Measurements of material refractive index with a circular heterodyne interferometer

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ABSTRACT

When a light coming from a circularly polarized heterodyne light source incidents on an optical material, a phase difference between s- and p- polarization components of the reflected light occurs. This phase difference can be measured accurately with the heterodyne interferometry. The measured data are substituted into the special equations derived from Fresnel equations, the refractive index can be estimated. This method bears both merits of a common-path interferometer and a heterodyne interferometer. The refractive indices of three optical glasses and two birefringent crystals were measured to show the validity of this method.

Keyword: refractive index, circular heterodyne interferometer, isotropic material, birefringent crystal, Fresnel equations.

1. INTRODUCTION

Optical materials such as optical glasses or birefringent crystals are often used to fabricate optical components. Recently, some devices, for example, birefringent laser cavity filters[1], poled-polymer electro-optic devices[2], liquid-crystal spatial light modulations[3], and magneto-optic recording media[4], have been used for many applications. To enhance their quality and performance, it is necessary to determine their refractive indices accurately. There are several methods[5-15] for measuring the refractive index of an optical material. They are usually divided into two types: the transmission type measurement method[5-10] and the reflection type measurement method[11-15]. In the former method, the phase variations of the light beam transmitted through an optical material are measured. So, the accuracy of thickness, flatness and parallelism of the two opposite sides of materials are strongly required. Hence, the measurement processes become tedious. The latter method such as ellipsometric technique is related with the light intensity variations. Consequently, it is easily influenced by the stability of the light source, the scattering light, the internal reflection, etc., and its resolution will be decreased.

To overcome these drawbacks, a circular heterodyne interferometer for measuring the refractive indices of an isotropic material and a birefringent crystal is proposed in this paper. It utilizes a common-path heterodyne interferometric technique and Fresnel equations. When a light coming from a circularly polarized heterodyne light source[16] incidents on an optical material, a phase difference between s- and p- polarization components of the reflected light occurs. This phase difference can be measured accurately with the heterodyne interferometry. The measured data are substituted into the special equations derived from Fresnel equations, the refractive index can be estimated. This method bears both merits of a common-path interferometer and a heterodyne interferometer.

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Optical Measurement Systems for Industrial Inspection IV, edited by Wolfgang Osten, Christophe Gorecki, Erik Novak, Proc. of SPIE Vol. 5856 (SPIE, Bellingham, WA, 2005)
0277-786X/05/$15 · doi: 10.1117/12.612480

2. PRINCIPLE

The schematic diagram of this method is shown in Fig. 1. A light beam coming from a circularly polarized heterodyne light source is incident at θ onto an optical material OM located on a rotation stage. The light beam reflected from OM passes through an analyzer AN_t and enters a photo detector D_t. If the amplitude of the light detected by D_t is E_t, then the intensity measured by D_t is $I_t = |E_t|^2$. Here, I_t acts as a test signal. On the other hand, the electronic modulated signal of the circularly heterodyne polarized light source is filtered and becomes the reference signal. Finally, these two signals are sent to a phase meter PM and the phase difference between them can be measured.

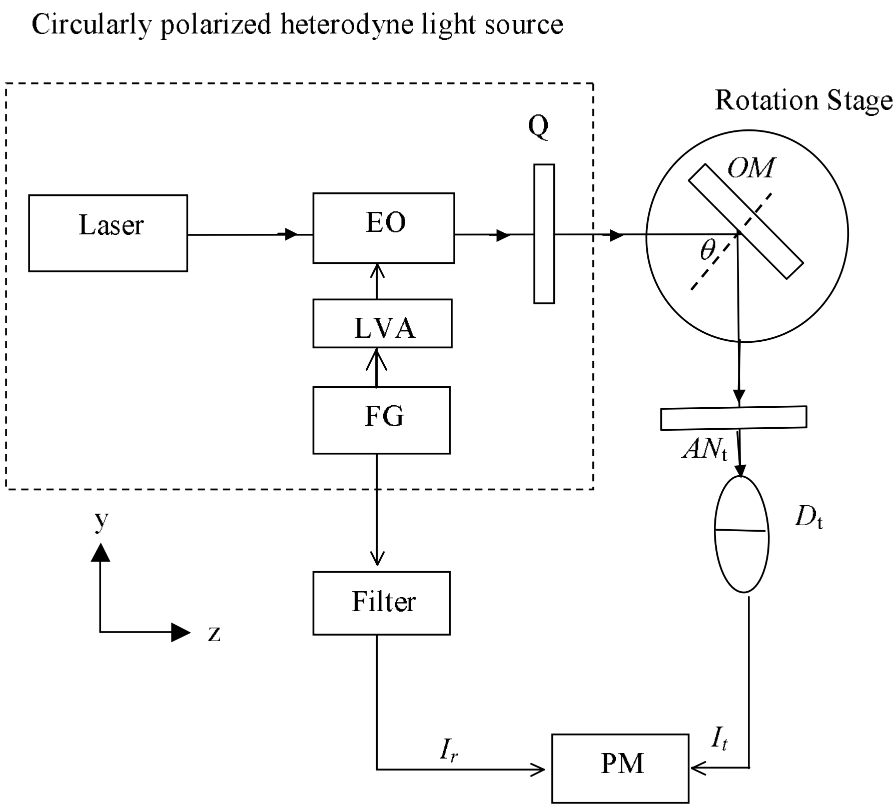

Fig. 1 Schematic structure for measuring the phase difference owing to reflection at an optical material. EO: electro-optic modulator; Q: quarter-wave plate; OM: optical material; AN_t: analyzer; D_t: photodetector; LVA: linear voltage amplifier; FG: function generator; PM: phase meter.

2.1 Circularly polarized heterodyne light source

The circularly polarized heterodyne light source consists of a linearly polarized laser light source, an electro-optic modulator EO and a quarter- wave plate Q as shown as shown in Fig. 1. EO is driven by a function generator FG and a linear voltage amplifier LVA. For convenience, the +z axis is chosen along the propagation direction and the y-axis is along the vertical direction. Let the laser light be horizontally linearly polarized, the fast axis of EO and Q be 45° and 0° with respect to the x-axis, respectively. If an external saw tooth voltage signal with angular frequency ω and amplitude $V_{\lambda/2}$, the half-voltage of EO, is applied to EO, then the phase retardation produced by EO can be expressed as ωt. The Jones vector of the light coming from the circularly polarized heterodyne light source can be written as

$$
E_i = Q(0°)\cdot EO(\omega t)\cdot E_0 = \begin{pmatrix} \cos(\frac{\omega t}{2}) \\ -\sin(\frac{\omega t}{2}) \end{pmatrix}
$$

$$
= \frac{1}{2}\begin{pmatrix} 1 \\ i \end{pmatrix}\exp(i\frac{\omega t}{2}) + \frac{1}{2}\begin{pmatrix} 1 \\ -i \end{pmatrix}\exp(-i\frac{\omega t}{2}). \tag{1}
$$

From Eq. (1), it is obvious that there is an angular frequency difference ω between the left- and the right- circular polarizations of the light beam.

2.2 Phase difference between s- and p- polarizations of reflected light

2.21 Birefringent crystal

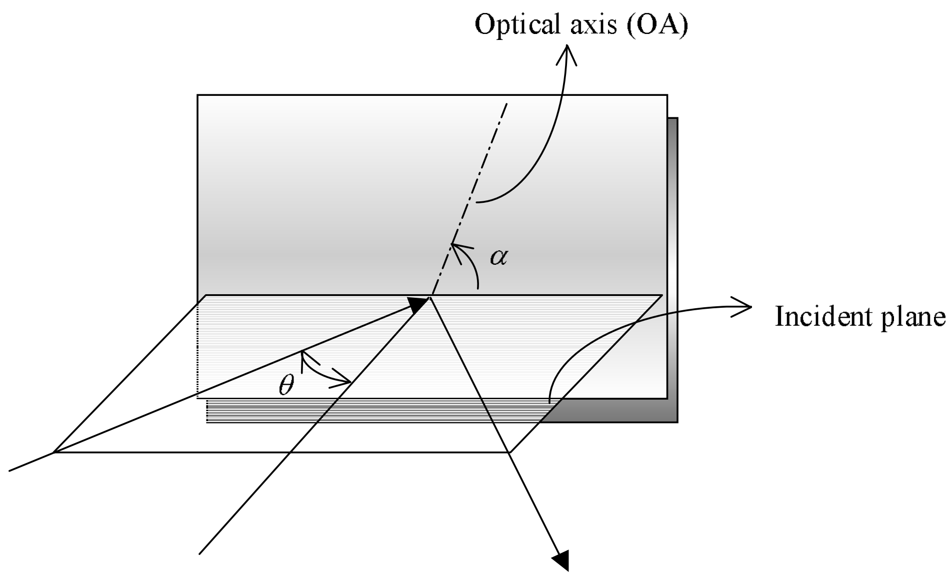

Fig. 2 Reflection at surface of an optical material. OA: optical axis.

Here OM is a birefringent crystal with the extraordinary index n_e and the ordinary index n_o, and its optical axis is located at α with the incident plane as shown in Fig. 2. If the transmission axis of AN_t is located at β with respect to the x-axis, then we have

$$
E_t = AN(\beta)\cdot S\cdot E_i = AN(\beta)\cdot\begin{bmatrix} r_{pp} & r_{ps} \\ r_{sp} & r_{ss} \end{bmatrix}\cdot E_i
$$

$$
= \left((r_{pp}\cos\beta + r_{sp}\sin\beta)\cos\frac{\omega t}{2} - (r_{ps}\cos\beta + r_{ss}\sin\beta)\sin\frac{\omega t}{2} \right)\begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix}, \tag{2}
$$

where S is the Jones matrix for OM, r_{pp} and r_{ss} are the direct-reflection coefficients, and r_{ps} and r_{sp} are the cross-reflection coefficients, respectively. They can be expressed as

$$r_{pp} = \frac{A_1 A_6 + A_2 A_5}{A_1 + A_2}, \tag{3a}$$

$$r_{ps} = \frac{A_1 A_2 (A_4 - A_3)}{A_1 + A_2}, \tag{3b}$$

$$r_{sp} = \frac{A_6 - A_5}{A_1 + A_2}, \tag{3c}$$

and

$$r_{ss} = \frac{A_1 A_3 + A_2 A_4}{A_1 + A_2}, \tag{3d}$$

where

$$A_1 = \frac{C}{(\sin^2\theta + C\cos\theta)\tan\alpha}, \tag{4a}$$

$$A_2 = \frac{n_o \tan\alpha (B + n_o \cos\theta)}{Bn_o \cos\theta + C^2}, \tag{4b}$$

$$A_3 = \frac{\cos\theta - C}{\cos\theta + C}, \tag{4c}$$

$$A_4 = \frac{n_o \cos\theta - B}{n_o \cos\theta + B}, \tag{4d}$$

$$A_5 = \frac{n_o{}^2 \cos\theta - C}{n_o{}^2 \cos\theta + C}, \tag{4e}$$

$$A_6 = \frac{Bn_o \cos\theta - C^2}{Bn_o \cos\theta + C^2}, \tag{4f}$$

$$B^2 = n_o{}^2 n_e{}^2 - \sin^2\theta (n_o{}^2 \sin^2\alpha + n_e{}^2 \cos^2\alpha), \tag{4g}$$

$$C^2 = n_o{}^2 - \sin^2\theta. \tag{4h}$$

Hence, we have

$$I_t = |E_t|^2 = I_0 [1 + \cos(\omega t + \phi)], \tag{5}$$

where

$$I_0 = \frac{(r_{pp}\cos\beta + r_{sp}\sin\beta)^2 + (r_{ps}\cos\beta + r_{ss}\sin\beta)^2}{2}, \tag{6}$$

and

$$\phi = \tan^{-1}\left(\frac{2(r_{pp}\cos\beta + r_{sp}\sin\beta)(r_{ps}\cos\beta + r_{ss}\sin\beta)}{(r_{pp}\cos\beta + r_{sp}\sin\beta)^2 - (r_{ps}\cos\beta + r_{ss}\sin\beta)^2} \right). \tag{7}$$

2.2.2 Isotropic material

For isotropic material, we have the relation $n_e = n_o = n$. Then, Eq. (3) can be expressed as

$$r_{sp} = r_{ps} = 0, \tag{8}$$

$$r_{\mathrm{pp}} = \frac{n^2\cos\theta - \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}, \tag{9}$$

and

$$r_{\mathrm{ss}} = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}}. \tag{10}$$

The average intensity I_o and the phase difference ϕ in Eqs. (5) ~ (7) can be rewritten as

$$I_0 = \frac{(r_{pp}\cos\beta)^2 + (r_{ss}\sin\beta)^2}{2}, \tag{11}$$

and

$$\phi = \tan^{-1}\left(\frac{(r_{pp}r_{ss}\sin 2\beta)}{(r_{pp}\cos\beta)^2 - (r_{ss}\sin\beta)^2}\right), \tag{12}$$

respectively.

2.3 Estimation of refractive index

The electronic signal generated by FG is filtered, and it uses as the reference signal. So the reference signal has the form of

$$I_r = I'[1 + \cos(\omega t + \phi_0)]. \tag{13}$$

Both the test signal I_t and the reference signal I_r are sinusoidal signals. They are sent to a phase meter PM, and ϕ can be measured accurately when ϕ_0 is known.

2.3.1 Birefringent crystal

From Eqs. (3) ~ (5) and Eq. (7), we can see that ϕ depends on n_e, n_o, α, θ, and β. In practical measurement processes, θ and β are obtained from the direct angle readouts of the division mark of the rotation stage. Consequently, only three factors n_e, n_o and α should be solved. That is, we have

$$\phi = \phi(n_e, n_o, \alpha). \tag{14}$$

Theoretically, the data of ϕ being corresponding to three different conditions should be measured. They are substituted into Eq. (14) and n_e, n_o and α can be obtained. However, these equations are very complicated, it is not easy solve them directly. For easier operations and estimations, θ and β could be so chosen that Eq. (7) can be simplified. As the condition β=0° is chosen, Eq. (7) can be rewritten as

$$\phi = \tan^{-1}\left(\frac{2r_{pp}r_{ps}}{r_{pp}^2 - r_{ps}^2}\right). \tag{15}$$

It can be seen from Eqs. (3) and (4) that either r_{ps} or r_{sp} equals zero as α equals either 0° or 90°, respectively. Hence, under the condition β=0°, the optical axis OA of the birefrigent crystal can be rotated until the condition ϕ=0° is satisfied. Then, the optical axis is located at either 0° or 90° with respect to the incidence plane.

Next, AN_t is rotated so that β is nonzero, and Eq. (7) is rewritten as

$$\phi = \tan^{-1}\left(\frac{\sin 2\beta \cdot r_{pp}r_{ss}}{r_{pp}^2\cos^2\beta - r_{ss}^2\sin^2\beta}\right). \tag{16}$$

Now we consider two particular conditions:

(i) if α=0°, then

$$r_{pp} = \frac{n_o n_e\cos\theta - \sqrt{n_o^{\,2} - \sin^2\theta}}{n_o n_e\cos\theta + \sqrt{n_o^{\,2} - \sin^2\theta}}, \tag{17a}$$

and

$$r_{ss}=\frac{\cos\theta-\sqrt{n_o{}^2-\sin^2\theta}}{\cos\theta+\sqrt{n_o{}^2-\sin^2\theta}}\,; \tag{17b}$$

(ii) if α=90°, then

$$r_{pp}=\frac{n_o{}^2\cos\theta-\sqrt{n_o{}^2-\sin^2\theta}}{n_o{}^2\cos\theta+\sqrt{n_o{}^2-\sin^2\theta}}\,, \tag{18a}$$

and

$$r_{ss}=\frac{\cos\theta-\sqrt{n_e{}^2-\sin^2\theta}}{\cos\theta+\sqrt{n_e{}^2-\sin^2\theta}}\,. \tag{18b}$$

Since three unknowns (n_e, n_o, and α) are to be solved, we need three equations. These can be obtained by measuring ϕ at three different incident angles θ_1, θ_2, and θ_3. Three corresponding phase differences ϕ_1, ϕ_2, and ϕ_3 are obtained, and represented as

$$\phi_1=\phi_1(n_e,n_o,\alpha)\,, \tag{19a}$$

$$\phi_2=\phi_2(n_e,n_o,\alpha)\,, \tag{19b}$$

$$\phi_3=\phi_3(n_e,n_o,\alpha)\,. \tag{19c}$$

Two of Eqs. (19a)~(19c) are combined to form a set of simultaneous equations, and three sets are obtained. Any set of the simultaneous equations can be solved under either condition (i) or (ii), and so two corresponding pairs of solutions for (n_e, n_o) are obtained. Therefore, there are six pairs for (n_e, n_o). Among them, three pairs are derived under condition (i) and form a group of solutions. The other three are derived under condition (ii) and form another group of solutions. Then, the justification of correct solutions can be achieved by the following approaches:

1. Rationality of the solution: In general, both n_e and n_o are within the range 1 and 5. If any estimated data of n_e and n_o is not within this range, it is obvious that the estimated data may be incorrect.
2. Comparison between n_e and n_o: Either a positive or negative crystal is tested, all three pairs of solutions of either group should meet with only either $n_e>n_o$ or $n_e<n_o$. If not, then that group is incorrect.

Hence, only one group of solutions is correct, and the corresponding data of α is the azimuth angle of its optical axis.

2.3.2 Isotropic material

From Eq.(10), it is easily seen that the case r_{ss}=0 should not exist. From Eq. (12) it is obvious that only when β is neither 0° nor 90°, then ϕ=0° as r_{pp}=0. Under this condition, the incident angle is equivalent to the Brewster's angle θ_B. Eqs. (9) and (10) are substituted into Eq. (12), we obtain

$$\phi=\tan^{-1}\left[\frac{((\sin^2\theta-n^2\cos^2\theta)\sin2\beta)}{[(2\sin^4\theta-\sin^2\theta+n^2\cos^2\theta)\cos2\beta-2\sin^2\theta\cos\theta\sqrt{n^2-\sin^2\theta}}\right]. \tag{20}$$

From Eq. (20), θ_B is capable to be determined with the experimental curve between ϕ and θ, so n can be evaluated by using the relation $n=\tan\theta_B$.

3. EXPERIMENTS AND RESULTS

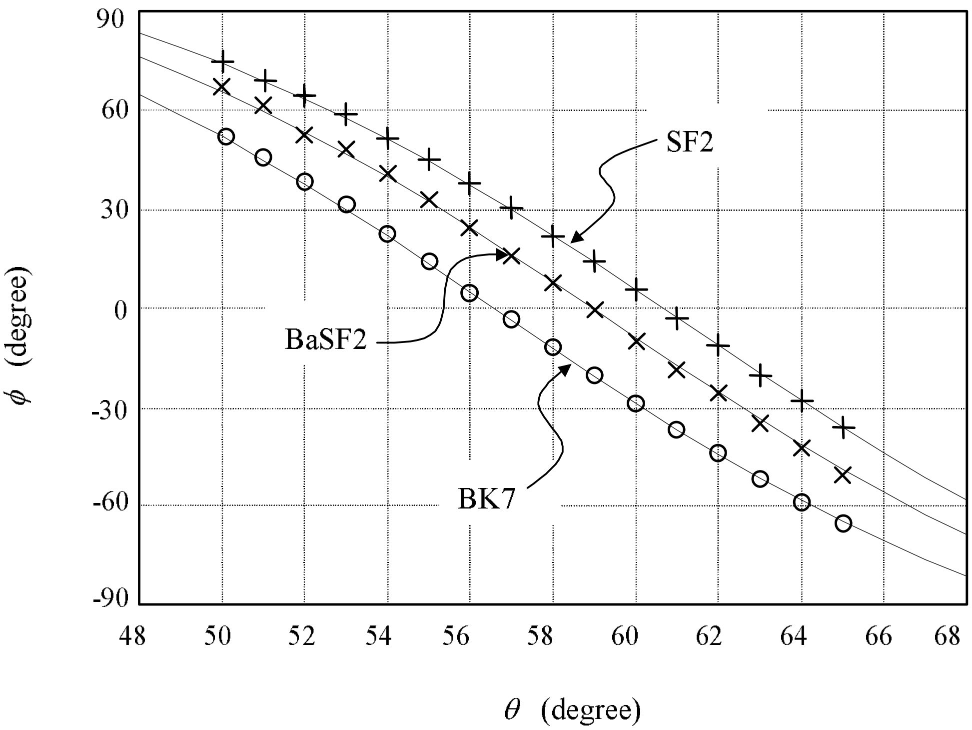

Fig. 3 Theoretical and experimental curves of ϕ versus θ for BK7, BaSF2, and SF2 as β=20°.

In order to demonstrate the feasibility of this method, the refractive indices of three glasses (BK7, BaSF2, and SF2), and two birefringent crystals (quartz and calcite) were measured. A He-Ne laser with 632.8 nm wavelength and an electro-optic modulator (PC/2; England Electro-Optics Development Ltd.) were used. The frequency of a sawtooth signal applied to the EO was 800 Hz. We used to a high-resolution rotation stage (PS-θ-90; Japan Chuo Precision Industrial Company, Ltd.) with an angular resolution 0.005° to mount and rotate the test material, and a high- resolution phasemeter with an angular resolution 0.01° to measure the phase difference. In addition, we used a personal computer to record and analyze the data. Firstly, the refractive indices of three glasses were measured. For easier operation in glasses measurement, β=20° was chosen. The theoretical and experimental curves of ϕ versus θ for these three glasses are shown in Fig 3. In this figure, the full curves represent the theoretical reference values which are obtained by introducing their reference refractive indices[17] into Eq. (20), and the symbols "O, x, and +" represent the direct readouts of division mark of rotation stage for BK7, BaSF2, and SF2, respectively. It is clear that these three curves show good correspondence. The Brewster angles of BK7, BaSF2, and SF2 were measured to be 56.574°, 58.942°, and 60.634°, their refractive indices are 1.51508, 1.66046, and 1.77717, respectively.

Next, the refractive indices of two birefringent crystals were measured. The data of the three incident angles and their corresponding phase differences are listed in Tab. 1. These simultaneous equations are solved with 2-D Newton's method and mathematics software "MATHEMATICA". And two groups of solutions are calculated and summarized in Tab. 2. The right column represents the judged results according to the above approaches, and marks O and × mean the groups of solutions are correct and incorrect. The measured data of (n_e, n_o) and their averages for calcite and quartz are listed at the first two rows in Tab. 3 and Tab. 4, respectively. α=90° exist in testing these two crystals.

Material	θ_1	θ_2	θ_3	ϕ_1	ϕ_2	ϕ_3
Calcite	55°	60°	65°	24.52°	-6.85°	-25.85°
Quartz	55°	60°	65°	17.46°	-24.40°	-62.56°

Table 1 Experimental conditions and measurement results.

Material	α	(n_e, n_o)			Justification
		(ϕ_1, ϕ_2)	(ϕ_2, ϕ_3)	(ϕ_3, ϕ_1)	
Calcite	0°	(1.6695,1.5453)	(0.5041,1.0007)	(580.71,-22.545)	×
	90°	(1.4333,1.6233)	(1.4267,1.6144)	(1.4333,1.6233)	O
Quartz	0°	(1.5522,1.5627)	(1.5128,1.4638)	(1.5293,1.5132)	×
	90°	(1.5552,1.5449)	(1.5560,1.5243)	(1.5647,1.5195)	O

Table 2 Calculated solutions and judged results.

	(ϕ_1, ϕ_2)	(ϕ_2, ϕ_3)	(ϕ_3, ϕ_1)	Average
n_e	1.4333	1.4267	1.4333	1.4311
n_o	1.6233	1.6144	1.6233	1.6203
$\lvert\Delta n_e\rvert$	9.977×10^{-4}	1.196×10^{-3}	6.178×10^{-4}	9.371×10^{-4}
$\lvert\Delta n_o\rvert$	1.947×10^{-4}	3.69×10^{-4}	2.248×10^{-4}	2.628×10^{-4}
$\lvert\Delta\alpha\rvert$	0.0043°	0.0076°	0.0043°	0.0162°

Reference values from Ref. 19: (n_e, n_o) are (1.4852,1.6559) at 627.8nm.

Table 3 Estimated results and their average for calcite.

	(ϕ_1, ϕ_2)	(ϕ_2, ϕ_3)	(ϕ_3, ϕ_1)	Average
n_e	1.5552	1.5560	1.5647	1.5586
n_o	1.5449	1.5243	1.5195	1.5295
$\lvert\Delta n_e\rvert$	1.626×10^{-3}	1.9763×10^{-3}	1.046×10^{-3}	1.549×10^{-3}
$\lvert\Delta n_o\rvert$	2.14×10^{-4}	5.90×10^{-4}	2.18×10^{-4}	3.406×10^{-4}
$\lvert\Delta\alpha\rvert$	0.1454°	0.0243°	0.0373°	0.069°

Reference values from Ref. 20: (n_e, n_o) are (1.5518, 1.5428) at 627.8nm.

Table 4 Estimated results and their average for quartz.

4. DISCUSSION

At first, measured resolutions of three glasses are discussed. From Eq. (20), we have

$$\Delta\theta \cong \left|\sin^3\theta_{\rm B}\cdot\cos\theta_{\rm B}\cdot\tan\beta\right|\times\Delta\phi\,, \tag{21}$$

where $\Delta\theta$ and $\Delta\phi$ are the errors in the incident angle and the phase meter, respectively. Our experimental conditions are substituted into Eq. (21), we can get $\Delta\theta \cong 0.001°$. Substituting the data of $\Delta\theta$ and θ_B into the equation

$$\Delta n = \sec^2\theta_B\cdot\Delta\theta\,, \tag{22}$$

the measured resolutions of BK7, BaSF2, and SF2 are 5.7×10^{-5}, 6.5×10^{-5}, and 7.2×10^{-5}, respectively.
Secondly, we discuss the measured resolutions of two birefringent crystals. From Eq. (7), we get

$$\left|\Delta\alpha\right| = \frac{1}{\left|{d\phi}/{d\alpha}\right|}\left|\Delta\phi\right|, \tag{23}$$

$$\left|\Delta\phi_1\right| = \frac{\partial\phi_1}{\partial n_e}\left|\Delta n_e\right| + \frac{\partial\phi_1}{\partial n_o}\left|\Delta n_o\right|, \tag{24}$$

and

$$\left|\Delta\phi_2\right| = \frac{\partial\phi_2}{\partial n_e}\left|\Delta n_e\right| + \frac{\partial\phi_2}{\partial n_o}\left|\Delta n_o\right|. \tag{25}$$

Eqs. (24) and (25) can be rewritten as

$$\left|\Delta n_e\right| = \frac{\left|\frac{\partial\phi_2}{\partial n_o}\right|\left|\Delta\phi_1\right| + \left|\frac{\partial\phi_1}{\partial n_o}\right|\left|\Delta\phi_2\right|}{\left|\frac{\partial\phi_1}{\partial n_e}\frac{\partial\phi_2}{\partial n_o} - \frac{\partial\phi_2}{\partial n_e}\frac{\partial\phi_1}{\partial n_o}\right|}, \tag{26}$$

and

$$\left|\Delta n_o\right| = \frac{\left|\frac{\partial\phi_1}{\partial n_e}\right|\left|\Delta\phi_1\right| + \left|\frac{\partial\phi_2}{\partial n_e}\right|\left|\Delta\phi_2\right|}{\left|\frac{\partial\phi_1}{\partial n_e}\frac{\partial\phi_2}{\partial n_o} - \frac{\partial\phi_2}{\partial n_e}\frac{\partial\phi_1}{\partial n_o}\right|}, \tag{27}$$

where $\Delta\alpha$, $\Delta n_{\rm e}$, and $\Delta n_{\rm o}$ are the errors in α, $n_{\rm e}$, and $n_{\rm o}$, and $\Delta\phi_{\rm i}$ and $\Delta\phi_{\rm j}$ are the errors in the phase differences at two different incident angles $\theta_{\rm i}$ and $\theta_{\rm j}$, respectively. Either i or j is an integer between 1 and 3, and i $\neq$ j. Considering the angular resolution of the phase meter, the second-harmonic error, and the polarization- mixing error, $\Delta\phi=\Delta\phi_{\rm i}=\Delta\phi_{\rm j}\cong0.03°$ can be estimated in our experiments[18]. Substituting this data and the experimental conditions into Eqs. (23), (26), and (27), the corresponding data of $\Delta\alpha$, $\Delta n_{\rm e}$, and $\Delta n_{\rm o}$ of three sets of simultaneous equations and their averages are calculated and listed at the last three rows in Tab. 3 and Tab. 4, respectively.

5. CONCLUSION

A novel method for determining the refractive indices of an optical material is presented with a common-path heterodyne interferometric technique and Fresnel equations. It has no drawbacks of the conventional methods. Besides, it has both merits of a common-path interferometer and a heterodyne interferometer. So, it has merits, such as, simple setup, high stability, easier operation and high-resolution.

5. ACKNOWLEDGEMENT

This study was supported in part by the National Science Council, Taiwan, under contract NSC-92-2215-E-009–052.

REFERENCES

1. X. Wang and J. Yao, "Transmitted and tuning characteristics of birefringent filters," Appl. Opt. 31, 4505-4508 (1992).
2. J. F. Valley, J.W. Wu, and C. L. Valencia, "Heterodyne measurement of poling transient effects in electro-optic polymer thin films," Appl. Phys. Lett. 57, 1084-1086 (1990).
3. I. Moreno, J. A. Davis, K. G. D'Nelly, and D. B. Allison, "Transmission and phase measurement for polarization eigenvectors in twisted-nematic liquid crystal spatial light modulators," Opt. Eng. 37, 3048-3052 (1998).
4. R. S. Weis and T. K. Gaylord, "Magnetooptic multilayered memory structure with a birefringent superstrate: a rigorous analysis," Appl. Opt. 28, 1926-1930 (1989).
5. W. R. Hunter, "Optical constant of metals in the extreme ultraviolet I. A midified critical- angle technique for measuring the index of refraction of metals in the extreme ultraviolet," J. Appl. Phys. **34**, 15-19 (1964).
6. R. M. A. Azzam, " Simple and direct determination of complex refractive index and thickness of unsupported or embedded thin thin films by combined reflection and transmission and ellipsometry at 45 ° angle of incidence," J. Opt. Soc. Am. **73**, 1080-1082 (1983).
7. S. F. Noe, and H. E. Bennett, "Accurate null polarimetry for measuring the refractive index of transparent materials," J. Opt. Soc. Am. A **10**, 2076-2083 (1993).
8. R. P. Shukla, G. M. Perera, M. C. George, and P. Venkateswarlu, "Measurement of birefringence of optical materials using a wedged plate interferometer," Opt. Commun. 78, 7-12 (1990).
9. M. H. Chiu, C. D. Chen, and D. C. Su, "Method for determining the fast axis and phase retardation of a wave plate," J. Opt. Soc.Am. A 13, 1924-1929 (1996).
10. Y. C. Huang, C. Chou, and M. Chang, "Direct measurement of refractive indices of a linear birefringent retardation plate," Opt. Commun. 133, 11-16 (1997).
11. M. Schubert, B. Rheinlander, J. A.Woollam, B. Johs, and C. M. Herzinger, "Extension of rotating-analyzer ellipsometry to generalized ellipsomery: determination of the dielectric function tensor from uniaxial TiO_2.," J. Opt. Soc. Am. A 13, 875-883 (1996).
12. J. D. Hecht, A. Eifler, V. Riede, M. Schubert, G. Krauss, and V. Kramer, "Birefringence and reflectivity of single-crystal $CdAl_2Se_4$ by generalized ellipsometry," Phys. Rev. B 57, 7037-7042 (1998).
13. G. E. Jellison, Jr., F. A. Modine, and L. A. Boatner, "The measurement of the optical functions of uniaxial materials using two-modulator generalized ellipsometry: rutile (TiO_2)," Opt. Lett. 22, 1808-1810 (1997).
14. G. E. Jellison, Jr, and F. A. Modine, "Two-modulator generalized ellipsometry: theory," Appl. Opt. 36, 8190-8198 (1997).
15. G. E. Jellison, Jr, and F. A. Modine, "Two-modulator generalized ellipsometry: experiment and calibration," Appl. Opt. 36, 8184-8189 (1997).
16. J. Y. Lee and D. C. Su, "A method for measuring Brewster's angle by circularly polarized heterodyne interferometry," J. Opt. 29, 349-353 (1998).
17. Schott Glass Technologies, Duryea, PA BK7 517642, $BaSF_2$ 664358 and SF_2 648339 Data Sheets.
18. M. H. Chiu, J. Y. Lee, and D. C. Su, "Complex refractive-index measurement based on Fresnel's equations and the uses of heterodyne interferometry," Appl. Opt. 38, 4047-4052 (1999).
19. E. D. Palik, ed., Handbook of Optical Constants of Solids Ⅲ (Academic, New York, 1998), P. 708.
20. E. D. Palik, ed., Handbook of Optical Constants of Solids Ⅲ (Academic, New York, 1998), P. 729.