Equiangular-Spiral Non-Loss Bent Lightpipe and its Applications

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ABSTRACT

A novel bending lightpipe operation scheme with an arbitrary bent angle without light leakage is illustrated in details. Possible applications to a light-splitting element that could split light flux with specified separation ratio, and a light-mixing element that could mix light fluxes from several ports of entrance without light leakage are also shown with case demonstration.

Keywords: non-loss bent lightpipe, light pipe, lightpipe, light-splitting element, light-mixing element

1. INTRODUCTION

Optical instruments and equipment demand high efficiency in managing optical throughput and current application transference. A lightpipe is one of such optical elements, which guides light from one place to another place with desired performance and simple implementation. Generally, light pipe is a self-contained optical device that traps and guides light without considering information propagation from the input to the output aperture. While light guide carries information content, lightpipe attracts much attention and has been used as an illumination design tool. Typical applications include projector engine illumination, [1] liquid crystal panel backlight systems, [2] automobile dashboards, [3] and in headlights, [4] etc. Lightpipe shape varies with designer's needs, and bending the lightpipe is generally necessary. Avoiding bend loss of light leakages caused by bending light pipe, has been a longstanding issue and remains a crucial problem in lightpipe development.

Gupta, Lee, and Koshel recently developed an approach to analyze light leakages of lightpipe by analyzing the principle section lightpipe ray. [5] The conditions of the acceptance angle, bend ratio, and refractive index of the leakage-free circular bend lightpipe with one-time bending were derived. The acceptance angle is the maximum ray input angle correspondence to the ray incidence angle inside the lightpipe surface equal to the total internal reflection angle, θ_c. [5] Bend ratio reduction and/or increase of the lightpipe refractive index could increase the acceptance angle, reducing bent loss. In other words, circular-bent lightpipe bent loss is source-dependent. However, in many common applications, light source has an angular distribution extending from -90^0 to 90^0, such as the light emitting diode (LED) with Lambertian or batwing angular distributions. [6] The first circular-bent lightpipe constraint is that bent loss caused by the limited acceptance angle may not be acceptable. Second, multiple bending operations on lightpipes are inevitable in many applications, and avoiding bent loss caused by multiple bending is needed in practical situations. In our early work, [8] a non-loss equiangular-spiral bent lightpipe capable of bending lightpipe at 90^0 and multiple times without light leakages or acceptance angle limitations was shown. Bending the lightpipe in any *arbitrary specified angle* without bent loss is more useful in practical applications however, and deserves investigation. The word "non-loss" notably means

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“no light leakages” in this paper because we are only concerned with design issues of a light-transferring optical element, avoiding crucial light leakage loss.

Moreover, splitting or mixing light is necessary to a variety of practical cases, such as concentrator design, [8] or projection device of illumination device. [9] Splitting light with an assigned ratio is needed in many applications. [10-11] In such applications, the optical design goal is to split or mix light without reducing coupling efficiency. The crucial issue is *whether any structures can split light in a specified ratio or mix light without light leakages.*

The formation of equiangular spiral non-loss bent lightpipe capable of bending the lightpipe in an arbitrary angle is detailed in this paper. Light-splitting elements and light-mixing elements exhibiting no bent-loss based on the non-loss equiangular-spiral bent lightpipe will then be proposed. The paper is organized as follows. The guiding ray in no bent-loss equiangular-spiral lightpipe will be discussed in Section 2. The detailed formation of the non-loss equiangular-spiral bent lightpipe will be presented in Section 3. The non-loss light-splitting/light-mixing element construction scheme based on the proposed equiangular-spiral bent lightpipe will be interpreted in Section 4. The application demonstration of the non-loss light-mixing elements as an approach to, a crucial problem in practical LED (light-emitting diode) illumination, the traditional RGB (red-green-blue) color mixing, will also be addressed in section 5. Conclusions are presented in Section 6.

2. RAY GUIDING IN NON-LOSS BENT EQUIANGULAR –SPIRAL LIGHTPIPE

We illustrate the basic requirement for guiding ray without bent loss and the considerations to the geometric form of the non-loss equiangular-spiral bent lightpipe in this section. We first consider how to avoid bent loss. The lightpipe guides light by the means of total internal reflection, and thus to avoid any light leakages, the guided ray incident angle at the lightpipe guiding surface should be greater than or equal to the total internal reflection angle, θ_c. Hence, the guided ray angular distribution inside the lightpipe is between $-\theta_c$ to θ_c. By the condition, we can limit our discussion to identify the basic non-loss bent lightpipe geometric unit ,which was connected to the lightpipe end with angular distribution between $-\theta_c$ to θ_c and could transfer the light to the other end of a lightpipe without light leakages. To avoid bent loss, the non-loss bent lightpipe must have a geometric form in which all the guiding rays will have an incident angles on the bent surface greater than or equal to θ_c, the basic non-loss bent lightpipe requirement.

The “principle section” concept of Gupta *et al.* [5] helps us to derive the non-loss bent lightpipe geometric form. Principal section rays have proven to determine the smallest incident angle of ray at the bent surface. [5] The *critical rays* striking the outer surface from the inner bend point of the principle section are most critical to bent loss, and the most critical ray is the first one to escape from the lightpipe when it strikes the bent lightpipe outer facet from the inner bending point on the principle plane. [7] For avoiding critical ray leakage, the basic requirement of the non-loss bent lightpipe outer surface was the angle between the critical ray incident ray direction and the outer surface tangent should be greater than or equal to the value $\pi/2+\theta_c$. Besides confining the non-loss bent lightpipe outer surface, the geometric form inner surface choice should not only guide all rays reflected from the bent lightpipe outer surface, but should also behave reasonably so that rays reflected from the inner surface can be further guided by the outer surface. As a result of all guiding rays could be guided in a straight lightpipe infinitely without any light leakage, the inner surface choice guide rule of the non-loss bent lightpipe should be that “the inner surface should behave similarly to the non-loss bent lightpipe outer surface.” Hence, all rays could be guided between these surfaces without any light leakage just like be guided in a straight lightpipe.

3. FORMATION OF THE NON-LOSS BENT LIGHTPIPE GEOMETRIC FORM

Geometric form formation detail of the proposed equiangular-spiral non-loss bent lightpipe that redirects arbitrary direction light flux is interpreted in this section. An equiangular-spiral bent lightpipe that redirects light in 90^0 without light leakage is proposed in our recent work, [7] fulfilling the non-loss bent lightpipe requirements and simulation-verifying its no bent loss property. The work has been extended to arbitrary bent angle, i.e., the non-loss bent lightpipe geometric form capable of bending in arbitrary bent angle with no light leakage; its formation detail is here illustrated.

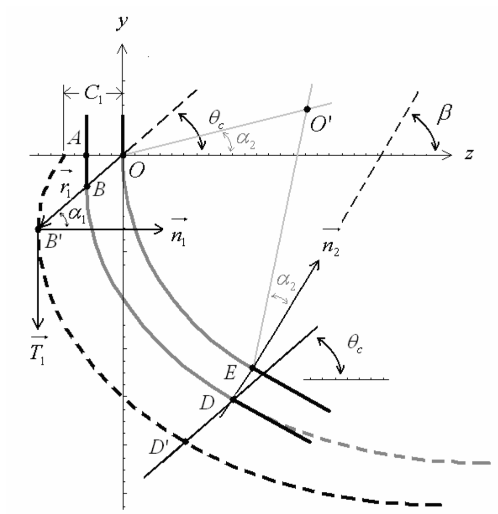

Figure1. The proposed non-loss bent lightpipe with bent angle δ .

As shown in Fig.1, except for the input and output lightpipe ports, the non-loss bent lightpipe can be treated as a three part combination: straight surface AB , outer bent surface BD , and inner bent surface OE . We illustrate the geometric form by three sections as follows: (1) The geometric form of the outer surface, (2) The geometric form of the inner surface, and (3) The numerical verification of the non-loss characteristic.

Sec. 3.1 The geometric form of outer surface

The outer surface contains two parts, straight surface AB , and outer bent surface BD . First, the surface AB is kept as a straight surface where the included angle of AO and BO is the total internal reflection angle, θ_c because all rays cannot escape in this portion as in a parallel lightpipe. Second, the outer bent surface of the non-loss bent lightpipe is chosen as a shifted equiangular-spiral, shifting an equiangular spiral

$$r_1(\theta_1) = -C_1 e^{-\theta_1 Cot\phi_1} , \tag{1}$$

whose polar is situated at the origin of the Cartesian coordinate, along an angle of elevation θ_c with a distance $l_1 = |r_1(\theta_c)| - W\sec(\theta_c)$. Where W is the width of the lightpipe, the fixed tangential angle of equiangular spiral, the included angle between the tangent $\vec{T_1}$ and radial line $\vec{r_1}$ of a equiangular spiral, is $\phi_1 = \pi/2 + \alpha_1$, and the angle α_1 is chosen as θ_c. C_1 is the length parameter which is reformulated as $C_1 = mC_0$, where m is a multiple factor and C_0 is the minimum length parameter such that the outer equiangular surface can be connected and tangential to the straight facet AB, i.e., when the length parameter is chosen as C_0, the length of $r(\theta_c)$ will be equal to the hypotenuse BO. It means that C_0 satisfies the following condition:

$$C_0 e^{-\theta_c Cot\phi} = W \sec(\theta_c), \tag{2}$$

Comparing the normal math form of the equiangular spiral that $r(\theta) = C_1 e^{-\theta Cot\phi_1}$, the minus sign in Eq.(1) is introduced by a simple change of variable that $\theta_1 = \theta - \pi$ for constructing the non-loss bent lightpipe in the third quadrant in the Cartesian coordinate. The two reasons for choosing the outer surface as a shifted equiangular spiral of larger length parameter are: (1) all critical rays are guided by the facet because all incident angles are equal to or greater than the angle of total internal reflection to fulfill the basic requirement of the outer surface of non-loss bent lightpipe (2) the incident angle of the ray reflected from the inner equiangular spiral facet increases because of choosing the shifted equiangular spiral which loosens the restriction of choosing the inner surface shape.

Sec. 3.2 The geometric form of inner surface

There are two geometric form selection considerations for the inner facet. The first is the non-loss bent lightpipe inner surface basic requirement that the inner surface geometric form be similar to the outer surface such that all rays could be guided as in a parallel lightpipe. The other consideration is that the inner surface form be chosen such that the lightpipe size after bending will behave the same size as before bending. To fulfill the two considerations, the inner bent surface OE is chosen as a shifted equiangular spiral which tangents to the input and output port of the bent lightpipe, i.e.,

$$r_2(\theta_2) = -C_2 e^{-\theta_2 Cot\phi_2}. \tag{3}$$

For constructing the non-loss bent lightpipe capable of bending in arbitrary bent angle β, we need to deduce four parameters of the *inner* bent surface of non-loss bent lightpipe, fulfilling the above two considerations: the polar origin position of the inner equiangular spiral in the Cartesian coordinate $O'(Z_o, Y_o)$, the fixed tangential angle $\phi_2 = \pi/2 + \alpha_2$, and the length parameter C_2.

For determining the *inner* bent surface geometric form of the non-loss bent lightpipe, we first derive the output inner bent point E position that the inner bent surface is tangent to. Owing to the fact that point E is derived from two steps of position-shift from point D', we determine the correspondence output outer bent point D' on the equiangular spiral surface BD' before shifting to connect to the straight surface AB. The radial line $\vec{r_1}$ of any point on the outer equiangular spiral surface BD' is represented in the Cartesian coordinate as $\vec{r_1} = r_1(\theta_1)\cos\theta_1\,\hat{x} + r_1(\theta_1)\sin\theta_1\,\hat{y}$, and its tangent $\vec{T_1}$ was that $\vec{T_1} = d\vec{r_1}/d\theta_1 = [r_1'(\theta_1)\cos\theta_1 - r_1(\theta_1)\sin\theta_1]\hat{z} + [r_1'(\theta_1)\sin\theta_1 + r_1(\theta_1)\cos\theta_1]\hat{y}$, where $\hat{z}$ and $\hat{y}$ are the unit vector of the Cartesian coordinate. While the non-loss bent lightpipe with a bent angle β, the included angle between the tangent $\vec{T_1}$ and the $\hat{z}$ at the output outer bent point D' will be $\pi/2 - \beta$, such that

$$\tan(\pi/2 - \beta) = -\left[\frac{r_1'(\theta_1)\sin\theta_1 + r_1(\theta_1)\cos\theta_1}{r_1'(\theta_1)\cos\theta_1 - r_1(\theta_1)\sin\theta_1}\right]. \tag{4}$$

Solving the Eq. (4), the polar angle of the output outer bent point D' is derived as

$$\theta_{D'} = \tan^{-1}\left[\frac{1 + \tan\alpha_1 \cot\beta}{\cot\beta - \tan\alpha_1}\right], \tag{5}$$

such that the position of the output outer bent point D' is specified as $(Z_{D'}, Y_{D'}) = [r_1(\theta_{D'})\cos\theta_{D'}, r_1(\theta_{D'})\sin\theta_{D'}]$. After determining the position of output outer bent point D', the position of output inner bent point E can be derived from two steps of position-shift from point D'. The first step shifts D' along an angle of elevation θ_c with a distance $l_1 = |r_1(\theta_c)| - W\sec(\theta_c)$, so we obtain the position of D. The second step shifts D along the normal surface direction, $\vec{n}_2$, of the outer bent surface with a lightpipe width W distance to get the output inner bent point E position. The two position-shift steps are presented as

$$\overrightarrow{r_E} = \overrightarrow{r_{D'}} + \Delta\vec{r}, \tag{6}$$

where $\Delta\vec{r} = (l\cos\theta_c + W\cos\beta, l\sin\theta_c + W\sin\beta)$, i.e., the position of the point E is $(Z_{D'} + l\cos\theta_c + W\cos\beta, Y_{D'} + l\sin\theta_c + W\sin\beta)$.

While we have determined the output inner bent point E position, we can now derive the exact inner bent surface form of the non-loss bent lightpipe, which is a shifted equiangular spiral surface that is tangent to both the input inner bent point O and the output inner bent point E. The polar position of the inner equiangular spiral O' could be easily derived because it is the intersection point of the two lines $\overleftrightarrow{OO'}$ and $\overleftrightarrow{EO'}$ that

$$\begin{cases} Z_O = [Y_E - Z_E\tan(\alpha_2 + \beta)]/[\tan\alpha_2 - \tan(\alpha_2 + \beta)] \\ Y_O = Z_O\tan\alpha_2 \end{cases}. \tag{7}$$

Applying the properties of the inner equiangular spiral that

$$\begin{cases} \overline{OO'} = |r_2(\alpha)| \\ \overline{EO'} = |r_2(\beta + \alpha)| \end{cases}, \tag{8}$$

and substituting Eq. (7) into Eq. (8), we obtain the following two equations,

$$2\beta\tan\alpha_2 = \ln[\overline{EO'}/\overline{OO'}], \tag{9}$$

and

$$C_2 = \overline{OO'}e^{-\alpha_2\tan\alpha_2}. \tag{10}$$

From the three derived equations of Eq. (7), Eq. (9) and Eq. (10), we obtain the exact geometric form of the inner equiangular spiral surface of the non-loss bent lightpipe. Solving the Eq. (7) and Eq. (9) combinations, we obtain the first three parameters of the inner equiangular spiral, the inner equiangular spiral polar origin in the Cartesian coordinate $O'(Z_o, Y_o)$ and the fixed tangential angle $\phi_2 = \pi/2 + \alpha_2$; substituting the value α_2 into Eq. (10) we obtain the value of C_2. With the origin position of the *inner* equiangular spiral surface $O'(Z_o, Y_o)$, the fixed tangent angle ϕ_2, and the length parameter C_2, the explicit inner bent surface geometrical form is derived, forming a non-loss bent lightpipe.

Sec. 3.3 Numerical verification of the non-loss characteristic

It is worthwhile to provide numerical verification of the non-loss characteristic of the equiangular spiral non-loss bent lightpipe. Three typical bent angles 30^0, 60^0, and 90^0, were considered in Fig. 2, and demonstrated that the proposed lightpipe can be bent with any bent angle β without any bent loss. There is notably not a problem in having different bent angle values. In all current article simulations, without loss of generality, a Lambertian-like source emitting 50000 rays with an angular distribution between -90^0 to 90^0 was used. However, only 20 rays were arbitrarily selected and shown in Fig. 2 for illustrating the guided ray path. A Lambertian-like Lambertian source combined with a straight piece was used instead of using a Lambertian-like Lambertian source alone, because we wanted to create a general condition, i.e., create the most crucial rays of critical rays, to verify the no bent-loss characteristic of the proposed non-loss bent lightpipe. We considered an acrylic lightpipe material, where the index for the simulated wavelength $0.571\,\mu m$ is 1.49189. We used the commercially available simulation package, TracePro (version 3.2.5), [12] and the equiangular lightpipe geometrical profile was created by Macro Scheme, a Marco commend language in TracePro. The emitting ray colors from the light source shown in Fig. 2 were selected by the TracePro simulation package default setting. In a monochromatic (, i.e. single wavelength) ray tracing, ray colors reflect the ray flux ratio compared to the ray peak flux. The Crucial loss, light leakages, during light transference in the bent lightpipe is our major concern here, and the Fresnel surface loss that source-to-lightpipe coupling are not considered here.

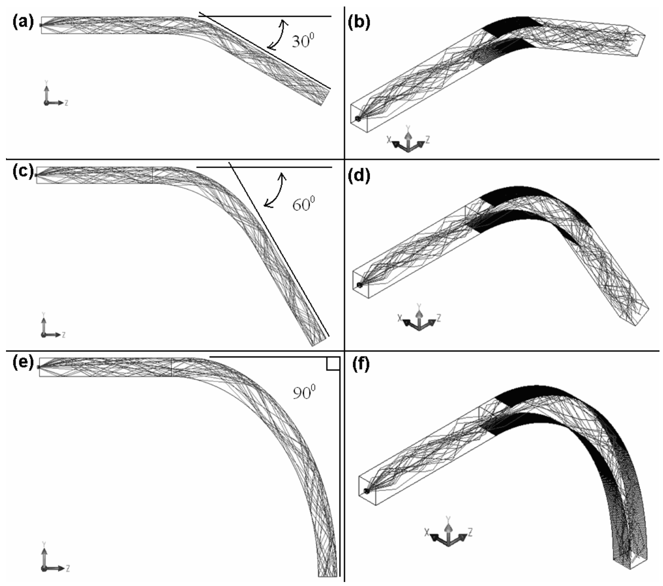

Figure 2. The proposed non-loss bent lightpipe with bent angle δ where (a): 30^0, (b) 60^0, and (c) 90^0.

4. CONSTRUCTION OF NON-LOSS LIGHT-SPLITTING-SPLITTING ELEMENTS

The basic construction scheme for a non-loss *light-splitting element (LSE)* to split light flux without any light leakage is depicted in this section. A non-loss bent lightpipe with a bent angle of 30 degrees, shown in Fig. 3, was the basic unit used to construct LSE and LME. As shown in Fig. 3(a), 500,000 rays were used to verify the non-loss characteristic of the basic unit; however, only 100 rays were arbitrarily selected and shown here to indicate the guided ray path. The proposed light-splitting elements are shown to behave in a non-loss characteristic way because of the non-loss bent lightpipe basic unit which behaves in a non-loss characteristic way. The non-loss bent lightpipe is formed in three parts: a straight surface AB, outer bent surface BD, and an inner bent surface OE, referred to in Fig. 3(b).

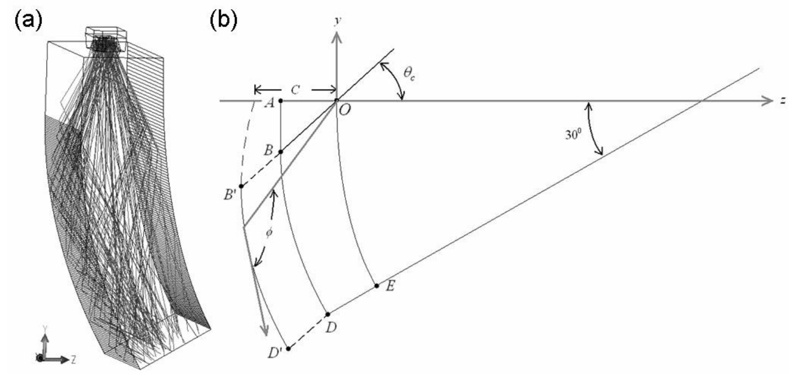

Figure 3. (a) Perspective diagram of the base element of non-loss bent lightpipe, and (b) Schematic diagram of the base element of non-loss bent lightpipe

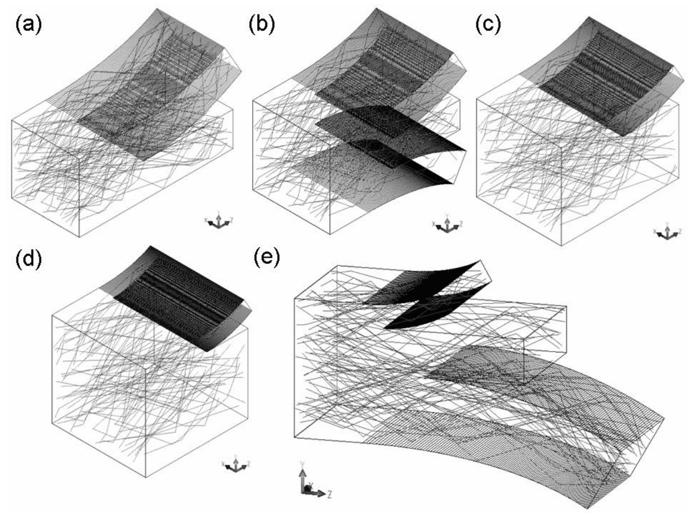

Figure 4: Perspective diagram of the non-loss light splitting elements: (a) one-to-two ELSE, (b) one-to-three ELSE, (c) ULSE with a flux ratio 1:3, and (d) ULSE with a flux ratio 1:5, and (e) ULSE with a flux ratio 1:2 :3.

Creating a non-loss light-splitting element *(LSE)* via the proposed equiangular spiral non-loss bent lightpipe is shown. Assuming that the entrance port flux distribution of the light-splitting element (LSE) plane is uniform, the entrance port divided area will be proportional to the separated exit flux. As a numerical illustration, the LSE is constructed by uniting a block, whose thickness in half of LSE entrance port diameter, to entrance ports of non-loss bent lightpipes OA and straight lightpipes. By choosing the connected area ratios of non-loss bent lightpipe and straight lightpipes, the light-splitting ratio of LSE can be controlled. The proposed non-loss LSE divides into two groups by the light flux ratio: equal light-splitting elements *(ELSE)*, and unequal light-splitting elements *(ULSE)*. The non-loss equal light-splitting elements *(ELSE)* with non-loss bent lightpipes are embedded, as shown in Fig. 4 (a) and (b). In Fig. 4 (a), a one-to-two ELSE divides the entrance light into two equal parts is shown, while a one-to-three ELSE is shown in Fig. 4 (b). On the other hand, three unequal light-splitting elements *(ULSE)* are shown in Fig. 4 (c), (d) and (e). In Fig. 4 (c), the light flux was separated into a 1:3 ratio at the output end, while a 1:5 ratio is shown for Fig. 4 (d). A specified flux ratio 1: 2: 3 is also shown in Fig. 4 (e). Two kinds of entrance light distributions verified the LSE splitting property: a uniform rectangular square surface source with a diameter 10 μm smaller than the LSE and separated from the LSE with distance 1 μm, and a Lambertian-like source with an angular distribution between -90^0 to $+90^0$ passing through a lightpipe length 10 times the diameter. The LSE diameter was chosen as $1.2\,mm$. Both sources could be guided, behaving in non-loss characteristics as shown in Fig. 4. 500,000 rays were used, but only 100 rays were arbitrarily selected and shown here to indicate the guided ray path and the non-loss proposed LSE characteristic. The light flux ratio at the LSE output ports are shown in Table 1, with high precision. The precision of light-splitting ratio of the proposed LSE notably depends only on the uniformity of entrance port light.

Table 1. Output Flux Ratio of Light Splitting Element (LSE)

Type of LSE	Output flux ratio (Lambertian-like source)	Output flux ratio (Square surface source)
ELSE (2 ports)	1 : 1.006	1: 0.995
ELSE (3 ports)	1 : 1.002 : 1.003	1: 0.991 : 0.996
ULSE (1:3)	1 : 3.007	1 : 2.994
ULSE (1:5)	1 : 5.015	1: 5.013
ULSE (1:2:3)	1 : 1.999 : 3.016	1 : 2.014 : 2.999

5. Construction of non-loss light-mixing elements

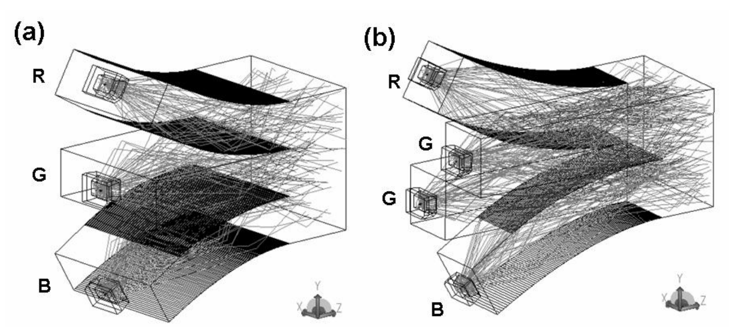

Figure 5: Proposed non-loss bent lightpipe with bent angle δ.

The basic construction scheme for a *light-mixing element (LME)* to mix light flux without loss is depicted in this section. The non-loss light-mixing element (LME) is also achieved by a similar LSE construction approach. The LME is constructed by uniting a block, whose thickness in half of the LME exit port diameter, to the exit ports of non-loss bent lightpipes DE and straight lightpipes. Two different LME geometries, a three-to-one LME, and a four-to-one LME, are shown in Fig. 5 (a) and (b) respectively. An air gap between the straight and bent part should be added in four-to-one LME to avoid light loss guiding from the pipe adjacent to oneself. It should be noted that if the two straight lightpipes were replaced by non-loss bent lightpipes that bent in other two directions, the tiny air gap could be avoided. These two proposed geometrics are only demonstrations in constructing LME and with suitable design, the non-loss LME with more entrance ports can be further constructed. As a numerical demonstration of the proposed non-loss LME characteristics, a practical RGB color mixing backlight is considered here.

In traditional backlight, lens and beam-splitters are used to achieve color mixing, and hence, light loss is introduced with decreased coupling efficiency. Coupling efficiency reduction in the RGB color-mixing system can be avoided by using the proposed non-loss LME. The RGB color-mixing system is a design using a multiple colorful light source, and the highest wavelength of the light source should be chosen as the design wavelength of LME. Shorter wavelengths have a smaller internal total reflectance angle than that of the design wavelength and can thus be guided. In other words, as long as the longest wavelength component of the light source can be guided, all other wavelength components will have an incident angle greater than the total reflectance angle, and will hence be guided without loss. Meanwhile, the, Lambertian-like source with the same radiation pattern used above, was used to show the non-loss characteristic of LME. Red, green, and blue colors with wavelengths of 0.4358 μm, 0.5461 μm, and 0.7 μm were used at the non-loss LME entrance, passing a straight lightpipe with a length 10 times the diameter to mix and form a white color. 500,000 rays were used, but only 100 rays were arbitrarily selected in each light source and are shown here to indicate the guided ray path and the non-loss characteristic of the proposed LME. The tiny air gap of the four-to-one LME was chosen as 1 μm in the simulation. The color uniformity of the output port was analyzed in the color space, CIE 1976 Uniform Colour spaces, [13] CIELUV. In the CIE1976 color spaces, the color is described according to three parameters, Y, u',and v'. In the CIE LUV color space, the difference between two colors is estimated by a parameter CLDS (Color-luminance differences) ΔE_{uv} , defined as

$$\Delta E_{uv} = \sqrt{\Delta L^{*2} + \Delta u^{*2} + \Delta v^{*2}} \,, \tag{11}$$

where $\Delta L^* = L^* - L^*_n$, $\Delta u^* = u^* - u^*_n$ and $\Delta v^* = v^* - v^*_n$. Parameters L^*, u^*, and v* are defined as

$$L^* = \begin{cases} 116(Y/Y_n) - 16; & \text{if } (Y/Y_n) > 0.008856 \\ 903.3(Y/Y_n); & \text{if } (Y/Y_n) \le 0.008856 \end{cases}, \tag{12}$$
$$u^* = 13L^*(u' - u'_n),$$
$$v^* = 13L^*(v' - v'_n),$$

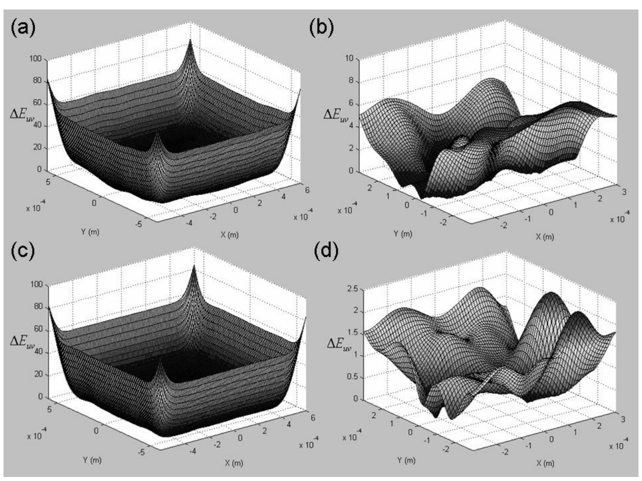

Figure 6: Color-luminance differences at exit surface after the operation of RGB color-mixing by a three-to-on LME, (a), whole plane, (b), center 1/4 plane, a four-to-one LME, (c), whole plane, (d), center 1/4 plane

where Y_n, u'_n and v'_n were the reference color parameter. When ΔE_{uv} equals to 3, the difference between two colors can be distinguished. [14] The referenced white color was chosen as the center average 1/4 output plane in our analysis. The CLDS at the straight lightpipe output port, has a length ten times larger than the proposed LME diameter, as shown in Fig. 6. The CLDS at whole output plane and at the center 1/4 output plane for the three-to-one LME case is shown in Fig. 6 (a) and (b) respectively, while the four-to-one LME case is shown in Fig. 6 (c) and (d) for comparison. The output surface suburb color difference is much higher than the center part because luminance is much lower there. Color uniformity of the four-to-one LME light-mixing result is better than the three-to-one LME. In the case of four-to-one LME, $\Delta E_{uv} < 3$ in the center 1/4 output plane, i.e., the difference between the two colors is hard to distinguish. The relative positions of the RGB source at the non-loss LME entrance ports are not significant in our simulation results.

6. CONCLUTIONS

In conclusions, an equiangular spiral non-loss bent lightpipe bent in arbitrary angle without light leakage and its detail formation have been shown. It has been also demonstrated that by using the proposed non-loss bent lightpipe of arbitrary bent angle as the basic unit, applications to two non-loss optical elements, non-loss light splitting elements, and non-loss light mixing elements, are available

It is, however, interesting to compare our bending scheme, an equiangular spiral non-loss bent lightpipe, to the most common bending scheme, a leakage-free circular bend lightpipe. The geometric form of the one time leakage-free circular bend lightpipe (i.e., its acceptance angle equals to 90^0) can be easily solved following Gupta's et al approach.[5] Considering that the lightpipe entrance port cross-section is rectangular with a 14 mm diameter, the lightpipe material is acrylic. The two kinds of non-loss bent lightpipes that bend twice in opposite directions are shown in Fig. 7. The relative positions between the entrance ports and exit ports of the two different double-bent lightpipes are nearly same, however, as shown from Fig. 7(b), the proposed equiangular spiral non-loss bent lightpipe could be further bent without light leakages.

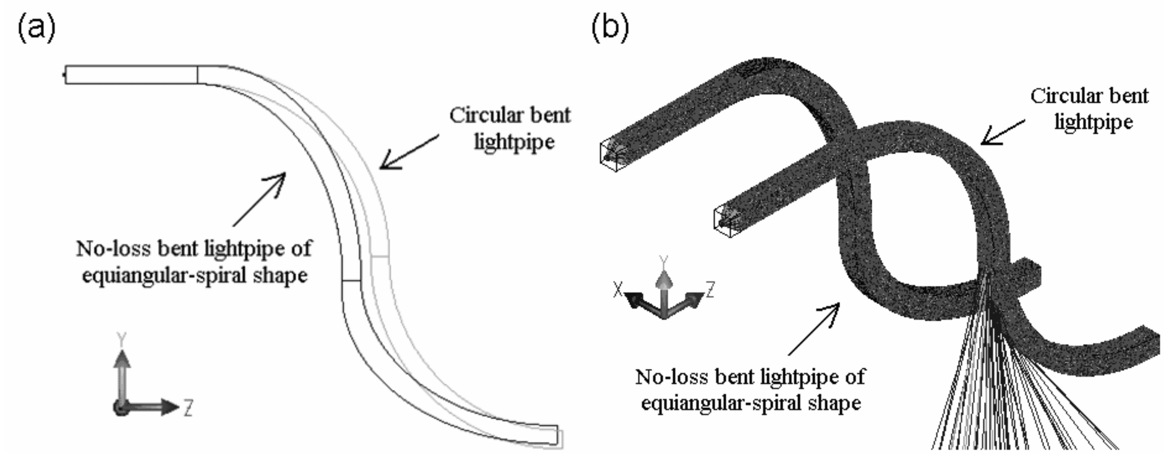

Figure 7 (a) cross-section of the non-loss bent lightpipe, and (b) simulation demonstration of the twice bends of bent lightpipes.

Nevertheless, it is worthwhile to point out the limitations of our current approach. First, the non-loss bent lightpipe proposed here was chosen with a rectangular cross-section. This choice simplified the analysis and practical implementation. The refractive index of most transparent materials is larger than $\sqrt{2}$ ensuring that rays distributed between $-\theta_c$ to θ_c will be guided in two *parallel* planes without loss, when ray incident angles are greater than critical angles at a parallel surface, while, obviously a rectangular cross-section is easier to connect in a perpendicular direction. Nevertheless, once the cross-section forms are different, more effort is needed to clarify the bending rules such that light leakage can be eliminated.

As a final note, when the lightpipe has a higher refractive index, the total internal reflection angle will be smaller and the bent lightpipe geometric size becomes more compact. As an example, a lightpipe made of glass material, SF15, where the index for the simulated wavelength 0.571 μm is 1.70098, provides a non-loss bent lightpipe, but with a smaller geometric size, and the multiple factor $m = 1.6$ is also smaller, as presented in Fig. 8.

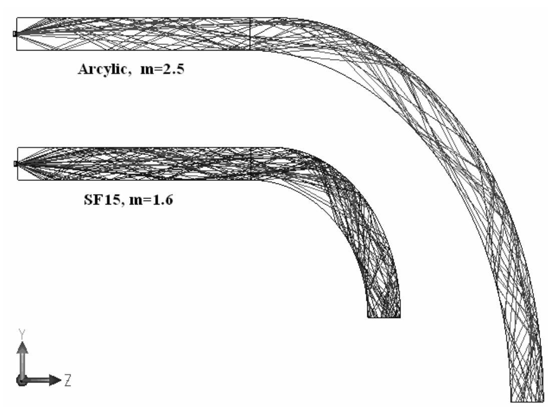

Figure 8: Two non-loss bent lightpipes made of different materials and multiple factors.

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