Hologram enhancement in photorefractive media

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Abstract. We consider the possibility of restoration and/or enhancement of decaying holograms in photorefractive media by using a simple optical readout in conjunction with a phase conjugator. The results indicate that extremely weak holograms can be enhanced provided that the two-beam coupling is sufficiently strong. Steady-state photorefractive holograms can be maintained continuously without decay by using a properly designed readout scheme. The result also provides an explanation for the formation of mutually pumped phase conjugation in terms of the amplification of initial noise gratings.

Subject terms: photorefractive nonlinear optics; holography; phase conjugation.

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1 Introduction

It is well known that volume index gratings and holograms can be recorded by using optical interferometric techniques in photorefractive media.[1] These index gratings and holograms can also be erased by illumination. The dynamic nature of these index gratings and holograms offers unique capability in many advanced applications, including real-time image processing, optical phase conjugation, reconfigurable interconnection, optical neural networks, etc. In many of the applications, several holograms must be recorded sequentially in a photorefractive medium. As a result of the optical erasure, the amplitudes of the previously recorded holograms may decay exponentially during the subsequent recording stages.[2] There have been proposals for the equalization of the amplitude of holograms by using a properly designed exposure schedule and even the sustainment of decaying holograms by using rerecording schemes.[3–7] In this paper, we propose and analyze a new optical method for the enhancement and restoration of decaying holograms in photorefractive media.

Referring to Fig. 1, consider the readout of a photoinduced volume index grating or hologram in a photorefractive medium by using a laser beam. A diffracted beam bearing the image information is produced provided the readout beam is incident along the Bragg angle. As a result of the photorefractive effect, the diffracted beam and the readout beam will jointly induce a new index grating or hologram which bears exactly the same information as the existing one. The photorefractive medium is oriented so that the photoinduced index grating or hologram produced by the simultaneous presence of the readout beam and the diffracted beam is in phase with the existing grating or hologram. In view of the two-wave mixing,[8] such an orientation will lead to a gain for the diffracted beam. In other words, the initially diffracted beam will be amplified by the readout beam via two-wave mixing for a short period of time. During the readout process, the existing hologram is being erased exponentially. On the other hand, the new hologram formed by the diffracted beam and the readout beam jointly is growing exponentially. The newly formed hologram is in phase with the existing one and is thus reinforcing the amplitude of the grating for a short period of time. This mechanism itself is responsible for the transient enhancement of the hologram.[9] The transient increase in the amplitude of the hologram manifests itself in an increase in the diffraction efficiency for a short period of time. Continued reading of the hologram by a single readout beam for a long period of time leads eventually to a decay of the hologram.

In what follows, we first analyze the temporal growth and the spatial variation of the hologram in the bulk of a photorefractive medium. Then we consider an optical method that utilizes the transient gain of two-wave mixing to restore a decaying hologram and thus achieve a steady-state enhancement of the hologram. For simplicity, we first consider the case of a single photoinduced volume index grating in a photorefractive medium.

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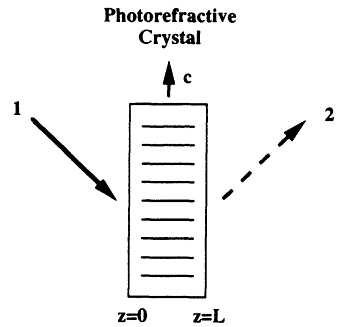

Fig. 1 Schematic diagram of a single-beam readout of a photorefractive grating.

2 Transient Enhancement of Photorefractive Gratings

Consider a volume index grating inside a photorefractive medium with grating wave vector **K**. A plane wave with amplitude A is incident on the photorefractive grating along a direction that exactly satisfies the Bragg condition (see Fig. 1). As a result of the Bragg scattering, a diffracted plane wave with amplitude B is generated. The electric field of the two waves is written as

$$E=A\ \exp[i(\omega t-\mathbf{k}_1\cdot\mathbf{r})]+B\ \exp[i(\omega t-\mathbf{k}_2\cdot\mathbf{r})]\ , \tag{1}$$

where ω is the angular frequency, and $\mathbf{k}_1$ and $\mathbf{k}_2$ are the wave vectors, which satisfy the conservation of momentum (Bragg condition)

$$\mathbf{K}=\mathbf{k}_2-\mathbf{k}_1\ . \tag{2}$$

The spatiotemporal equations describing the growth and decay of a photoinduced index grating[10,11] can be derived from the wave equation and the Kukhtarev equations.[1,12] In the following discussion we assume that the photorefractive time constant is much longer than the time of flight needed for a light beam to propagate through the thickness of the crystal (i.e., $\tau \gg L/c$, where τ is the photorefractive time constant, L is the thickness of the crystal, and c is the speed of light in vacuum) and that diffusion is the dominant mechanism for the transportation of charge carriers in the photorefractive medium. We also assume that the transverse dimension (xy plane) of the beam is of infinite extent and the slowly-varying-envelope approximation is valid. Under these assumptions, the spatiotemporal equations of the two beams in the photorefractive medium can be written approximately[11]

$$\frac{\partial A}{\partial z}=-\frac{\Gamma}{2}GB\ , \tag{3}$$

$$\frac{\partial B}{\partial z}=\frac{\Gamma^*}{2}G^*A\ , \tag{4}$$

$$\frac{\partial G}{\partial t}=-\frac{1}{\tau}\left(G-\frac{AB^*}{I_0}\right)\ , \tag{5}$$

where Γ is the photorefractive coupling constant, G is a measure of the relative amplitude of the photorefractive index grating, $I_0=|A|^2+|B|^2$, and τ is the time constant of the photorefractive crystal.

The first two equations describe the coupling between the reading beam and the diffracted beam. The third equation [Eq. (5)] describes the growth and decay of the index grating. The first term on the right-hand side of Eq. (5) is responsible for the decay of the index grating, while the second term is responsible for its transient growth. From Eq. (5) we note that the growth rate (or decay rate) of the amplitude of the photorefractive grating is determined by the time constant τ. We also note that a steady-state solution for G according to Eq. (5) is given by $G=AB^*/I_0$. In other words, if both A and B are provided externally, the photoinduced index grating can be written[1]

$$n=n_0+\frac{1}{2}[n_1G\ \exp(-i\phi+i\mathbf{K}\cdot\mathbf{r})+\text{c.c.}]\ , \tag{6}$$

where n_0 is the index of refraction of the medium, n_1 is a constant proportional to the saturated depth of modulation of the index grating, ϕ is the spatial phase shift between the interference fringe and the photoinduced index grating, and $\mathbf{K}=\mathbf{k}_2-\mathbf{k}_1$ is the grating wave vector. When the two writing beams are of equal amplitude, a saturated index grating is given by

$$n=n_0+\frac{1}{4}[n_1\ \exp(-i\phi+i\mathbf{K}\cdot\mathbf{r})+\text{c.c.}]\ . \tag{7}$$

In the case of reading out a photoinduced index grating by using a single beam, both A and B are functions of space and time. The relative amplitude of the index grating G is also a function of both space and time. Equations (3)–(5) must be solved in order to obtain the beams' amplitudes A, B and the relative amplitude of the index grating G.

In the case of diffusion-dominated medium, $\phi=\pi/2$, Γ is real, and the phase of G does not vary inside the photorefractive crystal.[1] We can assume that G is a real variable, i.e., the phase of G is 0. By using the method of the grating integral[1] and the boundary conditions

$$A(t,z=0)=A_0\ , \tag{8}$$

$$B(t,z=0)=0\ , \tag{9}$$

the solution of Eqs. (3)–(4) can be written as

$$A(t,z)=A_0\ \cos\left(\frac{\Gamma}{2}u\right)\ , \tag{10}$$

$$B(t,z)=A_0\ \sin\left(\frac{\Gamma}{2}u\right)\ , \tag{11}$$

where A_0 is a constant and the grating integral u is written

$$u\equiv\int_0^z G(z')\ \mathrm{d}z'\ . \tag{12}$$

The grating integral $U=u(t,z=L)$ is a measure of the relative grating strength. The diffraction efficiency is related to U by

$$\eta=\sin^2\left(\frac{\Gamma}{2}U\right)\ , \tag{13}$$

where maximum grating strength of a photoinduced grating is given by $U=L/2$. We note that G is a function of both space and time as described by Eq. (5).

To examine the temporal variation of the grating strength (i.e., its growth rate), we substitute Eqs. (10) and (11) into Eq. (5) and integrate the resulting equation over the thickness of the crystal. We obtain

$$\frac{\partial U}{\partial t}=-\frac{1}{\tau}\left(U-\frac{1}{2}\int_0^L \sin(\Gamma u)\,\mathrm{d}z\right) . \tag{14}$$

When the derivative $\partial U/\partial t$ is greater (less) than zero, the grating strength is increasing (decreasing). From Eq. (12) and Eq. (14), we also note that the growth rate (i.e., transient enhancement) of the grating strength depends on the spatial distribution of the index grating and the photorefractive coupling constant Γ.

We now consider the case of a uniform initial grating (which may be recorded with two ordinarily polarized plane waves), i.e.,

$$G(t=0,\ z)=G_0 \ , \tag{15}$$

where G_0 is a constant. At $t=0$, the temporal variation of the grating strength is given by

$$\frac{\partial U}{\partial t}=-\frac{1}{\tau}\left[G_0L-\frac{1}{\Gamma G_0}\sin^2\left(\frac{\Gamma L G_0}{2}\right)\right] . \tag{16}$$

Therefore, the condition for the grating strength to increase is given by

$$\frac{\Gamma L}{4}\frac{\sin^2(\Gamma L G_0/2)}{(\Gamma L G_0/2)^2}>1 . \tag{17}$$

From the above condition, we note that a necessary condition for the increase of grating strengths is $\Gamma L>4$.

To obtain the grating amplitude $G(t,z)$, we need to solve Eqs. (5) and (10)–(12) numerically. Figure 2 plots the grating strength $u(t,\ z=L)$ as functions of the readout time t (normalized to the photorefractive time constant τ), where the dependences of the initial grating amplitude G_0 and the photorefractive coupling strength ΓL are shown in Fig. 2(a) and 2(b), respectively. According to the results in Fig. 2, we find that the photorefractive index grating may be enhanced by the readout process for a short period of time provided the amplitude of the initial grating is weak and the coupling strength is large enough. The grating strength decays eventually if the readout beam remains on indefinitely. We also find that as a result of the nonreciprocal energy coupling in two-wave mixing[1] there is no enhancement of the grating strength if the direction of the c axis is reversed (see Fig. 1).

To further understand the growth of the index grating, let us examine Eq. (5) for the relative grating amplitude G. Near the entrance face ($z=0$) of the medium, B grows spatially from zero. Thus the right-hand side of Eq. (5) is always negative for small z, indicating a decay of the grating amplitude. At the exit face ($z=L$) of the medium, the diffracted beam B has grown to reach its maximum depleting energy from the readout beam. Depending on the amount of the depletion, the right-hand side of Eq. (5) may be positive,

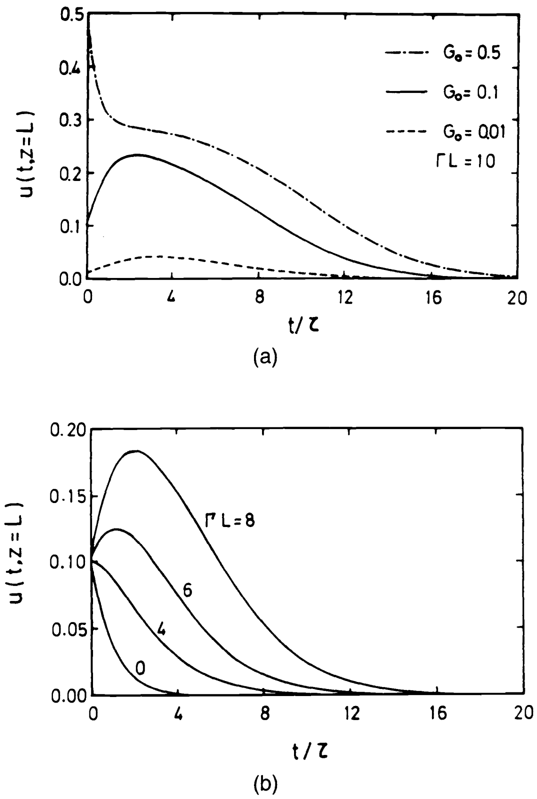

Fig. 2 (a) Temporal variation of the grating strength $u(t,\ z=L)$ for various initial grating amplitudes $G_0=0.01$, 0.1, 0.5, and the photorefractive coupling strength of $\Gamma L=10$. (b) Grating strength as a function of readout time for various coupling strength $\Gamma L=0$, 4, 6, and 8 for a given initial uniform grating amplitude $G_0=0.1$.

which indicates a temporal growth of the index grating near the exit face. Thus the grating amplitude distribution is modified as a result of the readout. This is illustrated in Fig. 3 for the case of an initial grating amplitude of $G_0=0.1$ and the photorefractive coupling strength $\Gamma L=10$. The results in Fig. 3 show that after a readout time $t=\tau$, the grating is slightly erased near the entrance face of the medium and strengthened near the exit face. At readout time $t=2.3\tau$, the grating strength (the area under the curve of the grating amplitude) reaches its maximum value [also see Fig. 2(a), solid line]. After a long readout time, e.g., at $t=10\tau$, the grating near the entrance face of the medium is almost completely erased and the resultant grating tends to concentrate near the exit face. We note that indeed the index grating is always pushed toward the exit face of the medium during a single beam readout process. In other words, the front portion of the volume index grating is partially erased, while the rear portion is enhanced. The change of grating amplitude profile has also been analyzed by other researchers.[13]

The redistribution and the temporal variation of the index grating also affects the amplitude and the spatial variation of the diffracted beam. Eventually, this leads to a decay of the grating amplitude in the whole medium for all $0<z<L$. Thus for the purpose of enhancing the amplitude of the index grating, the initial grating should be concentrated near the front of the medium. This ensures the generation of a diffracted beam in the bulk of the medium and the subsequent writing of a photoinduced grating in the rear portion of the medium. We note that the enhancement of the grating amplitude is

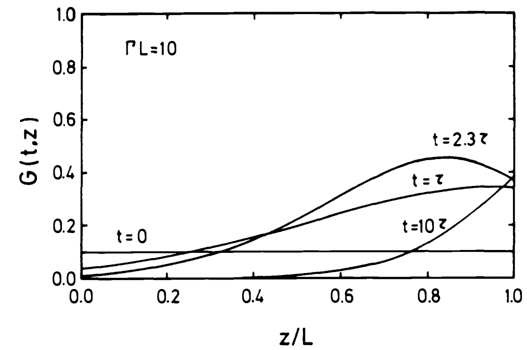

Fig. 3 Spatial distribution of the grating amplitude $G(t,z)$ at various readout times $t=0$, τ, 2.3τ, and 10τ.

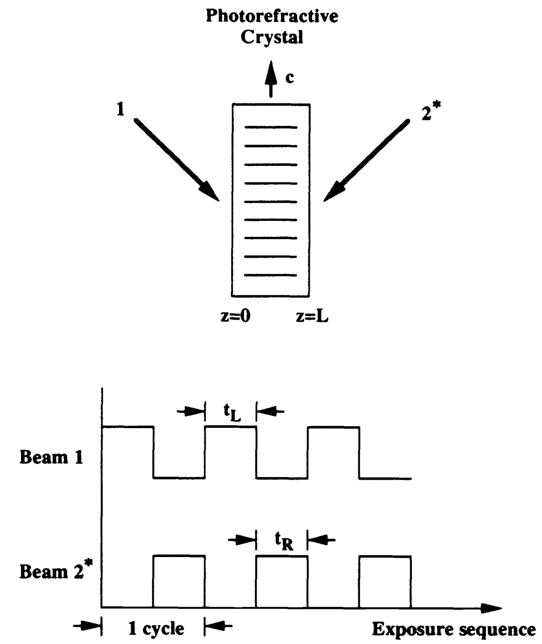

Fig. 4 Schematic diagram of the double-side alternating readout scheme. The graph below shows the exposure sequence.

accompanied by a spatial redistribution of the grating amplitude. To further increase the amplitude of the index grating, we must read the resultant index grating from the rear (i.e., $z=L$). This can be facilitated by the double-side alternating readout scheme as shown in Fig. 4. In the following section, we analyze and discuss this optical method.

3 Double-Side Alternating Readout

Referring to Fig. 4, we now consider the case of a single grating in a photorefractive medium for double-side alternating readout. The photorefractive grating is first read out by the incident laser pulse (beam 1 with the propagation wave vector $\mathbf{k}_1$) for a short time t_L from the front ($z=0$) of the crystal. The exposure time t_L is chosen so that the strength of the grating is enhanced at the end of the first readout. The grating is then read out by the alternating laser pulse (beam 2 with the propagation wave vector $-\mathbf{k}_2$) for another short period t_R. The second exposure time t_R is also chosen so that the grating is further enhanced at the end of the second readout. Thus there is a net gain in the amplitude of the hologram during the first cycle. If the process repeats, further increase in the amplitude of grating is possible.

Figure 5 shows the temporal variation of the spatial distribution of the grating amplitude $G(t,z)$ after several cycles of readout, where the initial uniform grating amplitude $G(t=0,\ z)=0.01$ and the photorefractive coupling strength is $\Gamma L=10$. The results in Fig. 5(a) indicate that at the end of the first readout ($t_L=\tau$), the grating is pushed away from the front face of the medium [solid line in Fig. 5(a), $n=1$]. At the end of the second readout ($t_R=\tau$), which is from the rear of the medium, the grating is pushed back toward the front of the medium [dashed line in Fig. 5(a), $n=1$]. After several cycles of pushing back and forth, the resultant photorefractive grating is concentrated near the center region of the crystal. In addition, the resultant index grating after several cycles of readout tends to form a steady-state grating amplitude. We note that the twin peaks in the figure [see Fig. 5(a), $n=4$] is symmetric with respect to the center of the crystal and the diffraction efficiencies (determined by the area under the curve of grating amplitude) are the same from both sides.

Figure 5(b) shows the case of unequal exposure time ($t_L=\tau$ and $t_R=5\tau$). As a result of the long exposure from the rear, the grating is overexposed during readout and the center of gravity of the index grating is pushed toward the front face of the medium, and this may lead to decrease of the total grating strength [see Fig. 5(b), $n=4$].

To further investigate the dependence of the steady-state value on the shape and level of the initial photorefractive grating, we have simulated various shapes of the initial grating amplitude (uniform, triangle, sinusoidal, etc.) with different values of grating amplitude. Our analysis indicates that the steady-state grating is dependent only on the photorefractive coupling strength and the exposure time, and independent of the shape and level of the initial photorefractive grating.

Figure 6 plots the diffraction efficiency η as a function of the number of exposure cycles, n, using the double-side alternating exposure scheme. The results in Fig. 6 indicate that after several cycles ($n=20$ for $G_0=0.01$, $\Gamma L=10$, and $t_L=t_R=0.1\tau$) the diffraction efficiency reaches a steady-state value (about 98% in Fig. 6) and the diffraction efficiencies read out from the front and rear of the medium are equal. We also find that the steady-state diffraction efficiencies from the two sides of the medium are unequal under the condition of exposure times $t_L \neq t_R$, and this is consistent with the results in Fig. 5.

We now consider the dependence of the steady-state diffraction efficiency η_s on the coupling strength ΓL and the exposure duration t (where $t=t_L=t_R$). Figure 7 shows the steady-state diffraction efficiency η_s as a function of the coupling strength ΓL. The results show that there exists a threshold value ΓL for a nonzero steady-state grating. In the case of an exposure duration $t=0.1\tau$, the threshold value is about $\Gamma L=4$, which is the limiting value for extremely short exposure duration ($t \rightarrow 0$). In other words, the curve for $t=0.1\tau$ approaches the asymptotic curve for extremely short exposure duration ($t \rightarrow 0$). Therefore, an exposure duration of 10% of the photorefractive time constant can be considered short exposure. The threshold coupling strength ΓL increases when the exposure duration becomes large, as shown in Fig. 7 for $t=\tau$ and $t=5\tau$.

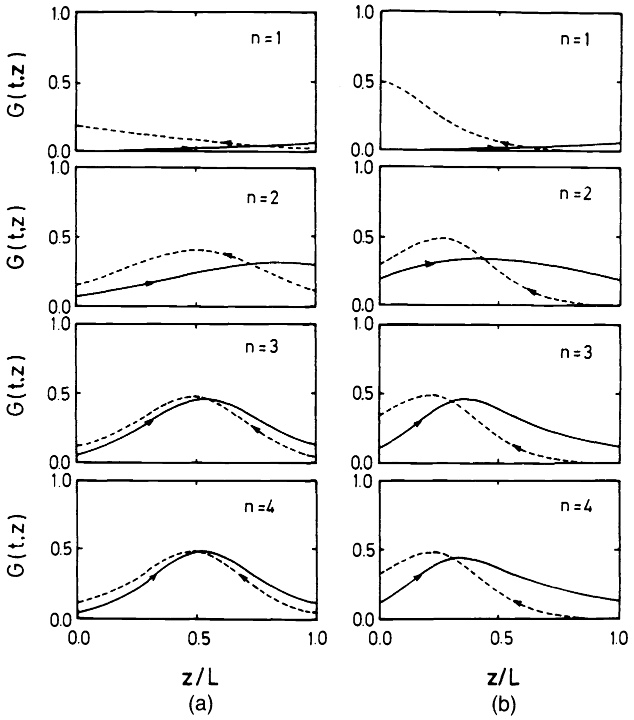

Fig. 5 Temporal and spatial variation of the grating amplitude $G(t,z)$ after n cycles of double-side alternating readout: (a) $t_L=t_R=\tau$ and (b) $t_L=\tau$, $t_R=5\tau$, where $G_0=0.01$ and $\Gamma L=10$. Solid curves (dashed curves) represent the amplitude distribution of the grating at the end of readout from the front (rear) face of the crystal.

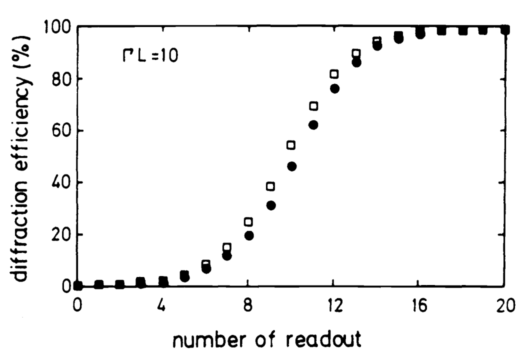

Fig. 6 Diffraction efficiency η as a function of the number of readout cycles, n, for a given initial uniform grating $G_0=0.01$, where the exposure duration per readout is $t=0.1\tau$ and the photorefractive coupling strength $\Gamma L=10$. Circles and squares represent the diffraction efficiencies of readout from front and rear faces of the crystal, respectively.

Figure 8 plots the steady-state diffraction efficiency η_s as a function of the exposure duration t in each readout. The results in Fig. 8 indicate that the steady-state diffraction efficiency decreases when the exposure duration increases, due to the erasure during readout. When the exposure time $t<0.1\tau$, the diffraction efficiency asymptotically becomes a constant. We also note that there is a cutoff exposure time beyond which the grating will eventually be erased by the reading beams, leading to a steady-state diffraction efficiency of 0. This is illustrated in Fig. 8.

By examining Fig. 7, we further note that the diffraction efficiency as a function of ΓL bears a strong resemblance to

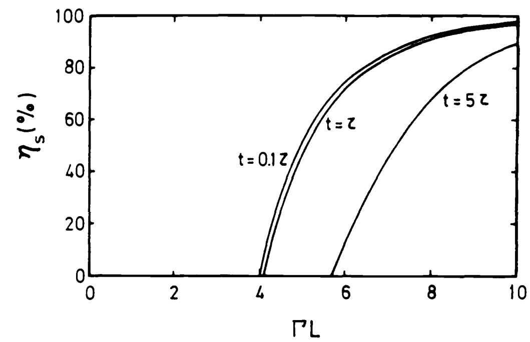

Fig. 7 Steady-state diffraction efficiency η_s as a function of the coupling strength ΓL, for values of the exposure duration $t=0.1\tau$, τ, and 5τ.

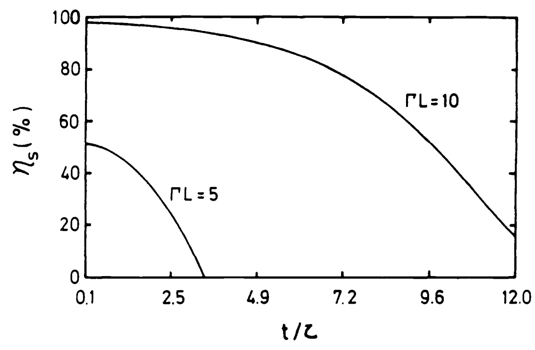

Fig. 8 Steady-state diffraction efficiency η_s versus the exposure time per readout, for values of the photorefractive coupling strength $\Gamma L=5$ and 10.

that of a mutually pumped phase conjugator (MPPC).[1,14–19] In fact, for an extremely small exposure time ($t\ll\tau$), the diffraction efficiency becomes identical to that of an MPPC[1,19,20] with a threshold of $\Gamma L=4$ for equal pump intensities. In addition, the steady-state index grating as shown in Fig. 5(a) is also similar to that of an MPPC.[1] The process of alternating readout of an index grating from both sides of the crystal is equivalent to an MPPC with pulsed pump beams. It is known that steady-state MPPC exists even with pulsed pump beams.[19,20] This is often achieved by first initiating the process of MPPC with cw laser beams. Upon reaching the steady state, the pump beams can then be modulated temporally so that only one of the pump beams is on at any given time. This is exactly identical to our alternating readout scheme for the enhancement of the gratings. The only difference is that we start the process from the very beginning with an extremely weak grating. Thus our results can be employed to explain the initiation and the growth of the MPPC process from an extremely weak grating (or hologram), which may be a small portion of a noisy fanning grating.

We also consider the double-side alternating readout schedule for restoring an extremely weak photoinduced grating in a photorefractive medium to reach a maximum value of the grating strength. Referring to Fig. 9, in each exposure, the self-enhancement of the grating strength lasts for a short time after the reading beam is turned on. Overexposure during the readout will decrease the grating strength due to optical erasure [also see Fig. 1(a), solid line]. To avoid overexposure during the readout process, the exposure time t_N of each readout ($N=1, 2, 3, ...$) can be chosen so that the grating strength reaches its maximum value at the end of each exposure. Since the temporal enhancement and decay of the grating strength depend on the spatial distribution of the grating amplitude, which varies with the exposures, the exposure time of each readout must be adjusted accordingly.

Table 1 shows the results of the grating optimization in the case of an initial uniform grating amplitude $G_0=10^{-5}$ and the photorefractive coupling strength $\Gamma L=10$. Figure 10 shows the increase of the diffraction efficiency during the grating enhancement. The results indicate that this optimized exposure schedule leads to the maximum value of the grating strength with the minimum number of exposures (in this example the diffraction efficiency reaches 98.77% with only six exposures). Our analysis further indicates that the exposure time t_N of the optimized readout schedule satisfies

$$\lim_{N\to\infty} t_N = 0$$

and

$$T=\sum_{N=1}^{\infty} t_N = \text{finite value} \ .$$

These results are due to the fact that the grating strength cannot be increased indefinitely.

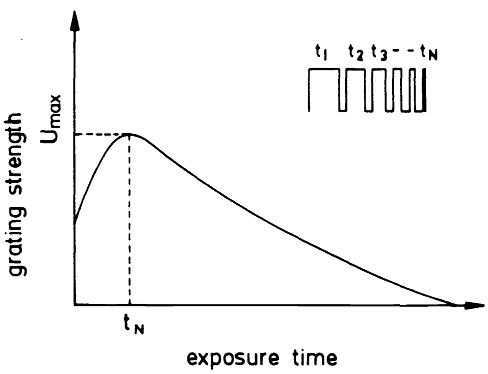

Fig. 9 Schematic diagram of the temporal variation of the diffraction efficiency during a single-beam readout. To optimize the double-side alternating readout, the exposure time of each readout is chosen to maximize the diffraction efficiency.

Table 1 Optimized grating enhancement schedule for an initial uniform grating with $G_0=10^{-5}$ and the photorefractive coupling strength $\Gamma L=10$.

exposure time (unit: τ)	grating strength (U)	diffraction efficiency (%)
t_1=3.475	4.2×10^{-5}	4.4×10^{-6}
t_2=3.963	4.3×10^{-4}	4.6×10^{-4}
t_3=3.963	4.35×10^{-3}	4.7×10^{-2}
t_4=3.938	4.36×10^{-2}	4.68
t_5=2.825	0.2356	85.35
t_6=1.225	0.2919	98.77

4 Optical Restoration of Photorefractive Holograms

We now consider that an image is recorded by the interferometric technique in a photorefractive medium and this forms a hologram that consists of many grating components. For the case of optical restoration of a photorefractive hologram, a phase-conjugate mirror (PCM) is essential to ensure the readout from the rear of the medium. Referring to Fig. 11, we consider a readout of a photoinduced hologram in a photorefractive medium by using a pulsed laser (beam 1). The diffracted beam (beam 2), which bears the image information, is retroreflected by the PCM. The pulse length (or exposure time) and the repetition rate are selected so that there is no physical overlap between the incident pulse and the phase-conjugated pulse inside the photorefractive medium. The hologram is first read out by the incident laser pulse for a short duration of time t, producing an image-bearing beam that is enhanced by the simultaneous presence of the two beams in the photorefractive medium. When the diffracted beam is retroreflected by the phase conjugator, the hologram is then read out by the retroreflected beam for another short period of time t, producing a phase conjugate version of the original laser pulse that is further enhanced by the simultaneous presence of the two beams in the photorefractive medium. The exposure time t is chosen so that there is a net gain in the diffraction efficiency of the hologram during the first cycle. Due to the self-enhancement of the hologram, which is similar to that for a single grating, if the double-side alternating readout process continues, further increase in the strength of the hologram is possible until a steady-state value of the hologram diffraction efficiency is reached.

To further understand the process of hologram restoration, we consider that an image with N resolution pixels is recorded in a photorefractive crystal. Suppose that the hologram is formed by the interference of a reference beam and the Fourier transform of the image. The image pixels and the orientation of the photorefractive crystal are appropriately chosen so that there is no energy coupling between the image pixels. Thus a photorefractive hologram with N grating components exists inside the photorefractive medium. For the readout of multiple gratings, the spatial-temporal equations

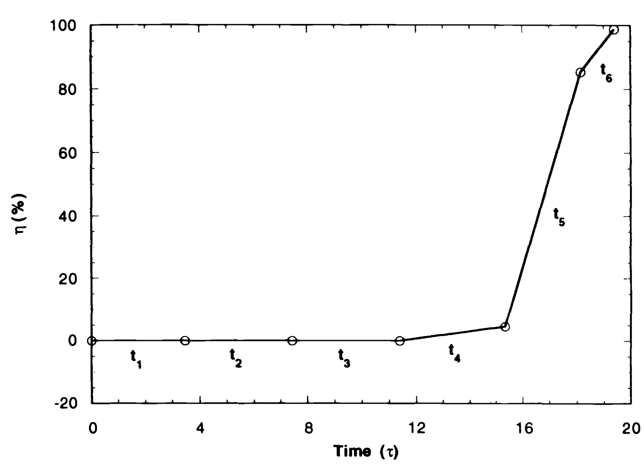

Fig. 10 Increase of the diffraction efficiency during the optimized grating enhancement, where time $=\sum_{i=1}^{N} t_i$.

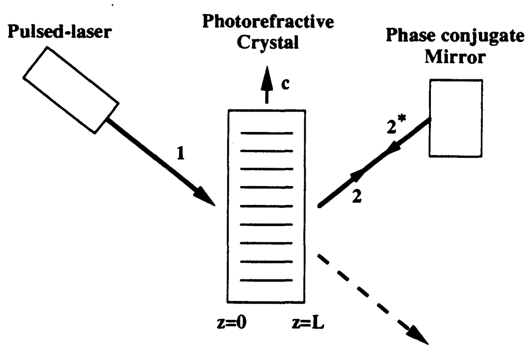

Fig. 11 Schematic diagram of the proposed readout of a photo-induced hologram in a photorefractive medium by using a pulsed laser in conjunction with a phase-conjugate mirror.

of the interacting beams in the photorefractive medium can be written approximately as

$$\frac{\partial A}{\partial z} = -\frac{\Gamma}{2}\left(\sum_{j=1}^{N} G_j B_j\right) , \tag{18}$$

$$\frac{\partial B_j}{\partial z} = \frac{\Gamma^*}{2} G_j^* A, \qquad \text{for } j=1, 2, 3, ..., N , \tag{19}$$

$$\frac{\partial G_j}{\partial t} = -\frac{1}{\tau}\left(G_j - \frac{AB_j^*}{I_0}\right) \qquad \text{for } j=1, 2, 3, ..., N, \tag{20}$$

where the total beam intensity

$$I_0 = |A|^2 + \sum_{j=1}^{N} |B_j|^2 , \tag{21}$$

A is the amplitude of the reference beam, B_j is the amplitude of the jth diffracted plane wave corresponding to the jth image pixel, and G_j is the amplitude of the jth grating component.

For heuristic purposes, we now consider the case of a hologram that consists of only two grating components with their initial grating amplitudes G_{10} and G_{20}, respectively, which are constants. Equations (18)–(20) can be written as

$$\frac{\partial A}{\partial z} = -\frac{\Gamma}{2}(G_1 B_1 + G_2 B_2) , \tag{22}$$

$$\frac{\partial B_1}{\partial z} = \frac{\Gamma^*}{2} G_1{}^* A , \tag{23}$$

$$\frac{\partial B_2}{\partial z} = \frac{\Gamma^*}{2} G_2{}^* A , \tag{24}$$

$$\frac{\partial G_1}{\partial t} = -\frac{1}{\tau}\left(G_1 - \frac{AB_1{}^*}{I_0}\right) , \tag{25}$$

$$\frac{\partial G_2}{\partial t} = -\frac{1}{\tau}\left(G_2 - \frac{AB_2{}^*}{I_0}\right) , \tag{26}$$

where the total intensity $I_0 = |A|^2 + |B_1|^2 + |B_2|^2$. For the given initial grating distributions and the boundary conditions, Eqs. (22)–(26) must be solved in order to obtain the beam amplitudes A, B_1, B_2 and the relative grating amplitudes G_1 and G_2.

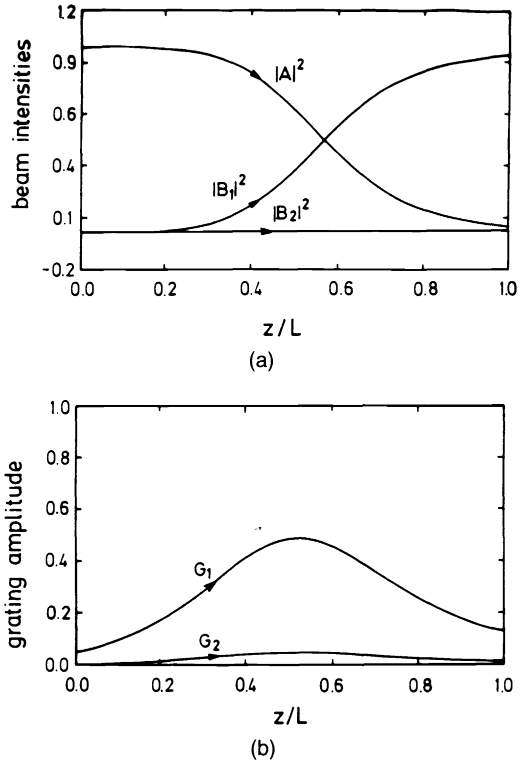

Fig. 12 Spatial dependence of (a) steady-state beam intensities $|A|^2$, $|B_1|^2$, $|B_2|^2$ and (b) the grating amplitudes, where $G_{10}=0.1$, $G_{20}=0.01$, $\Gamma L=10$, and $t=\tau$.

To evaluate the performance of the optical restoration method, we define a parameter, the contrast of the two reconstructed image pixels, as a measure of the image quality:

$$C \equiv \frac{|B_1|^2 - |B_2|^2}{|B_1|^2 + |B_2|^2} . \tag{27}$$

As an example, suppose a photorefractive hologram with initial grating components $G_{10}=0.1$ and $G_{20}=0.01$ is given. By Eqs. (22)–(26), we obtain the output beam intensities $|B_1|^2(t=0,\ z=L)=0.2307$ and $|B_2|^2(t=0,\ z=L)=0.0023$ (with relative amplitude) as the readout starts, and according to Eq. (27) the contrast is $C=0.9802$. During the cycles of double-side alternating readout, we assume that the gain of the PCM is adjusted so that the intensities of the two reading beams (beam 1 and beam 2* in Fig. 11) are equal. Note that the phase-conjugated beam (beam 2*) consists of two plane waves, B_1^* and B_2^*, whose relative amplitude information is preserved during the phase conjugation. In hologram restoration, we observe that the spatial redistribution of the grating profiles and the enhancement of hologram strength are similar to those observed in the case of a single grating. After several cycles of readout, the amplitudes of the photorefractive gratings tend to reach their steady-state distributions

Figure 12 plots the spatial dependence of the steady-state beam intensities $|A|^2$, $|B_1|^2$, $|B_2|^2$ and the grating amplitudes G_1 and G_2, where the photorefractive coupling strength is $\Gamma L=10$. By examining the results in Fig. 12(a), we note that at the exit face ($z=L$) of the crystal the output intensities are

$|B_1|^2 = 0.9606$ and $|B_2|^2 = 0.0096$ and the value of contrast remains 0.9802. In fact, the contrast remains a constant during the entire process of hologram restoration. In other words, the contrast of the reconstructed image pixels is sustained in the optical restoration of the photorefractive hologram. The sustainment of contrast ensures the quality of the reconstructed image.

The method of hologram restoration has been demonstrated experimentally.[21] In the experiment, the photorefractive crystal used is 45-deg-cut $BaTiO_3$ with $\Gamma L \approx 5$. Results with various exposure times are presented. In addition, the spatial profile of the relative grating amplitude of a single grating has been measured. The hologram restoration of a stored image pattern has also been successfully demonstrated. The experimental results are in good agreement with our theoretical predictions. The image resolution observed experimentally reached 200 lines/mm. It was also observed experimentally that the image quality was preserved during the hologram enhancement process until the saturation of the diffraction efficiency was reached. Continued toggling after saturation eventually led to a degradation of image quality as noise levels due to internal scattering and reflections grew to match those of the signal. This eventual result was virtually identical to that achieved when the hologram was generated in a double PCM arrangement, in agreement with theory. Thus, to preserve the original hologram's image quality while allowing for a significant enhancement it is preferable to stop the enhancing process when the diffraction efficiency reaches its saturation value.

5 Conclusions

In conclusion, we have proposed and analyzed a new optical method for the enhancement and restoration of decaying holograms in photorefractive media. The results indicate that extremely weak holograms can be enhanced provided that the two-beam coupling is sufficiently strong. Steady-state photorefractive holograms can be maintained continuously without decay by using a sequential readout schedule in conjunction with a phase conjugator. Our analysis shows that the optical restoration method provides a possibility of sustaining the contrast of the reconstructed image in the process of hologram enhancement. The result also explains the formation of mutually pumped phase conjugation (MPPC) in terms of the enhancement of initial noise gratings.

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References

1. See, for example, P. Yeh, *Introduction to Photorefractive Nonlinear Optics*, Wiley, New York (1993).
2. K. Y. Hsu, S. H. Lin, C. J. Cheng, T. C. Hsieh, and P. Yeh, "An optical neural network for pattern recognition," *J. Opt. Comput.* **2**, 409–423 (1991).
3. D. Psaltis, D. Brady, and K. Wagner, "Adaptive optical networks using photorefractive crystals," *Appl. Opt.* **27**, 1752–1759 (1988).
4. D. Brady, K. Hsu, and D. Psaltis, "Periodic refreshed multiply exposure photorefractive holograms," *Opt. Lett.* **15**, 817–819 (1990).
5. Y. Taketomi, J. E. Ford, H. Sasaki, J. Ma, Y. Fainman, and S. H. Lee, "Incremental recording for photorefractive hologram multiplexing," *Opt. Lett.* **16**, 1774–1776 (1991); H. Sasaki, Y. Fainman, J. E. Ford, Y. Taketomi, and S. H. Lee, "Dynamic photorefractive optical memory," *Opt. Lett.* **16**, 1874–1876 (1991).
6. Y. Qiao, D. Psaltis, C. Gu, J. Hong, P. Yeh, and R. R. Neurgaonkar, "Phase-locked sustainment of photorefractive holograms using phase conjugation," *J. Appl. Phys.* **70**, 4646–4648 (1991).
7. Y. Qiao and D. Psaltis, "Sampled dynamic holographic memory," *Opt. Lett.* **17**, 1376–1378 (1992).
8. See, for example, P. Yeh, "Two-wave mixing in nonlinear media," *IEEE J. Quantum Electron.* **25**, 484–519 (1989).
9. T. K. Gaylord, T. A. Rabson, F. K. Tittel, and C. R. Quick, "Self-enhancement of $LiNbO_3$ holograms," *J. Appl. Phys.* **44**, 896–897 (1973).
10. D. M. Lininger, D. D. Crouch, P. J. Martin, and D. Z. Anderson, "Theory of bistability and self pulsing in ring resonator with saturable photorefractive gain and loss," *Opt. Commun.* **76**, 89–96 (1990).
11. C. Gu, J. Hong, and P. Yeh, "Diffraction properties of momentum-mismatched gratings in photorefractive media," *J. Opt. Soc. Am. B* **9**, 1473–1479 (1992).
12. N. V. Kukhtarev, V. B. Markov, S. G. Odulov, M. S. Soskin, and V. L. Vinetskii, "Holographic storage in electrooptic crystals, I. Steady state. II. Beam coupling—light amplification," *Ferroelectrics* **22**, 949–964 (1979).
13. M. Jeganathan, M. C. Bashaw, and L. Hesselink, "Trapping the grating envelope in bulk photorefractive media," *Opt. Lett.* **15**, 1415–1417 (1994).
14. M. Cronin-Golomb, B. Fischer, J. O. White, and A. Yariv, "Theory and application of four-wave mixing in photorefractive media," *IEEE J. Quantum Electron.* **QE-20**, 12–30 (1984).
15. P. Yeh, T. T. Chang, and M. D. Ewbank, "Model for mutually pumped phase conjugation," *J. Opt. Soc. Am. B* **5**, 1743–1749 (1988).
16. M. D. Ewbank, "Mechanism for photorefractive phase conjugation using incoherent beams," *Opt. Lett.* **13**, 47–49 (1988); M. D. Ewbank, R. A. Vazquez, R. R. Neurgaonkar, and J. Feinberg, "Mutually pumped phase conjugation in photorefractive strontium barium niobate: theory and experiment," *J. Opt. Soc. Am. B* **7**, 2306–2316 (1990).
17. S. Weiss, S. Sternklar, and B. Fischer, "Double phase-conjugation mirror: analysis, demonstration, and applications," *Opt. Lett.* **12**, 114–116 (1987); S. Sternklar, S. Weiss, M. Segev, and B. Fischer, "Beam coupling and looking of lasers using photorefractive four-wave mixing," *Opt. Lett.* **11**, 528–530 (1986).
18. M. Segev, D. Engin, A. Yariv, and G. C. Valley, "Temporal evolution of photorefractive double phase-conjugate mirrors," *Opt. Lett.* **18**, 1828–1830 (1993).
19. S. A. Korol'kov, A. V. Mamaev, and V. V. Shkunov, "Mutual phase conjugation of temporally non-overlapping optical beams," *Sov. J. Quantum Electron.* **22**, 861–864 (1992).
20. M. D. Ewbank, private communication.
21. S. Campbell, P. Yeh, C. Gu, S.-H. Lin, C.-J. Cheng, and K.-Y. Hsu, "Optical restoration of photorefractive holograms via self-enhanced diffraction," *Opt. Lett.* **20**, 330–332 (1995).

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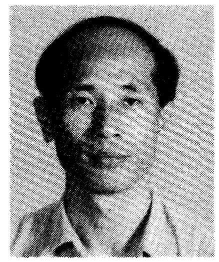

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