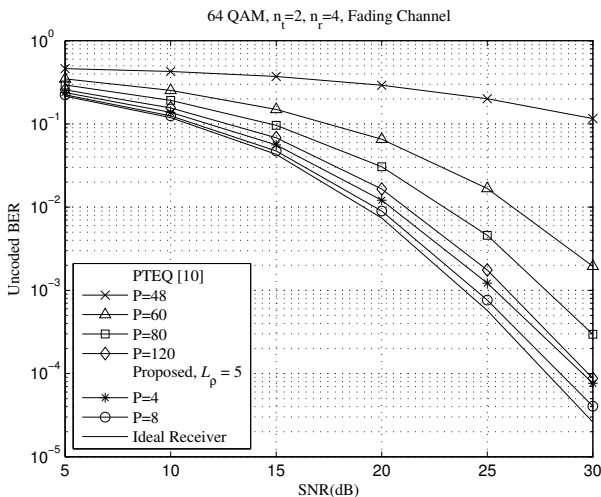


Fig. 4. Performance comparisons between the proposed method and per-tone equalization [10] (spatial-multiplexing MIMO).

the BER performance of the proposed receiver with the PTEQ in [10], as an example of the spatial-multiplexing MIMO systems. Since there is no method for frequency and dc-offset estimation in [10], ideal frequency compensation with no dc-offset (both sides) is assumed for comparison purpose. As can be seen, the new receiver significantly outperforms PTEQ in terms of BER and the required training symbols. Figure 5 is the comparison of the proposed method with the one in [9] as an example of STBC-MIMO systems. Again, ideal frequency compensation with no dc-offsets is assumed. Clearly, the method [9] does not work properly in the presence of frequency-offset which ranges from -0.5 to 0.5 subcarrier spacing in our simulations.

## VI. CONCLUSION

In this paper, a new estimation and compensation method is proposed for the transmitter and receiver radio impairments in the liner-dispersion coded MIMO-OFDM systems. The radio

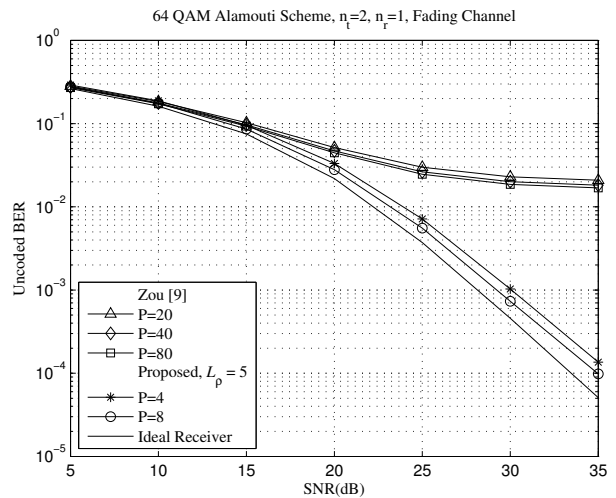


Fig. 5. Performance comparisons between the proposed method and the one in [9] (STBC MIMO).

impairments and channel are jointly estimated under the least-squares criterion. A novel two-stage method consisting of time and frequency-domain compensation is proposed that is applicable to a general form of MIMO operations with any number of transmit and receive antennas. Numerical results show that significant performance improvement is observed for both the spatial-multiplexing and STBC MIMO systems.

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