Fault-Tolerance Analysis of a Wireless Sensor Network with Distributed Classification Codes

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Abstract—**In this work, we analyze the performance of a wireless sensor network with distributed classification codes, where independence across sensors, including local observations, local classifications and sensor-fusion link noises, is assumed. In terms of large deviations technique, we establish the necessary and sufficient condition under which the minimum Hamming distance fusion error vanishes as the number of sensors tends to infinity. With the necessary and sufficient condition and the upper performance bounds, the relation between the fault-tolerance capability of a distributed classification code and its pair-wise Hamming distances is characterized.**

I. Introduction

Consider a wireless sensor network (WSN) that consists of N sensors, N wireless and hence noisy one-way communication links, and a fusion center as shown in Fig. 1. The WSN is tasked with the solution of a M-ary hypothesis testing or classification problem. Compression on the local observation is assumed to be performed at each sensor before information is sent to the fusion center. In this work, we are specifically concerned with the case where the sensor nodes only send out binary decisions to the fusion center at which they are fused to produce the final M-ary decision.

An issue that may be encountered in the WSNs considered is that the wireless binary-output sensor that is supposed to be manufactured by a simple and low-cost technology may suffer from hardware as well as software malfunctions after deployment over a harsh environment [1]. Therefore, the fault-tolerance capability to protect against unexpected sensor failures is of great importance in such cases to maintain an acceptable level of performance in a WSN.

To achieve the desired robustness against sensor faults, a distributed classification code has been proposed to be used in the wireless sensor network to provide a good fault-tolerance capability under feasible system complexity [3]. It was shown in [3] that with adequately high probability, the decision made by the minimum Hamming distance fusion rule can fall into the correct acceptance region even if several sensor faults that are unknown to the fusion center are present.

In [2], we had characterized the asymptotic performance of the minimum Hamming distance fusion rule under some restrictive assumptions. In this work, we extend our analysis in [2] by relaxing the assumptions of common distribution for all local observations and identical local classification rule for all sensors. Also, only independence across sensors is assumed for the additive noises over the wireless links. Contrary to the requirement of sufficiently large number of sensors in [2], the probability bounds obtained in this work are now valid for any finite number of sensors. In particular, the necessary and sufficient condition under which the minimum Hamming distance fusion error vanishes as the number of sensors tends to infinity is established. With the necessary and sufficient condition and the upper bounds on the error probability, the relation between the fault-tolerance capability of a distributed classification code and its pair-wise Hamming distances is characterized.

II. System model

As depicted in Fig. 1, the distributed M-ary classification system assumes that the local observations $\{y_j\}_{j=1}^N$ are conditionally independent given each hypothesis, and each local sensor classifies its own observation, independent of all others, to one of the M hypotheses using its own decision rule. Denote by $h_{\ell|i}^{(j)}$ the probability of classifying H_ℓ given that H_i is the true hypothesis at sensor j. Also assume that the prior probability of each hypothesis is equal, and the event of link error, i.e, $[u_j \neq u_j^*]$, is not only independent across sensors but independent of the local observation as well as the true hypothesis H_i.

Based on the assumed statistics, an $M \times N$ code matrix $\boldsymbol{C}$ is then designed in advance, of which element $c_{\ell,j}$ lies in $\{0, 1\}$ for $\ell = 0, \ldots, M-1$ and $j = 1, \ldots, N$. In the code matrix, each hypothesis is associated with a row, and each column stands for the local binary outputs corresponding to the classified hypotheses at the respective sensor. Thus, sensor j transmits $c_{\ell,j}$, if H_ℓ is declared locally. For notational convenience, $\boldsymbol{c}_\ell \triangleq (c_{\ell,1}, c_{\ell,2}, \ldots, c_{\ell,N})$ is used to denote the row of $\boldsymbol{C}$ corresponding to the hypothesis H_ℓ.

After the observation is locally processed, the local output code bit u_j^* is transmitted to the fusion center. The fusion center receives the word $\boldsymbol{u} = (u_1, u_2, \ldots, u_N)$, where u_j and u_j^* form a binary symmetric channel (BSC) with crossover probability ϵ_j. The minimum Hamming distance fusion rule,

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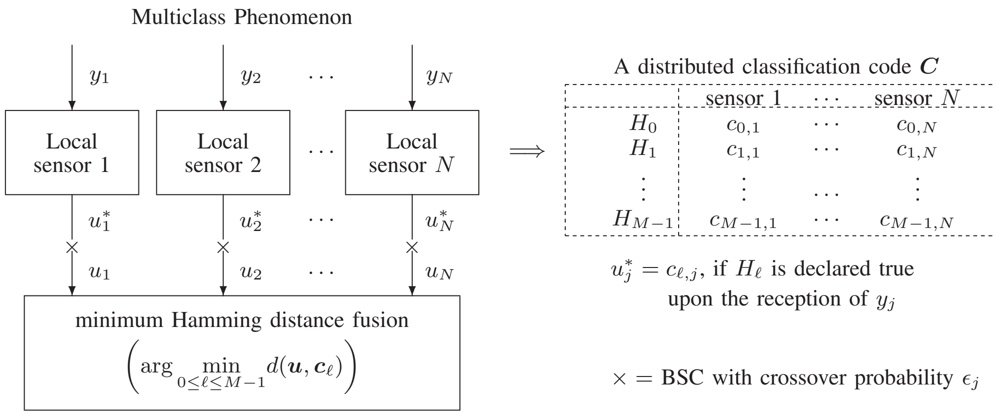

Fig. 1. System model for a WSN with distributed classification code.

or specifically, $\omega = \arg\min_{0\leq \ell\leq M-1} d(\boldsymbol{u}, \boldsymbol{c}_\ell)$, is then employed to obtain the multiclass decision ω, where $d(\cdot,\cdot)$ is the Hamming distance.

III. Performance analysis

Lemma 1: Let $\{Z_j\}_{j=1}^{\infty}$ be independent binary variables with $\Pr[Z_j = 1] = q_j$ and $\Pr[Z_j = -1] = 1 - q_j$. Then, if $\lambda_m \triangleq E[Z_1 + \cdots + Z_m]/m < 0$,

$$\Pr\{Z_1 + \cdots + Z_m \geq 0\} \leq e^{-m\cdot I_m(0)}, \quad (1)$$

where

$$I_m(x) \triangleq \sup_{\theta\geq 0}[\theta x - \varphi_m(\theta)]$$

and

$$\varphi_m(\theta) \triangleq \frac{1}{m}\log E\left[e^{\theta(Z_1+\cdots+Z_m)}\right].$$

Proof: The lemma can be proved by following the fundamental large deviations argument. It is omitted due to page limitations. □

The probability bound in (1) does not exhibit any apparent relation with λ_m, namely the average of the means of $\{Z_i\}_{i=1}^m$. This can be amended by the next lemma.

Lemma 2: If $\lambda_m \triangleq E[Z_1 + \cdots + Z_m]/m < 0$, then

$$\Pr\{Z_1 + \cdots + Z_m \geq 0\} \leq (1 - \lambda_m^2)^{m/2}.$$

Proof: Let $\bar{q}_m = (1/m)\sum_{i=1}^m q_j$, and note that $\lambda_m = 2\bar{q}_m - 1$. So, the assumption of the lemma is equivalent to $\bar{q}_m < 1/2$.

The validity of the lemma for $0 < \bar{q}_m < 1/2$ can be proved by Jensen's inequality in terms of the upper bound in (1) as follows.

$$\begin{aligned}
e^{-m\cdot I_m(0)} &= \inf_{\theta\geq 0}\exp\left\{\sum_{j=1}^m \log\left(q_j e^{\theta} + (1-q_j)e^{-\theta}\right)\right\} \\
&= \inf_{\theta\geq 0}\exp\left\{m\left(\sum_{j=1}^m \frac{1}{m}\log\left(q_j e^{\theta} + (1-q_j)e^{-\theta}\right)\right)\right\} \\
&\leq \inf_{\theta\geq 0}\exp\left\{m\cdot\log\left(\sum_{k=1}^m \frac{1}{m}\left(q_j e^{\theta} + (1-q_j)e^{-\theta}\right)\right)\right\} \\
&= \inf_{\theta\geq 0}\exp\left\{m\cdot\log\left(\bar{q}_m e^{\theta} + (1-\bar{q}_m)e^{-\theta}\right)\right\} \\
&= \left(4\bar{q}_m(1-\bar{q}_m)\right)^{m/2},
\end{aligned}$$

where the last equality takes the optimizer

$$\theta^* = \log\sqrt{(1-\bar{q}_m)/\bar{q}_m} > 0$$

for $0 < \bar{q}_m < 1/2$.

In case $\bar{q}_m = 0$, we have $\left(4\bar{q}_m(1-\bar{q}_m)\right)^{m/2} = 0$, and

$$\begin{aligned}
&\inf_{\theta\geq 0}\exp\left\{\sum_{j=1}^m \log\left(q_j e^{\theta} + (1-q_j)e^{-\theta}\right)\right\} \\
&\quad\leq \inf_{\theta\geq 0}\exp\left\{m\log\left(\bar{q}_m e^{\theta} + (1-\bar{q}_m)e^{-\theta}\right)\right\} \\
&\quad= \inf_{\theta\geq 0}\exp\{-m\theta\} = 0.
\end{aligned}$$

□

Based on the probability bounds obtained in Lemmas 1 and 2, we can upper-bound the minimum Hamming distance fusion error for a WSN with distributed classification codes by the following theorem.

Theorem 1: If

$$\lambda_{\max} < 0, \quad (2)$$

then the minimum Hamming distance fusion error satisfies:

$$P_e \leq (M-1)(1-\lambda_{\max}^2)^{d_{\min}/2}, \quad (3)$$

where

$$P_e \triangleq \frac{1}{M}\sum_{i=0}^{M-1}\Pr(\text{fusion decision} \neq H_i | H_i),$$

$$d_{\min} \triangleq \min_{0\le \ell,i\le M-1,\ell\neq i} d(\boldsymbol{c}_\ell, \boldsymbol{c}_i),$$

$$q_{i,j} \triangleq \epsilon_j + (1-2\epsilon_j)\sum_{k=0}^{M-1}(c_{i,j}\oplus c_{k,j})h_{k|i}^{(j)}, \quad (4)$$

and

$$\lambda_{\max} \triangleq \max_{\substack{0\le \ell,i\le M-1,\\ \ell\neq i}} \frac{1}{d(\boldsymbol{c}_\ell,\boldsymbol{c}_i)}\sum_{j=1}^{N}(c_{\ell,j}\oplus c_{i,j})(2q_{i,j}-1). \quad (5)$$

Proof:

$$\begin{aligned}
&\Pr(\text{fusion decision} \neq H_i|H_i)\\
&\le \Pr\left(d(\boldsymbol{u},\boldsymbol{c}_i)\ge \min_{0\le\ell\le M-1,\ell\neq i} d(\boldsymbol{u},\boldsymbol{c}_\ell)\middle| H_i\right)\\
&\le \sum_{0\le\ell\le M-1,\ell\neq i}\Pr\left(d(\boldsymbol{u},\boldsymbol{c}_i)\ge d(\boldsymbol{u},\boldsymbol{c}_\ell)\middle| H_i\right)\\
&= \sum_{\substack{0\le\ell\le M-1,\\ \ell\neq i}}\Pr\left(\sum_{\substack{\{j\in[1,\cdots,N]:\\ c_{\ell,j}\neq c_{i,j}\}}}(z_{i,j}-\bar{z}_{i,j})\ge 0\middle| H_i\right),
\end{aligned}$$

where $z_{i,j} \triangleq u_j \oplus c_{i,j}$ and $\bar{z}$ represents the complement of the binary 0-1 variable z. Observe that

$$\begin{aligned}
&\Pr(z_{i,j}=1|H_i)\\
&= \Pr(u_j\oplus c_{i,j}=1|H_i)\\
&= \Pr(u_j=u_j^* \text{ and } u_j\oplus c_{i,j}=1|H_i)\\
&\quad +\Pr(u_j\neq u_j^* \text{ and } u_j\oplus c_{i,j}=1|H_i)\\
&= \Pr(u_j=u_j^* \text{ and } u_j^*\oplus c_{i,j}=1|H_i)\\
&\quad +\Pr(u_j\neq u_j^* \text{ and } u_j^*\oplus c_{i,j}=0|H_i)\\
&= \Pr(u_j=u_j^*)\Pr(u_j^*\oplus c_{i,j}=1|H_i)\\
&\quad +\Pr(u_j\neq u_j^*)\Pr(u_j^*\oplus c_{i,j}=0|H_i)\\
&= \epsilon_j+(1-2\epsilon_j)\Pr(u_j^*\oplus c_{i,j}=1|H_i)\\
&= \epsilon_j+(1-2\epsilon_j)\sum_{k=0}^{M-1}(c_{i,j}\oplus c_{k,j})h_{k|i}^{(j)} = q_{i,j},
\end{aligned}$$

and $\{z_{i,j}\}_{j=1}^N$ is independent across sensors given H_i is true. Therefore, (3) can be obtained by applying the upper bound in Lemma 2. □

With the above theorem, we figure that if for some $\delta>0$, $\lambda_{\max} < -\delta$ for all sufficiently large N, the decoding error vanishes exponentially fast as $d_{\min}$ approaches infinity. Since under a fixed number of hypotheses, $d_{\min}$ can be made to grow linearly with the number of sensors N, we conclude that the average error probability for a WSN with distributed classification code and minimum Hamming distance fusion can be made zero asymptotically as N goes to infinity, and the error exponent is bounded below by

$$\liminf_{N\to\infty} -\frac{1}{N}\log P_e \ge \liminf_{N\to\infty} -\frac{d_{\min}}{2N}\log(1-\lambda_{\max}^2)$$

as long as $\limsup_{N\to\infty}\lambda_{\max} < 0$. Next, we will show that the assumption that $\limsup_{N\to\infty}\lambda_{\max} > 0$ leads to a non-vanishing P_e, and hence, establish the necessary and sufficient condition under which P_e vanishes.

Theorem 2: P_e is bounded away from zero infinitely often, if $\limsup_{N\to\infty}\lambda_{\max} > 0$.

Proof: The assumption that $\limsup_{N\to\infty}\lambda_{\max} > 0$ implies the existence of $\delta > 0$ such that $\lambda_{\max} > \delta$ for infinitely many N. Hence, for any N validating $\lambda_{\max} > \delta$, there exists $\ell = \ell(N)$ and $i = i(N)$ such that

$$\sum_{j=1}^{N}(c_{\ell,j}\oplus c_{i,j})(2q_{i,j}-1) > \delta\cdot d(\boldsymbol{c}_\ell,\boldsymbol{c}_i). \quad (6)$$

By defining $z_{i,j}$ and $\bar{z}_{i,j}$ the same as in the proof of Theorem 1, we obtain:

$$\begin{aligned}
\mu_{\ell,i} &\triangleq E\left[\sum_{\{j\in[1,\cdots,N]\ :\ c_{\ell,j}\neq c_{i,j}\}}(z_{i,j}-\bar{z}_{i,j})\right]\\
&= \sum_{j=1}^{N}(c_{\ell,j}\oplus c_{i,j})(2q_{i,j}-1) > \delta\cdot d(\boldsymbol{c}_\ell,\boldsymbol{c}_i).
\end{aligned}$$

As a result,

$$\begin{aligned}
&\Pr(\text{fusion decision}\neq H_i|H_i)\\
&\ge \Pr\left(d(\boldsymbol{u},\boldsymbol{c}_i) > \min_{0\le\ell\le M-1,\ell\neq i} d(\boldsymbol{u},\boldsymbol{c}_\ell)\middle| H_i\right)\\
&\ge \Pr\left(d(\boldsymbol{u},\boldsymbol{c}_i) > d(\boldsymbol{u},\boldsymbol{c}_\ell)\middle| H_i\right)\\
&= \Pr\left(\sum_{\{j\in[1,\cdots,N]\ :\ c_{\ell,j}\neq c_{i,j}\}}(z_{i,j}-\bar{z}_{i,j}) > 0\middle| H_i\right)\\
&\ge \Pr\left(\sum_{\{j\in[1,\cdots,N]\ :\ c_{\ell,j}\neq c_{i,j}\}}(z_{i,j}-\bar{z}_{i,j})-\mu_{\ell,i} > 0\middle| H_i\right)\\
&\to \frac{1}{2},\ \text{if } d(\boldsymbol{c}_\ell,\boldsymbol{c}_i) \text{ approaches infinity},
\end{aligned}$$

where the last step follows the central limit theorem for the sum of independent bounded variables. Thus, the claim of the theorem holds for the case that $d(\boldsymbol{c}_\ell,\boldsymbol{c}_i)$ tends to infinity.

In situations when $d(\boldsymbol{c}_\ell,\boldsymbol{c}_i)$ is bounded as N approaches infinity in which case a bad code design results, the theorem is trivially valid. □

IV. Analysis of Pessimistic Fault-tolerance Capability

As mentioned earlier, the wireless sensor network considered in this paper is likely to contain faulty sensors. Faults may include all misbehaviors, ranging from *stuck-at faults* to sensors that behave arbitrarily. Observe that when sensor faults (SF) occur, $q_{i,j}$ is no longer given by (4), but becomes a function of the new statistics of u_j^* owing to sensor faults. For example, when stuck-at-one fault occurs at sensor j, $\Pr\{u_j^*=1|H_i\}=1$ for $0\le i\le M-1$. Hence,

$$\begin{aligned}
q_{i,j}^{(\mathrm{SF})} &= \epsilon_j+(1-2\epsilon_j)\Pr(u_j^*\oplus c_{i,j}=1|H_i)\\
&= \epsilon_j c_{i,j}+(1-\epsilon_j)(1-c_{i,j}).
\end{aligned}$$

Similarly, for stuck-at-zero fault,

$$\begin{aligned}
q_{i,j}^{(\mathrm{SF})} &= \epsilon_j+(1-2\epsilon_j)\Pr(u_j^*\oplus c_{i,j}=1|H_i)\\
&= \epsilon_j(1-c_{i,j})+(1-\epsilon_j)c_{i,j}.
\end{aligned}$$

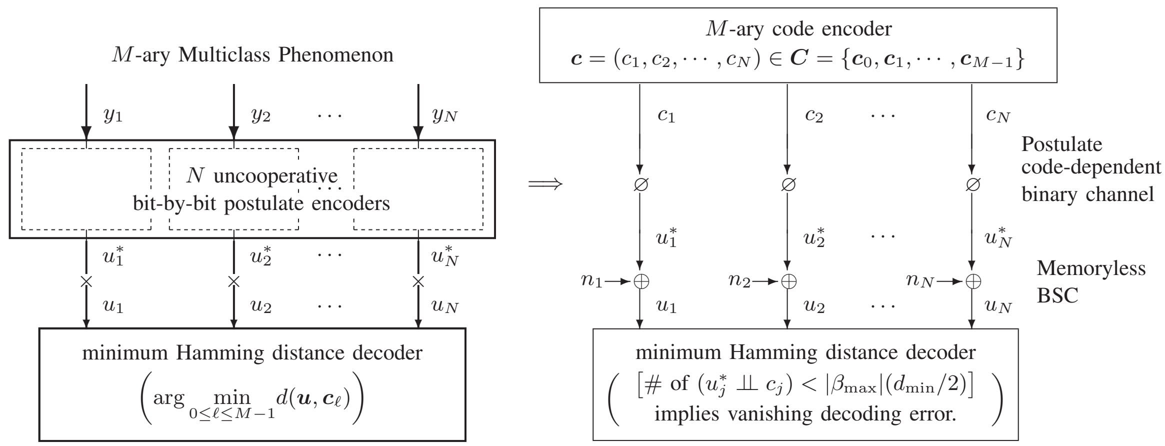

Fig. 2. Equivalent serial-connected binary channel model specifically for wireless sensor networks.

In case a random fault occurs, in which $\Pr\{u_j^* = 0|H_i\} = \Pr\{u_j^* = 1|H_i\}$,

$$q_{i,j}^{(\mathrm{SF})} = \epsilon_j + (1-2\epsilon_j)\Pr(u_j^* \oplus c_{i,j} = 1|H_i) = \frac{1}{2}.$$

In fact, $q_{i,j}^{(\mathrm{SF})}$ ranges from $\min\{\epsilon_j, 1-\epsilon_j\}$ to $\max\{\epsilon_j, 1-\epsilon_j\}$. As no prior information on the sensor fault type, as well as the faulty sensor number, is assumed known at the fusion center, it is safer to consider the fault-tolerance capability of the system by the worst case scenario. Then, the next corollary, which is a straightforward extension of Theorem 1, can be used to characterize the fault-tolerance capability of a distributed classification coding system.

Corollary 1: Suppose that the fusion center knows the set of faulty sensor indices, $\mathcal{F}$, and also knows the respective $q_{i,j}^{(SF)}$ of those $j \in \mathcal{F}$. Then, if $\lambda_{\max}(\mathcal{F}) < 0$, we have:

$$P_e \le (M-1)(1-\lambda_{\max}^2(\mathcal{F}))^{d_{\min}/2},$$

where the superscript "c" denotes the set complement operation and

$$\lambda_{\max}(\mathcal{F}) \triangleq \max_{\substack{0\le \ell, i \le M-1,\\ \ell \ne i}} \frac{1}{d(\boldsymbol{c}_\ell, \boldsymbol{c}_i)} \left(\sum_{j\in\mathcal{F}^c} (c_{\ell,j}\oplus c_{i,j})(2q_{i,j}-1) + \sum_{j\in\mathcal{F}} (c_{\ell,j}\oplus c_{i,j})(2q_{i,j}^{(\mathrm{SF})}-1) \right).$$

By $\min\{\epsilon_j, 1-\epsilon_j\} \le q_{i,j}^{(\mathrm{SF})} \le \max\{\epsilon_j, 1-\epsilon_j\}$, we can verify based on the above corollary that:

$$\begin{aligned}
&\lambda_{\max}(\mathcal{F}) - \lambda_{\max} \\
&= \max_{0\le \ell,i\le M-1, \ell\ne i} \frac{1}{d(\boldsymbol{c}_\ell,\boldsymbol{c}_i)} \left(\sum_{j=1}^{N} (c_{\ell,j}\oplus c_{i,j})(2q_{i,j}-1) + 2\sum_{j\in\mathcal{F}} (c_{\ell,j}\oplus c_{i,j})(q_{i,j}^{(\mathrm{SF})} - q_{i,j}) \right) - \lambda_{\max} \\
&\le 2 \max_{0\le \ell,i\le M-1, \ell\ne i} \frac{1}{d(\boldsymbol{c}_\ell,\boldsymbol{c}_i)} \sum_{j\in\mathcal{F}} (c_{\ell,j}\oplus c_{i,j})(q_{i,j}^{(\mathrm{SF})} - q_{i,j}) \\
&\le \begin{cases} 2 \max\limits_{\substack{0\le \ell,i\le M-1,\\ \ell\ne i}} \dfrac{1}{d(\boldsymbol{c}_\ell,\boldsymbol{c}_i)} \sum\limits_{j\in\mathcal{F}} (c_{\ell,j}\oplus c_{i,j})(1-2\epsilon_j) \sum\limits_{k=0}^{M-1} [1-(c_{i,j}\oplus c_{k,j})] h_{k|i}^{(j)}, & \text{if } \epsilon_j \le \dfrac{1}{2} \\ 2 \max\limits_{\substack{0\le \ell,i\le M-1,\\ \ell\ne i}} \dfrac{1}{d(\boldsymbol{c}_\ell,\boldsymbol{c}_i)} \sum\limits_{j\in\mathcal{F}} (c_{\ell,j}\oplus c_{i,j})(2\epsilon_j-1) \sum\limits_{k=0}^{M-1} (c_{i,j}\oplus c_{k,j}) h_{k|i}^{(j)}, & \text{if } \epsilon_j > \dfrac{1}{2} \end{cases} \\
&\le 2 \max_{0\le \ell,i\le M-1, \ell\ne i} \frac{1}{d(\boldsymbol{c}_\ell,\boldsymbol{c}_i)} \sum_{j\in\mathcal{F}} |1-2\epsilon_j| \sum_{k=0}^{M-1} h_{k|i}^{(j)} \\
&= 2 \max_{0\le \ell,i\le M-1, \ell\ne i} \frac{1}{d(\boldsymbol{c}_\ell,\boldsymbol{c}_i)} \sum_{j\in\mathcal{F}} |1-2\epsilon_j| \\
&= \frac{2}{d_{\min}} \sum_{j\in\mathcal{F}} |1-2\epsilon_j|. \qquad (7)
\end{aligned}$$

In order to guarantee a vanishing P_e with the maximal allowable number $|\mathcal{F}|$ of faulty sensors, it suffices to have

$$\lambda_{\max}(\mathcal{F}) \le \lambda_{\max} + \frac{2}{d_{\min}} \sum_{j=1}^{|\mathcal{F}|} |1-2\epsilon_j| < 0. \qquad (8)$$

For an identical sensor system where $\epsilon_j = \epsilon$ and $h_{k|i}^{(j)} = h_{k|i}$ for $0 \le k, i \le M-1$ and $1 \le j \le N$, this condition reduces to

$$d_{\min} > -2|1-2\epsilon|\frac{|\mathcal{F}|}{\lambda_{\max}} = 2\frac{|\mathcal{F}|}{|\beta_{\max}|}, \quad (9)$$

where

$$\beta_{\max} \triangleq \max_{0\le\ell,i\le M-1,\ell\ne i} \frac{\sum_{k=0}^{M-1} h_{k|i}\left[d(\boldsymbol{c}_i,\boldsymbol{c}_k)-d(\boldsymbol{c}_\ell,\boldsymbol{c}_k)\right]}{d(\boldsymbol{c}_\ell,\boldsymbol{c}_i)}.$$

Since

$$\begin{aligned}\lambda_{\max} &\ge \min_{0\le i\le M-1,1\le j\le N}(2q_{i,j}-1)\\ &= -(1-2\epsilon)\left(1-2\sum_{k=0}^{M-1}(c_{i,j}\oplus c_{k,j})h_{k|i}^{(j)}\right)\\ &\ge -|1-2\epsilon|\end{aligned}$$

for an identical sensor system, we have:

$$d_{\min} > -2|1-2\epsilon|\frac{|\mathcal{F}|}{\lambda_{\max}} = 2\frac{|\mathcal{F}|}{|\beta_{\max}|} \ge 2|\mathcal{F}|. \quad (10)$$

Note that the condition of $d_{\min} > 2|\mathcal{F}|$ that was formerly used as a heuristic code search requirement in [3] resembles the interpretation for conventional coding techniques, which states that a code with minimum pair-wise Hamming distance $d_{\min}$ can tolerate around $d_{\min}/2$ errors. However, inequality (10) hints that a larger $d_{\min}$ than $(2|\mathcal{F}|)/|\beta_{\max}|$ instead of $2|\mathcal{F}|$ may be necessary for an identical fault-tolerant sensor network system. By examining those codes that minimize (3) for $M = 8$ and $N \in \{50, 100, 150, \cdots, 600\}$, we found that $\beta_{\max}$ is around -0.66. In other words, in the worst case where the fusion center has no information on both the sensor fault types and faulty sensor indices, the number of faulty sensors allowable for these codes is only two-third of $d_{\min}/2$. Inequality (10) also interestingly indicates that under an identical sensor system, the worst-case fault-tolerance requirement has nothing to do with the link noise as we have anticipated. Inequality (10) will reduce to the heuristic constraint of $d_{\min} > 2|\mathcal{F}|$ when all the misclassification probabilities become zero (in which case $h_{i|i} = 1$ for $0 \le i \le M-1$, and hence $\beta_{\max} = -1$ regardless of the codes adopted).

V. Concluding remarks

The coding problem considered in this paper can actually be transformed into one for the memoryless binary symmetric channel (BSC) with *unreliable bit-by-bit postulate encoders* as shown in Fig. 2, when the link noises have common marginal distribution. We can further consider the memoryless BSC channel with unreliable bitwise postulate encoders as a serial connection of two binary channels, in which the first channel suffers *code-dependent* noises that give

$$\Pr(u_j^*|c_j) = \frac{\sum_{i=0}^{M-1}\left\{\overline{(c_j\oplus c_{i,j})}\sum_{k=0}^{M-1}\overline{1-(u_j^*\oplus c_{k,j})}h_{k|i}^{(j)}\right\}}{\sum_{i=0}^{M-1}\overline{1-(c_j\oplus c_{i,j})}},$$

where an over bar represents a complement operation, and the second channel is the memoryless BSC channel. The case of sensor faults under the equivalent channel model becomes that u_j^* *turns independent of* c_j *(and hence, code-independent) without notifying the fusion center*. Our results then indicate that the constraint that the number of code-independent bits in $\boldsymbol{u}^*$ (i.e., the number of faulty sensors) is less than $|\beta_{\max}| \times (d_{\min}/2)$ is sufficient to guarantee a vanishing decoding error for such a serially connected binary channel. This bound is derived based on the pessimistic view when both faulty sensor indices and sensor fault types are unknown to the fusion center, or equivalently, the decoder is aware of neither the index of every *faulty* bit u_j^* nor its resultant *code-independent* distribution. In the extreme case that $\boldsymbol{u}^*$ and $\boldsymbol{c}$ are completely dependent, which should occur when $h_{k|i}^{(j)} = 1$ for every $0 \le k = i \le M-1$, the constraint reduces to the conventional $|\mathcal{F}| < d_{\min}/2$ for the coding technique since $\beta_{\max} = -1$, and the serially connected binary channel reduces to a memoryless BSC channel. This observation hints that in a channel suffering from code-dependent noises, a code that makes $\boldsymbol{u}^*$ (channel output) and $\boldsymbol{c}$ (channel input) more "dependent" (and thus, the channel output has more information about the input) is still expected to be a better and more robust code, which is exactly the underlying concept behind the Shannon baptized "channel capacity". It would be interesting to conduct research along this line, and determine the capacity of the postulate code-dependent channels.

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