PHYSICAL REVIEW D VOLUME 52, NUMBER 5 1 SEPTEMBER 1995

Light-front heavy-quark effective theory and heavy-meson bound states

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(Received 21 March 1995)

The heavy-quark effective theory is developed on the light front. Based on this effective theory, a light-front heavy-meson bound state with definite spin and parity is constructed. Within the effective theory, the Isgur-Wise function is derived in terms of the asymptotic light-front bound state amplitudes in the limit $m_Q \to \infty$; the result is a general expression for arbitrary recoil velocities. With the asymptotic form of the BSW amplitudes, the Isgur-Wise function is given by $\xi(v \cdot v') = 1/v \cdot v'$. The slope at the zero-recoil point is $\rho^2 = -\xi'(1) = 1$, in excellent agreement with the recent CLEO result of $\rho^2 = 1.01 \pm 0.15 \pm 0.09$.

PACS number(s): 12.39.Hg, 11.10.Ef, 14.40.Lb, 14.40.Nd

I. INTRODUCTION

The discovery of heavy-quark symmetry (HQS) [1] and the subsequent construction of heavy-quark effective theory (HQET) [2,3] have led to intense activities in the study of heavy-hadron physics, and much progress has been made in recent years [4]. The so-called HQET is an effective theory of quantum chromodynamics (QCD) valid in situations where the gluon momenta ($\sim \Lambda_{\rm QCD}$) are much smaller than the heavy-quark masses (m_Q). In effect, HQET provides us with a systematic expansion of the QCD Lagrangian in terms of the dimensionless parameter $\Lambda_{\rm QCD}/m_Q$ [5–9]. In the symmetry limit ($m_Q \to \infty$), the coupling between a heavy quark and gluon becomes independent of the spin and flavor of the heavy quark. Thus the leading order effective Lagrangian possesses a new spin-flavor symmetry, which is not manifest in the original QCD Lagrangian.

Since HQS is a symmetry of QCD for heavy quarks at the confinement scale, it can therefore be used to extract model-independent dynamical consequences of the theory at a scale where perturbative calculations are not possible. In practical applications, HQS is most useful in reducing the number of independent form factors in various heavy-hadron decays, and thereby greatly simplifying the complexity of theoretical analyses. For instance, in the symmetry limit, all of the form factors in $B \to D$ and $B \to D^*$ are related by spin symmetry to a single universal function, called the Isgur-Wise function. Moreover, the normalization of this universal function at the zero-recoil point is also fixed by flavor symmetry, which then permits a model-independent means of extracting the important Kobayashi-Maskawa matrix element $|V_{cb}|$ from experimental data. Similarly in heavy-baryon decays, such as $\Lambda_b \to \Lambda_c$ [10], the application of HQS also leads to tremendous simplifications. Although HQS was first discovered in the weak decays of heavy hadrons, it has since found applications in many other areas of heavy-hadron physics. For example, by combining HQS with chiral symmetry it is possible to construct a chiral Lagrangian for the low energy interactions of heavy hadrons with Goldstone bosons [11–13]. This theory has been extended to include heavy-flavor-conserving weak decays [14], as well as electromagnetic interactions [15,16]. Furthermore, HQS has also been applied in inclusive B meson decays, where the main thrust was to reliably extract the Kobayashi-Maskawa (KM) matrix element V_{ub} from the end point spectrum of the charge lepton [17].

Beyond the symmetry limit, HQET serves as a theoretical framework for the systematic computation of $1/m_Q$ corrections. However, in order to make definite predictions, it is also necessary to construct explicitly the heavy-hadron bound state wave functions within HQET. This is of course a difficult task, to which a satisfactory solution does not exist. Nevertheless, we do expect that HQS will lead to considerable conceptual and calculational simplifications. One of the purposes of this paper is to lay the groundwork for solving this important problem on the light front.

In order to better motivate the work of this paper as well as to be self-contained, we present below a brief description of HQET in the equal-time form, and point out the issues to be addressed in this paper as we proceed. Let us start with the QCD Lagrangian for a heavy quark:

$$\mathcal{L} = \bar{Q}(i \not{D} - m_Q)Q, \tag{1.1}$$

where Q is the heavy-quark field operator, m_Q the heavy-quark mass, and $D^\mu = \partial^\mu - igT_a A_a^\mu$ the QCD covariant derivative. The pure gauge part of the QCD Lagrangian has not been included because it is irrelevant for our discussions. HQET in the usual equal-time formalism is

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obtained simply by redefining the heavy-quark fields as [3]

$$Q(x) = e^{-im_Q v\cdot x}[h_v(x) + H_v(x)], \tag{1.2}$$

where v^μ is the four-velocity of the heavy hadrons $v^2 = 1$; $h_v(x)$ and $H_v(x)$ are, respectively, the large and small components of $Q(x)$, such that

$$\not{v} h_v(x) = h_v(x),$$
$$\not{v} H_v(x) = -H_v(x). \tag{1.3}$$

This phase redefinition amounts to a splitting of the heavy-quark momentum: $p = m_q v + k$, where $k \sim \Lambda_{\rm QCD}$ is called the residual momentum which measures the fluctuation around the mass shell. With such a redefinition, the heavy-quark Dirac equation is reduced to

$$i\not{D} h_v + i(\not{D} - 2m_Q)H_v = 0, \tag{1.4}$$

which can be further decomposed into two coupled equations: viz.,

$$-iv\cdot D h_v = i\not{D}_\perp H_v,$$
$$(iv\cdot D + 2m_Q)H_v = i\not{D}_\perp h_v, \tag{1.5}$$

where $D_\perp^\mu \equiv D^\mu - v^\mu v\cdot D$. Thus, one can express $H_v(x)$ in terms of $h_v(x)$ and show that $H_v(x)$ is suppressed by $1/m_Q$ compared to $h_v(x)$. Using the relation between $h_v(x)$ and $H_v(x)$ obtained from the Dirac equation, one can then rewrite the QCD Lagrangian in powers of $1/m_Q$:

$$\begin{aligned}\mathcal{L} &= \overline{h}_v iv\cdot D h_v + \overline{h}_v(i\not{D}_\perp)\frac{1}{2m_Q + iv\cdot D - i\epsilon}(i\not{D}_\perp)h_v \\ &= \overline{h}_v iv\cdot D h_v + \sum_{n=1}^{\infty}\left(\frac{1}{2m_Q}\right)^n \overline{h}_v(i\not{D}_\perp) \\ &\quad \times(-iv\cdot D)^{n-1}(i\not{D}_\perp)h_v.\end{aligned} \tag{1.6}$$

This is the effective Lagrangian for the heavy quark. An equivalent derivation of HQET via the QCD generating functional can be found in Ref. [9]. In the heavy-mass limit ($m_Q \to \infty$), only the first term in Eq. (1.6) survives. This leading order Lagragian is obviously spin and flavor independent, which is the origin of the heavy-quark spin-flavor symmetry. Note that, in Eq. (1.6), the nonleading contributions contain high order time derivatives. Consequently the quantization of HQET beyond leading order is rather cumbersome [18]. As we will see later, this unpleasant feature does not exist in the light-front formulation.

In order to gain a deeper understanding of heavy-quark dynamics, it is both necessary and important to study the symmetry-breaking effects caused by the higher order terms in the effective Lagrangian. Similar terms can also arise in the $1/m_Q$ expansion of heavy-quark currents. These higher order interactions would either spoil relations established by HQS or introduce new transition form factors. With HQET, one can in principle investigate these symmetry-breaking effects systematically. However, in order to evaluate the various matrix elements involved, one needs a detailed knowledge of the structures of heavy-hadron bound states. To date, except for lattice simulation, a direct QCD approach to the hadronic bound states does not exist, and one has to rely on various phenomenological models, such as the constituent quark model [19], the bag model [20], and the QCD sum rules [21], to estimate these matrix elements. Since one does not know how to properly boost a constituent quark bound state or a bag wave function to arbitrary velocities, these models are, strictly speaking, applicable only at the zero-recoil point. However, to compare with experimental data, matrix elements at various momentum transfer are required in general.

In the past few years, a boost-free relativistic approach to the hadronic bound state problem of QCD on the light front has attracted much attention [22–24]. One of the advantages for the light-front QCD approach is that light-front Hamiltonian field theory provides a direct way of calculating relativistic bound states by solving Schrödinger-type eigenstate equations in a truncated Fock space [25]. It is well known that boost operations on the light front are kinematic and therefore it is easy to boost a hadron state to any frame of reference when its wave function is known in a particular Lorentz frame. Moreover, the special behavior of the light-front infrared singularity may also lead to a possible understanding of nontrivial QCD dynamics, such as color confinement and dynamical chiral symmetry breaking [22]. Nevertheless, light-front quantization for heavy quarks has only been briefly explored in the (1+1)-dimensional model [26]. Recently some light-front hadronic wave functions have been constructed either phenomenologically [27] or from the light-front QCD sum rule [28,29]. They have been used quite sucessfully in the calculations of the Isgur-Wise function and other heavy-hadron form factors. Furthermore, inclusive heavy-meson decays have also been discussed on the light front [30].

In order to better understand these light-front wave functions and their applications in various heavy-hadron processes, we have recently reformulated HQET on the light-front [31]. In the present paper, apart from providing a more detailed account of the derivation of light-front heavy-quark effective theory (LFHQET), the construction of heavy-meson bound states is also formulated. We then derive the Isgur-Wise function using the light-front wave functions so constructed; the resulting expression is compatible with HQS and valid for arbitrary recoil velocities. The paper is organized as follows. In Sec. II, LFHQET is derived, and its advantages over the equal-time formulation are discussed. In Sec. III, the quantization procedure for LFHQET is described. In Sec. IV, based on LFHQET, heavy-meson bound states are constructed in the heavy-mass limit. An explicit calculation of the Isgur-Wise function is given in Sec. V. Finally, a summary is presented in Sec. VI.

II. LIGHT-FRONT HEAVY-QUARK EFFECTIVE THEORY

In this section, HQET is formulated on the light front. We shall use the following light-front notation: The light-front coordinate is denoted by $x^\mu = (x^+, x^-, x_\perp)$ where $x^+ = x^0 + x^3$ is the light-front timelike component and $x^- = x^0 - x^3$ and $x^i_\perp$ $(i = 1,2)$ the light-front longitudinal and transverse components, respectively. With this notation, the product of 2 four-vectors is given by $a \cdot b = \frac{1}{2}(a^+b^- + a^-b^+) - a_\perp \cdot b_\perp$, and the light-front derivatives are written as $\partial^- = 2\frac{\partial}{\partial x^+}$ (the light-front time derivative), $\partial^+ = 2\frac{\partial}{\partial x^-}$, and $\partial^i = \frac{\partial}{\partial x_i}$ (the longitudinal and transverse derivatives, respectively). In order to express the final results in covariant forms, we will also need the light-front unit vector $n^\mu = (0, 1, 0_\perp)$, such that the "+" component of any four-vector a can be written covariantly as $n \cdot a$.

In the conventional formulation of HQET, the first step is to separate the full heavy-quark field $Q(x)$ into large and small components, by means of the projection operators $\Lambda_\pm = \frac{1}{2}(1 \pm \not{v})$. The situation is somewhat different in the framework of light-front field theory [24]. Here, before the $1/m_Q$ expansion is introduced, the heavy-quark field is first divided into two parts: $Q(x) = Q_+(x) + Q_-(x)$, with $Q_\pm(x) = \Lambda^\pm Q(x) = \frac{1}{2}\gamma^0\gamma^\pm Q(x)$. The equation of motion for Q can then be rewritten as two coupled equations for $Q_\pm$:

$$iD^- Q_+(x) = (i\alpha_\perp \cdot D_\perp + \beta m_Q) Q_-(x), \tag{2.1}$$
$$iD^+ Q_-(x) = (i\alpha_\perp \cdot D_\perp + \beta m_Q) Q_+(x), \tag{2.2}$$

where $\alpha_\perp = \gamma^0\gamma_\perp$ and $\beta = \gamma^0$. The above equations show that only the plus component $Q_+(x)$ is the dynamical field. The equation of motion for the minus component $Q_-(x)$ does not contain a light-front time derivative and therefore is a light-front constraint that determines $Q_-(x)$ from $Q_+(x)$. In terms of $Q_+(x)$, the heavy-quark part of the QCD Lagrangian (1.1) can be rewritten as

$$\mathcal{L} = Q_+^\dagger iD^- Q_+ - Q_+^\dagger (i\alpha_\perp \cdot D_\perp + \beta m_Q) Q_-, \tag{2.3}$$

where Q_- can be eliminated by Eq. (2.2).

To derive the light-front HQET, we use the same redefinition for the heavy-quark field as in the equal-time case,

$$Q(x) = e^{-im_Q v \cdot x} \mathcal{Q}_v(x), \tag{2.4}$$

but without imposing the separation of large and small components. It follows that

$$Q_+(x) = e^{-im_Q v \cdot x} \mathcal{Q}_{v+}(x), \quad Q_-(x) = e^{-im_Q v \cdot x} \mathcal{Q}_{v-}(x). \tag{2.5}$$

Substituting these equations into Eq. (2.2), we obtain

$$\mathcal{Q}_{v-}(x) = \frac{1}{m_Q v^+ + iD^+} \Big[i\alpha_\perp \cdot D_\perp + m_Q(\alpha_\perp \cdot v_\perp + \beta) \Big] \times \mathcal{Q}_{v+}(x). \tag{2.6}$$

It is worth noting that in the ordinary light-front formulation of quantum field theory, the elimination of the dependent component Q_- requires the choice of the light-front gauge $A^+ = 0$ and a specification of the operator $1/\partial^+$ which leads to severe light-front infrared problem that has still not been completely understood [32]. However, for the heavy-quark field with the redefinition of Eq. (2.4), the above problem does not occur since the elimination of the dependent component $\mathcal{Q}_{v-}$ now depends on the operator $1/(m_Q v^+ + iD^+)$ which has no infrared problem. Moreover, it has a well-defined series expansion in powers of iD^+/m_Q:

$$\frac{1}{m_Q v^+ + iD^+} = \sum_{n=1}^{\infty} \left(\frac{1}{m_Q v^+} \right)^n (-iD^+)^{n-1}. \tag{2.7}$$

Thus,

$$\begin{aligned} \mathcal{Q}_{v-}(x) &= \Big\{ \frac{\alpha_\perp \cdot v_\perp + \beta}{v^+} + \sum_{n=1}^{\infty} \left(\frac{1}{m_Q v^+} \right)^n (-iD^+)^{n-1} (i\vec{\alpha} \cdot \vec{D}) \Big\} \mathcal{Q}_{v+}(x) \\ &= \Big\{ \frac{\alpha_\perp \cdot v_\perp + \beta}{v^+} + \frac{1}{m_Q v^+ + iD^+} (i\vec{\alpha} \cdot \vec{D}) \Big\} \mathcal{Q}_{v+}(x), \end{aligned} \tag{2.8}$$

where we denote

$$\vec{\alpha} \cdot \vec{D} = \alpha_\perp \cdot D_\perp - \frac{\alpha_\perp \cdot v_\perp + \beta}{v^+} D^+. \tag{2.9}$$

In the following, we show that an alternative derivation based on the conventional way of eliminating the dependent quark field component Q_- gives the same result as above. We shall work with the light-front gauge, in which $A^+ = 0$, so that Eq. (2.2) becomes

$$Q_-(x) = \frac{1}{i\partial^+} (i\alpha_\perp \cdot D_\perp + \beta m_Q) Q_+(x). \tag{2.10}$$

Using the integral definition [32] of the operator $1/\partial^+$,

$$\left(\frac{1}{\partial^+} \right) f(x^-) = \frac{1}{4} \int_{-\infty}^{\infty} dx'^- \varepsilon(x^- - x'^-) f(x'^-), \tag{2.11}$$

where $\varepsilon(x) = -1,\ 0,\ 1$ for $x < 0,\ = 0,\ > 0$, respectively, we have

$$Q_-(x) = \frac{1}{i4}\int_{-\infty}^{\infty} dx'^- \varepsilon(x^- - x'^-) e^{-im_Q v\cdot\tilde{x}'}\Big[i\alpha_\perp \cdot D_\perp + m_Q(\alpha_\perp\cdot v_\perp + \beta)\Big]\mathcal{Q}_{v+}(\tilde{x}'), \tag{2.12}$$

where $\tilde{x}' \equiv (x^+, x'^-, x_\perp)$. By repeated integration by parts, and ignoring the surface terms [which are proportional to $\exp(-im_Q v\cdot x)|_{x^-=\pm\infty}$, a highly oscillating term that can be dropped], we finally find

$$\mathcal{Q}_{v-}(x) = \Big\{\frac{\alpha_\perp\cdot v_\perp + \beta}{v^+} + \sum_{n=1}^{\infty}\Big(\frac{1}{m_Q v^+}\Big)^n (-i\partial^+)^{n-1}(i\vec{\alpha}\cdot\vec{D})\Big\}\mathcal{Q}_{v+}(x), \tag{2.13}$$

which is the same as Eq. (2.8) in the light-front gauge.

Using Eq. (2.8), one can rewrite the equation of motion for $\mathcal{Q}_{v+}(x)$, i.e., Eq. (2.1), as

$$2(iv\cdot D)\mathcal{Q}_{v+}(x) = (i\vec{\alpha}\cdot\vec{D})\frac{v^+}{m_Q v^+ + iD^+}(i\vec{\alpha}\cdot\vec{D})\mathcal{Q}_{v+}(x). \tag{2.14}$$

Likewise, the heavy-quark QCD Lagrangian (2.3) can be reexpressed in terms of $\mathcal{Q}_{v+}$ alone. The complete $1/m_Q$ expansion is then given by

$$\begin{aligned}\mathcal{L} &= \frac{2}{v^+}\mathcal{Q}^\dagger_{v+}(iv\cdot D)\mathcal{Q}_{v+} - \mathcal{Q}^\dagger_{v+}(i\vec{\alpha}\cdot\vec{D})\frac{1}{m_Q v^+ + iD^+}(i\vec{\alpha}\cdot\vec{D})\mathcal{Q}_{v+}(x)\\ &= \frac{2}{v^+}\mathcal{Q}^\dagger_{v+}(iv\cdot D)\mathcal{Q}_{v+} - \sum_{n=1}^{\infty}\Big(\frac{1}{m_Q v^+}\Big)^n\mathcal{Q}^\dagger_{v+}\Big\{(i\vec{\alpha}\cdot\vec{D})(-iD^+)^{n-1}(i\vec{\alpha}\cdot\vec{D})\Big\}\mathcal{Q}_{v+}(x)\\ &= \mathcal{L}_0 + \sum_{n=1}^{\infty}\mathcal{L}_n.\end{aligned} \tag{2.15}$$

This is the light-front effective heavy-quark Lagrangian. One can readily check that the equation of motion, Eq. (2.14), is consistent with this Lagrangian. The dimensionless expansion parameter in the above Lagrangian is indeed $\Lambda_{\rm QCD}/m_Q$ as advertised earlier, since the operator $(-iD^+)$ picks up the "residual" momentum of the heavy quark, $k^+ = p^+ - m_Q v^+$, which is of the order $\Lambda_{\rm QCD}$.

As mentioned earlier, in the above derivation of light-front HQET, unlike the equal-time case, no constraint is imposed from the start to separate the large and small components of the heavy-quark field. In the present formalism, this separation of the large and small components is automatic. To see this point more clearly, we rewrite the above results in covariant forms. First let us define

$$\mathcal{Q}_v = \mathcal{Q}_{v+} + \mathcal{Q}_{v-} \equiv h_v^L + H_v^L, \tag{2.16}$$

where h_v^L is m_Q independent and H_v^L contains all the $1/m_Q$ correction terms, viz.,

$$h_v^L = \Big\{1 + \frac{\alpha_\perp\cdot v_\perp + \beta}{v^+}\Big\}\mathcal{Q}_{v+}, \tag{2.17}$$

$$H_v^L = \frac{1}{m_Q v^+ + iD^+}(i\vec{\alpha}\cdot\vec{D})\mathcal{Q}_{v+} = -\frac{\not{n}}{2(m_Q n\cdot v + in\cdot D)}(i\not{D})h_v^L, \tag{2.18}$$

where $n^\mu = (0, 1, 0_\perp)$ as defined earlier. The superscript L represents the fact that the large and small components of the heavy-quark field are separated on the light front. One can readily prove that the zeroth order field operator h_v^L has the desired property

$$\not{v} h_v^L = h_v^L, \tag{2.19}$$

whereas H_v^L satisfies $\Lambda_+ H_v^L = 0$. Thus all $1/m_Q$ corrections are contained in the light-front "bad" component $Q_-(x)$. This fact provides a direct connection of the $1/m_Q$ correction terms to high-twist operators, as noticed in a QCD sum rule calculation of the Isgur-Wise function [29]. In terms of h_v^L, the covariant form of the light-front effective heavy-quark Lagrangian reads

$$\begin{aligned}\mathcal{L} &= \bar{h}_v^L(iv\cdot D)h_v^L - \bar{h}_v^L(i\not{D})\frac{\not{n}}{2(m_Q n\cdot v + in\cdot D)}(i\not{D})h_v^L\\ &= \bar{h}_v^L(iv\cdot D)h_v^L - \frac{1}{2}\sum_{l=1}^{\infty}\Big(\frac{1}{m_Q\; n\cdot v}\Big)^l\,\bar{h}_v^L(i\not{D})\;\not{n}(-in\cdot D)^{l-1}(i\not{D})h_v^L.\end{aligned} \tag{2.20}$$

From Eq. (1.3), we see that formally light-front HQET has a very similar structure as equal-time HQET. In the heavy-mass limit, the lowest-order Lagrangian reads

$$\mathcal{L}_0 = \frac{2}{v^+} \mathcal{Q}^\dagger_{v+} (iv \cdot D) \mathcal{Q}_{v+} = \bar{h}^L_v (iv \cdot D) h^L_v, \qquad (2.21)$$

which is the same as the leading order equal-time effective theory and clearly exhibits the familiar flavor and spin symmetries. Note that spin symmetry on the light front is actually the same as helicity symmetry.

However, beyond the heavy-mass limit, LFHQET has its advantages. It is well known that, in the equal-time formulation, the nonleading part of HQET contains high-order time derivatives. This noncanonical structure of HQET causes certain difficulties in solving the theory [18]. For instance, it is very difficult to write down the Hamiltonian to all orders in $1/m_Q$. It is remarkable to see that, in LFHQET, only the linear time derivative appears, and it resides in $\mathcal{L}_0$ only. The factor $\not{n}$ in the nonleading terms of LFHQET eliminates all light-front time derivative contributions, as can be seen clearly from Eq. (2.15). Therefore, there is no difficulty in writing down the canonical conjugate field and hence the Hamiltonian from the effective Lagrangian on the light front. Explicitly, the canonical conjugate of the dynamical variable $\mathcal{Q}_{v+}$ is

$$\Pi_{\mathcal{Q}_{v+}} = \frac{\partial \mathcal{L}}{\partial(\partial^- \mathcal{Q}_{v+})} = i\mathcal{Q}^\dagger_{v+}, \qquad (2.22)$$

which does not involve any terms of order $1/m_Q$ or higher. The light-front heavy-quark effective Hamiltonian density is then given by

$$\begin{aligned} \mathcal{H} &= \Pi_{\mathcal{Q}_{v+}} \partial^- \mathcal{Q}_{v+} - \mathcal{L} \\ &= \frac{1}{iv^+} \mathcal{Q}^\dagger_{v+} (v^- \partial^+ - 2 v_\perp \cdot \partial_\perp) \mathcal{Q}_{v+} \\ &\quad - \frac{2g}{v^+} \mathcal{Q}^\dagger_{v+} (v \cdot A) \mathcal{Q}_{v+} + \mathcal{H}_{m_Q}, \qquad (2.23) \end{aligned}$$

with

$$\mathcal{H}_{m_Q} = \sum_{n=1}^{\infty} \mathcal{H}_n = -\sum_{n=1}^{\infty} \mathcal{L}_n, \qquad (2.24)$$

and the light-front Hamiltonian is defined as

$$H = P^- = \int dx^- d^2 x_\perp \mathcal{H}. \qquad (2.25)$$

This light-front heavy-quark effective Hamiltonian can serve as a useful basis for constructing the heavy-hadron bound states. It is also interesting to note that the light-front effective Hamiltonian $\mathcal{H}_n$ is precisely the minus of the light-front effective Lagrangian $\mathcal{L}_n$ given by Eq. (2.15). This simple relation does not exist in equal-time HQET. The reason is that, due to the existence of high-order time derivatives in equal-time HQET, the effective Hamiltonian is minus of the effective Lagrangian *plus* some noncanonical terms coming from the unusual conjugate field.

This concludes the derivation of LFHQET. In order to apply this theory to practical problems, one must first quantize it on the light front. We will turn to this subject in the next section.

III. LIGHT-FRONT QUANTIZATION OF HQET

As shown earlier, the equal-time heavy-quark effective Lagrangian contains higher-order time derivatives, so that it is very difficult to perform a consistent canonical quantization beyond the limit $m_Q \to \infty$ [18]. However, as we have seen, the light-front heavy-quark effective Lagrangian only contains a linear light-front time derivative term which resides in $\mathcal{L}_0$. Thus the full light-front effective Lagrangian can be easily quantized canonically. By the light-front phase space quantization procedure [32], the basic anticommutation relation is

$$\begin{aligned} \{\mathcal{Q}_{v+}(x) \ , \ \Pi_{\mathcal{Q}_{v'+}}(y)\}_{x^+=y^+} &= i\Lambda^+ \delta_{vv'} \delta(x^- - y^-) \\ &\quad \times \delta^2(x_\perp - y_\perp), \qquad (3.1) \end{aligned}$$

which is valid to all orders in $1/m_Q$.

In the limit $m_Q \to \infty$, the light-front heavy-quark field $\mathcal{Q}_+$ can be expanded in momentum space as

$$\mathcal{Q}_{v+}(x) = \sum_\lambda \int \frac{dk^+ d^2 k_\perp}{2(2\pi)^3} \omega_\lambda b_v(k,\lambda) e^{-ik\cdot x}, \qquad (3.2)$$

where k is the residual momentum of the heavy quark, $p = m_Q v + k$, with $v \cdot k = 0$ (mass-shell condition); ω_λ is the plus-component of the heavy-quark spinor which can be chosen to be momentum independent in a particular representation of the Dirac matrices [32], and it is normalized according to $\omega^\dagger_\lambda \omega_{\lambda'} = \delta_{\lambda\lambda'}$, $\sum_\lambda \omega_\lambda \omega^\dagger_\lambda = \Lambda^+$ [from Eq. (3.1)]. $b_v(k,\lambda)$ is the heavy-quark annihilation operator, satisfying the basic anticommutation relation

$$\begin{aligned} \{b_v(k,\lambda), b^\dagger_{v'}(k',\lambda')\} &= 2(2\pi)^3 \delta_{vv'} \delta(k^+ - k'^+) \\ &\quad \times \delta^2(k_\perp - k'_\perp) \delta_{\lambda\lambda'}, \qquad (3.3) \end{aligned}$$

where $\delta_{vv'}$ gives rise to the so-called velocity superselection rule [3]. Note that the antiquark part in Eq. (3.2) is dropped because heavy-quark–antiquark pair production is kinematically suppressed at the scale we are interested in.

Feynman rules for the effective heavy-quark field Q_{v+} are

$$\begin{aligned} S_{\mathcal{Q}_{v+}}(k) &= \tfrac{i}{2} \tfrac{v^+}{v\cdot k}, \\ \Gamma_{\mathcal{Q}_{v+}\mathcal{Q}'_{v+}g} &= i\tfrac{2}{v^+} g T^a v^\mu \end{aligned} \qquad (3.4)$$

for the heavy-quark propagator and the quark-gluon vertex, respectively.

For practical calculations, it is sometimes more convenient to work with the effective field $h^L_v(x)$, introduced in Sec. II, since it represents the full leading order part of the heavy-quark field $Q(x)$. The momentum space expansion of h^L_v is given by

$$h^L_v(x) = \sum_\lambda \int \frac{dk^+ d^2 k_\perp}{2(2\pi)^3} u(v,\lambda) b_v(k,\lambda) e^{-ik\cdot x}, \qquad (3.5)$$

where the corresponding heavy-quark spinor u is defined as

$$u(v,\lambda)=\Big\{1+\frac{\alpha_\perp\cdot v_\perp+\beta}{v^+}\Big\}\omega_\lambda \tag{3.6}$$

and satisfies the normalization conditions

$$\overline{u}(v,\lambda)u(v,\lambda')=\frac{2}{v^+}\delta_{\lambda\lambda'}\ ,\qquad \sum_\lambda u(v,\lambda)\overline{u}(v,\lambda)=\frac{1+\not v}{v^+}. \tag{3.7}$$

The corresponding Feynman rules for the h_v^L field are given by

$$\begin{array}{l} S_{h_v^L}(k)=\frac{i}{2}\frac{1+\not v}{v\cdot k},\\ \Gamma_{h_v^L h_v^L g}=igT^a v^\mu. \end{array} \tag{3.8}$$

This completes our discussion on the quantization of LFHQET.

IV. LIGHT-FRONT HEAVY-MESON BOUND STATES

In this section, we outline the procedure for constructing a heavy-meson bound state wave function on the light front [25]. In general, a hadronic bound state on the light front can be expanded in the Fock space composed of states with a definite number of particles. Explicitly, a hadronic bound state with the total longitudinal and transverse momenta $P^+, P_\perp$, and helicity λ can be written as

$$|\Psi(P^+,P_\perp,\lambda)\rangle=\sum_{n,\lambda_i}\int\left(\prod_i\frac{d^3\tilde p_i}{2(2\pi)^3}\right)2(2\pi)^3\delta^3\left(\tilde P-\sum_i\tilde p_i\right)|n,\tilde p,\lambda_i\rangle\Phi_n(x_i,\kappa_{\perp i},\lambda_i), \tag{4.1}$$

where $\tilde p\equiv(p^+,p_\perp)$, so that $d^3\tilde p=dp^+d^2p_\perp$, and $\delta^3(\tilde p-\tilde p')=\delta(p^+-p'^+)\delta^2(p_\perp-p'_\perp)$; $|n,\tilde p,\lambda_i\rangle$ is the Fock state consisting of n constituents, each of which carries momentum $\tilde p_i$ and helicity λ_i ($\sum_i\lambda_i=\lambda$); $\Phi(x_i,\kappa_{\perp i},\lambda_i)$ is the corresponding amplitude which depends on λ_i, the longitudinal momentum fraction x_i, and the relative transverse momentum $\kappa_{\perp i}$:

$$x_i=\frac{p_i^+}{P^+}\ ,\quad \kappa_{i\perp}=p_{i\perp}-x_iP_\perp. \tag{4.2}$$

The eigenstate equation that the wave functions obey on the light front is obtained from the operator Einstein equation $P^2=P^+P^--P_\perp^2=M^2$:

$$H_{\rm LF}|P^+,P_\perp,\lambda\rangle=\frac{P_\perp^2+M^2}{P^+}|P^+,P_\perp,\lambda\rangle, \tag{4.3}$$

where $H_{\rm LF}=P^-$ is the light-front Hamiltonian. Explicitly, for a meson wave function, the corresponding light-front bound state equation is

$$\left(M^2-\sum_i\frac{\kappa_{i\perp}^2+m_i^2}{x_i}\right)\begin{bmatrix}\Phi_{q\bar q}\\ \Phi_{q\bar q g}\\ \vdots\end{bmatrix}=\begin{bmatrix}\langle q\bar q|H_{\rm int}|q\bar q\rangle & \langle q\bar q|H_{\rm int}|q\bar q g\rangle & \cdots\\ \langle q\bar q g|H_{\rm int}|q\bar q\rangle & \cdots & \\ \vdots & \ddots & \end{bmatrix}\begin{bmatrix}\Phi_{q\bar q}\\ \Phi_{q\bar q g}\\ \vdots\end{bmatrix}, \tag{4.4}$$

where $H_{\rm int}$ is the interaction part of P^-.

Obviously to solve the above equation from QCD with the whole Fock space is impossible. Nevertheless, HQS can still bring great simplification to the problem. First of all we note that, on the light front, the total helicity of a heavy meson is simply the sum of the helicity of the heavy quark and the total helicity of the light-quark sector (the so-called brown muck which carries total spin 1/2; the brown muck of a baryon is more complex, and will not be discussed here.) HQS implies that, in the limit $m_Q\to\infty$, the spin of the heavy quark is decoupled from that of the light-quark part, because the heavy quark interacts with the light-quark part only through spin-independent soft-gluon exchanges. Thus, for a heavy meson, we can approximate the general expression of the light-front bound states, Eq. (4.1), as

$$|\Psi(P^+,P_\perp,\lambda)\rangle=\sum_{\lambda_Q\lambda_q}\int\frac{d^3\tilde p_Q d^3\tilde p_q}{2(2\pi)^3}\delta^3(\tilde P-\tilde p_Q-\tilde p_q)\Phi_{Q\bar q}(x,\kappa_\perp,\lambda_Q,\lambda_q)|Q(p_Q,\lambda_Q),\bar q(p_q,\lambda_q)\rangle, \tag{4.5}$$

where $P=Mv$, M is the mass of meson, and v^μ is its four-velocity, while

$$|Q(p_Q,\lambda_Q),\bar q(p_q,\lambda_q)\rangle=b_Q^\dagger(p_Q,\lambda_Q)d_q^\dagger(p_q,\lambda_q)|0\rangle, \tag{4.6}$$

and $d_q^\dagger$ should be regarded as the creation operator of a constituent light antiquark (brown muck), consisting of the valence current antiquark and a sea of gluons and quark-antiquark pairs. Consequently, contribution from the higher Fock states may be replaced by an effective two-body interaction kernel, so that Eq. (4.4) is reduced to a light-front

Bethe-Salpeter equation

$$\Big(M^2 - M_0^2\Big)\Phi_{Q\bar{q}}(x,\kappa_\perp) = \int \frac{dx' d^2\kappa'_\perp}{2(2\pi)^3} V_{\rm eff}(x,\kappa_\perp,x',\kappa'_\perp)\Phi_{Q\bar{q}}(x',\kappa'_\perp) \tag{4.7}$$

and

$$M_0^2 = \frac{\kappa_\perp^2 + m_q^2}{x} + \frac{\kappa_\perp^2 + m_Q^2}{1-x}. \tag{4.8}$$

In principle, the two-body effective interaction kernel $V_{\rm eff}$ should be derived from the leading order light-front heavy-quark effective Hamiltonian, plus the full QCD Hamiltonian for the light quarks at the hadronic scale. As is well known, the latter is very complicated even in the naive canonical case [32], and to derive $V_{\rm eff}$ is beyond the scope of this paper. We will leave this subject for future investigation. Until a way is found to solve the light-front bound state dynamics, we will have to be content with a phenomenological amplitude for $\Phi_{Q\bar{q}}$. One example that has been often used in the literature is the so-called Bauer-Stech-Wirbel (BSW) amplitude [27]:

$$\Phi_{\rm BSW}(x,k_\perp) = \mathcal{N}\sqrt{x(1-x)}\exp\Big(-\frac{\kappa_\perp^2}{2\omega^2}\Big)\exp\Big(-\frac{M^2}{2\omega^2}(x-x_0)^2\Big), \tag{4.9}$$

where $\mathcal{N}$ is the normalization constant, x is the longitudinal momentum fraction carried by the light quark, $x_0 = \Big(\frac{1}{2} - \frac{m_Q^2 - m_q^2}{2M^2}\Big)$, and ω is a parameter related to the physical size of the meson. Other forms, such as the Gaussian-type [33,34], are also possible, but we shall not dwell on this matter further.

Spin is always a troublesome issue in the light-front approach. For example, the heavy-meson light-front bound state we have constructed is labeled by helicity rather than spin. However, for practical applications physical states with definite spins are needed. This discrepancy is usually remedied by introducing the so-called Melosh rotation [35], which transforms a single particle state from the light-front helicity basis to the ordinary spin basis,

$$R(x_i,k_\perp,m_i) = \frac{m_i + x_i M_0 - i\sigma\cdot(\mathbf{n}\times\kappa_\perp)}{\sqrt{(m_i + x_i M_0)^2 + \kappa_\perp^2}}, \tag{4.10}$$

where $\mathbf{n} = (0,0,1)$. With the Melosh transformation incorporated, the light-front heavy-meson bound state with a definite spin can be expressed as [36]

$$|\Psi(P^+,P_\perp,S,S_z)\rangle = \sum_{\lambda_Q\lambda_q}\int \frac{d^3\tilde{p}_Q d^3\tilde{p}_q}{2(2\pi)^3}\delta^3(\tilde{P} - \tilde{p}_Q - \tilde{p}_q)\Phi_{Q\bar{q}}(x,\kappa_\perp)R^{SS_z}_{\lambda_Q\lambda_q}(x,\kappa_\perp)|Q(p_Q,\lambda_Q),\bar{q}(p_q,\lambda_q)\rangle, \tag{4.11}$$

where

$$R^{SS_z}_{\lambda_Q\lambda_q}(x,\kappa_\perp) = \sum_{s_1 s_2}\langle\lambda_Q|R^\dagger(1-x,-\kappa_\perp,m_Q)|s_1\rangle\langle\lambda_q|R^\dagger(x,\kappa_\perp,m_q)|s_2\rangle\langle\tfrac{1}{2}s_1\tfrac{1}{2}s_2|SS_z\rangle, \tag{4.12}$$

and $\langle\frac{1}{2}s_1\frac{1}{2}s_2|SS_z\rangle$ is the Clebsch-Gordan coefficient. A covariant form of Eq. (4.12) has been derived by Jaus [37], which makes practical calculations very convenient:

$$R^{SS_z}_{\lambda_Q\lambda_q}(x,\kappa_\perp) = \sqrt{\frac{n\cdot p_Q n\cdot p_q}{2[M_0^2 - (m_Q - m_q)^2]}}\,\overline{u}(p_Q,\lambda_Q)\Gamma v(p_q,\lambda_q), \tag{4.13}$$

where

$$\Gamma = \gamma^5 \qquad (\text{for pseudoscalar}, S=0), \tag{4.14}$$

$$\Gamma = -\not{\epsilon}(S_z) + \frac{\epsilon\cdot(p_Q - p_q)}{M_0 + m_Q + m_q} \qquad (\text{for vector}, S=1), \tag{4.15}$$

with

$$\epsilon^\mu(\pm1) = \Big(\frac{2}{P^+}\epsilon_\perp\cdot P_\perp, 0, \epsilon_\perp\Big), \quad \epsilon_\perp(\pm1) = \mp(1,\pm i)/\sqrt{2},$$

$$\epsilon^\mu(0) = \frac{1}{M}\Big(\frac{-M^2 + P_\perp^2}{P^+}, P^+, P_\perp\Big), \tag{4.16}$$

and the spinor $u(p,\lambda)$ has the same form as Eq. (3.6). Equation (4.11) is the phenomenological light-front heavy-meson bound state that has been widely used in the study of heavy-hadron dynamics [27,38].

However, the heavy-meson bound state so constructed still explicitly depends on the heavy-quark mass m_Q, and so is inconvenient from the view point of HQET. To calculate heavy-hadron matrix elements, we would like to use wave functions constructed in the heavy-mass limit, and then $1/m_Q$ corrections can be treated order by order within the framework of LFHQET. From Eq. (4.11), a heavy-meson bound state in the heavy-quark limit is given by

$$|\Psi(v,S,S_z)\rangle = \sum_{\lambda_Q\lambda_q}\int \frac{d^3\tilde{k}d^3\tilde{p}_q}{2(2\pi)^3}\delta^3(\Lambda_Q\tilde{v}-\tilde{k}-\tilde{p}_q)\Phi_{Q\bar{q}}(x,\kappa_\perp)R^{SS_z}_{\lambda_Q\lambda_q}|Q_v(k,\lambda_Q),\bar{q}(p_q,\lambda_q)\rangle, \tag{4.17}$$

where $\Lambda_Q = M - m_Q, x = p_q^+/(Mv^+), \kappa_\perp = p_{q\perp} - x(Mv_\perp)$, and the Melosh transformation matrix element is reduced to

$$R^{00}_{\lambda_Q\lambda_q} = \sqrt{\frac{v^+p_q^+}{4(\Lambda_Q+m_q)}}\,\overline{u}(v,\lambda_Q)\gamma^5 v(p_q,\lambda_q) \tag{4.18}$$

for a pseudoscalar meson and

$$R^{1S_z}_{\lambda_Q\lambda_q} = -\sqrt{\frac{v^+p_q^+}{4(\Lambda_Q+m_q)}}\,\overline{u}(v,\lambda_Q)\,\not\epsilon(S_z)v(p_q,\lambda_q) \tag{4.19}$$

for a vector meson, and the polarization vector becomes

$$\epsilon^\mu(\pm1) = \left(\frac{2}{v^+}\epsilon_\perp\cdot v_\perp, 0, \epsilon_\perp\right)\ ,\quad \epsilon(0) = -\left(\frac{v_\perp^2-1}{v^+}, v^+, v_\perp\right), \tag{4.20}$$

where we have approximately let $p_q = (M - m_Q)v = \Lambda_Q v$ in the Melosh transformation matrix elements. This is because in the symmetry limit the heavy-quark spinor in the Melosh transformation matrix element is independent of the residual momentum k (or the relative momentum $x, k_\perp$), as can be seen from Eqs. (4.18) and (4.19). Thus the residual momentum dependence in the light-quark spinor should also be very weak in order that the light-front heavy-meson state carry a fixed spin. The normalization condition for the state $|\Psi(v,S,S_z)\rangle$ is taken to be

$$\langle\Psi(v',S',S_z')|\Psi(v,S,S_z)\rangle = 2(2\pi)^3P^+\delta^3(\tilde{v}-\tilde{v}')\delta_{S'S}\delta_{S_z'S_z}, \tag{4.21}$$

which leads to

$$\int\frac{dxd^2\kappa_\perp}{2(2\pi)^3}|\Phi_{Q\bar{q}}(x,\kappa_\perp)|^2 = 1. \tag{4.22}$$

Thus we have constructed a light-front heavy-meson bound state in the symmetry limit ($m_Q \to \infty$) which has definite spin and parity. In the next section, we shall derive the Isgur-Wise function from this light-front wave function.

V. ISGUR-WISE FUNCTION

In LFHQET, as in the equal-time formulation, one can readily show that there exists a universal function describing weak transitions between heavy mesons. To do so, we first expand the weak heavy-quark current in $1/m_Q$ on the light-front: namely,

$$\begin{aligned}\overline{Q}^j(x)\Gamma Q^i(x) = e^{i(m_{Q^j}v'-m_{Q^i}v)\cdot x}Q^{j\dagger}_{v'^+}\left[1+\frac{\alpha_\perp\cdot v'_\perp+\beta}{v'^+}+(-i\vec{\alpha}\cdot\overleftarrow{D}_\perp)\sum_{n=1}^{\infty}\left(\frac{1}{m_Qv'^+}\right)^n(i\overleftarrow{D}^+)^{n-1}\right]\\ \times\gamma^0\Gamma\left[1+\frac{\alpha_\perp\cdot v_\perp+\beta}{v^+}+\sum_{n=1}^{\infty}\left(\frac{1}{m_Qv^+}\right)^n(-iD^+)^{n-1}(i\vec{\alpha}\cdot\vec{D}_\perp)\right]Q^i_{v^+}(x),\end{aligned} \tag{5.1}$$

where Γ stands for an arbitrary Dirac matrix (γ_5, γ_μ, etc.). In the heavy-mass limit, it reduces to the familiar form

$$\overline{Q}^j(x)\Gamma Q^i(x) \to e^{i(m_{Q^j}v'-m_{Q^i}v)\cdot x}\overline{h}^{jL}_v(x)\Gamma h^{iL}_v(x), \tag{5.2}$$

which shows that, apart from a trivial exponential factor, the effective current does not depend on the heavy-quark masses, and hence is flavor independent. The consequences of the spin and flavor symmetries can be readily derived using this zeroth order heavy-quark current. Consider the following matrix elements, for example:

$$\langle P_{Q^j}(v')|\overline{h}^{jL}_{v'}\Gamma h^{iL}_v|P_{Q^i}(v)\rangle \quad\text{and}\quad \langle P^*_{Q^j}(v')|\overline{h}^{jL}_{v'}\Gamma h^{iL}_v|P_{Q^i}(v)\rangle, \tag{5.3}$$

where P_Q and P^*_Q represent, respectively, a pseudoscalar meson and a vector meson containing a single heavy quark Q. Formally the heavy-meson states can be represented by the interpolating fields $|P_{Q^i}(v)\rangle = \sqrt{M_i}\ \overline{h}^{iL}_v\gamma_5\ell_v|0\rangle$, $|P^*_{Q^i}(v)\rangle = \sqrt{M^*_i}\ \overline{h}^{iL}_v\ \not\epsilon\ \ell_v|0\rangle$, where the mass factors are introduced for normalization purpose only, and ℓ_v stands for the fully interacting light antiquark (or brown muck) inside a heavy meson moving with velocity v. ℓ_v carries the quantum numbers of the valence light antiquark, but is independent of the spin and flavor of the associated heavy quark. As we have seen in Sec. III, the propagator for the h^L_v field is proportional to $(1+\not v)/2$. It is then easy to show that, in the symmetry limit, the heavy-meson transition matrix elements take the familiar forms [39]

$$\langle P_{Q^j}(v')|\overline{h}^{jL}_{v'}\Gamma h^{iL}_v|P_{Q^i}(v)\rangle = \sqrt{M_iM_j}\,\mathrm{Tr}\Big\{\gamma_5\Big(\frac{1+\not v'}{2}\Big)\Gamma\Big(\frac{1+\not v}{2}\Big)\gamma_5\mathcal{M}\Big\}, \tag{5.4}$$

$$\langle P^*_{Q^j}(v')|\overline{h}^{jL}_{v'}\Gamma h^{iL}_v|P_{Q^i}(v)\rangle = \sqrt{M_iM^*_j}\,\mathrm{Tr}\Big\{\not\epsilon^*\Big(\frac{1+\not v'}{2}\Big)\Gamma\Big(\frac{1+\not v}{2}\Big)\gamma_5\mathcal{M}\Big\}, \tag{5.5}$$

where $\mathcal{M}$ is the transition matrix element for the light antiquark (brown muck):

$$\mathcal{M} = \langle 0|\overline{\ell}_{v'}\ell_v|0\rangle \to \xi(v\cdot v')I. \tag{5.6}$$

Thus HQS implies that the transition matrix elements (5.3) are described by a single form factor $\xi(v\cdot v')$, known as the Isgur-Wise function.

Next, we explicitly derive Eq. (5.4) and (5.5) from light-front bound state wave functions of the general form (4.17), and thereby extract the Isgur-Wise function in terms of the light-front amplitudes. The hadronic matrix element for the B to D transition is given by

$$\begin{aligned}\langle D(v',0,0)|\overline{h}^{cL}_{v'}\Gamma h^{bL}_v|B(v,0,0)\rangle = \int \frac{d^3\tilde p_q d^3\tilde p'_q}{[2(2\pi)^3]^2}\Phi^*_D(x',\kappa'_\perp)\Phi_B(x,\kappa_\perp)R^{\dagger 00}_{\lambda_c\lambda'_q}R^{00}_{\lambda_b\lambda_q}\\ \times\langle c_{v'}(\Lambda_c v'-p'_q,\lambda_c)|\overline{h}^{cL}_{v'}\Gamma h^{bL}_v|b_v(\Lambda_b v-p_q,\lambda_b)\rangle\langle\bar q(p'_q,\lambda'_q)|\bar q(p_q,\lambda_q)\rangle.\end{aligned} \tag{5.7}$$

Since $\Lambda_b = \Lambda_c$ in the heavy-quark limit, and

$$\langle\bar q(p'_q,\lambda'_q)|\bar q(p_q,\lambda_q)\rangle = 2(2\pi)^3\delta^3(\tilde p_q-\tilde p'_q)\delta_{\lambda_q\lambda'_q}, \tag{5.8}$$

$$\langle c_{v'}(\Lambda_c v'-p'_q,\lambda_c)|\overline{h}^{cL}_{v'}\Gamma h^{bL}_v|b_v(\Lambda_b v-p_q,\lambda_b)\rangle = \overline{u}(v',\lambda_c)\Gamma u(v,\lambda_b), \tag{5.9}$$

making use of relation (3.7), we obtain

$$\langle D(v',0,0)|\overline{h}^{cL}_{v'}\Gamma h^{bL}_v|B(v,0,0)\rangle = \sqrt{M_BM_D}\ \zeta(v,v')\ \mathrm{Tr}\Big\{\gamma_5\Big(\frac{1+\not v'}{2}\Big)\Gamma\Big(\frac{1+\not v}{2}\Big)\gamma^5\Big\}; \tag{5.10}$$

similarly for the B to D^* transition, we have

$$\langle D^*(v',S,S_z)|\overline{h}^{cL}_{v'}\Gamma h^{bL}_v|B(v,0,0)\rangle = \sqrt{M_BM_{D^*}}\ \zeta(v,v')\ \mathrm{Tr}\Big\{\not\epsilon^*\Big(\frac{1+\not v'}{v^+}\Big)\Gamma\Big(\frac{1+\not v}{v^+}\Big)\gamma^5\Big\}, \tag{5.11}$$

where ϵ is given by Eq. (4.20). The Isgur-Wise function appearing in the above expressions is given by

$$\zeta(v,v') = \sqrt{\frac{M_B}{M_D}z}\int\frac{dxd^2\kappa_\perp}{2(2\pi)^3}\Phi^*_D(x',\kappa'_\perp)\Phi_B(x,\kappa_\perp), \tag{5.12}$$

where $z \equiv v^+/v'^+$, $x' = \frac{M_B}{M_D}zx$, and $\kappa'_\perp = \kappa_\perp + xM_B(v_\perp - zv'_\perp)$. To see the covariant structure of $\zeta(v,v')$, without the loss of generality, we can choose a frame where $v_\perp = v'_\perp = 0$; this is the most natural choice for light-front calculations. In such a frame ζ is a function of z only, and z can be expressed in terms of $v\cdot v'$ as

$$z_\pm = v\cdot v' \pm \sqrt{(v\cdot v')^2-1}, \tag{5.13}$$

where the + (−) sign corresponds to v^3 greater (less) than v'^3, and $z_+ = 1/z_-$. In the rest frame of the B meson, this sign ambiguity corresponds to whether one chooses the velocity of the $D(D^*)$ meson, $\vec v\,'$, to be in the negative or positive z direction. Since physically these two situations are indistinguishable, we must have

$$\zeta(z) = \zeta(1/z) = \xi(v\cdot v'), \tag{5.14}$$

which puts a constraint on the light-front amplitudes. Furthermore, it is interesting to note that this constraint condition can also be derived by demanding that altering the order of the integrations in Eq. (5.7) does not change the final result.

In the symmetry limit, the Isgur-Wise function, Eq. (5.12), should be independent of all heavy-meson masses. This property can be explicitly checked by observing that, when $m_Q \to \infty$, the light-front amplitude must have the scaling behavior

$$\Phi_{Q\bar{q}}(x,\kappa_\perp) \to \sqrt{M}\ \tilde{\Phi}(Mx,\kappa_\perp), \tag{5.15}$$

where the factor $\sqrt{M}$ (M being the meson mass) comes from the particular normalization we have assumed for the physical state in Eq. (4.21). The reason why the light-front heavy-meson wave function should have such an asymptotic form is as follows. Since x is the longitudinal momentum fraction carried by the light quark, the meson wave function should be sharply peaked near $x \sim \Lambda_{\rm QCD}/M$. It is then clear that only terms of the form "Mx" survive in the wave function as $M(m_Q) \to \infty$.[1] With Eq. (5.15), Eq. (5.12) can be rewritten as

$$\zeta(z) = \sqrt{z}\int_0^\infty dX \int \frac{d^2\kappa_\perp}{2(2\pi)^3}\tilde{\Phi}^*(X',\kappa_\perp)\tilde{\Phi}(X,\kappa_\perp), \tag{5.16}$$

where $X \equiv M_B x$, $X' \equiv M_D x'$, and $X' = Xz$. Now it is evident that the Isgur-Wise function $\zeta(z)$, or $\xi(v\cdot v')$, is totally independent of the heavy-meson masses, not even their ratio [38]. Furthermore, we also see that, at the zero-recoil point ($v\cdot v' = 1$), Eq. (5.16) reduces to the normalization condition (4.22) in the symmetry limit; hence $\xi(1) = \zeta(1) = 1$ as required.

In other works which also use light-front wave functions, hadronic form factors are usually evaluated either at the maximum recoil point $(P-P')^2 = 0$ or for $(P-P')^2 \le 0$, and special techniques are required to cover the whole kinematic region of interest [37,38]. This is not the case here. In this paper, the Isgur-Wise function is derived without assuming a particular value for $(P-P')^2$. Hence Eq. (5.16) is quite general and valid for arbitrary momentum transfers.

In the following, we explicitly calculate the Isgur-Wise function for model light-front amplitudes. In the heavy-quark limit, one can easily show that the phenomenological wave function given in Eq. (4.9) does have the correct asymptotic form (5.15), with

$$\tilde{\Phi}(x,\kappa_\perp) = \sqrt{32}\left(\frac{\pi}{\omega^2}\right)\sqrt{Mx}\ \exp\left(\frac{-\kappa_\perp^2}{2\omega^2}\right) \times\exp\left(\frac{-M^2x^2}{2\omega^2}\right). \tag{5.17}$$

Combining this expression and Eq. (5.16), we find

$$\zeta(z) = \frac{2z}{1+z^2}, \tag{5.18}$$

which indeed satisfies the consistency condition (5.14). With relation (5.13), the Isgur-Wise function in the symmetry limit can be expressed in terms of $v\cdot v'$, viz.,

$$\xi(v\cdot v') = \frac{1}{v\cdot v'}. \tag{5.19}$$

One can also check that the slope of $\xi(v\cdot v')$ at the zero-recoil point ($v\cdot v' = 1$),

$$\rho^2 \equiv -\xi'(1) = 1, \tag{5.20}$$

satisfies the Bjorken constraint of $\rho^2 > 1/4$ [40]. Moreover, it is in excellent agreement with the recent experimental result from CLEO, $\rho^2 = 1.01 \pm 0.15 \pm 0.09$ [41].

[1]Note that $Mx = p_q^+$ in the rest frame of the heavy meson.

VI. SUMMARY

To summarize, in this paper, we have explored in details HQET and the $1/m_Q$ expansion on the light front. In the heavy-quark limit, the light-front formulation reproduces the heavy-quark spin-flavor symmetry, as in the equal-time case. However, the structure of LFHQET is rather simple, so that canonical quantization presents no difficulty, and the Hamiltonian is well defined to all orders in $1/m_Q$, which is in contrast with the equal-time approach where since the nonleading terms contain high-order time derivatives, the canonical procedures are not valid for quantizing the theory and constructing the Hamiltonian. In Sec. IV, we construct the light-front heavy-meson bound states in the $m_Q \to \infty$ limit for performing practical evaluations of heavy-hadron dynamics within LFHQET. Finally, the Isgur-Wise function is derived from the light-front heavy-meson wave functions, and the result is a general expression valid for arbitrary recoil velocities. For the asymptotic form of the BSW amplitude in the $m_Q \to \infty$ limit, we find that the Isgur-Wise function $\xi(v\cdot v') = 1/v\cdot v'$ and its slope at the zero-recoil point is $\rho^2 = -\xi'(1) = 1$ which is in excellent agreement with the recent CLEO result of $\rho^2 = 1.01 \pm 0.15 \pm 0.09$.

ACKNOWLEDGMENTS

This work was supported in part by the National Science Council of the Republic of China under Grant Nos. NSC84-2112-M-001-036, NSC84-2816-M-001-012L, and NSC84-2112-M-009-024.

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