A Novel Application of LDPC-Based Decoder for WiMAX Dual-mode Inner Encoder

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***Abstract*— LDPC codes have recently received a lot of attention by their excellent error-correcting capability and have been adopted as an optional error correct coding scheme in WiMAX (802.16e) standard for mobility. In order to the low-complexity SOC (System-on-a-chip) design and multi-mode transmitter compatible, we proposed a novel application of LDPC decoder to substitute Viterbi decoder for convolutional encoder. The main character presented in this study is that the general two-decoder structure is replaced by one decoder. Our results indicate that the proposed scheme processes nice performance for mobile WiMAX system.**

***Index Terms*— LDPC decoder, WiMAX, convolutional code, Viterbi decoder, SoC design.**

I. Introduction

The performance of error correcting code was bounded by Shannon in 1948 [1]. However, until the arrival of turbo codes in 1993 [2], practical coding schemes for most non-trivial channels fell far short of the Shannon limit. Followed MacKay and Neal [3] rediscovered Gallager's low-density parity-check codes (LDPC) [4] and showed that. The performance on the additive white Gaussian noise (AWGN) channel is very close to Shannon capacity. LDPCs however have recently begun to gain attention because they are parallelizable, and VLSI technology has gotton to the point where LDPCs over fairly large code word sizes can be implemented in hardware.

WiMAX, for the mobile WirelessMAN standard 802.16e has been proposed last year. The 802.16e modifies from 802.16-2004 standard to improve its performance on mobility. In the forward error correction (FEC) part, besides the convolutional code and turbo code, LDPC is a new optional candidate for inner codec.

In this study, we proposed a novel application of LDPC decoder; it substitutes for more complexity Viterbi decoder to decode the convolutional code with its simple architecture. With this feature, it is quietly useful to reduce the receiver hardware design area for System-on-a-chip (SOC) to different transmitter's inner code.

In this contest, the paper presents the whole system simulation via a LDPC-Based decoder deal with the default and optional FEC scheme, LDPC, in WiMAX. After the comparison of convolutional code and LDPC Code, a brief introduction to the LDPC decoding algorithm and a concise description of our proposed WiMAX transceiver architecture is represented. Then, the system performance simulation results under AWGN channel will be shown.

II. Complation Of Convolutional Code And LDPC Code

In the WiMAX (802.16e) standard, the convolutional code is a default FEC inner encoder, that the decoding by the Viterbi algorithm is recommended. The Viterbi proposed a maximum likelihood (ML) decoding algorithm that was relatively easy to implement for soft-decision decoding of convolutional codes. On the other hand, Bahl, Cocke, Jelinek, and Raviv (BCJR) [5] introduced a maximum a posteriori probability (MAP) decoding algorithm for convolutional codes with unqual a priori probability for information bits. The BCJR algorithm has been applied in recent years to soft-decision iterative decoding schemes in which the a priori probability of the information bits change from iteration to iteration. According to this concept, we proposed a new methodology to utilize the LDPC decoding algorithm (see Fig. 1), such as log-BP (belief-propagation) algorithm [7] and so on, which has the similar significant kernel of iteration steps to decoding the coding sequence.

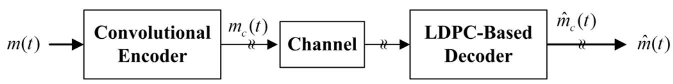

Fig. 1. The proposed methodology for a codec of convolutional encoder and LDPC-Based decoder.

A. Convolutional code

In the general case of an (n, k, l) convolutional code that the generator matrix is given by

$$\mathbf{G} = \begin{bmatrix} G_0 & G_1 & G_2 & \cdots & G_m & & \\ & G_0 & G_1 & G_2 & \cdots & G_m & \\ & & G_0 & G_1 & G_2 & \cdots & G_m \\ & & & \vdots & \vdots & & \vdots \end{bmatrix} \quad (1)$$

where each G_l (l=0, 1, 2,$\cdots$, m) is a $k \times n$ submatrix whose entries are

$$G_l = \begin{bmatrix} g^0_{1,l} & g^1_{1,l} & g^2_{1,l} & \cdots & g^{n-1}_{1,l} \\ g^0_{2,l} & g^1_{2,l} & g^2_{2,l} & \cdots & g^{n-1}_{2,l} \\ \vdots & \vdots & \vdots & & \vdots \\ g^0_{k,l} & g^1_{k,l} & g^2_{k,l} & \cdots & g^{n-1}_{k,l} \end{bmatrix} \quad (2)$$

where $g^i_{j,l}$ represent the generator sequence corresponding to input i and output j. Again note a each set of k rows of $\mathbf{G}$

is identical to the previous set of rows but shifted n places to the right. From (2), the convolutional code is a kind of infinite linear block code in fact. Therefore, we could use a linear block decoder algorithm to decode it. The decoding method will be described in the section IV.

B. Low-Density Parity-Check code

Low-density parity-code (LDPC) codes are a class of linear error-correcting codes. Linear codes use a generator matrix **G** to map message **s** to transmitted blocks **t**, also known as *codeword*. They have an equivalent description in terms of a related parity-check matrix **H**. All codewords satisfy **H**·**t**=0.

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & \cdots & 1 & \cdots & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 1 & \cdots & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ 1 & 0 & \cdots & h_{i,j} & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix} \quad (3)$$

The parity check matrix **H** for LDPC codes is a sparse binary matrix. The general structure of **H** is shown in (3). Each row of **H** corresponds to a parity check and a set element $h_{i,j}$ indicates that data symbol j participates in parity check i. In a block of n bits or symbols, there are m redundant parity symbols and the code rate is r given by $r = (n - m)/n$. The set row and column elements of **H** are chosen to satisfy a desired row and column weight profile, where the row and column weights are defined as the number of set elements in a given row and column, respectively [6]. In a *regular* LDPC code, all rows are of uniform weight, as are all columns. An *irregular* LDPC code is defined by a parity-check matrix **H** with multiple column weights and multiple row weights.

Therefore, based on the *irregular* LDPC definition, the convolutional code can be look as a kind of *irregular* LDPC, it apply that it could decode by the corresponding decoding algorithm. We will demonstrate the decoding performance with the 802.16e standard in the following section.

III. Belief Propagation of LDPC Code Decoding Algorithm

LDPC codes can be effectively decoded by the iterative belief-pripagation (BP) (also known as sum-product) algorithm [7]. A powerful representation tool of BP decoding algorithm is the Tanner graph, which compute the decoding message on each variable node (VN) and check node (CN) and iteratively exchanged through the edges between the neighboring nodes (see Fig. 2). In fact, for reducing the hardware complexity, which leads to the log-BP algorithm: both algorithms realize the same decoding rule.

Before presenting the log-BP decoding algorithm [7]-[8], we need to some definitions as follows: Let **H** denotes the $M \times N$ parity check matrix and $h_{i,j}$ denotes the entry at position (i, j). $\alpha_{m,n}$ defined as the message from the variable node n to the check node m and $\beta_{m,n}$ defined as the message from the check node m to the variable node n. $M(n)/m$ denotes the n row position of any 1's except the m column in **H**, $N(m)/n$ denotes the m column position of any 1's except the n row in **H**, we also define:

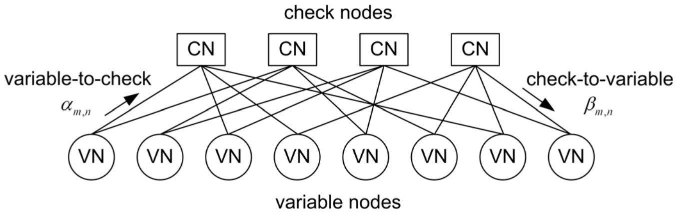

Fig. 2. Tanner graph representation of an LDPC parallel decoder structure.

$$L(u_n) = \ln \frac{P(u_n = 0|z, c, s)}{P(u_n = 1|z, c, s)} \quad (4)$$

where z is the initial input sequence, c means meeting all check equations, and s denotes the symbol including u_n message bits. The message passing from the variable nodes to the check nodes is defined as follows:

$$\begin{aligned} L(\alpha_{m,n}) &= \ln \prod_{m' \in M(n)/m} \frac{P(u_n = 0|z, c_{m'}, s_{m'})}{P(u_n = 1|z, c_{m'}, s_{m'})} \quad (5) \\ &= L^a(u_n) + \sum_{m' \in M(n)/m} L(\alpha_{m',n}) + L^s(s_k^j) \end{aligned}$$

where

$$\begin{aligned} L^a(u_n) &= \ln(P(u_n = 0)/P(u_n = 1)) \\ L(\alpha_{m',n}) &= \ln \frac{P(c_{m'}|z, u_n = 0)}{P(c_{m'}|z, u_n = 1)} \\ L^s(s_k^j) &= \sum_{m' \in s_m/m} \ln \frac{P(s_{m'}|z, u_n = 0)}{P(s_{m'}|z, u_n = 1)} \end{aligned}$$

$c_{m'}$ means that the m'-*th* check equation is satisfied, and $s_{m'}$ is the extrinsic information coming from the demapping device. The message passing from the check nodes to the variable nodes can be calculated as follows:

$$L(\beta_{m,n}) = \prod_{n' \in N(m)/n} a_{m,n'} \cdot \ln\left(\frac{J+1}{J-1}\right) \quad (6)$$

where

$$\begin{aligned} J &= exp\left(\sum_{n' \in N(m)/n} G(b_{m,n'})\right) \\ G(x) &= (1 - e^x)/(1 + e^x) \\ a_{m,n} &= sign(\ln((1 - \alpha_{m,n})/\alpha_{m,n})) \\ b_{m,n} &= |\ln((1 - \alpha_{m,n})/\alpha_{m,n})| \end{aligned}$$

The physical meaning of a, b and $G(x)$ function have been analysis by Gallager [4].

IV. System Design And Simulation Results

According to IEEE 802.16e standard, we construct the mobile WirelessMAN WiMAX simulation platform, including Randomizer, GF(256) Reed-Solomon code, Convolutional code, Interleaver, 16-QAM(or QPSK) and 256-FFT. The system block diagram is shown in Fig. 3

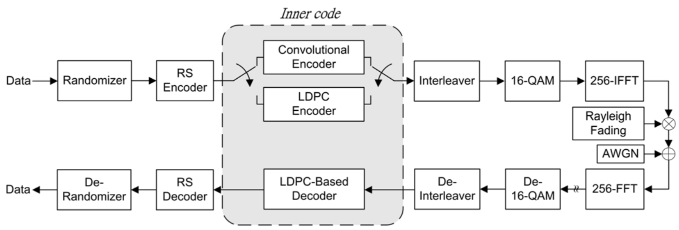

Fig. 3. Functional blocks of the proposed WiMAX transceiver.

The platform has special part in the receiver that it is used the LDPC-Based decoder to substitute for conventual's Viterbi decoder. In this study, our proposed method only uses the AWGN as a channel condition; hence, it is unnecessary to design the equalizer for channel estimation in the simulation processing.

From our simulation result, one fact is found that the LDPC-Based decoder can only be used to error correction of codeword of convolutional code. The messages $\hat{\mathbf{s}}$ still utilizes the linear block decoding algorithm. In this study, we demonstrated a decoding method, in the case of a binary-symmetric channel, as following.

In the LDPC-Based decoding procedure, we have to choose a suitable size of generator matrix $\mathbf{G}$ via transmission signal frame length and the code rate. Because the generator matrix $\mathbf{G}$ is a rectangular matrix, an inverse $\mathbf{G}^{-1}$ of the matrix in modulo 2 arithmetic has been computed during the Gaussian elimination which produced the matrix $\mathbf{R}$= $\mathbf{G}^T(\mathbf{G}\cdot\mathbf{G}^T)^{-1}$ such that

$$\mathbf{G}\cdot\mathbf{R} = \mathbf{G}\cdot\mathbf{G}^T(\mathbf{G}\cdot\mathbf{G}^T)^{-1} = \mathbf{I} \qquad (7)$$

According to the linear block code encoder $\mathbf{s}\cdot\mathbf{G}$=$\mathbf{t}$, the messages $\hat{\mathbf{s}}$ in the receiver could estimate by $\hat{\mathbf{s}}$=$\mathbf{t}\cdot\mathbf{R}$. Consequently, the original messages can be easily estimated. The simulation performance of the proposed LDPC-Based decoding algorithm is shown in Fig. 4 and Fig. 5. They include comparisons of different modulation, including 16-QAM and QPSK, and demonstrate the realistic performances of WiMAX systems with LDPC-Based decoder under AWGN and Rayleigh fading channels, respectively. In Fig. 5, the user moving with the velocity 30km/hr, central frequency 2.5GHz, and subcarrier bandwidth 3.5MHz. From the simulation results, it can be found that the proposed method still possesses a high error correction capability under different noise interferences.

Apparently, from the indications of the above simulation results, the proposed method can substitute for Viterbi decoder with its simple structure although it sacrifices some perfomance. Based on this design procedure, it is no doubt that the SOC design for WiMAX system in the future can effectively ultilize one decoder for decoding two kinds of encoder. Therefore, the proposed method is very suitable for practical applications in the WiMAX communication systems.

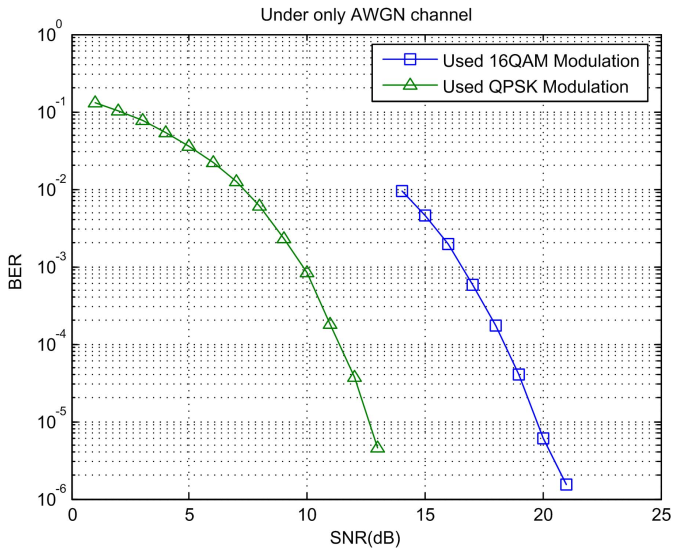

Fig. 4. The BER of the LDPC-Based decoder with different modulations under AWGN channel.

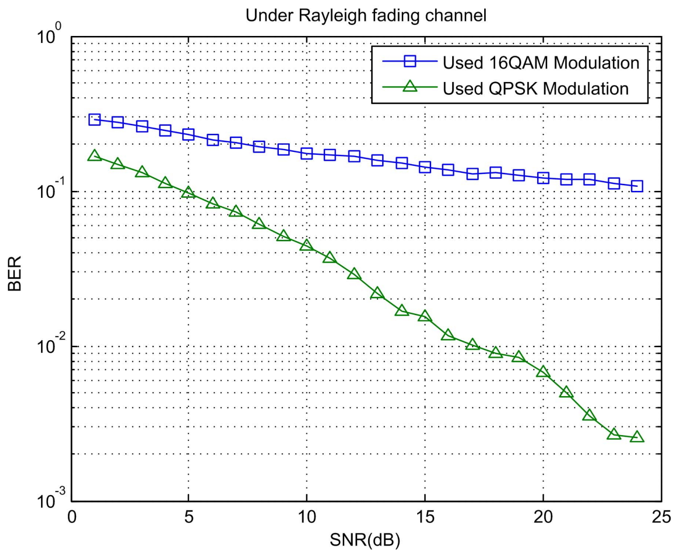

Fig. 5. The BER of the LDPC-Based decoder with different modulations under Rayleigh fading channel, in which the user condition is the velocity v = 30km/hr, central frequency f_c = 2.5GHz, and subcarrier bandwidth BW_{sub} = 3.5MHz.

V. Conclusion

This paper introduced a novel application of LDPC-Based decoder algorithm for the estimation of convoltinoal encoder. From the simulation results, it is easy to find the proposed method shown us the potential for SOC design to subsitituted for conventional Viterbi decoder.

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REFERENCES

[1] C. E. Shannon, “A mathematical theory of communication,“ *The Bell System Technical Journal*, vol. 27, no. 6, pp. 379–423, 623–656, 1948.

[2] C. Berrou, A. Glavieux, and P. Thitimajshima, “Near Shannon limit error-correcting coding and decoding: Turbo-codes,“ in *Proceedings of IEEE International Communications Conference*, vol. 2, Geneva, Switzerland, May 1993, pp. 1064–1070.

[3] D.J.C. MacKay and R.M. Neal, “Near Shannon limit performance of low density parity check codes,“ *IEE Electronics Letters*, vol. 32, pp. 1645–1646, Aug 1996.

[4] G. Gallager, “Low density parity check codes,“ *IRE Transactions on Information Theory*, vol. 8, pp. 21–28, 1962.

[5] L. Bahl, J. Cocke, F. Jelinek, and J. Raviv, “Optimal decoding of linear codes for minimizing symbol error rate,“ *IEEE Transactions on Information Theory*, vol. 20, pp. 284–287, 1974.

[6] T.J. Richardson, M.A. Shokrollahi, R.L.J. Urbanke, and A. Richter, “Design of capacity-approaching irregular low-density parity-check codes,“ *IEEE Transactions on Information Theory*, vol. 47, pp. 619–637, 2001

[7] D.J.C. MacKay, “Good error-correcting codes based on very sparse matrices,“ *IEEE Transactions on Information Theory*, vol. 45, pp. 399–431, Mar 1999.

[8] H. Rui, X.F. Zhang, and D.Z. Xu, “Performance of belief propagation coded modulation with iterative decoding,“ in *Proceedings of IEEE International Conference on Communications, Circuits and Systems*, vol. 1, Sichuan, China, June 2004, pp. 71–74.