An intuitive view of the origin of orbital angular momentum in optical vortices

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ABSTRACT

A modulated laser beam by a phase pattern $\exp(il\theta)$ can be focused by an objective into a ring-like optical vortex, where l is a constant and θ is the azimuth angle. The vortex is capable of trapping the particles nearby and circulating them along the ring. This phenomenon is often explained involving Fourier optics and the transfer of orbital angular momentum (OAM). Although Fourier optics transforms the electric field distribution of the modulated laser beam behind the phase pattern to that of the vortex, it does not include both the path and OAM of the photons of the electromagnetic wave. Therefore, it is difficult to further trace the transfer of OAM from the photons to the particles in the vortex. In this paper, we propose a simple and intuitive view to the origin of optical vortex. By analyzing the relationship of the intensity distributions between the phase of the phase pattern and the intensity of the vortex by utilizing Fourier transform, we propose that the phenomenon of vortex also involve the transfer of linear momentum on the vortex plane transversely.

Keywords: Optical vortex, orbital angular momentum, holographic optical tweezers.

1. INTRODUCTION

An optical vortex can be generated by focusing a laser beam modulated with a phase pattern, $\exp(il\theta)$, where l is a constant and θ is the azimuthal angle[1]. At the back focal plane of the focusing lens, an optical vortex is formed. Optical vortices have ring-shaped intensity distributions. In addition, particles trapped in optical vortices move along the circumference. This phenomenon is explained by orbital angular momentum (OAM) [2]. Every photon in optical vortices can carry an OAM of $l\hbar$, like the photons in Lagarre-Gaussian beam[3] or Bessel beam[4] do. As the particles in optical vortices absorb or scatter these photons, the orbital angular momentum is transferred to particles.

Many researchers have been trying to utilize this property of optical vortices for transporting particles. Some researchers used an array of optical vortices to drive particles flow in a micro-fluid channel[5]. Some other researchers developed methods for producing optical vortices with desired intensity distributions[6,7]. However, within these methods, the OAM carried by a photon is hard to define. Since the phase patterns calculated by these methods is no longer the form, $\exp(il\theta)$. Thus, using the OAM carried by photons to describe the OAM in optical vortices becomes complicated.

In this paper, we use linear momentum of photons to describe the OAM in optical vortices instead the OAM of photons. A method is proposed to provide a clarity concept of the origin of OAM in optical vortices. In this method, the initial propagating directions of the photons on the phase plane are qualitatively obtained from the localized gratings of the phase plane. Then, a ray-tracing approach is used to describe the photon propagation from the phase plane to optical vortices. A simpler and more intuitive picture of OAM can be obtained.

2. THEORY

2.1 Generation of optical vortices (holographic optical tweezers)

In this section, the apparatus we used to generate optical vortices is introduced and the principle of it is also reviewed.

Optical vortices can be easily generated by holographic optical tweezers (HOT) [1]. HOT are basically optical tweezers (OT) with a phase-only spatial light modulator. The laser intensity distribution in the OT can be controlled by the spatial

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light modulator. This gives HOT the capability of manipulating multiple objects at the same time, and creating some special laser modes, such as Bessel beam, Laguerre-Gaussian beam, or optical vortices.

The simplified setup is shown in Fig. 1. The light comes from the left then is incident on the phase plane with transmittance t(x,y). The phase plane is where the spatial light modulator is placed and the transmittance of it can be controlled by a computer. After the light passes through the phase plane, the modulated light is focused by the lens.

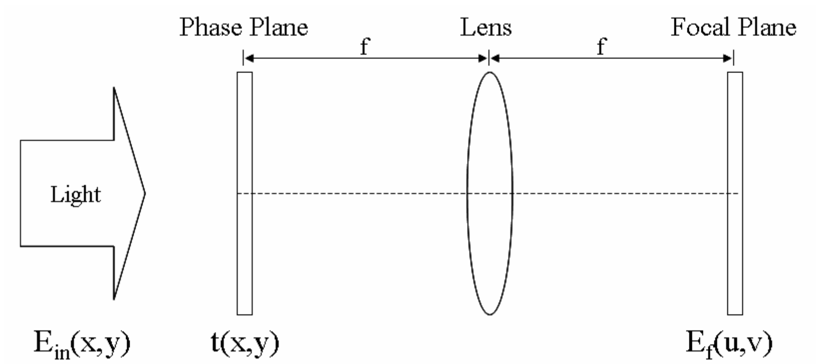

Fig. 1. The simplified setup of HOT.

There are two ways to see how HOT control the intensity distribution on the focal plane. One way is from the basic property of a converging lens[8]. Since the phase plane is placed in the front focal plane of the lens. The light field at the back focal plane is the scaled Fourier transform of the light field just behind the phase plane. The light field just behind the phase plane is the product of the incident light field and the transmittance of the phase plane, that is $E_{in}(x,y)t(x,y)$. Thus by proper control of t(x,y), any desired intensity distribution can be obtained.

Another way is from the concept of angular spectrum[8]. The angular spectrum of the light filed on the phase plane tells us the composition of the plane waves that propagate in different directions away form the phase plane. The plane waves propagating in different directions focus to different points on the focal plane, as shown in Fig. 2. Thus, the intensity distribution on the focal plane can be controlled by controlling the angular spectrum.

These two points of view are equivalent. Since the angular spectrum of the light field just behind the phase plane is simply the Fourier transform of the light field just behind the phase plane. The relation between the light field at the back focal plane and the light field just behind the phase plane is still a Fourier transform.

The last one, however, can give us extra information, how photons travel from the phase plane to the focal plane. With this information, the transverse momentum distribution on the focal plane contributed by the photons can be known. Then the OAM distribution on the focal plane can be obtained. This is also the approach we used to describe the OAM in optical vortices.

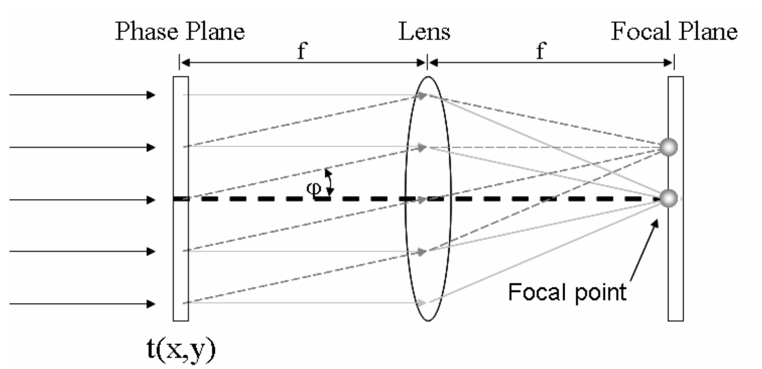

Fig. 2. The convergence of plane waves propagating in different directions.

To generate an optical vortex, the transmittance of the phase plane is chosen to be

$$t_v(r,\theta) = e^{i(l\theta)}, \tag{1}$$

where the r and θ are polar coordinates of the phase plane and the l is a constant. Then, an optical vortex is generated in the focal plane. The l in equation (1) controls some properties of an optical vortex, such as the radius of an optical vortex and OAM. It has been shown that the radius of an optical vortex linearly depends on the magnitude of l [2, 9]. A larger magnitude of l produces an optical vortex with larger radius. The OAM carried by the photon in optical vortices also depends on l. It has also been shown that every photon carries an OAM of $l\hbar$ [2].

The phase pattern of equation (1) with different l and the corresponding simulated intensity distributions on the focal plane are shown in Fig. 3. In these and latter simulations, we assume the phase plane is illuminated by a unit-amplitude plane wave. The field just behind the phase plane is equal simply to $t_v(r,\theta)$. Since the light field on the focal plane is the Fourier transform of the light field just behind the phase plane. The intensity distribution on the focal plane is obtained from the Fourier transform of $t_v(r,\theta)$. The Fourier transform is performed by fast Fourier transform (FFT).

From Fig. 3, we can see that the optical vortices have ring-shaped intensity distribution and the optical vortex with a larger l has larger radius.

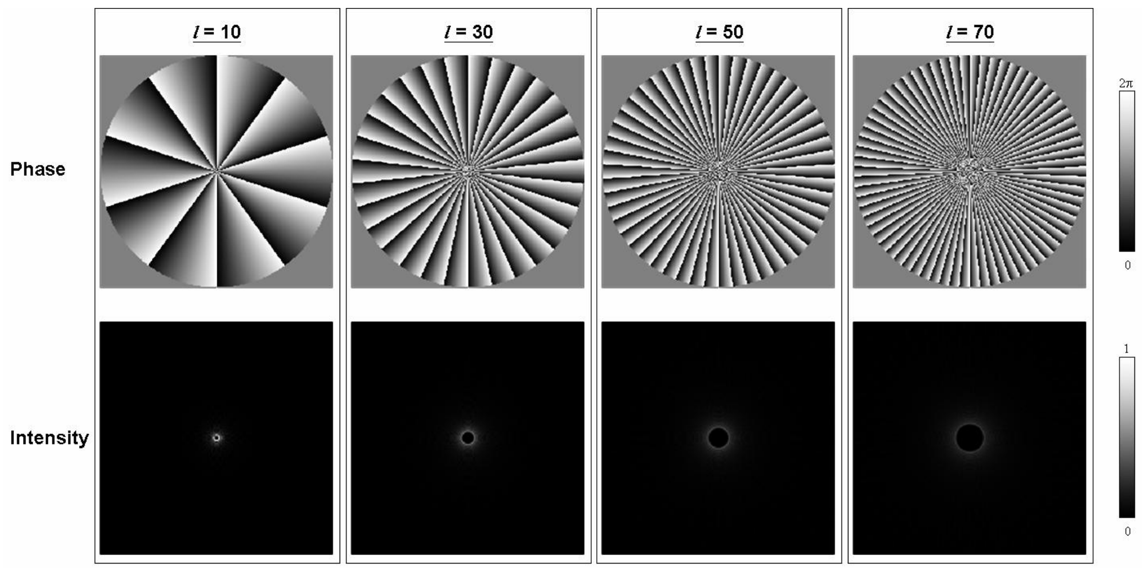

Fig. 3. The phase pattern with different l and the corresponding intensity distributions of optical vortices. The figures in the first row are the phase pattern. The figures in the second row are the corresponding intensity distributions, which are normalized by the maximum intensity of the l = 10 optical vortex.

2.2 Properties of optical vortices

In addition to these l-dependent properties, it is worth noting that there exits an intensity correspondence between the phase plane and optical vortices. If certain range of the azimuth angle, θ, of the phase plane is blocked, a range of the azimuth angle, ϕ, of the optical vortex disappears. It is also noted that these two ranges of the azimuth angle have 90^o shift, as shown in Fig. 4.

Fig. 4 shows the results of computer simulations. Each figure in Fig. 4 displays the phase and intensity distributions of the light field on the phase plane and the focal plane. From Fig. 4, we can see that the blocked angle range, $\triangle\theta$, in the phase plane approximately equals to the disappearing angle range, $\triangle\phi$, of the optical vortex, even for different l. Besides, the center angle, ϕ_0, of the disappearing angle range shifts by 90^o from the center angle, θ_0, of the blocked angle range for a positive l. For a negative l, the angle shift between ϕ_0 and θ_0 is –90^o.

The angle shift, $\phi_0 - \theta_0$, and $\triangle\phi$ at different l and different $\triangle\theta$ are shown in Fig. 5. From Fig. 5 (a), we can see that $\triangle\phi$ is approximately equal to $\triangle\theta$. From Fig. 5 (b), we can see that $\phi_0 - \theta_0$ are equal to 90^o for positive l and –90^o for negative l, even at different $\triangle\theta$.

These properties imply that the light at certain position in an optical vortex only comes from a specific region of the phase plane. Thus, it is also why a ray-tracing method could be used to find the propagating paths of photons from the phase plane to an optical vortex on the focal plane.

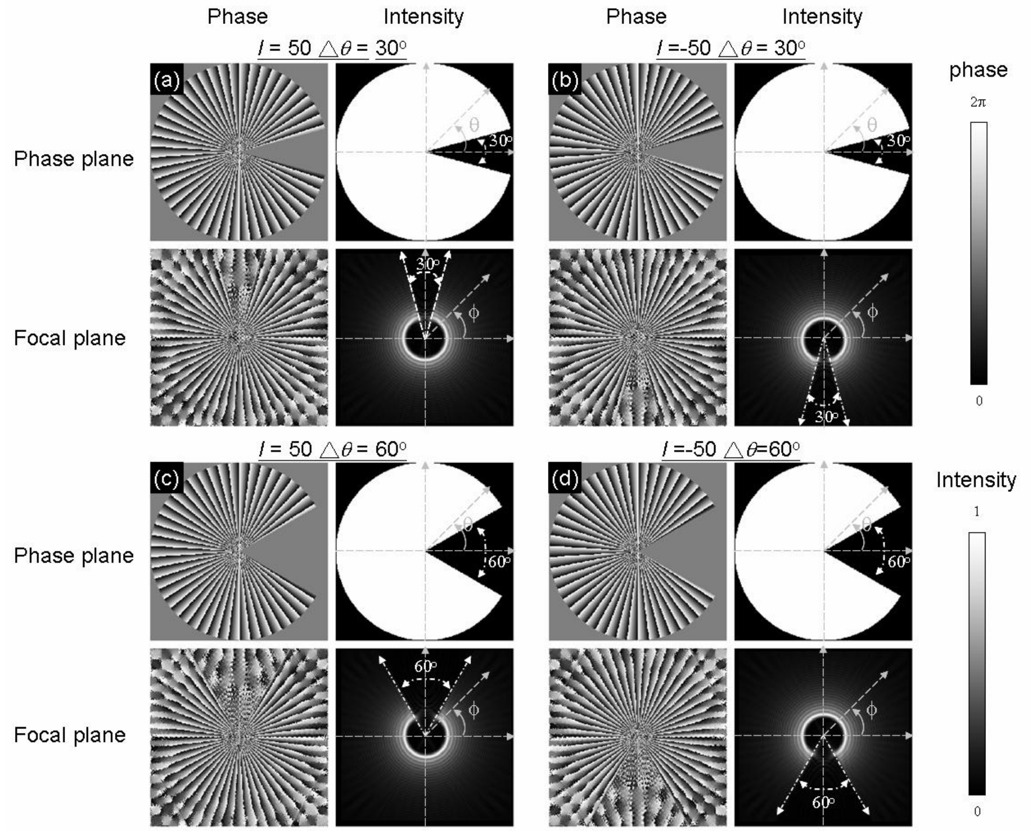

Fig. 4. The phase and normalized intensity distributions of the light field on the phase plane and on the focal plane, for (a) l=50 and $\triangle\theta$= 30°; (b) l = -50 and $\triangle\theta$ = 30°; (c) l = 50 and $\triangle\theta$ = 60°; (d) l = -50 and $\triangle\theta$ = 60°.

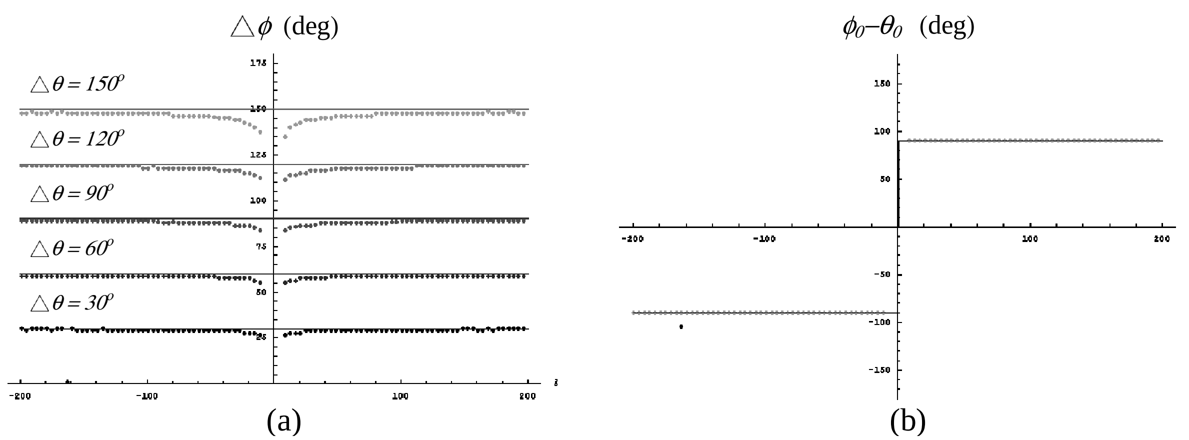

Fig. 5. (a) The relation between $\triangle\phi$ and l at different $\triangle\theta$. The solid line represents the value of $\triangle\theta$. The points are the simulated results. (b) The relation between $\phi_0 - \theta_0$ and l at different $\triangle\theta$. In both figures, the range of l is from -199 to 200.

2.3 Localized gratings

From the properties of optical vortices described in previous section, we known that a ray-tracing method could be used to trace the paths of photons from the phase plane to the optical vortex. If the position and propagating direction of photons on the phase plane are known, how and where the photons are incident on the focal plane can be obtain. To find the initial condition of photons on the phase plane, let us consider the phase pattern of gratings first. The transmittance of the grating has the form

$$t(x,y) = e^{i\vec{k}\cdot\vec{r}} \quad . \tag{2}$$

Here, $\vec{r}$ is the position vector of the phase plane. $\vec{k}$ is the direction vector. It determines the direction and the period of the grating. It also determines the propagating direction of the photons after passing through it.

Consider the phase pattern of equation (1) with a positive l around $\theta = 0^{\circ}$ region, as shown in Fig. 6. From the properties of optical vortices mentioned before, the photons in this region focus to $\phi = 90^{\circ}$ region of the focal plane. However, after passing through the grating with $\vec{k} = k\hat{y}$, where k is a positive constant, the light also focuses to $\phi = 90^{\circ}$ region of the focal plane. Since this grating deflects the light upward. The upward propagating wave focuses to $\phi = 90^{\circ}$ region of the focal plane. Thus $\theta = 0^{\circ}$ region of equation (1) have a similar effect on the incident photons as the grating with $\vec{k} = k\hat{y}$ does.

For the region around $\theta = 180^{\circ}$, we can also find a corresponding grating that has similar effects on the incident photons. From the properties of optical vortices, we know that the photons on this region focus to $\phi = 270^{\circ}$ region of the focal plane. The grating with $\vec{k} = -k\hat{y}$ also makes light propagate downward and focus to $\phi = 270^{\circ}$ region of the focal plane.

Thus, in a small region of the phase pattern of equation (1), we use the phase pattern of the corresponding grating to approximate the original phase pattern. Since the phase pattern of the grating with different $\vec{k}$ only exist in a small region. We call these gratings as localized grating. In such way, the initial directions of the photons on the phase plane can be easily qualitatively determined by these localized gratings.

Therefore, the phase distributions of equation (1) can be considered as the assembly of many localized gratings orientate in different directions at different position.

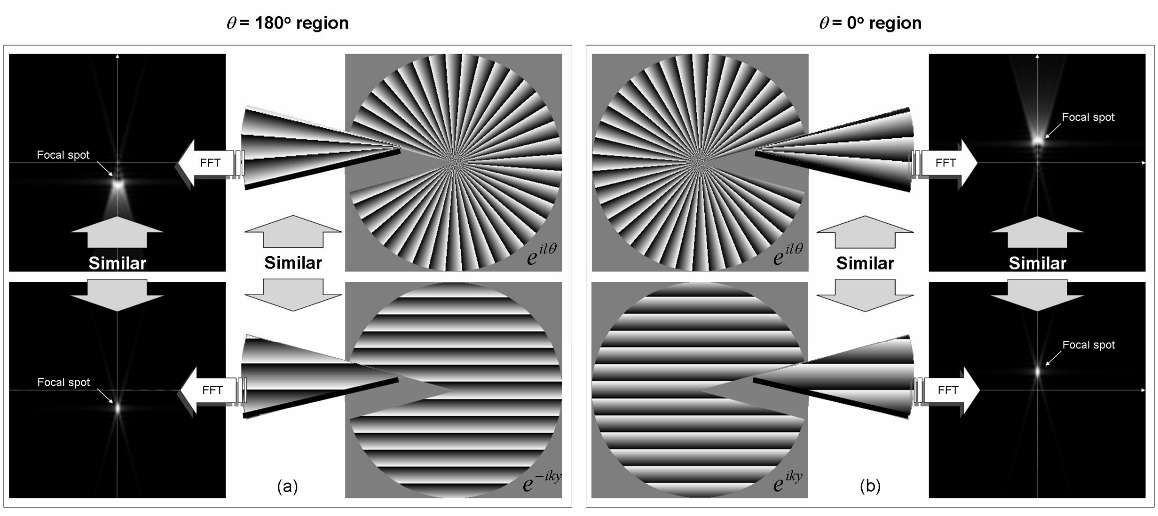

Fig. 6. The comparison between the phase pattern of exp($il\theta$) and that of gratings. (a) The phase pattern around the $\theta = 0^{\circ}$ region of exp($il\theta$) is similar to that of the grating, exp(iky). The corresponding intensity distributions are also similar. (b) The phase pattern around $\theta = 180^{\circ}$ region is similar to the phase pattern of exp($-iky$). The corresponding intensity distributions are also similar. Thus, the phase pattern, exp($il\theta$), can be considered as many localized gratings orientate in different directions at different position.

2.4 An intuitive view of the OAM in optical vortices

OAM is defined as the cross product of a position vector and the transverse momentum of light at that position. If the transverse momentum distribution is known, the OAM distribution can be obtained. In addition, since photons carry momentum. If how photons are incident on the focal plane is known, the transverse momentum on the focal plane can be known.

From previous section, we said that the phase pattern of equation (1) can be considered as the assembly of many localized gratings orientate in different directions at different position. The orientation and period of each grating can qualitatively decide the propagating direction of photons after they leave the phase plane. With these initial conditions of the photons, the ray-tracing method is applied to decide the paths of these photons. Then, the incident angles of the photons at the focal spot on the focal plane can be known. Thus the OAM in optical vortices can be obtained

Fig. 8 shows the path of photons that travel from the phase plane to the optical vortex with positive *l*. In the right of Fig. 8 are the projections of the photon path on *y*-*z* plane and *x*-*z* plane. From Fig. 8, the photons travel upward after passing through the $\theta = 0^{\circ}$ region of the phase plane. Then, the photons are focused to point A on the focal plane by the lens. From the incident angle of the photons, we know that the direction of the total transverse momentums, $\vec{P}_t$, on the focal plane is along $-\hat{u}$. The direction of the position vector of point A is along $\hat{v}$. Thus, the direction of OAM at point A is along $\hat{z}$. The directions of the OAM in other positions of the optical vortex are also along $\hat{z}$, since the phase pattern has azimuth symmetry. Thus, when the particles trapped in the optical vortex absorb the photons, they obtain the OAM and move around the optical vortex.

Fig. 9 shows the case of negative *l*. By the same process, we can know that the direction of the OAM in the optical vortices with negative *l* is along $-\hat{z}$. This is the opposite direction of the OAM in the optical vortices with positive *l*. Thus the particles trapped in the optical vortex with negative *l* move around the optical vortex in different direction.

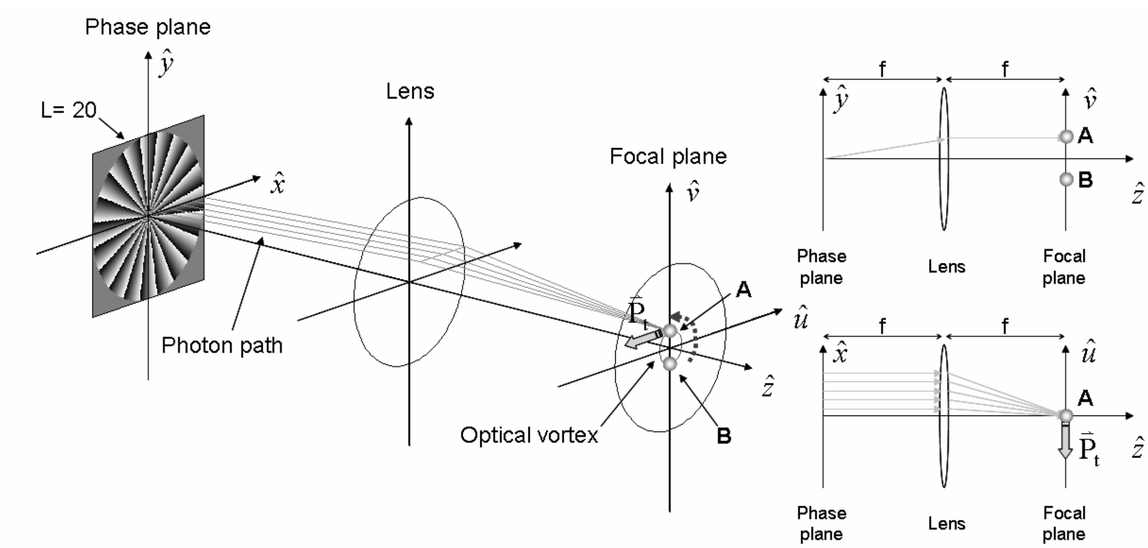

Fig. 8. The path of photons traveling from phase plane to the optical vortex with positive *l*. The projection of the path on the *y*-*z* plane is shown in right top. The projection of the path on the *x*-*z* plane is shown in right bottom.

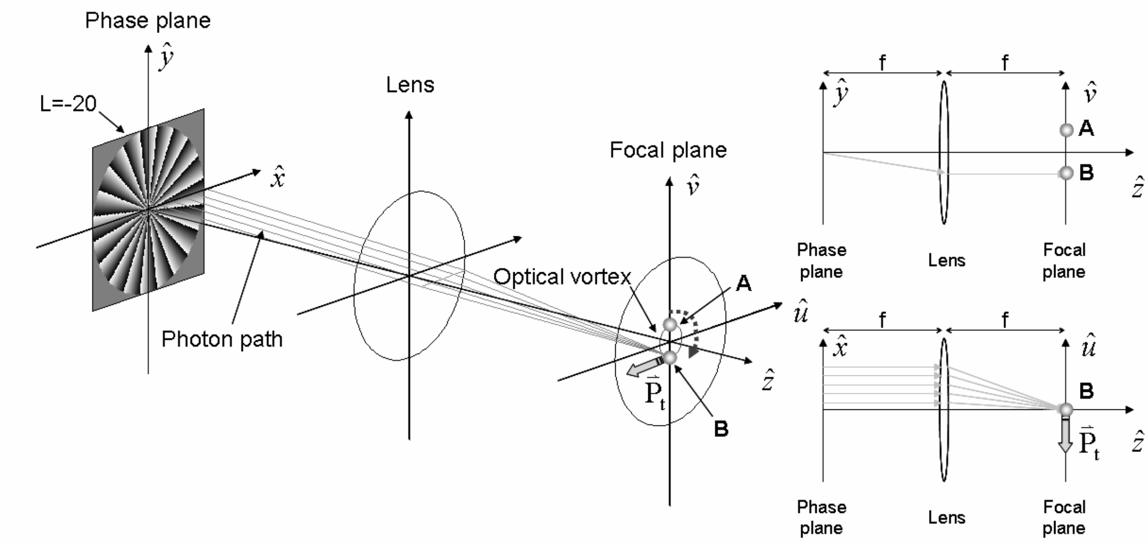

Fig. 9. The path of photons traveling from phase plane to the optical vortex with negative *l*. The projection of the path on the *y-z* plane is shown in right top figure. The projection of the path on the *x-z* plane is shown in right bottom figure.

3. CONCLUSIONS

In this paper, a method that uses the linear momentum of photons to describe the OAM in optical vortices is proposed. Although it only gives the qualitative results of the OAM in optical vortices, it provides us a much simpler way to obtain some properties of optical vortices. By simply looking at the phase pattern and then the direction of OAM and the shape of optical vortices can be estimated.

In addition, we also found that the transporting property of optical vortices can be simply described by the linear momentum transfer form the inclined incident photons. Thus we might be able to create an optical field which has the transporting property without using optical vortices.

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