Surface plasmon resonance heterodyne interferometry for measuring physical parameters

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ABSTRACT

A light beam is incident on the boundary surface between the thin metal film of a surface-plasmon-resonance (SPR) apparatus and the test medium. If the incident angle is equal or very near to the resonant angle, then the phase difference between p- and s- polarizations of the reflected light is related to the associated physical parameter. The phase difference can be measured accurately by the heterodyne interferometry. If the relation between the phase difference and the associated physical parameter is specified, the associated physical parameter can be estimated with the data of the phase difference. This method has the advantages of both common-path interferometry and heterodyne interferometry.

Keywords: Surface-plasmon resonance, heterodyne interferometry, physical parameter

1. INTRODUCTION

A light beam is incident on the boundary of a surface plasmon resonance (SPR) [1~3] apparatus. As the incident angle is equal or very near to the resonant angle, the phase difference between p- and s- polarizations of the reflected light is changed with the variation of the refractive index of either the thin metal film or the test medium. Consequently, the relative refractive index can be estimated with the data of the phase difference. The relative refractive index is related to some other physical parameters, such as wavelength, pressure, temperature, and concentration [4~6]. If the relation between the phase difference and a physical parameter is specified, then the physical parameter can also be estimated with the data of the phase difference.

In a heterodyne interferometer [7], two light beams with slightly different frequencies interfere. The interference signal being modulated at the difference frequency between these two beams are converted to the electronic signal. The signal can be processed more easily and accurately. Hence the heterodyne interferometry can be measured the phase difference between two interfering beams with high resolution and in real-time. In this paper, a surface plasmon resonance heterodyne interferometry is presented by combining the surface plasmon resonance effect and the heterodyne interferometry. This alternative method can be applied to measure associated physical parameters. In this method, the phase difference between p- and s- polarizations of the reflected light at the boundary of the SPR can be measured accurately with the heterodyne interferometry. The associated physical parameter can be estimated with the measured data of the phase difference and their corresponding relation. To show its validity, the concentrations of several different solutions and the wavelength of a tunable laser are measured. It has the advantages of both common-path interferometry and heterodyne interferometry, such as simple structure, high stability, high resolution, easier operation, and rapid real-time measurement.

2. PRINCIPLE

2.1 Phase difference resulting from reflection of SPR apparatus

A linearly polarized light beam enters one surface of the SPR apparatus, as shown in Fig. 1. This apparatus has a

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Kretschmann configuration [8] and is an isosceles right-prism with a thin metal film of thickness d_2 deposited on the hypotenuse surface. The test medium is contacted with the thin metal film. The light beam is transmitted through the surface and is incident at an angle θ on the thin metal film. For convenience, the numbers 1, 2, and 3 are labeled in Fig. 1 to represent the media of glass (prism), metal (film), and the test medium or air. Their refractive indices are n_1, $n_2 = n + ik$, and n_3, respectively. If θ is equal or very near to the resonant angle θ_{sp}, then surface plasmons are excited. So the phase difference between the p- and s- polarizations of the reflected light coming from the boundary surface under the condition of SPR can be expressed as

$$\phi = \phi_p - \phi_s, \tag{1}$$

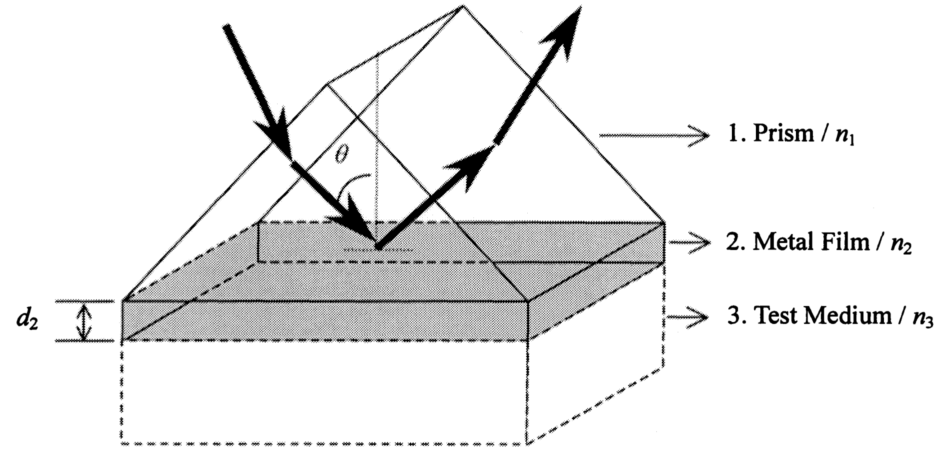

Fig. 1 Reflection in a surface plasmon resonance apparatus.

Where

$$\phi_q = \tan^{-1}\left[\frac{q_1.\sin\phi_{q1}.(1-q_2^2.e^{-4bd_2}) + q_2.e^{-2bd_2}.\sin(\phi_{q2}+2ad_2).(1-q_1^2)}{q_1.\cos\phi_{q1}.(1+q_2^2.e^{-4bd_2}) + q_2.e^{-2bd_2}.\cos(\phi_{q2}+2ad_2).(1+q_1^2)}\right], \quad q = p,s. \tag{2}$$

Each symbol in Eq. (2) can be written as

$$p_1 = \frac{\sqrt{[(n^2+k^2)^2A^2 - n_1^4(a^2+b^2)]^2 + 4n_1^4A^2C^2}}{[(n^2-k^2)A + n_1^2a]^2 + (2An\ k\ + n_1^2b)^2}, \tag{3a}$$

$$p_2 = \frac{\sqrt{[n_3^4(a^2+b^2) - (n^2+k^2)^2B^2]^2 + 4n_3^4B^2C^2}}{[(n^2-k^2)B + n_3^2a]^2 + (2Bn\ k\ + n_3^2b)^2}, \tag{3b}$$

$$\phi_{p1} = \tan^{-1}\left[\frac{2n_1^2AC}{(n^2+k^2)^2A^2 - n_1^4(a^2+b^2)}\right], \tag{3c}$$

$$\phi_{p2} = \tan^{-1}\left[\frac{-2n_3^2BC}{n_3^4(a^2+b^2)-(n^2+k^2)^2B^2}\right], \tag{3d}$$

$$s_1 = \frac{\sqrt{(A^2-a^2-b^2)^2+4A^2b^2}}{(A+a)^2+b^2}, \tag{3e}$$

$$s_2 = \frac{\sqrt{(a^2+b^2-B^2)^2+4B^2b^2}}{(B+a)^2+b^2}, \tag{3f}$$

$$\phi_{s1} = \tan^{-1}\left[\frac{-2Ab}{A^2-a^2-b^2}\right], \tag{3g}$$

$$\phi_{s2} = \tan^{-1}\left[\frac{2Bb}{a^2+b^2-B^2}\right], \tag{3h}$$

$$A = k_0(n_1^2 - n_1^2.\sin^2\theta_1)^{1/2}, \tag{3i}$$

$$B = k_0(n_3^2 - n_1^2.\sin^2\theta_1)^{1/2}, \tag{3j}$$

$$C = 2ank - b(n^2-k^2), \tag{3k}$$

$$a = \mathrm{Re}[k_0n_2.\cos(\sin^{-1}(\frac{n_1}{n_2}.\sin\theta_1))], \tag{3l}$$

and

$$b = \mathrm{Im}[k_0n_2.\cos(\sin^{-1}(\frac{n_1}{n_2}.\sin\theta_1))]. \tag{3m}$$

It is obvious from Eqs. (1)~(3) that the phase difference ϕ is strongly dependent on the relative refractive index $n'=n_2/n_3$. So the relative refractive index can be estimated with the measured data of the phase difference. Similarly, the physical parameter p being related to the relative index may also be estimated. Hence, we can infer that if the relation curve of the phase difference ϕ versus the physical parameter p is specified, then the physical parameter p can be estimated with the measurement of the phase difference ϕ.

2.2 Phase difference measurements with heterodyne interferometry

A schematic diagram of the optical arrangement of our method, which is based on Chiu's [9] considerations, was designed and is shown in Fig. 2. For convenience, the +z axis is chosen to be along the light propagation direction and +x axis is along the horizontal axis. The light beam coming from a heterodyne light source has an angular frequency difference ω between s- and p- polarizations. It is incident on the boundary surface between an SPR apparatus and a test medium. The incident angle θ is equal or very near to the resonant angle θ_{sp}. The reflected light passes through an analyzer AN with the transmission axis at α to the horizontal axis and it is detected by a photodetecor D. The detected intensity measured by D can be derived as [9]

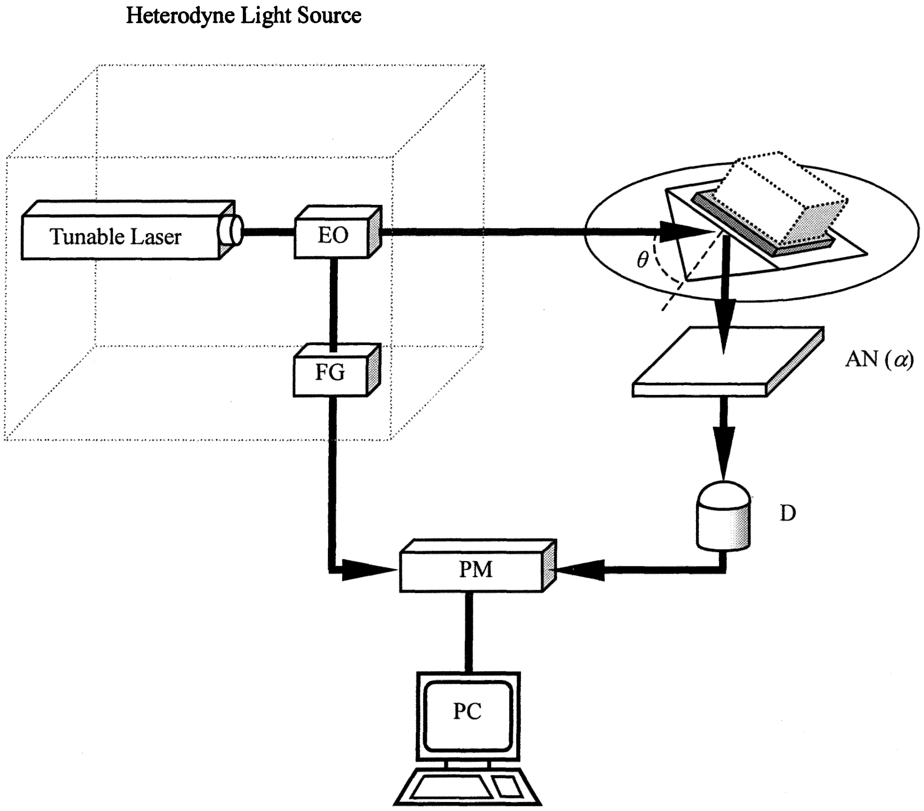

Fig. 2 Schematic diagram for measuring the phase difference of the reflected light: EO, electro-optic modulator; FG, function generator; PM, phase meter; AN, analyzer; D, detector; PC, personal computer.

$$I_t = \frac{1}{4}\left[r_p^2 \cos^2\alpha + r_s^2 \sin^2\alpha + 2r_p r_s \cos\alpha \sin\alpha \cos(\omega t + \phi)\right], \tag{4}$$

and I_t is the test signal. Where r_p and r_s are the reflections coefficients of p- and s- polarizations, respectively. They can be expressed as [10]

$$r_q = \frac{r_{12}^q + r_{23}^q e^{i2k_{z2}d_2}}{1 + r_{12}^q r_{23}^q e^{i2k_{z2}d_2}}, \quad q = p, s, \tag{5}$$

where r_{ab}^p and r_{ab}^s are the Fresnel reflection coefficients of p- and s-polarizations as the light beam propagates from medium a to medium b, and k_{za} is the component of the wave vector in medium a along the z direction. They can be written as

$$r_{ab}^{p}=\frac{n_b^2 k_{za}-n_a^2 k_{zb}}{n_b^2 k_{za}+n_a^2 k_{zb}},\quad a,b=1,2,3, \tag{6a}$$

$$r_{ab}^{s}=\frac{k_{za}-k_{zb}}{k_{za}+k_{zb}}, \tag{6b}$$

and

$$k_{za}=k_0(n_a^2-n_1^2\sin^2\theta)^{1/2}, \tag{6c}$$

respectively, where k_0 is the free-space wave vector. On the other hand, the electronic signal coming from the driver of the heterodyne light source is filtered and becomes the reference signal. It has the form as

$$I_r=\frac{1}{2}[1+\cos(\omega t)]\cdot \tag{7}$$

Both of these two sinusoidal signals are sent to a phase meter PM, hence ϕ can be measured accurately. By substituting the data of ϕ into the specified relation curve of phase difference ϕ versus physical parameter p, the associated physical parameter p can be estimated.

3. EXPERIMENTS AND RESULTS

To show the feasibility of this method, we measure the concentrations of several different solutions and the wavelength of a tunable laser. The heterodyne light source consists of a tunable laser (Model 6304, New Focus) with central wavelength 632.8nm and an electro-optic modulator EO (Mode 4002, New Focus) with a half-wave voltage of 125V. The electro-optic modulator EO is driven with a sawtooth signal generated by a function generator FG. Its amplitude is the half-wave voltage of EO and its frequency is 1 kHz. So the frequency difference between p- and s- polarization components is 1 kHz. The SPR apparatus consists of a BK7 prism and a thin gold film of thickness 35nm. The refractive indices of a BK7 prism and a thin gold film are measured with an ellipsometer (Model eta, Stag Inc.) and they are 1.5151 and 0.1973+i3.5631 at wavelength 632.8nm, respectively. A high-resolution rotation stage (Model URM 80, Newport) with angular resolution 0.001° is used to mount the SPR apparatus and the test medium. The azimuth angle α of the analyzer is 10° with respect to the horizontal axis. A phase meter with angular resolution 0.01° is used to measure the phase difference. A personal computer is used to record and analyze the data.

(1) Measurement of concentration of a solution

In our experiments, salt-water, glucose solution, acetone, and ethanol are tested at room temperature 20°C. At first, we estimate the resonant angle for each solution of concentration 5 mg/ml by measuring the critical minimum reflectance. They are 71.1°, 71.09°, 71.04°, and 71.03° for salt-water, glucose solution, acetone, and ethanol, respectively. Then we set the incident angles to be 71.1°, 71.09°, 71.04°, and 71.03° for these four test solutions, respectively. The measured results and their fitting curves are shown in Fig. 3. In this figure, the symbols ■, ▲, *, and ● represent the measurement data of salt-water, glucose solution, acetone, and ethanol, respectively. It is seen that the four fitting curves are nearly straight lines with slopes 1.092 deg·ml/mg, 0.917 deg·ml/mg, 0.474 deg·ml/mg, and 0.364 deg·ml/mg, respectively. Consequently, they can be expressed with the equations

$$\phi_s\cong 70.641+1.092c, \tag{8a}$$

$$\phi_g \cong 71.405+0.917c, \tag{8b}$$

$$\phi_a \cong 74.163+0.474c, \tag{8c}$$

and

$$\phi_e \cong 74.498+0.364c, \tag{8d}$$

respectively, where c is the concentration of the test solution.

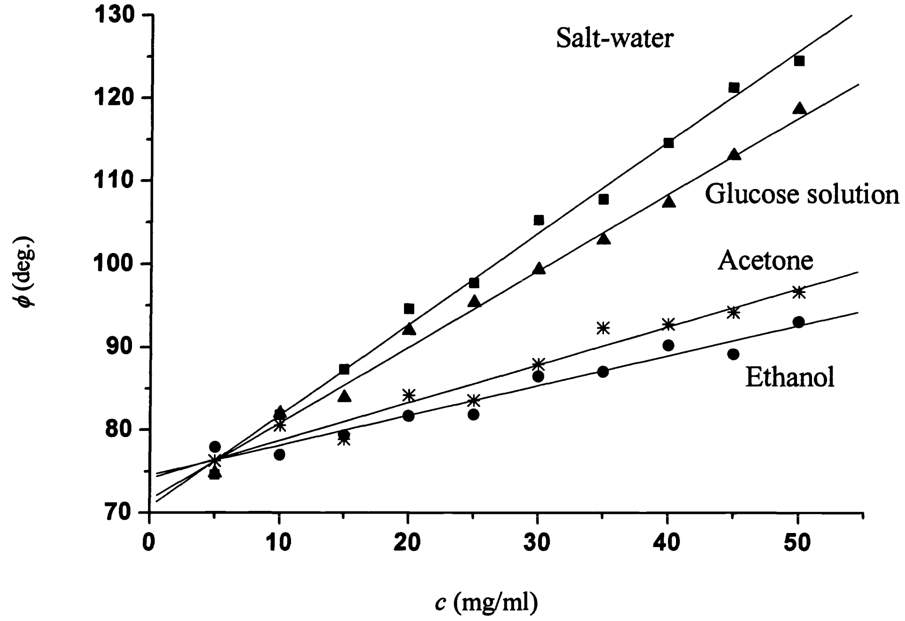

Fig. 3 Measurement data and the fitting curves of ϕ versus c for four different solution.

(2) Measurement of wavelength of a tunable laser

In these experiments,, the test medium is removed and the thin metal film is contacted with air directly. The wavelength variation in the range 632.7nm and 633.9nm is measured. The θ_{sp} = 43.9° can be obtained by measuring the critical minimum reflectance, so we set the incident angles to be 43.9°. The experimental results of the phase difference ϕ versus the wavelength λ are shown in Fig. 4. In this figure, it is seen that the fitting curve is nearly a straight line with slope –3.514 deg./nm. It can be expressed as

$$\phi \cong 2231.917-3.514\lambda \cdot \tag{9}$$

Hence this method can be realized that if the relation curve of phase difference ϕ versus physical parameter p for another test medium is specified in advance, then the physical parameter p can be estimated from the measurement of ϕ.

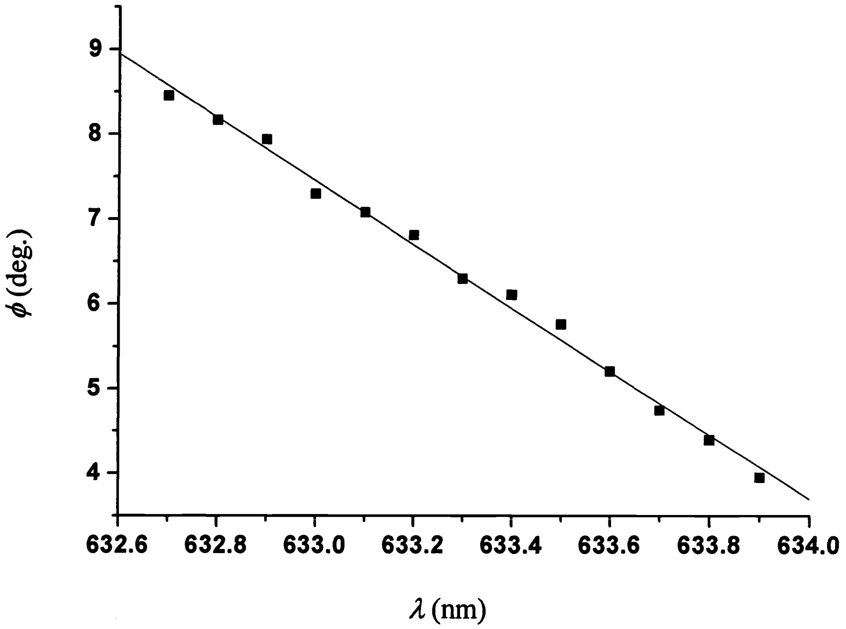

Fig. 4 Measurement data and the fitting curve of ϕ versus λ.

4. DISCUSSION

As θ is equal or very near to the resonant angle θ_{sp}, hence the reflection coefficient r_p is very small. To enhance the contrast of the test signal, the azimuth angle of the analyzer should be chosen moderately. In our experiments, it is set to 10°, the contrast of the test signal is about 0.88.

Considering the second harmonic error, the polarization-mixing errors, and the angular resolution of the phase meter, the total phase difference errors $\Delta\phi$ can be decreased to 0.03° in our experiments [11]. Substituting this data and the corresponding slope of each fitting curve into the equation

$$\Delta\phi = s\Delta p, \tag{10}$$

the error Δp of the associated physical parameter can be calculated. They are 2.75×10^{-2} mg/ml, 3.27×10^{-2} mg/ml, 6.33×10^{-2} mg/ml, 8.24×10^{-2} mg/ml, and 8.53×10^{-3} nm for the concentration measurements and the wavelength measurements, respectively.

Although the measurement resolution enhances as the thin gold film becomes thicker, both the intensity and the contrast of the test signal will be decreased rapidly [12]. To compromise for these conditions, d = 35 nm is chosen in our experiments.

5. CONCLUSION

In this paper, an alternative method for measuring physical parameters has been proposed by using the surface plasmon resonance effect and the heterodyne interferometry. A linearly polarized light enters a surface plasmon resonance apparatus of Kretschmann configuration. If the incident angle is equal or near the resonant angle, the surface plasmons are excited. At this time, the phase difference between p- and s- polarizations of the reflected light relates to the physical parameter. It can be measured accurately with the heterodyne interferometry. A surface plasmon resonance apparatus with thin gold film of thickness 35 nm is used to measure the concentrations of salt-water, glucose solution, acetone, ethanol, and the wavelengths of a tunable laser. Their resolutions are 2.75×10^{-2} mg/ml, 3.27×10^{-2} mg/ml, 6.33×10^{-2} mg/ml, 8.24×10^{-2} mg/ml, and 8.53×10^{-3} nm, respectively. This method can be realized that if the relation curve of phase difference versus physical parameter for another test medium is specified in advance, then the physical parameter can be estimated from the measurement of the phase difference. It has the advantages of both common-path interferometry and heterodyne interferometry, such as high stability, high resolution, and real-time operation.

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