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Spatio-temporal solitary pulses in graded-index materials with Kerr nonlinearity

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Abstract

We analyze spatio-temporal pulse propagation in graded-index Kerr materials with the variational method, and find that if the path of propagation provides negative group-velocity dispersion, stable solitary pulses exist for pulse energy smaller than a critical value. Spatial and temporal degrees of freedom cannot be analyzed separately due to their coupling through the Kerr nonlinarity. Our unified analysis not only elucidates the hidden coupling, but also clarifies relations between parameters of the solitary pulse. The analysis is verified by direct numerical simulation of the paraxial wave equation.

It is well known that optical Kerr nonlinearity gives rise to interesting soliton phenomena. In the space domain the combined effects of self-focusing and diffraction in two-dimensional media produce spatial solitons [1]. In the time domain the combined effects of self-phase modulation and group-velocity dispersion (GVD) produce temporal solitons [1,2]. An interesting question is whether there exist solitons in both space and time domains simultaneously, and if yes, what are the characteristics of them. Such solitons look like light bullets with its energy localized in both space and time domains, and may be called spatio-temporal solitons. If the Kerr medium is two-dimensional, the answer to the above question is simply negative because the wave equation is equivalent to self-focusing in three-dimensional Kerr media, which is known to have no stable stationary solutions [3]. Light bullets in three-dimensional Kerr media have also been studied recently [4]. They are also found unstable. The study reveals that the optical power large enough to make the beam self-guided is also large enough to cause wavefront instability.

Recently, cw beam propagation in three-dimensional graded-index (GRIN) Kerr materials has been studied by several authors [5,6]. It is found that there exist stationary solutions when the optical power is less than a critical value. This is because GRIN helps counteract diffraction, so that the beam profile can maintain stationary below the critical power. However, it is still not clear whether GRIN Kerr materials can support stationary, or quasi-stationary propagation of optical pulses, because unlike a cw beam, in which the optical power is constant, in an optical pulse the instantaneous power varies greatly from the pulse peak to the pulse wings.

In this letter we investigate the propagation of spatio-temporal pulses through GRIN Kerr materials with the variational method. We find that stable spatio-temporal solitary pulses exist if the path of propagation provides negative GVD and the pulse

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energy is less than a critical value.

A GRIN material is one with a parabolic refractive index.

$$n(x,y,z,\omega) = n(\omega)\left(1 - \frac{G(\omega)}{2}(x^2+y^2)\right). \quad (1)$$

The propagation of an optical pulse on the axis of a GRIN waveguide is described by the following paraxial wave equation:

$$\mathrm{i}\frac{\partial u}{\partial z} = \frac{1}{2k_o}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{k_o''}{2}\frac{\partial^2 u}{\partial t^2} - \frac{k_o G_o}{2}(x^2+y^2)u + \frac{n_2 k_o}{n_o}|u|^2 u. \quad (2)$$

Here $u(x,y,z,t)$ is the pulse envelope, k_o is the propagation constant in the center of the GRIN material at the carrier frequency ω_o, $n_o = n(\omega_o)$, $G_o = G(\omega_o)$, and n_2 is the nonlinear refractive index.

To simplify notations, we introduce the following normalization units: (1) distance z: $2/\sqrt{G_o}$; (2) spatial dimensions x and y: $[k_o^2 G_o]^{-1/4}$; (3) time t: $\sqrt{2|k_o''|/\sqrt{G_o}}$; (4) intensity $|u|^2$: $n_o\sqrt{G_o}/(2n_2k_o)$. Here the dispersion parameter k_o'' has been assumed to be negative. Under these normalization units,

$$\mathrm{i}\frac{\partial u}{\partial z} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{1}{2}\frac{\partial^2 u}{\partial t^2} - (x^2+y^2)u + |u|^2 u, \quad (3)$$

$$L = \iiiint \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}t\left[\frac{\mathrm{i}}{2}\left(u\frac{\partial u^*}{\partial z} - u^*\frac{\partial u}{\partial z}\right) - |\nabla_T u|^2 - \frac{1}{2}\left|\frac{\partial u}{\partial t}\right|^2 - (x^2+y^2)|u|^2 + \frac{1}{2}|u|^4\right], \quad (4)$$

where ∇_T is the transverse gradient operator. By using the standard variational approach based on the Ritz optimization procedure [7], and by assuming that the pulse can be described by the following solution ansatz:

$$u(x,y,z,t) = A(z)\exp(\mathrm{i}\theta(z)) \times \exp\left(-\frac{x^2+y^2}{2w^2(z)}\right)\mathrm{sech}\left(\frac{t}{w_t(z)}\right), \times \exp\left(\mathrm{i}\frac{p(z)}{2}(x^2+y^2) + \mathrm{i}\frac{p_t(z)}{2}t^2\right), \quad (5)$$

we obtain the following evolution equations for the four pulse parameters w, p, w_t and p_t.

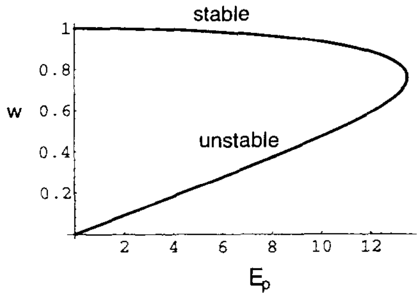

Fig. 1. Beam width w versus pulse energy E_p.

$$\frac{\mathrm{d}w}{\mathrm{d}z} = -2pw, \quad (6)$$

$$\frac{\mathrm{d}p}{\mathrm{d}z} = 2\left(p^2 - \frac{1}{w^4}\right) + 2 + \frac{E_p}{6\pi w^4 w_t}, \quad (7)$$

$$\frac{\mathrm{d}w_t}{\mathrm{d}z} = -p_t w_t, \quad (8)$$

$$\frac{\mathrm{d}p_t}{\mathrm{d}z} = \left(p_t^2 - \frac{4}{\pi^2 w_t^4}\right) + \frac{1}{\pi^3}\frac{E_p}{w^2 w_t^3}. \quad (9)$$

In the solution ansatz Eq. (5), the pulse envelope $u(x,y,z,t)$ is assumed to be separable in the x, y, t dimensions and is circularly symmetric in the x and y dimensions. The spatial beam profile is assumed to be Gaussian and the temporal profile sech. The pulse parameter p is the wavefront curvature, w is the transverse spatial beam width, p_t is the temporal chirp of the pulse, and w_t is the pulse duration. All the four parameters are functions of the propagation distance z, and $E_p = 2\pi A^2 w^2 w_t$ is the pulse energy.

Eqs. (6)-(9) have stationary solutions that are given by $p = p_t = 0$:

$$w_t = \frac{4\pi}{E_p}w^2, \quad (10)$$

$$(w^2)^3 - (w^2) + \frac{E_p^2}{48\pi^2} = 0. \quad (11)$$

Eq. (11) is a third order polynomal equation of w^2 and has meaningful solutions (positive w^2) onlu when

$$E_p \le 2^{5/2}3^{-1/4}\pi \approx 13.5034. \quad (12)$$

The stationary solutions for the normalized beam width and pulse duration are plotted in Figs. 1 and 2. There are two branches of solution. After linearizing

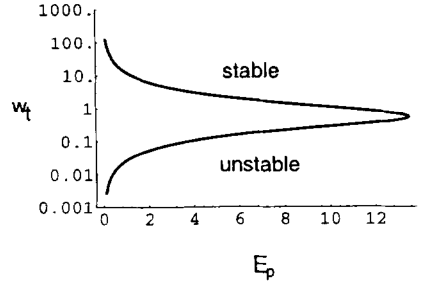

Fig. 2. Pulse duration w_t versus pulse energy E_p.

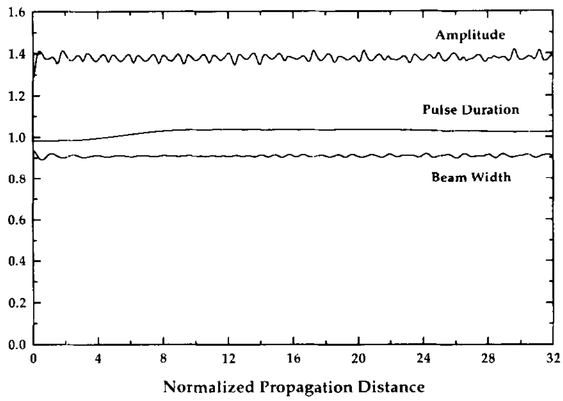

Fig. 3. Amplitude, pulse duration and beam width from numerical simulation.

Eqs. (6)–(9) near the stationary solutions and examining the eigensolutions of the linearized equations, we find that only the upper branch of the solution is stable. It should be noted that the normalized beam width and pulse duration of the solitary pulse are controlled by a single parameter, the normalized pulse energy. At the critical energy given by Eq. (12), the system is on the edge of stability and the peak power of the pulse is exactly equal to the value obtained from the cw analysis [5].

To verify the predictions from the variational method, we have performed direct numerical simulation of the paraxial wave equation (3) We take advantage of the cylindrical symmetry to reduce the computational complexity by one dimension and use finite difference beam propagation method to propagate the pulse. The solutions from the variational method are used as the initial conditions. We have calculated the peak amplitude, pulse duration and beam width for solitary pulses with the normalized pulse energy E_p equal to 2, 4, 6, 8, 10, 12. The pulse

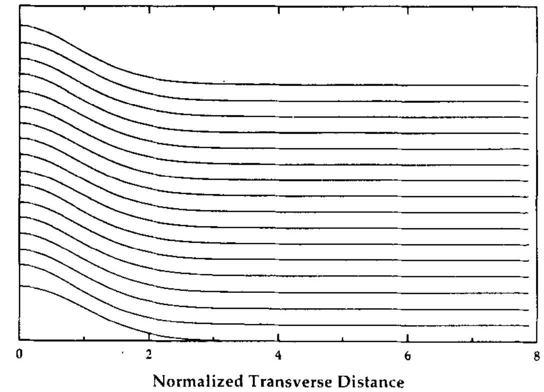

Fig. 4. Spatial pulse shapes sampled at $z = 0, 2, 4, \ldots, 32$ (from bottom to top).

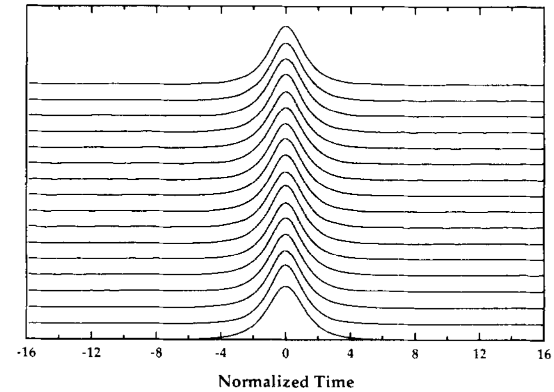

Fig. 5. Temporal pulse shapes sampled at $z = 0, 2, 4, \ldots, 32$ (from bottom to top).

parameters for $E_p = 10$ are shown in Fig. 3. The evolutions of spatial and temporal pulse shapes are shown in Figs. 4 and 5. The pulse duration plotted in Fig. 3 is defined by the second moment: $[\int t^2 |u(0,0,z,t)|^2 dt / \int |u(0,0,z,t)|^2 dt]^{1/2}$. For a sech pulse, it is equal to 0.9069 w_t. The beam width is defined in a similar way. We find that the solution near the critical energy is sensitive to the initial conditions. To get quasi-stationary solutions shown in Fig. 3, the actual initial conditions we use are $A = 1.280$, $w = 0.9329$ and $w_t = 1.083$, of which A and w_t are 1% smaller than the variational solution. For one-dimensional solitons with a pulse duration equal to 1, one normalized distance unit is equal to $2/\pi$ soliton period. Our simulation is carried out up to 32 normalized distance units, which should be long enough to verify the existence of solitary pulses with a pulse duration around 1. The accuracy of our numerical simulation is checked by monitoring the pulse energy and the error is found to be of the order of 10^{-5}. From Fig. 3, it can be seen that the solution is stable and behaves almost like real solitons. From Fig. 4 and 5, it can be seen that the pulse shapes during propagation stay close to our solution ansatz. The small fluctuations of the pulse parameters in Fig. 3 indicate that the initial pulse parameters and pulse shapes we assume are close to, but not exactly the real stationary solution. The small mismatch causes the solution to oscillate around the stationary solution. If we do not make the 1% adjustment of the initial A and w_t, the solution is still stable but the pulse parameters will oscillate with a somewhat larger amplitude. For smaller pulse energies $E_p \leq 8$, the discrepancy between the variational and numerical solution is even smaller. However, for a larger pulse energy $E_p = 12$, the solution seems to be unstable. This is because when the pulse energy is close to the critical energy, the solution ansatz Eq. (5) may not be accurate enough. From our calculation, we numerically prove the existence of solitary pulses in GRIN Kerr materials and find that our variational method accurately (within 1%) predicts the pulse parameters of solitary pulses for $E_p \leq 10$. We have also tried the Gaussian temporal profile in the solution ansatz. The discrepancy between the variational and numerical solutions will be much larger even when the pulse energy is small. This indicates that the choice of the solution ansatz is very crucial in the variational method in order to achieve quantitative accuracy.

To generate spatial-temporal solitary pulses experimentally, the group-velocity dispersion (GVD) has to be negative. One possibility is to work in the long wavelength regime, where it is easier to find materials that have negative GVD. Another possibility is by off-axially propagating the optical pulse along a helix or side-winding trajectory in GRIN materials [8,9]. Negative dispersion comes from the spatial dispersion which accompanies the winding optical path. In this way, one should be able to generate solitary pulses at a wide range of wavelengths [10]. As a numerical example, assuming GVD in units of square seconds per meter $D = -2 \times 10^{-26} s^2/m$, a typical value that can be obtained in commercial GRIN materials by i.e., a SML-W2.0 (Selfoc Micro Lens-W type of 2.0 mm diameter from NSG America Inc.), at the 630 nm wavelength, stable spatio-temporal solitary pulses exist for pulse energy smaller than 15 nJ. For a pulse energy of 1 nJ, the full-width-half-maximum (fwhm) beam width is 23.7 μm and the fwhm pulse duration is 281 fs. Longer pulse duration can be achieved by reducing the pulse energy. The numbers above indicate that it is not difficult to generate such solitary pulses with popular femtosecond lasers.

In conclusion, using both a variational approach and direct numerical simulation, we have investigated the propagation of spatio-temporal pulses in GRIN materials with Kerr nonlinearity. We found that stable solitary pulses exist when GVD is negative and pulse energy less than a critical value. The variational approach can accurately predict the pulse parameters when the pulse energy is not very close to the critical energy. In wavelength regions where material dispersion is positive, the required negative GVD can be produced by off-axial propagation. The characteristics of solitary pulses, i.e., the normalized beam width and pulse duration, are controlled by a single parameter, the normalized pulse energy. Our analysis points out a simple way to produce spatio-temporal solitary pulses in a wide wavelength range.

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