Solving Consensus Measure of Ambiguous MPDM Problems Using Vague Sets — An Application to of Risk Assessment

C-C Lo* and P Wang+
Institute of Information Management,
National Chiao Tung University, Taiwan
**cclo@faculity.nctu.edu.tw;*
+ping.wang88@msa.hinet.net

K-M Chao,
DSM Research Group
School of MIS, Coventry University, UK
k.chao@coventry.ac.uk

ABSTRACT

The traditional consensus-evaluation method determines the solution by fuzzy set, but it cannot treat the negative evidence for membership function. In this paper, we present a new method for consensus measure in the risk assessment process by relaxing many assumptions on existing hesitation situations. A new similarity measure of vague sets is introduced. A fuzzy synthetic evaluation method is employed to attain the consensus interval of the group via the agreement matrix for Group Decision Making (GDM) problems. The proposed solution algorithm is presented and two consensus policies are given to consensus analysis of risk assessment guided by BS7799. The proposed method improves the soft consensus method proposed by Kacprzyk and Fedrizzi and analyzes the variation trend of group consensus using similarity measures of vague sets and consensus index. From numerical illustrations, the usefulness and efficiency of the proposed method has shown, particularly in a situation with vague and ill-defined data.

Index Terms— **MPDM, Vague sets, Similarity Measure, Consensus, BS7799.**

1. Introduction

This paper investigates a number of aspects of consensus reaching process for GDM problems. In many complex situations, it is difficult for an expert to make a right decision since GDM often considers many criteria and factors. A wrong decision for GDM problem often arises, due to the limitations of individual human ability. According to a number of criteria, a committee is formed to evaluate the results of works in order to exploit group wisdom.

According to Herrera (1997), there are two critical problems to solve: 1) alternative selection problem, i.e., how to select an alternative, and 2) consensus measure problem, i.e., how to achieve an acceptable or maximum consensus degree to a group of experts when they have diverging opinions.

The objective of GDM is to obtain preference of major opinions and group consensus. In [6], the consensus measure process is divided into three steps as shown in Fig. 1., and described as follows: 1) counting process, i.e., to count the individuals' opinions about preference values, 2) coincidence process, i.e., to calculate the agreement degree between two experts' opinions, and 3) computing process, i.e., to determine the consensus degree of group by aggregating previous agreement degrees for all experts. In the process of obtaining a group consensus, there arise situations of conflicts and partial agreements among the experts with respect to different evaluated objects. Hence reaching consensus is one of major goal of group decision-making problems.

In the GDM approach, the solving methods can either be classified as quantitative methods or qualitative methods depending on the nature of experts' preferences. Quantitative methods [2] include eigenvector function, utility function and Borda score, etc, which neglect the human behavior and only can be applied to the case of rating data that is definite and complete information. In this study, we develop a qualitative method, which focuses on solving the consensus measure problem under uncertainty situation using linguistic variables.

Several fuzzy methods for qualitative consensus measures have been studied [10-11, 15-16, 19]. Hererra et al. (1997) [6] proposed a linguistic-consensus measures based on fuzzy theory and defined in three levels of aggregation action. Chiclana and Herrera (1998) [6] studied the process of the consensus reaching for GDM. Kacprzyk and Fedrizzi [8-9] introduced the soft consensus concept based on the fuzzy majority and developed some models for drawing the group consensus.

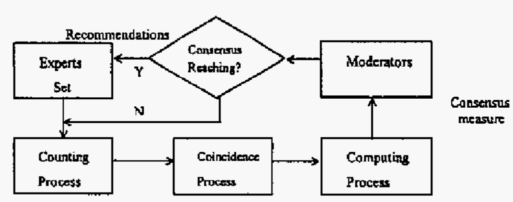

Fig. 1. The reaching process of consensus measure.

However, fuzzy sets cannot disclose the negative evidence of membership function and the hesitation degree of unknown objects [4]. In the fuzzy environments, experts often give the answer "I prefer" or "I do not prefer". They can not treat the hesitation situation. For example, the expert answers the question with "I am not sure" or "I can not justify" during the decision making process, because they did not have enough certain knowledge or historical information on unfamiliar objects. Gau and Buehrer (1993) [14] pointed out that this single value (u_A) combines the evidence $u_A \in X$ and the evidence against $u_A \in X$, without indicating their degrees.

In this paper, we present a new consensus method to improve the soft consensus method proposed by Kacprzyk and Fedrizzi [8-9] and analyze the tendency of group consensus through the use of similarity measures of vague sets and consensus index. Besides, we introduce an index of consensus to assess the consensus degree of group based on the optimism degree of expert. Finally, the proposed solution algorithm is presented and two cases of the similarity measures of vague sets for different consensus policies are given for consensus analysis.

2. Preliminary description of vague set

The vague sets, which is a generalization of the concept of a fuzzy set, has been introduced by Gau and Buehrer [14] as follows:

A vague set $A'(x)$ in X, $X=\{x_1,x_2,\ldots,x_n\}$, is characterized by the truth-membership t_A and a false-membership function f_A of the element $x_k \in X$ to $A'(x) \in X$, (k=1,2,…,n); $t_A: X \to [0,1]$ and $f_A: X \to [0,1]$, where the functions $t_A(x_k)$ and $f_A(x_k)$ are constrained by $0 \le t_A(x_k)+f_A(x_k) \le 1$, where $t_A(x_k)$ is a lower bound on the grade of membership of the evidence for x_k, $f_A(x_k)$ is a lower bound on the negation of x_k derived from the evidence against x_k. The grade of membership of x_k in the vague set A' is bounded to a subinterval $[t_A(x_k), 1-f_A(x_k)]$ of [0,1]. Fig 2. shows a vague set in the universe of discourse X.

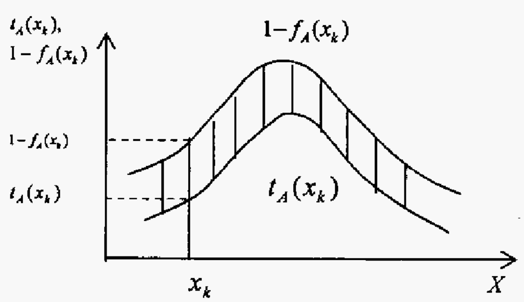

Fig. 2. A vague set.

When X is continuous, a vague set A' can be written as [13]

$$A' = \int_X [t_A(x_k), 1-f_A(x_k)]/x_k, \quad x_k \in X. \tag{1}$$

When X is discrete, a vague set A' can be written as

$$A' = \sum_{k=1}^{n} [t_A(x_k), 1-f_A(x_k)]/x_k, \quad x_k \in X. \tag{2}$$

In the sequel, we will omit the argument x_k of $t_A(x_k)$ and $f_A(x_k)$ throughout unless they are needed for clarity.

Definition 1. The intersection of two vague sets, A' and B' is a vague set C', written as $C' = A' \wedge B'$, where truth-membership function and false-membership function are t_C and f_C, respectively,

where $t_C = Min(t_A, t_B)$, and

$1-f_C = Min(1-f_A, 1-f_B)$.

That is, $[t_C, 1-f_C] = [t_A, 1-f_A] \wedge [t_B, 1-f_B] = [Min(t_A, t_B), Min(1-f_A, 1-f_B)]$.

Definition 2. The union of vague sets A' and B' is a vague set C', written as $C' = A' \vee B'$, where truth-membership function and false-membership function are t_C and f_C, respectively,

where $t_C = Max(t_A, t_B)$ and

$1-f_C = Max(1-f_A, 1-f_B)$.

That is, $[t_C, 1-f_C] = [t_A, 1-f_A] \vee [t_B, 1-f_B] = [Max(t_A, t_B), Max(1-f_A, 1-f_B)]$.

Next, let us define the similarity measures between two vague values in order to represent the agreement between experts' opinions as follows: [22]

Let $A' = [t_A(x_k), 1-f_A(x_k)]$ be a vague value, where $t_A(x_k) \in [0,1]$, $f_A(x_k) \in [0,1]$, and $0 \le t_A(x_k)+f_A(x_k) \le 1$.

Definition 3. Let A' be a vague value in X, $X=\{x_1,\ldots,x_n\}$, $A' = [t_A(x_k), 1-f_A(x_k)]$. The median value of A' is [3]

$$\varphi_A(x_k) = \frac{t_A(x_k)+1-f_A(x_k)}{2}. \tag{3}$$

Definition 4. For two vague values A' and B' in X, $X=\{x_1,\ldots,x_n\}$, $S(A',B')$ is the degree of similarity between A' and B' which preserves the properties (P1)-(P4). [3]

(P1) $0 \le S(A',B') \le 1$;

(P2) $S(A',B')=1$ if $A'=B'$; (4)

(P3) $S(A',B')=S(B',A')$.

(P4) $S(A',C') \le S(A',B')$ and $S(A',C') \le S(B',C')$ if $A' \subseteq B' \subseteq C'$, C' is a vague set in X.

3. The Proposed Method

In a GDM process, the experts have to form a committee. Each expert has to evaluate alternatives according to the well-defined criteria, and then assign performance ratings (or ranking) to the alternatives individually based on each criterion. The experts allocate ratings based on their own preferences and subjective judgments. The explicit representation of their preference and judgment with precise numerical values may not be simple, whereas the use of linguistic terms is more natural to human experts. This formulation is imprecise, ambiguous and often leads to an increasing complexity in the decision making process. In our evaluation process of group decision-making, the evaluation criteria are pre-defined. Hence GDM can be regarded as a fuzzy MPDM problem.

3.1 The problem formulation

A consensus measure of fuzzy MPDM problem can be expressed concisely in agreement matrix [2] as follows: Suppose that a decision group has m experts have to give linguistic ratings on q evaluated targets,

$$A(t_i)=\begin{bmatrix} 1 & \tilde{a}_{12} & \cdots & \tilde{a}_{1m} \\ \tilde{a}_{21} & 1 & \cdots & \tilde{a}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & 1 \end{bmatrix}, \quad (5)$$

$W=[w_1 \;\; w_2 \;\; \cdots \;\; w_m]$, and $\sum_{i=1}^{m} w_i = 1$,

where A is an agreement matrix of the group, $e_1, e_2,\ldots, e_m$ are a finite set of experts, $t_1, t_2,\ldots, t_q$ are possible evaluated targets from which experts have to select, $\tilde{a}_{ii'}$ ($i,i' = 1,\ldots,m$) is the agreement degree between the opinion of expert e_i and expert $e_{i'}$ which can be calculated by similarity measure of two fuzzy opinions; and w_i is the importance weights of expert e_i.

3.2 Similarity measures

We present three similarity measures between two vague sets which may be continuous or discrete form as follows.

According to Def. 3, we use the median value of A' and B' to represent the mean of truth-membership and false-membership function. The agreement between two experts can be represented by the proportion of the consistent area to the total area [11].

Definition 5. Using median of vague value, $S^m(A',B')$ is defined as the similarity measure between two vague values

$$S^m(A',B')=\frac{\int\{\varphi_A(x)\wedge\varphi_B(x)\}dx}{\int\{\varphi_A(x)\vee\varphi_B(x)\}dx}=\frac{\int\min\{\varphi_A(x),\varphi_B(x)\}dx}{\int\max\{\varphi_A(x),\varphi_B(x)\}dx}. \quad (6)$$

3.3 Solution process

In the following, we apply the new similarity measures of vague sets to compute group consensus degree based on the consensus reaching process defined by Herrera [5] as follows.

Suppose that there exist a set of experts $E=\{e_1,\ldots e_m\}$ and a finite set of evaluated targets. $T=\{t_1,\ldots t_q\}$. Let X be the universe of discourse, $X=\{x_1,\ldots,x_n\}$. Each expert $e_i \in E$ provides his/her opinion on an evaluated target by linguistic terms which can be transformed into a vague set.

A. *Counting process*

We calculate the agreement degree of two experts' opinions expressed by Eq.(6) and denote $S^m(i,i')$ as $a_{ii'}$, $i,i'=1,\ldots,m$, where two vague sets i, i' represents the linguistic opinion of expert $e_i, e_{i'}$. The agreement matrix A for evaluated targets $t_1 .. t_q$ is

$$A(t_1)=\begin{bmatrix} 1 & \tilde{a}_{12} & \cdots & \tilde{a}_{1m} \\ \tilde{a}_{21} & 1 & \cdots & \tilde{a}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & 1 \end{bmatrix},\ldots A(t_q)=\begin{bmatrix} 1 & \tilde{a}_{12} & \cdots & \tilde{a}_{1m} \\ \tilde{a}_{21} & 1 & \cdots & \tilde{a}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & 1 \end{bmatrix}. \quad (7)$$

Remark. For $a_{ii'} = S^m(i,i')$ if $i \neq i'$, and $a_{ii}=1$ if $i=i'$; It means that if two experts fully agree on an evaluated target, and they have $\tilde{a}_{ii'}=1$; it implies: $t_i(x)=t_{i'}(x)$, $1-f_i(x)=1-f_{i'}(x)$. By contrast, if they have completely different estimations, then $\tilde{a}_{ii'}=0$ is true.

B. *Coincidence process*

Once all the agreement vectors are measured, we then aggregate those pairs of agreement vectors based on two distinct consensus policies—average consensus policy and strict consensus policy [5] to derive the consensus of the group as Case 1 and Case 2.

Case 1: Average consensus policy:

By applying simple additive aggregation rule, we have the average consensus of all the experts on an evaluated target as

$$C(t_j)=\frac{2}{m(m-1)}\sum_{i=1}^{m-1}\sum_{i'=i+1}^{m} a_{ii'}(t_j). \quad (8)$$

Case 2: Strict consensus policy:

By applying soft consensus formula [8], we measure the soft consensus of group as follows: If the proposition of group consensus is defined as, "<u>most of important experts have many relevant opinions and have agreed on the evaluated targets</u>", where Q_1="many", Q_2="most", E="important".

So the truth of the proposition "expert i and i' agree to respect Q_1 (many) linguistic quantifier [10] on target t_j, can be calculated as

$$a_{ii'}^{Q1}(t_j) = u_{Q_1}(\tilde{a}_{ii'}(t_j)), \quad (9)$$

where $Q_1(r) = \begin{cases} 1 & r > 0.7 \\ 2r - 0.4 & 0.2 \le r \le 0.7 \\ 0 & r \le 0.2 \end{cases}$.

If the different importance of the individuals is considered, then a fuzzy set E can be defined as $\forall i \in n$, $w(i) \in [0,1]$ a weighting of an expert i. The average importance of two experts i and i' can be formulated as

$$w_{ii'}^E = (w(i) + w(i'))/2. \quad (10)$$

A measure of consensus based on the importance of the individuals can be defined as

$$C_{Q_1\backslash E}(t_j) = \sum_{i'=i+1}^{m}(a_{ii'}^{Q_1}(t_j).w_{ii'}^E) / \sum_{i'=i+1}^{m} w_{ii'}^E. \quad (11)$$

Similarly, the degree of consensus of Q_2(most) pairs of individuals with respect to Q_1 (many) opinions on target t_j is given by

$$C_s = C_{Q_1\backslash E\backslash Q_2}(t_j) = u_{Q_2}(C_{Q_1\backslash E}(t_j)), \quad (12)$$

where $Q_2(r) = \begin{cases} 1 & r \ge 0.8 \\ 2r - 0.6 & 0.3 \le r \le 0.8 \\ 0 & r \le 0.3 \end{cases}$.

C. *Computing process*

In order to obtain the degree of group consensus on a specific mission, a general compensation operator proposed by Zimmermann and Zysno (1983) is adopted as the consensus operator in this work [7]. The consensus index ($C_{Q_1\backslash E\backslash Q_2}(t)$) synthesizes the agreement of the group on all evaluated targets ($t_1,\ldots, t_q$) which are a global measure of consensus and is calculated as

$$C(t) = \left(\prod_{j=1}^{q} C_j\right)^{1-r}\left(1 - \prod_{j=1}^{q}(1 - C_j)\right)^{r}. \quad (13)$$

As the compensation parameter γ is varied from 0 to 1, the operator describes the aggregation properties of "AND" and "OR", that is,

$$\max_{j=1,\ldots,q}(t_j) \ge F(t_1,\ldots,t_q) \ge \min_{j=1,\ldots,q}(t_j). \quad (14)$$

where F is an aggregation function of Eq.(13).

The compensation parameter γ indicates the degree of optimism of expert. A small γ implies the higher degree of optimism. Finally, the moderator can estimate the degree of optimism depending on average value of individual confidence level and to decide whether a group consensus has been reached using $C_{Q_1\backslash E\backslash Q_2}(t)$ and γ.

4. Illustrative Example: Risk Assessment

In this section an example for risk assessment of Internet data center (IDC) is used as a demonstration of the application of the proposed method in a realistic scenario. Four types of equipments were taken as examples in this empirical experiment: a database server (a_1), a mail server (a_2), a firewall device (a_3), and a portal web server (a_4).

A linguistic model of aggregative risk includes five important risk criteria which are excerpted from ten major control items of BS7799 Information Security Management Standard [1,11] as: c1) security policy c2) assets classification & control, c3) personnel security, c4) physical & environment security c5) communication management & access control.

The risk assessment process in this case includes two stages. In the first stage, the risk management system is reviewed and each individual expert needs to provide an evaluation of the related documents with respect to security policy, standard operation procedure (SOP), and working instruction (WI) for information security management system (ISMS). In the second stage, an examination takes place for assessing the operation consistency with the related documents for each information asset. Finally, all of the risk ratings from the experts are aggregated in order to obtain an aggregative risk for each information asset.

Step 1: Suppose that an assessment committee consisting of a set of six experts, $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$, have to evaluate the risk of a set of information assets of IDC, $T = \{t_1, t_2, t_3, t_4\}$, including a database server (t_1), a mail server (t_2), an application server (t_3), and a web server (t_4) according to the five risk criteria (c1~ c5).

Step 2: Let a vague set A' in X= {VL, L, M, H, VH} present a set of linguistic variables of access control items, which are shown in Table 1.

Table 1. Linguistic variables for the risk criteria

Very Low (VL)	$[t_A(x), 1 - f_A(x)]/1$
Low (L)	$[t_A(x), 1 - f_A(x)]/2$
Medium (M)	$[t_A(x), 1 - f_A(x)]/3$
High (H)	$[t_A(x), 1 - f_A(x)]/4$
Very High (VH)	$[t_A(x), 1 - f_A(x)]/5$

Step 3: For evaluated target t_1, we calculate the preference agreement vectors between d_1, d_2 using Eq.(6) as

$$a_{12} = \frac{\int_2^3[\min\{t_{11},t_{21}\},\min\{1-f_{11},1-f_{21}\}]dx}{\int_2^3[\max\{t_{11},t_{21}\},\max\{1-f_{11},1-f_{21}\}]dx} = \frac{\int_2^3[0.5,0.8]dx}{\int_2^3[0.6,0.9]dx} = \frac{\int_2^3 0.65dx}{\int_2^3 0.75dx} = \frac{0.65}{0.75} = 0.867.$$

Following the same procedure, we can obtain the others elements $a_{13}, a_{14}, \ldots, a_{65}$ for targets t_1, t_2, t_3 and t_4.

Step 4: Construct the preference-agreement matrixes on $c1$ for all targets as

$$A(t_1)=\begin{bmatrix}1.00 & 0.87 & 0.93 & 0.00 & 0.87 & 0.86\\ 0.87 & 1.00 & 0.80 & 0.00 & 0.94 & 0.86\\ 0.93 & 0.80 & 1.00 & 0.00 & 0.80 & 0.92\\ 0.00 & 0.00 & 0.00 & 1.00 & 0.00 & 0.00\\ 0.87 & 1.00 & 0.80 & 0.00 & 1.00 & 0.86\\ 0.86 & 0.86 & 0.92 & 0.00 & 0.86 & 1.00\end{bmatrix}\quad A(t_2)=\begin{bmatrix}1.00 & 0.00 & 0.93 & 0.73 & 0.87 & 1.00\\ 0.00 & 1.00 & 0.00 & 0.00 & 0.00 & 0.00\\ 0.93 & 0.00 & 1.00 & 0.79 & 0.93 & 0.93\\ 0.73 & 0.00 & 0.79 & 1.00 & 0.85 & 0.73\\ 0.87 & 0.00 & 0.93 & 0.85 & 1.00 & 0.87\\ 1.00 & 0.00 & 0.93 & 0.73 & 0.87 & 1.00\end{bmatrix}$$

$$A(t_3)=\begin{bmatrix}1.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00\\ 0.00 & 1.00 & 0.92 & 0.73 & 0.80 & 0.73\\ 0.00 & 0.92 & 1.00 & 0.80 & 0.87 & 0.80\\ 0.00 & 0.73 & 0.80 & 1.00 & 0.81 & 1.00\\ 0.00 & 0.80 & 0.87 & 0.81 & 1.00 & 0.81\\ 0.00 & 0.73 & 0.80 & 1.00 & 0.81 & 1.00\end{bmatrix}\quad A(t_4)=\begin{bmatrix}1.00 & 0.85 & 0.92 & 0.92 & 0.79 & 0.79\\ 0.85 & 1.00 & 0.92 & 0.92 & 0.80 & 0.93\\ 0.92 & 0.92 & 1.00 & 0.85 & 0.86 & 0.86\\ 0.92 & 0.92 & 0.85 & 1.00 & 0.73 & 0.86\\ 0.79 & 0.80 & 0.86 & 0.73 & 1.00 & 0.87\\ 0.79 & 0.93 & 0.86 & 0.86 & 0.87 & 1.00\end{bmatrix}$$

Similarly, c2~c5 of the preference-agreement matrixes of all targets are also constructed.

Step 5: Aggregate the preference-agreement vectors for targets t_1, t_2, t_3 and t_4 to obtain the average group preference using Eq.(8) as

Case 1: Average consensus of group:

	t_1	t_2	t_3	t_4
C_{avg}	0.474	0.552	0.491	0.642

Case 2: Soft consensus of group:

Assume we have priori information about the importance degree of six experts from work experiences, $W=[0.15,0.2,0.10,0.15,0.15,0.25]$. From Eq.(10), we have

$$W_E=\begin{bmatrix}0.150 & 0.175 & 0.125 & 0.150 & 0.150 & 0.200\\ 0.175 & 0.200 & 0.150 & 0.175 & 0.175 & 0.225\\ 0.125 & 0.150 & 0.100 & 0.125 & 0.125 & 0.175\\ 0.150 & 0.175 & 0.125 & 0.150 & 0.150 & 0.200\\ 0.150 & 0.175 & 0.125 & 0.150 & 0.150 & 0.200\\ 0.200 & 0.225 & 0.175 & 0.200 & 0.200 & 0.250\end{bmatrix}.$$

Then, we aggregate the agreement vectors to obtain the soft consensus of group using Eq.(11)~ Eq.(12) as

	t_1	t_2	t_3	t_4
$C_{Q_1/E/Q_2}(t_j)$	0.615	0.742	0.660	0.763

The consensus solution $C_{Q_1\backslash E\backslash Q_2}$ satisfies the proposition that "most" of the important experts have "many" similar relevant opinions and agreed on evaluated targets." Clearly, the highest consensus set is $t^{Q_1/E/Q_2}=\{t_4\}$ and the consensus ranking of the evaluated targets is $t_4>t_2>t_3>t_1$.

Step 6: Calculate the group-preference index on all targets for $\gamma=0$, $\gamma=0.5$, $\gamma=1$, respectively

	$\gamma=0$	$\gamma=0.5$	$\gamma=1$
$C(t)$	0.300	0.546	0.996

Obviously, the consensus interval of group is [0.30, 0.983], we will analyze the deviation tendency of consensus interval in the Sec. 5.

Step 7: The moderator takes the mean value of three different levels of confidences: low, moderate, and high, $C(t)=0.614$ to judge that group preferences have been reached due to the fact $C(t)=0.614\geq 0.5$.

Step 8: If a group has reached a consensus over the preferences, then the alternative selection procedures can be executed. If not, it goes back to step 1.

5. Discussions

Without any comparison of the proposed method with other well-established methods, the resulting decision may be questionable.

A. *Methods Comparison*

In this section, we will compare the distance-based similarity measure of intuitionistic fuzzy sets (IFS), which developed by Szmidt and Kacprzyk [4], to treat the same problem. It has been proven that IFS is equivalent to vague sets. The more detailed information can be found in Fuzzy sets and systems, Vol. 79, pp.403-405, 1996. The computational procedure of similarity of vague sets is applied to calculate the agreement of experts through the use of metric distance between two vague sets.

Let A' and B' be two vague sets, the similarity measure $M_H(A',B')$ between the vague values A' and B' is [4]

$$M_H(A',B')=1-\frac{|t_A-t_B|+|f_A-f_B|}{2}. \tag{15}$$

Furthermore, the similarity measure $S_H(A',B')$ between the vague sets A' and B' is given by

$$S_H(A',B')=\frac{1}{n}\sum_{k=1}^{n}1-\frac{|t_A(x_k)-t_B(x_k)|+|f_A(x_k)-f_B(x_k)|}{2}. \tag{16}$$

Similarly, the agreement matrix of all experts based on the similarity measure of the vague sets can be expressed as Eq. (6). Using Eq. (8), the average consensus of a group on an evaluated target, respectively is as follows.

	t_1	t_2	t_3	t_4
C_{avg}	0.611	0.723	0.635	0.742

The soft consensus of group on an evaluated target can be obtained using Eqs. (9) ~ (12) as

	t_1	t_2	t_3	t_4
$C_{Q_1\backslash E\backslash Q_2}$	0.747	0.827	0.805	0.872

Obviously, the highest consensus evaluated target is $\{t_4\}$, and the consensus ranking of the evaluated targets is $t_4>t_2>t_3>t_1$. The solutions of two methods using two different similarity measure of vague sets are the same. Notably, part of the consensus ranking may be changed when the distinct consensus policy is selected or a different weighting (importance) of the experts is used.

B. *Consensus Interval*

In order to identify the consensus interval of the group, we discuss the solution of Case 1 and Case 2 in details as follows: The partial results of two consensus measures (i.e., t_1 and t_3) is less than 0.7. Obviously, the results might not be accepted by the moderator, if the moderator sets the threshold degree of consensus, as C_t=0.70. In the following, there are eight discussions using the Delphi procedure to attain group consensus as shown in Fig. 3. At the initial step, the soft consensus for evaluated targets is (0.615, 0.742, 0.660, 0.763) respectively. After 8 interactive discussions within a group, the average consensus for the evaluated targets is (0.750, 0.827, 0.701, 0.886), which satisfies the threshold degree of consensus. From Fig. 3, we find that the consensus degree of group increases as the experts adjust their risk ratings accordingly. In addition, Fig. 4 shows that a consensus of the group can be reached via a dynamic and iterative process through the exchange of information and rational arguments.

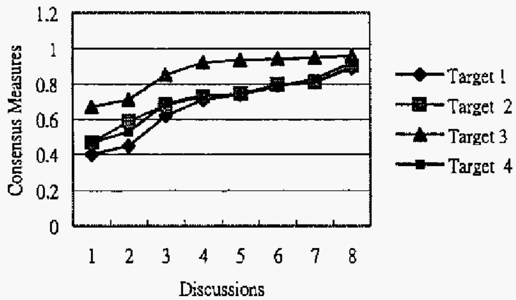

Fig 3. The consensus reaching process.

Next, let γ=0.0, 0.5, 1.0, we obtain the dynamic feature of consensus interval of Case 2, as shown in Fig. 4. From Fig. 4, the consensus interval of the group slowly converges in an acceptable interval [0.47, 0.998] after 8 discussions. Clearly, our method can reveal the decreasing tendency of uncertainty associated with experts' subject judgements. By contrast, the traditional consensus methods [8,9,14,17] are neither can illustrate the confidence level of experts' altitude on risk assessment nor can reveal the variation trend of group consensus.

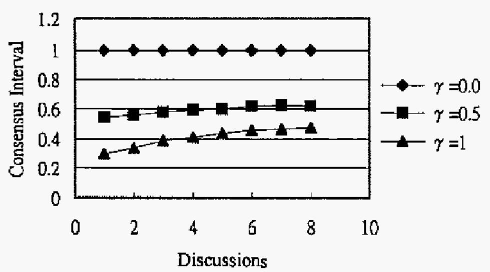

Fig. 4. The dynamic feature of consensus interval.

6.Conclusions

This paper presents a new fuzzy approach to solve consensus measure problems of GDM. Since information security risk itself contains certain degrees of ambiguity, the authors use similarity measures of vague sets to derive a group consensus degree for risk assessment. Consequently, the proposed approach can not only effectively improve the soft consensus approach proposed by Kacprzyk and Fedrizzi, but also can reveal the variation tendency of consensus reaching process. By examples verification, the usefulness and effectiveness of proposed method has been demonstrated.

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