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The pricing of deposit insurance considering bankruptcy costs and closure policies

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ABSTRACT

The paper aims to study the pricing issue of deposit insurance with explicit consideration of bankruptcy costs and closure policies. Full coverage from deposit insurance is imposed by many regulators to stabilize the banking system in the current financial crisis, despite of the potential moral hazard problems. We argue that bankruptcy cost is an important factor in pricing deposit insurance, especially when the insured institution is insolvent. Applying the isomorphic relationship between deposit insurance and put option, we first derive a closed-form solution for the pricing model with bankruptcy costs and closure policies. Then, we modify the barrier option approach to price the deposit insurance in which the bankruptcy cost is set as a function of asset return volatility and more realistic closure policies considering possible forbearance can be accounted for. The properties of the models are supported by numerical simulations and are consistent with the risk-based pricing scheme.

1. Introduction

Deposit insurance is an important scheme that many countries implement to prevent banks runs, to control financial crises, and to stabilize the financial system. According to Demirgüç-Kunt and Kane (2002), by 1999 more than 70 countries had established deposit insurance systems in which governments provide a safety net for depositors. As of 2008, based on the statistics from the International Association of the Deposit Insurers, 103 countries have instituted an explicit deposit insurance system (EDI), and another 16 countries are studying or considering implementing one.

Laeven (2002) describes two advantages if EDI is used. First, an EDI scheme sets the rules of the game concerning coverage, participants, and funding. Second, the existence of this system not only protects small depositors without immediate impact on the government budget, but also shields the economy and society from major fluctuations in the markets.

One of the reasons why bank runs often cause the contagion effect is due to information asymmetry in the financial markets. It can make event risk evolve into systematic risk, which leads not only to losses for depositors but also to recessions for the economy. EDI, in effect, creates a firewall that can shield the economy from a financial crisis. Its significance is further evidenced during the recent financial turmoil which underscores the need to find ways to make the financial system more resilient and stable. Many governments thus impose the EDI system by extending the full coverage of deposits to all depositors in order to stabilize the banking system.

However, as Laeven (2002) also points out, EDI brings about moral hazard problems. Under the protection of EDI, depositors have little incentives to exert market discipline on banks, and thus banks are encouraged to take excessive risks. Concerns of moral hazard issues have been discussed repeatedly within the deposit insurance literature, and the insurer needs to discourage banks' risk-taking behavior and should be able to close problematic banks in a timely fashion. VanHoose (2007) reviews theories of bank behavior under capital regulation considering moral hazard issue and the role of deposit insurance. Researchers have found that institutional environments and fair pricing of deposit insurance are crucial in alleviating the moral hazard problems.

With regard to environmental factors, the effectivity of EDI has been considered as country-specific. Moral hazards and other incentive problems created by existing governmental deposit insurance schemes differ among various countries. Demirgüç-Kunt and Detragiache (2002) provide cross-country evidence that EDI increases the probability of banking crises in countries with weak institutional environments. Demirgüç-Kunt and Kane (2002) as well as Laeven (2002) infer a similar conclusion. The financial regulatory environment is also affected by difference in market

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discipline which increases bank incentives to maintain adequate capital positions and avoid excessive risk (Goldberg and Hudgins, 2002; Spiegel and Yamori, 2007). On the other hand, Hulzinga and Nicodeme (2006) suggest that international competition in the area of deposit insurance design will not affect external liabilities of banks.

For the consideration of fair pricing, how to make the premium of the deposit insurance properly reflect the risk of the insured bank has long been the focus in controlling the bank's risk-taking behavior. The record losses in the late 1980s for the insurance funds of banks and savings and loans further reveal the weakness of the flat-rate pricing system, which has led to the consensus that risk-based deposit insurance is more equitable and economically supportable (Bloecher et al., 2003). The Federal Deposit Insurance Corporation Improvement Act of 1991 (FDICIA) requires that a risk-based premium system be implemented. Recent practice in deposit insurance pricing has utilized supervisory ratings (e.g. CAMELS rating), statistical models (e.g. failure-prediction model) and the combination of the two for risk classification. Moreover, the FDIC uses market measure of risk like credit ratings from Moody's, S&P, and Fitch, or default frequency from KMV to categorize the large banks. Bloecher et al. (2003) claim that combining these models with a scorecard can achieve the right balance among policy objectives and attributes aimed by FDIC such as accuracy, simplicity, flexibility, appropriate incentives and fairness.

In theory, deposit insurance has typically been modeled as a put option since Merton (1977). It is written on the bank's assets by the deposit insurer and held by bank shareholders under the Black-Sholes option pricing framework (Merton, 1978; Marcus and Shaked, 1984; Pyle, 1986; Ronn and Verma, 1986; Thomson, 1987; Episcopos, 2004). Using a structural approach, these models treat default as an event in which the market value of the bank's assets, typically modeled as a diffusion process, is insufficient to meet its liabilities, so bankruptcy is determined endogenously.

In contrast to the structural approach, Duffie et al. (2003) propose a reduced-form model, treating default as a stopping time whose arrival intensity may depend, in an exogenously parameterized fashion, on such covariates as leverage, credit rating, or macroeconomic conditions. Their results show that the approximation of fair-market deposit insurance rates can be the product of a bank's short-term credit spread and the ratio of the insurer's expected loss at failure per dollar of assessed deposit to the bond investors' expected loss at failure per dollar of principal.

However, these models have usually ignored the possibility of regulatory forbearance, assumed a limited term for the option contract, and treated the risk as an exogenous variable. In general, these models overlook the important agency issues that arise from the regulatory environment in a multilateral nexus of contracts. For example, these pricing models for deposit insurance have been shown to underestimate the benefits acquired by bank stockholders (Kane, 1995; Laeven, 2002). Subsequent studies, therefore, have exerted efforts to reflect the various features of bank and regulator behavior (Ronn and Verma, 1986; Pennacchi, 1987a,b; Allen and Saunders, 1993; Hovakimian and Kane, 2000).

In particular, Allen and Saunders (1993) take into account insurer's closure policy and insolvent bank's self-closure policy, and model the deposit insurance as a callable perpetual American put. When it comes to the right to exercise, the deposit insurance is actually not a standard put. That is, even if the option expires in the money, bank shareholders may choose not to exercise because it implies voluntary bank closure. In their model, the closure decision is used to control the timing to exercise. Forbearance is considered as forfeiture of the call value of the deposit insurance compound option. This model, though complete and comprehensive, contains the unappealing result that beyond a certain level the riskier bank may be charged with lower deposit insurance premium.

When the bank cannot pay off its liabilities, bankruptcy costs occur due to the loss of its franchise value or its charter value (Cummins et al., 1995). In this paper, we believe that the bankruptcy cost should be explicitly incorporated in the pricing framework of deposit insurance. Buser et al. (1981) have remarked that both the risks of bank closure and the costs of bankruptcy are what deposit insurers have to face. Dreyfus et al. (1994) claim that to transfer a bank's assets or to enforce an early liquidation may be costly if the bank is insolvent. Though forbearance and closure policy are implicitly affected by the bankruptcy cost consideration, its impact is not fully explored in Allen and Saunders (1993).

Williamson (1988) points out that bankruptcy costs are related to special assets such as intangible assets including brands, R&D, advertisements, etc., which could increase the firm's value when it is normally operating. Various studies (Warner, 1977; Altman, 1984; Weiss, 1990; Franks and Torous, 1994; Betker, 1997; Branch, 2002) have shown that this kind of direct bankruptcy costs is about 3% to 4.5% of the firm's market value. However, the indirect impact of bankruptcy on related stakeholders, though difficult to measure, may cost more. The evidence in the literature shows it to be higher than 10% (Altman, 1984; Andrade and Kaplan, 1998; Cutler and Summers, 1988; Opler and Titman, 1994).

It is generally considered that the overall costs of bank failures are higher than those in other industries. This is due to the existence of negative externalities of bank failures, which may lead to systemic risk. According to Gendreau and Prince (1986), direct costs of bankruptcy in large US banks during the period 1929–1933 amounted to 6% of the liabilities and were higher than those in non-financial firms. James (1991) has examined the losses realized in 412 bank failures during the period from 1985 to mid-year 1988. The losses are measured as the difference between the book value of a bank's assets at the time of its closure and the value of the assets in an FDIC receivership or the value of the assets to an acquirer. He finds the loss on assets is substantial, averaging 30% of the failed bank's assets. Direct expenses associated with bank closures average 10% of the assets. Bordo et al. (1996) have calculated the loss rates, defined as the ratio of total losses ultimately experienced by depositors of the failing banks in a given year to the total deposits during that year, as higher than 40% in Canada for 4 out of 55 years between 1870 and 1925. With these substantial bankruptcy costs, it is interesting to note that few studies have considered this factor in the pricing of deposit insurance. The key role of bankruptcy cost in deposit insurance pricing, therefore, is worth exploring and should be further studied.

With respect to the closure policy, Allen and Saunders (1993) assume that the FDIC's closure rule is strictly observed and that no additional forbearance, except in the case of the largest 'too big to fail' banks, is ever granted beyond the regulatory closure point. This is not in accordance with our experience in practice. Kane (1986), who considers the cost of supervision, suggests that the deposit insurer cannot help but forbear to put the insured bank, which is insolvent, in a closure decision or bankruptcy enforcement. Brockman and Turtle (2003) argue that, on the other hand, creditors seldom wait for debt to mature but force the firm into bankruptcy before the value of the assets disappears. No matter what occasion the reality would be, it is beyond controversy that the deposit insurer has the right to enforce premature exercise of the option at any time by calling the put option and thus closing the bank. Since the put writer has the power to set closure policy which makes the option expire whenever the regulatory closure point is knocked, the deposit insurance could be priced as a down and out barrier option, in which the lower barrier is defined by the regulatory closure rule. Episcopos (2008) recently applies barrier option framework and show how the FDIC can use the model as an insurance management tool in addition to a deposit insurance premium schedule.

The purpose of this paper is to study the pricing issue of deposit insurance with explicit consideration of bankruptcy costs and more realistic closure rules. We first extend the callable perpetual American put option model in Allen and Saunders (1993) to price deposit insurance, with the essential bankruptcy cost being an important factor in the pricing framework. A new closed-form solution is derived and numerical simulations are conducted to explore its properties. One of the results from this model is that the value of the deposit insurance, after incorporating the bankruptcy costs, increases monotonically with the risk of the bank. In addition, we further adopt the barrier option approach to price deposit insurance while assuming more realistic closure policies, and the variable bankruptcy cost as a function of asset return volatility. After taking into account the practice in closure rules, the results still coincide with our intuition that the more risks the banks hold, the higher deposit insurance premium the banks will pay. The results from both models solve the inconsistency in Allen and Saunders (1993) as discussed earlier.

The remainder of this paper is organized as follows: Section 2 extends the put option deposit insurance model to incorporate the bankruptcy costs. The barrier option is directly applied in Section 3 to price the deposit insurance while considering more practical closure practice. Section 4 provides numerical simulations for the properties of the deposit insurance developed in earlier sections. Our conclusion is given in Section 5 and some proofs are detailed in Appendix A.

2. Deposit insurance pricing with bankruptcy costs

2.1. Basic setup

In the past three decades, it has become increasingly popular to analyze the deposit insurance using the put option model (Merton, 1977; Marcus and Shaked, 1984; Ronn and Verma, 1986; Pennacchi, 1987a,b; Duan et al., 1992). In this approach, the option value is exercised as soon as the insured institution becomes insolvent. Considering the fact that the insuring agent could terminate the put prematurely, Allen and Saunders (1993) have developed a model in which the deposit insurance is evaluated as a callable perpetual American put.

We assume that the value of a bank's assets, which is normalized by its deposits[1] and denoted by a, follows a logarithmic diffusion process. Without bankruptcy costs, the payout by the deposit insurer is $\max(0, 1-a)$ when the put is exercised. If it is not called in advance, we can derive the value of this non-callable perpetual American put according to Allen and Saunders (1993):

$$p(a,\infty;1)=\frac{1}{1+\gamma}\left[\frac{(1+\gamma)a}{\gamma}\right]^{-\gamma}, \tag{1}$$

where

$$\gamma = 2r/\sigma^2 > 0,$$

r: the risk-free rate, σ: the standard deviation of the market value of the bank's assets.

However, Eq. (1) does not reflect the true value of the deposit insurance because the insurer actually has the right to call. The put writer will call the put at an asset ratio denoted by $\bar{a}$ which is larger than bank's optimal exercise point. Only if the bank does not exercise the put in deposit insurance will the writer's call provision exhibit value. That is, the call provision has positive value only if the deposit insurer exercises it at a point $\bar{a} > x$, where x is the optimal self-closure point for banks. If the bank's self-closure point is optimal, the value of assets per dollar of deposits will reach the maximum. If the bank exercises the put option before the deposit insurer calls, i.e. $\bar{a} \leqslant x$, the deposit insurer will lose the right to call the put option.

Therefore, the FDIC's call provision has value only if the bank chooses not to prematurely exercise the deposit insurance put. In practice, the time to claim the closure of the bank is determined by the deposit insurer, and not by the stockholders. Thus $\bar{a} > x$ is the most likely case.

Similarly, as in Allen and Saunders (1993), we obtain the value of the call provision of the perpetual put in the case $\bar{a} > x$ as follows:

$$c(a,\infty;1)=a^{-\gamma}[\bar{a}^{\gamma+1}-\bar{a}^{\gamma}+(1-x)x^{\gamma}], \quad x<a\leqslant\bar{a}. \tag{2}$$

Finally, consider the case in which the deposit insurer retains the right to call. Determining the premium of the deposit insurance is then just like pricing a callable perpetual American put option. Subtracting the value of the call provision from the value of the non-callable put, we obtain the value of the callable perpetual American put option. The premium of the deposit insurance is then:

$$i(a,\infty;1)=(1-\bar{a})\left(\frac{a}{\bar{a}}\right)^{-\gamma}, \tag{3}$$

which is the value of a callable perpetual American put option with a premature exercise price of $\bar{a}$.

However, when a bank encounters bankruptcy, the value of its assets becomes lower due to the difficult circumstances. It will be hard for the deposit insurer to sell the assets at their book values, and thus its payouts to the insured depositors will exceed $\max(0, 1-a)$. Therefore, the deposit insurer will bear the bankruptcy costs.

Taking bankruptcy cost into consideration, we assume that k_x is the discount factor at the optimal exercise point or the self-closure point, x, and $0<k_x\leqslant 1$. Thus, the insurer's total payoff with bankruptcy discount is $1-k_x x$ at this time which is higher than the case without bankruptcy cost, i.e., $1-x$. The bankruptcy cost is therefore $(1-k_x)x$. Similarly $k_{\bar{a}}$ is the discount factor at the regulatory closure point, $\bar{a}$, and $0<k_{\bar{a}}\leqslant 1$. We will consider the variable rather than the constant bankruptcy costs at different closure points because the bank's self-closure point and the regulatory closure point normally occur at different times. Also, different institutions may choose different closure points which will give rise to different bankruptcy costs for their own considerations.

Denote $p_{bc}(a,\infty;1)$ as the value of a non-callable perpetual American put with bankruptcy costs. Following Merton (1973), $p_{bc}(a,\infty;1)$ satisfies the following differential equation:

$$0.5\sigma^2a^2p''_{bc}+rap'_{bc}-rp_{bc}=0, \tag{4}$$

subject to the following boundary conditions:

$$p_{bc}(\infty,\infty;1)=0, \tag{5}$$
$$p_{bc}(x,\infty;1)=1-k_x x. \tag{6}$$

Boundary condition (6) expresses the difference between our bankruptcy cost model and that of Allen and Saunders (1993). We assume that the option will be exercised as banks "fail" in our model. When the bankruptcy cost is incurred, the asset value is reduced to $k_x x$ at the exercise point. Therefore, the bankruptcy cost is incorporated in the boundary condition (6).

2.2. The value of non-callable deposit insurance with bankruptcy costs

With the setup as described in the previous section, we can obtain the value of a non-callable perpetual American put with bankruptcy costs.

[1] In the normalized model, the exercise price for the put option is one.

Lemma 1

$$p_{bc}(a,\infty;1) = (1-k_x x)\left(\frac{a}{x}\right)^{-\gamma}, \tag{7}$$

where $\gamma = 2r/\sigma^2$.

Proof. See Appendix A.

Since x denotes the optimal asset value for the bank to exercise its deposit insurance put, we have the following result:

Lemma 2

$x = \gamma/[(1+\gamma)k_x]$.

Proof. See Appendix A.

From Lemmas 1 and 2, the value of the non-callable perpetual American put with bankruptcy costs is:

Lemma 3

$$p_{bc}(a,\infty;1) = \frac{1}{1+\gamma}\left[\frac{(1+\gamma)k_x a}{\gamma}\right]^{-\gamma}. \tag{8}$$

Proof. See Appendix A.

Comparing Eq. (1) with Eq. (8), given $0 < k_x \leqslant 1$, we can find that

$P_{bc}(a,\infty;1) > p(a,\infty;1)$.

The relationship between p_{bc} and k_x can be presented as follows:

Proposition 1

$$\frac{\partial p_{bc}}{\partial k_x} = -\left(\frac{1+\gamma}{\gamma}k_x\right)^{-(1+\gamma)} a^{-\gamma} \leqslant 0. \tag{9}$$

The negative slope indicates that p_{bc} is increasing for a given decrease in k_x, or for a given increase in $1-k_x$. So the value of the deposit insurance as a put option increases with higher bankruptcy costs.

Now, consider the case in which the regulator calls the put at the regulatory closure point $\bar{a}$ with $k_{\bar{a}}$ as the discount factor for the bankruptcy costs. The call provision of the perpetual put with bankruptcy costs [denoted as $c_{bc}(a,\infty;1)$] will satisfy the following differential equation:

$$0.5\sigma^2 a^2 c''_{bc} + rac'_{bc} - rc_{bc} = 0. \tag{10}$$

subject to the following boundary conditions:

$$c_{bc}(\infty,\infty;1) = 0 \tag{11}$$

$$c_{bc}(\bar{a},\infty;1) = k_{\bar{a}}\bar{a} - 1 + (1-k_x x)\left(\frac{\bar{a}}{x}\right)^{-\gamma}. \tag{12}$$

Thus,

$$c_{bc}(a,\infty;1) = a^{-\gamma}[k_{\bar{a}}\bar{a}^{\gamma+1} - \bar{a}^{\gamma} + (1-k_x x)x^{\gamma}], \quad x \leqslant \bar{a},$$

where $\gamma = 2r/\sigma^2$ and $k_{\bar{a}} > [1-(1-k_x x)(x/\bar{a})^{\gamma}]/\bar{a}$. Substituting bank's optimal $x = \gamma/[(1+\gamma)k_x]$, we can get:

Lemma 4

$$c_{bc}(a,\infty;1) = a^{-\gamma}\left[k_{\bar{a}}\bar{a}^{\gamma+1} - \bar{a}^{\gamma} + \frac{\gamma^{\gamma}}{(1+\gamma)^{1+\gamma}k_x^{\gamma}}\right]. \tag{13}$$

Two comparative statics results can be derived. First, as Buser et al. (1981) point out, banks will transfer the bankruptcy cost to the insurer through deposit insurance. When the bankruptcy cost is higher, i.e., as k_x becomes smaller, *ceteris paribus*, the deposit insurer pays more to the banks. Therefore, the value of the call provision of the deposit insurance increases monotonically with the increase in the bankruptcy costs at optimal exercise point x.

Proposition 2

$$\frac{\partial c_{bc}}{\partial k_x} = \begin{cases} -\left(\frac{1+\gamma}{\gamma}k_x\right)^{-(1+\gamma)} a^{-\gamma} < 0, & k_x < \frac{1-(1-k_{\bar{a}}\bar{a})(\bar{a}/x)^{\gamma}}{x} \\ 0, & \text{otherwise}. \end{cases} \tag{14}$$

Secondly, if the deposit insurer chooses to close a bank, it will try to lower the bankruptcy costs in the process, i.e. the smaller the value of $1-k_{\bar{a}}$, the smaller the subsidy provided by the deposit insurer to the bank's stockholders. In other words, the amount the deposit insurer pays to the bank is smaller. Therefore, the value of the call provision of the deposit insurance subsidy increases monotonically with decreases in the bankruptcy costs at regulatory closure point $\bar{a}$. That is,

Proposition 3

$$\frac{\partial c_{bc}}{\partial k_{\bar{a}}} = \begin{cases} \frac{\bar{a}^{1+\gamma}}{a^{\gamma}} > 0, & k_{\bar{a}} > \frac{1-(1-k_x x)(x/\bar{a})^{\gamma}}{\bar{a}} \\ 0, & \text{otherwise}. \end{cases} \tag{15}$$

Therefore, we have a positive slope that is totally different from the one shown in Allen and Saunders (1993).

2.3. The value of callable deposit insurance with bankruptcy costs

The value of the federal deposit insurance to bank stockholders with explicit consideration of bankruptcy costs [denoted as $i_{bc}(a,\infty;1)$] is obtained by subtracting the value of the call provision with bankruptcy costs from the value of the non-callable perpetual American put with bankruptcy costs. That is, $i_{bc}(a,\infty;1) = p_{bc}(a,\infty;1) - c_{bc}(a,\infty;1)$. Then

Lemma 5

$$i_{bc}(a,\infty;1) = (1-k_{\bar{a}}\bar{a})\left(\frac{a}{\bar{a}}\right)^{-\gamma}. \tag{16}$$

We next analyze the relationship between the value of the deposit insurance and the asset risk of the bank as follows:

Proposition 4

$$\frac{\partial i_{bc}}{\partial \sigma} = (1-k_{\bar{a}}\bar{a})\left(\frac{a}{\bar{a}}\right)^{-\gamma}\left(\frac{4r}{\sigma^3}\right)\ln\left(\frac{a}{\bar{a}}\right) \geqslant 0. \tag{17}$$

All the terms being positive as $\bar{a} < a$, Eq. (17) is greater than zero. This result is consistent with that of Allen and Saunders (1993). The more risk the bank has, the higher the deposit insurance premium is. This conforms to the spirit of risk-adjusted premium and the principle of fairness.

Furthermore, to consider the impact of bankruptcy cost on the deposit insurance premium, we compare Eq. (3) with Eq. (16). It is easy to see that $i_{bc}(a,\infty;1) \geqslant i(a,\infty;1)$ under $0 < k_{\bar{a}} \leqslant 1$. The deposit insurance premium Allen and Saunders (1993) describe is our special case with $k_{\bar{a}} = 1$, which is also our minimum value. So the firm exhibiting higher bankruptcy costs should be charged a higher deposit insurance premium.

From Eq. (16), we can obtain the following result:

Proposition 5

$$\frac{\partial i_{bc}}{\partial k_{\bar{a}}} = \begin{cases} -\frac{\bar{a}^{1+\gamma}}{a^{\gamma}} < 0, & k_{\bar{a}} > \frac{1-(1-k_x x)(x/\bar{a})^{\gamma}}{\bar{a}} \\ 0, & \text{otherwise}. \end{cases} \tag{18}$$

It is interesting to note that Eq. (16) seems irrelevant to k_x. Nevertheless, the deposit insurance premium is obtained by subtracting $c_{bc}(a,\infty;1)$ from $p_{bc}(a,\infty;1)$, so we have,

Proposition 6

$$\frac{\partial i_{bc}}{\partial k_x}=\frac{\partial p_{bc}}{\partial k_x}-\frac{\partial c_{bc}}{\partial k_x}=\begin{cases}-\left(\frac{1+\gamma}{\gamma}k_x\right)^{-(1+\gamma)}a^{-\gamma}<0, & k_x>\frac{1-(1-k_{\bar{a}}\bar{a})(\bar{a}/x)^{\gamma}}{x}\\ 0, & \text{otherwise}.\end{cases} \tag{19}$$

Thus, a higher deposit insurance premium is required for higher bankruptcy costs.

Finally, this paper resolves the puzzling result in Allen and Saunders (1993). That is, the deposit insurance premium decreases as the bank risk increases beyond a certain level. Suppose the bankruptcy cost can be expressed as a function of the bank risk, i.e., $k_{\bar{a}}(\sigma)$. We call it "risk-based bankruptcy cost". Define $k_{\bar{a}}\equiv k_{\bar{a}}(\sigma)$ where $k'_{\bar{a}}(\sigma)<0$. It is reasonable to assume that the bankruptcy cost increases with the bank risk. Thus, from Eq. (17), we have:

$$\frac{\partial i_{bc}}{\partial\sigma}=(-k'_{\bar{a}}(\sigma)\bar{a})\left(\frac{a}{\bar{a}}\right)^{-\gamma}+\left(\frac{a}{\bar{a}}\right)^{-\gamma}(1-k_{\bar{a}}\bar{a})\left(\frac{a}{\bar{a}}\right)^{-\gamma}\left(\frac{4r}{\sigma^3}\right)\ln\left(\frac{a}{\bar{a}}\right)\geqslant 0. \tag{20}$$

3. Pricing deposit insurance considering closure policies

3.1. Down-and-out put option and deposit insurance

In this section, we further apply the down-and-out put option to elaborate the regulatory closure point defined as the lower barrier of the option for the deposit insurance. If the value of the underlying asset knocks this lower barrier, the option issuer, i.e., the FDIC, has the right to close the bank.

Lemma 6. Suppose X_t follows a Brownian motion, i.e.

$$dX_t=\mu dt+\sigma dZ_t^Q, \tag{21}$$

where Q represents the risk neutral probability measure, Z_t is a white noise, and μ, σ are constants. Let $m_t=\min_{0\leqslant s\leqslant t}X_s$. We get the following distributions for pricing the knockout put option:

$$\begin{aligned}\Pr(X_T\leqslant X, m_T\geqslant B)&=\left[\Phi\left(\frac{-B+\mu T}{\sigma\sqrt{T}}\right)-\Phi\left(\frac{-X+\mu T}{\sigma\sqrt{T}}\right)\right]\\&\quad-e^{2\alpha B}\left[\Phi\left(\frac{B+\mu T}{\sigma\sqrt{T}}\right)-\Phi\left(\frac{-X+2B+\mu T}{\sigma\sqrt{T}}\right)\right],\end{aligned} \tag{22a}$$

$$\Pr(m_T<B)=\Phi\left(\frac{B-\mu T}{\sigma\sqrt{T}}\right)+e^{2\alpha B}\Phi\left(\frac{B+\mu T}{\sigma\sqrt{T}}\right), \tag{22b}$$

where B is the barrier and τ is the first passage time for X_t to hit B.

Also,

$$E(e^{-r\tau})=\left(\frac{B}{X_0}\right)^{\alpha+\beta}\Phi(z_1+2\beta\sigma\sqrt{T})+\left(\frac{B}{X_0}\right)^{\alpha-\beta}\Phi(z_1), \tag{22c}$$

where $X_0\geqslant B$. Here $\Phi(\bullet)$ is the cumulative standard normal distribution function, $\alpha=\mu/\sigma^2$, $\beta=\sqrt{\mu^2+2r\sigma^2}/\sigma^2$ and $z_1=\ln(B/X_0)/\sigma\sqrt{T}-\beta\sigma\sqrt{T}$.

Suppose that the value of a bank's assets follows a logarithmic diffusion process. Denote the value of the normalized assets (assets/deposits) at time t as a_t, and $\tilde{a}_t=\min_{0\leqslant s\leqslant t}a_s$. The value of the down-and-out put option (DOP) can be expressed as

$$\text{DOP}=e^{-rT}E^Q[\max(1-a_T,0)\cdot 1_{\{\tilde{a}_T\geqslant\bar{a}\}}], \tag{23}$$

where a_T is the value of the normalized assets at time T, E^Q denotes the expectation conditional on all information at time 0 with respect to the risk neutral probability measure Q, r is the continuously compounded risk-free rate over the period considered, $1_{\{\}}$ is the indicator function, $\bar{a}$ is the regulatory closure point (i.e. the value of the barrier), and T is the time to maturity of the option.

However, the value of the down-and-out put option (DOP) as in Eq. (23) does not fully express the value of the deposit insurance premium. As a standard DOP, if the barrier is touched and the option is still not exercised, not only the option is valueless, but also the right to sell the underlining assets is lost. This is not quite the same as the case for the deposit insurance. Once the asset value of a bank reaches the insurer's regulatory closure point, FDIC will exercise its right to close the bank. FDIC then takes over the bank and implements the bankruptcy process.[2] However, the depositor can still get a rebate based on the difference between the regulatory closure point and the deposit insurance amount. For the insurer, this rebate should also be reflected in the premium. Eq. (23) does not include this part, and thus we use Eq. (24) to describe the premium of the deposit insurance:

$$\tilde{i}^{\text{DOP}}=e^{-rT}E^Q[\max(1-a_T,0)\cdot 1_{\{\tilde{a}_T\geqslant\bar{a}\}}]+E^Q[e^{-r\tau}(1-\bar{a})\cdot 1_{\{\tilde{a}_T<\bar{a}\}}], \tag{24}$$

where $\tilde{i}^{\text{DOP}}$ is the value of the deposit insurance premium using DOP model, and τ is the first passage of time for a_t to hit $\bar{a}$. Using (22a) and (22c) in Lemma 6 and applying Rubinstein and Reiner (1991), we can derive a closed-form solution to Eq. (24):

$$\begin{aligned}\tilde{i}^{\text{DOP}}&=[-a_0e^{-qT}\Phi(x_1+\sigma\sqrt{T})+e^{-rT}\Phi(x_1)]\\&\quad-[-a_0e^{-qT}\Phi(x_2+\sigma\sqrt{T})+e^{-rT}\Phi(x_2)]\\&\quad+[-a_0e^{-qT}(\bar{a}/a_0)^{2(\gamma+1)}\Phi(x_3+\sigma\sqrt{T})+e^{-rT}(\bar{a}/a_0)^{2\gamma}\Phi(x_3)]\\&\quad-[-a_0e^{-qT}(\bar{a}/a_0)^{2(\gamma+1)}\Phi(x_4+\sigma\sqrt{T})+e^{-rT}(\bar{a}/a_0)^{2\gamma}\Phi(x_4)]\\&\quad+(1-\bar{a})[(\bar{a}/a_0)^{\gamma+\delta}\Phi(z_2+2\delta\sigma\sqrt{T})+(\bar{a}/a_0)^{\gamma-\delta}\Phi(z_2)],\end{aligned} \tag{25}$$

where q is the continuous dividend yield paid by the bank, σ is the standard deviation of the market value of the bank assets, and

$$\begin{aligned}x_1&=\frac{\ln(a_0/\bar{a})+(r-q-0.5\sigma^2)T}{\sigma\sqrt{T}},\\x_2&=\frac{\ln a_0+(r-q-0.5\sigma^2)T}{\sigma\sqrt{T}},\\x_3&=\frac{\ln(\bar{a}^2/a_0)+(r-q-0.5\sigma^2)T}{\sigma\sqrt{T}},\\x_4&=\frac{\ln(\bar{a}/a_0)+(r-q-0.5\sigma^2)T}{\sigma\sqrt{T}},\\z_2&=\frac{\ln(\bar{a}/a_0)}{\sigma\sqrt{T}}-\delta\sigma\sqrt{T},\\\gamma&=\frac{r-q}{\sigma^2}-\frac{1}{2},\\\delta&=\frac{\sqrt{(r-q-0.5\sigma^2)^2+2r\sigma^2}}{\sigma^2}\end{aligned}$$

Eq. (24) presumes that FDIC will take over the bank and close it when the bank's asset value is lower than the regulatory closure point. However, in practice, the FDIC often does not close the bank, but keeps it in operation to stabilize the financial system. Therefore, in the following discussions we shall refer to $\bar{a}$ as the "asset regulatory point", and not the regulatory closure point. Since the main function of the deposit insurance is to enable the FDIC to absorb the bankruptcy cost of banks, this cost should be included in the deposit insurance premium.

[2] We assume that the bank is closed and that the rebate applies when the FDIC takes over the bank.

3.2. Pricing deposit insurance under realistic closure policy

The FDIC has to consider both the bank closure risk and the bankruptcy cost (Buser et al., 1981). Insolvent financial institutions give rise to costly public policy problems (Allen and Saunders, 1993). Deposit insurance is therefore different from a traditional option. We first take the bankruptcy cost into account for deposit insurance pricing. Besides that, on the part of forbearance, as Kane (1986) has suggested, the FDIC considers the cost of supervision in relation to the insured bank's closure point which is one type of capitalization forbearance. Kaufman (1992) remarks that the government and the supervisory authorities do not think it wise to close a bank because of the need to keep the economy stable. Allen and Saunders (1993) mention in their footnote 12 that "Forbearance is granted whenever the FDIC fails to enforce its known regulatory closure point". With Eq. (24), we have attempted to include these considerations in our model.

First of all, once the asset value of a bank is lower than the closure point set by the insurer, the FDIC may not close the bank right away but will cancel its exercise right by taking over the bank. We already refer to this point as the asset regulatory point. We assume that the FDIC will close the bank and provide the rebates when the option is due, but not when the value of its assets hits the asset regulatory point. Let $(1-k_{\bar{a}})$ be the asset regulatory point chosen by the deposit insurer. This means that bankruptcy costs include all the costs during the whole process of closing the bank. Furthermore, $(1-k_a)$ is the self-closure point chosen by the bank.[3] We have to emphasize that our self-closure point is different from that of Allen and Saunders (1993). The self-closure point in Allen and Saunders (1993) is a function of the risk-free rate and the standard deviation of the market value of the bank's assets. In accordance with Eq. (24), we assume that the bank's asset/deposit ratio is higher than the asset regulatory point before the option is due, and less than one on the maturity date, i.e., $\bar{a} < a_t < 1, t \leqslant T$. We call this interval the "self-closure region". Under this environment we can show that the value of the deposit insurance premium is as follows:

$$\tilde{i}_{bc}^{\mathrm{MDOP}} = e^{-rT}E^{Q}[\max(1-k_a a_T, 0)\cdot 1_{\{\tilde{a}_T \geqslant \bar{a}\}}] + e^{-rT}E^{Q}[(1-k_{\bar{a}}\bar{a})\cdot 1_{\{\tilde{a}_T < \bar{a}\}}], \tag{26}$$

where MDOP refers to the modified down-and-out put option embedded in the rule regarding the rebate, and $\tilde{i}_{bc}^{\mathrm{MDOP}}$ is the value of the deposit insurance premium evaluated using MDOP approach with consideration of the bankruptcy cost. Similarly, based on (22a) and (22b) in Lemma 6, we can derive a closed-form solution for Eq. (26) as follows:

$$\begin{aligned}\tilde{i}_{bc}^{\mathrm{MDOP}} &= [-k_a a_0 e^{-qT}\Phi(x_1+\sigma\sqrt{T}) + e^{-rT}\Phi(x_1)] \\ &\quad - [-k_a a_0 e^{-qT}\Phi(x_2+\sigma\sqrt{T}) + e^{-rT}\Phi(x_2)] \\ &\quad + [-k_a a_0 e^{-qT}(\bar{a}/a_0)^{2(\gamma+1)}\Phi(x_3+\sigma\sqrt{T}) + e^{-rT}(\bar{a}/a_0)^{2\gamma}\Phi(x_3)] \\ &\quad - [-k_a a_0 e^{-qT}(\bar{a}/a_0)^{2(\gamma+1)}\Phi(x_4+\sigma\sqrt{T}) + e^{-rT}(\bar{a}/a_0)^{2\gamma}\Phi(x_4)] \\ &\quad + (1-k_{\bar{a}}\bar{a})e^{-rT}[\Phi(x_5) + (\bar{a}/a_0)^{2\gamma}\Phi(x_4)],\end{aligned} \tag{27}$$

where

$$x_5 = \frac{\ln(\bar{a}/a_0) - (r-q-0.5\sigma^2)T}{\sigma\sqrt{T}},$$

and other notations are the same as indicated earlier.

Both Eqs. (24) and (26) assume that a bank's asset/deposit ratio is higher than the asset regulatory point before the option expires, and that the asset/deposit ratio is less than one on the maturity date. Therefore the bank will close by itself on maturity. However, this assumption is not acceptable. There is no incentive for a bank with deposit insurance to close by itself. When the bank exercises the option with an asset/deposit ratio less than one, all the assets are used to pay the depositors and nothing is left for the shareholders of the bank. It is thus better for the stockholders of the bank to continue the business rather than close it. Besides, it is more difficult to determine the self-closure region than the regulatory closure point. This is against our common understanding.[4] Owing to these reasons and having considered the bankruptcy cost, we extend Eqs. (24) and (26), respectively, to Eqs. (28) and (29):

$$i_{bc}^{\mathrm{DOP}} = E^{Q}[e^{-r\tau}(1-k_{\bar{a}}\bar{a})\cdot 1_{\{\tilde{a}_T<\bar{a}\}}], \tag{28}$$

$$i_{bc}^{\mathrm{MDOP}} = e^{-rT}E^{Q}[(1-k_{\bar{a}}\bar{a})\cdot 1_{\{\tilde{a}_T<\bar{a}\}}]. \tag{29}$$

And their closed-form solutions are:

Lemma 7

$$i_{bc}^{\mathrm{DOP}} = (1-k_{\bar{a}}\bar{a})[(\bar{a}/a_0)^{\gamma+\delta}\Phi(z_2+2\delta\sigma\sqrt{T}) + (\bar{a}/a_0)^{\gamma-\delta}\Phi(z_2)], \tag{30}$$

and

$$i_{bc}^{\mathrm{MDOP}} = (1-k_{\bar{a}}\bar{a})e^{-rT}[\Phi(x_5) + (\bar{a}/a_0)^{2\gamma}\Phi(x_4)]. \tag{31}$$

Because the bankruptcy time point for i_{bc}^{DOP} will be earlier than the time point for i_{bc}^{MDOP}, assuming a constant bankruptcy cost, we can get $i_{bc}^{\mathrm{DOP}} > i_{bc}^{\mathrm{MDOP}}$.

We next discuss some of the properties of i_{bc}^{MDOP}, and compare it with i_{bc}^{DOP} under the same assumptions. For ease of analysis, we assume $q = 0$. First, we analyze the relationship between the value of the deposit insurance premium and the bankruptcy cost as follows:

Proposition 7

$$\frac{\partial i_{bc}^{\mathrm{MDOP}}}{\partial(1-k_{\bar{a}})} = -\frac{\partial i_{bc}^{\mathrm{MDOP}}}{\partial k_{\bar{a}}} = \bar{a}e^{-rT}[\Phi(x_5) + (\bar{a}/a_0)^{2\gamma}\Phi(x_4)] > 0. \tag{32}$$

Eq. (32) is consistent with our intuition. The bank exhibiting higher bankruptcy costs should be charged with a higher deposit insurance premium.

Secondly, we analyze the relationship between the value of the deposit insurance premium and the asset risk of the bank as follows:

Proposition 8

$$\begin{aligned}\frac{\partial i_{bc}^{\mathrm{MDOP}}}{\partial\sigma} &= (1-k_{\bar{a}}\bar{a})e^{-rT}\left\{\left(\frac{\ln(a_0/\bar{a}) + (r+0.5\sigma^2)T}{\sigma^2\sqrt{T}}\right)\phi(x_5)\right. \\ &\quad + \left[(\bar{a}/a_0)^{2\gamma}\frac{4r\ln(a_0/\bar{a})}{\sigma^3}\right]\Phi(x_4) \\ &\quad \left. + \left[(\bar{a}/a_0)^{2\gamma}\frac{\ln(a_0/\bar{a}) - (r+0.5\sigma^2)T}{\sigma^2\sqrt{T}}\right]\phi(x_4)\right\} > 0,\end{aligned} \tag{33}$$

where $\varphi(\cdot)$ is the standard normal probability density function. The sign of Eq. (33) depends on the last term only. However, with the value of the cumulative standard normal distribution function being much greater than that of its probability density function, it can be found that the sign of Eq. (33) is positive.

Finally, as in Section 2.3, we set the risk-based bankruptcy cost function $k_{\bar{a}}$ as $k_{\bar{a}}(\sigma)$, which is a function of the volatility of bank's asset value. Thus,

[3] It is difficult to determine the value of this variable. However, this is not important in our pricing model.

[4] Most likely the self-closure point is less than the regulatory closure point (Allen and Saunders, 1993).

$$\frac{\partial i_{bc}^{\text{MDOP}}}{\partial\sigma} = -k'_{\bar{a}}(\sigma)\bar{a}e^{-rT}[\Phi(x_5) + (\bar{a}/a_0)^{2\gamma}\Phi(x_4)] + (1 - k'_{\bar{a}}(\sigma)\bar{a})e^{-rT}\left\{\left(\frac{\ln(a_0/\bar{a}) + (r + 0.5\sigma^2)T}{\sigma^2\sqrt{T}}\right)\phi(x_5) + \left[(\bar{a}/a_0)^{2\gamma}\frac{4r\ln(a_0/\bar{a})}{\sigma^3}\right]\Phi(x_4) + \left[(\bar{a}/a_0)^{2\gamma}\frac{\ln(a_0/\bar{a}) - (r + 0.5\sigma^2)T}{\sigma^2\sqrt{T}}\right]\phi(x_4)\right\} > 0. \quad (34)$$

Eq. (33) indicates that the sum of the last three terms should be positive. With $k_{\bar{a}}(\sigma) < 0$, the first term of Eq. (34) is also positive. So Eq. (34) is positive.

4. Simulation results

In this section we conduct simulations to test the models developed above. The constant risk-free rate, r, is 0.0649 and the standard deviation of the market value of the bank's asset, σ, is 0.0963, both of which are taken from Ibbotson et al. (1985). The exogenously determined regulatory closure rule, $\bar{a} = 0.97$, is from Ronn and Verma (1986). The results are summarized in Table 1 and from Figs. 1 to 7.

In Fig. 1, we examine the relationship between the value of deposit insurance and the bankruptcy cost. The bankruptcy cost is interpreted by k_x and $k_{\bar{a}}$, which are the discount factors at the optimal self-closure point x and the regular closure point $\bar{a}$, respectively. The higher the values of k_x and $k_{\bar{a}}$, the less the bankruptcy costs. From Fig. 1, we observe that, *ceteris paribus*, the value of the non-callable perpetual put considering bankruptcy costs increases monotonically with the bankruptcy cost at the optimal exercise point x, indicating that the value of the deposit insurance increases with higher bankruptcy costs. This result is consistent with Eq. (9) in Proposition 1.

When the bankruptcy cost is higher, i.e., k_x becomes smaller, *ceteris paribus*, the deposit insurer pays more to the banks, and thus the value of the call provision in the deposit insurance increases monotonically with higher bankruptcy costs at optimal exercise point x. In Fig. 2, we can observe this property, which is consistent with Eq. (14) in Proposition 2. On the contrary, the value of the call provision decreases monotonically with increases in the bankruptcy cost[5] which is represented by the decreasing $k_{\bar{a}}$ at the regulatory closure point $\bar{a}$ as indicated by Eq. (15) in Proposition 3. Note that only if $x < \bar{a}$[6] and $k_{\bar{a}} > [1 - (1 - k_x x)(x/\bar{a})^{\gamma}]/\bar{a}$, will this call have a value greater than zero, as observed in Fig. 2.

Fig. 3 plots the relationship between the net value of deposit insurance, $i_{bc}(a,\infty;1)$, and the bankruptcy cost. The value, $i_{bc}(a,\infty;1)$, is obtained by subtracting the value of the call provision with bankruptcy costs, $c_{bc}(a,\infty;1)$, from the value of the non-callable perpetual American put with bankruptcy costs, $p_{bc}(a,\infty;1)$. It can be observed that the insurance premium gets higher when the values of $k_{\bar{a}}$ and k_x are close to zero, representing that the higher the bankruptcy costs, the higher the deposit insurance premium, which coincides with the scenarios of Eqs. (18) and (19) in Propositions 5 and 6.

In Fig. 4 we consider three functional forms for $k_{\bar{a}}(\sigma)$, which are (i) $k_{\bar{a}}(\sigma) = 1 - 0.05\sigma$; (ii) $k_{\bar{a}}(\sigma) = 1 - 0.25\sigma$; and (iii) $k_{\bar{a}}(\sigma) = 1 - 0.5\sigma$, respectively, to simulate how the value of deposit insurance with bankruptcy costs varies with bank asset risk as discussed in Eq. (20). Moving from (i) to (iii) represents increases of the bankruptcy cost at the regulatory closure point $\bar{a}$. From this figure, three valuable implications can be provided as follows. First, the value of the non-callable perpetual American put with bankruptcy costs, $p_{bc}(a,\infty;1)$, increases monotonically with the increase in the bank asset risk, yet displays the similar shape from (i) to (iii). Since they are not incorporated with the closure policy of the deposit insurer, the changes of $k_{\bar{a}}(\sigma)$ have no effects on them. Second, the value of the call provision of the deposit insurance also increases monotonically with the increase in the bank asset risk, but decreases monotonically from (i) to (iii), which means the call value will be lower as $k_{\bar{a}}$ decreases, and thus the bankruptcy cost increases at the exogenously determined regulatory closure point $\bar{a} = 0.97$. This result coincides with the property shown in Fig. 2 and Eq. (15) in Proposition 3. Third, the value of deposit insurance, with risk-based bankruptcy costs and with fixed $k_x = 1$, increases monotonically with the increase in the bank asset risk, which eliminates the inconsistency in Allen and Saunders (1993) that the riskier bank may be charged with a lower deposit insurance premium beyond a certain level. Furthermore, the value of deposit insurance, with risk-based bankruptcy costs and with fixed $k_x = 1$, increases monotonically from (i) to (iii), which represents that as $k_{\bar{a}}$ decreases and thus the bankruptcy cost increases, the value of the deposit insurance subsidy will increase. In sum, the features that these graphs display meet the expectation of our models.

In addition, setting four parameters fixed: $r = 0.0649$, $\sigma = 0.0963$, $x = 0.9332$, $\bar{a} = 0.97$, Table 1 depicts the simulated values of deposit insurance compound options under different bankruptcy costs. In Panel A or B, it is shown that with decreasing $k_{\bar{a}}$ and fixed k_x, the value of the call provision with bankruptcy costs, $c_{bc}(a,\infty;1)$, is decreasing, and the value of deposit insurance with bankruptcy costs, $i_{bc}(a,\infty;1)$, is increasing; yet the value of the non-callable perpetual American put with bankruptcy costs, $p_{bc}(a,\infty;1)$ remains unchanged. Turning to Panel C, as $k_{\bar{a}}$ is fixed but k_x is decreasing, $p_{bc}(a,\infty;1)$, $c_{bc}(a,\infty;1)$, and $i_{bc}(a,\infty;1)$ are all increasing. The major implications Table 1 presents are the following: First, the value of the non-callable perpetual American put with bankruptcy costs, $p_{bc}(a,\infty;1)$ has a positive relationship with the bankruptcy cost factor, $1 - k_x$. Next, the value of the call provision increases

Table 1
Simulated value of the compound option for deposit insurance with different bankruptcy costs.

x	k_x	$k_{\bar{a}}$	$p_{bc}(a,\infty;1)$	$c_{bc}(a,\infty;1)$	$i_{bc}(a,\infty;1)$
Panel A					
0.93332	1.00000	1.00000	0.02538	0.00579	0.01959
0.93332	1.00000	0.99950	0.02538	0.00548	0.01990
0.93332	1.00000	0.99900	0.02538	0.00516	0.02022
0.93332	1.00000	0.99850	0.02538	0.00484	0.02054
0.93332	1.00000	0.99800	0.02538	0.00453	0.02085
0.93332	1.00000	0.99750	0.02538	0.00421	0.02117
Panel B					
0.93332	0.60000	1.00000	0.16749	0.14790	0.01959
0.93332	0.60000	0.90000	0.16749	0.08457	0.08292
0.93332	0.60000	0.70000	0.16749	0.00000	0.16749
0.93332	0.60000	0.50000	0.16749	0.00000	0.16749
0.93332	0.60000	0.30000	0.16749	0.00000	0.16749
0.93332	0.60000	0.10000	0.16749	0.00000	0.16749
Panel C					
0.93332	1.00000	0.60000	0.02538	0.00000	0.02538
0.93332	0.90000	0.60000	0.06091	0.00000	0.06091
0.93332	0.70000	0.60000	0.13196	0.00000	0.13196
0.93332	0.50000	0.60000	0.20301	0.00000	0.20301
0.93332	0.30000	0.60000	0.27406	0.00115	0.27291
0.93332	0.10000	0.60000	0.34512	0.07220	0.27291

The parameters are set as follows: $r = 0.0649$ and $\sigma = 0.0963$ are taken from Ibbotson et al. (1985). The exogenously determined regulatory closure rule, $\bar{a} = 0.97$, is taken from Ronn and Verma (1986).

[5] If the deposit insurer chooses to close a bank, it will try to lower the bankruptcy cost in the process, i.e. the smaller the value of $1 - k_{\bar{a}}$, the smaller the subsidy to the bank. In other words, the amount the deposit insurer pays to the bank is smaller. Therefore, the value of the call provision of the deposit insurance subsidy increases monotonically with decreases in the bankruptcy costs at regulatory closure point $\bar{a}$.

[6] See the discussion in Section 3.c. of Allen and Saunders (1993).

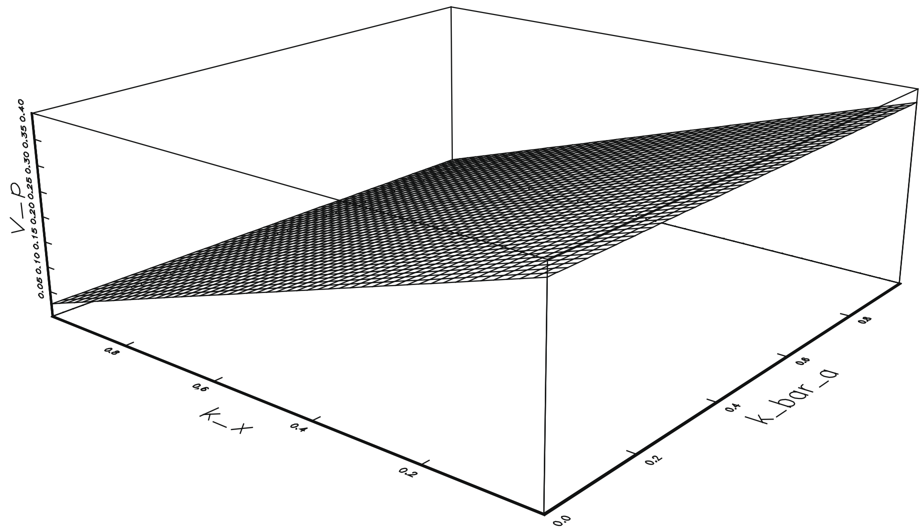

Fig. 1. The value of non-callable perpetual put with bankruptcy costs. This figure plots the values of non-callable perpetual puts with different bankruptcy costs. The parameters are set with the following values: $r = 0.0649$ and $\sigma = 0.0963$, which are taken from Ibbotson et al. (1985). The exogenously determined regulatory closure rule, $\bar{a} = 0.97$, is taken from Ronn and Verma (1986).

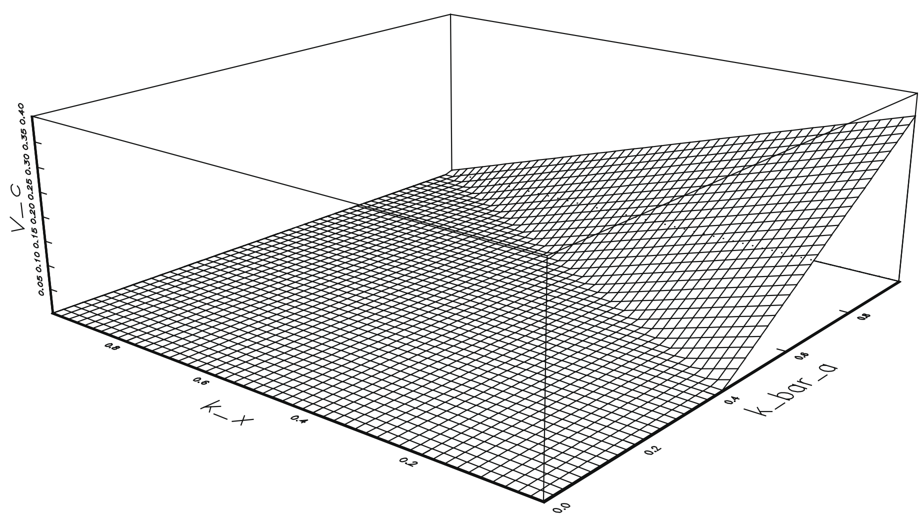

Fig. 2. The value of the call provision of deposit insurance with bankruptcy costs. This figure plots the values of the call provisions in deposit insurance with different bankruptcy costs. The parameters are set with the following values: $r = 0.0649$ and $\sigma = 0.0963$, which are taken from Ibbotson et al. (1985). The exogenously determined regulatory closure rule, $\bar{a} = 0.97$, is taken from Ronn and Verma (1986). The option has value only if $x < \bar{a}$ and $k_{\bar{a}} > [1 - (1 - k_x x)(x/\bar{a})^{\gamma}]/\bar{a}$.

monotonically with the increase in the self-closure bankruptcy cost, yet decreases monotonically with the increase in the regulatory-closure bankruptcy cost. Third, higher bankruptcy costs lead to higher deposit insurance premium. These are consistent with the results shown in the earlier Figures.

Fig. 5 shows the positive relationship between the bankruptcy cost and the value of the deposit insurance premium evaluated by the MDOP approach, indicating that the bank with higher bankruptcy costs will be charged with a higher deposit insurance premium. It is consistent with Eq. (32) in Proposition 7.

Fig. 6, based on Eq. (33), presents the relationship between the asset risk and the differential value of the deposit insurance premium, $\partial i_{bc}^{\mathrm{MDOP}}/\partial\sigma$, under various risk-free rates, i.e., $r = 0.1$, 0.3, and 0.5, respectively. From this figure, we can see that no matter what the scenario is, all the differential values are not negative, indicating the positive relationship between the deposit insurance value and the asset risk. Moreover, it can be observed that the deposit insurance value will be more sensitive (or volatile) to the variation of asset risk when the interest rate is lower or the asset risk is in the moderate level. Intuitively, this is reasonable because, when the interest rate goes down, the interest rate spread will shrink and will lower the profits of the bank. Then the probability of bank failures will increase at a growing rate. Therefore, the deposit insurance premium should increase accordingly.

Fig. 7 simulates how the value of deposit insurance varies with risk under three functional forms: (i) $k_{\bar{a}}(\sigma) = 1 - 0.05\sigma$; (ii) $k_{\bar{a}}(\sigma) = 1 - 0.25\sigma$; and (iii) $k_{\bar{a}}(\sigma) = 1 - 0.5\sigma$, while considering the bankruptcy cost and more realistic closure policy, as discussed in Eq. (34). No matter which approach (MDOP or DOP) is used, the insurance premium increases monotonically with the increase in the bank asset risk, consistent with Proposition 8. These results further ensure the required property for the risk-based premium, in contrast to that in Allen and Saunders (1993). In sum, the features that these graphs display meet the expectation of our models.

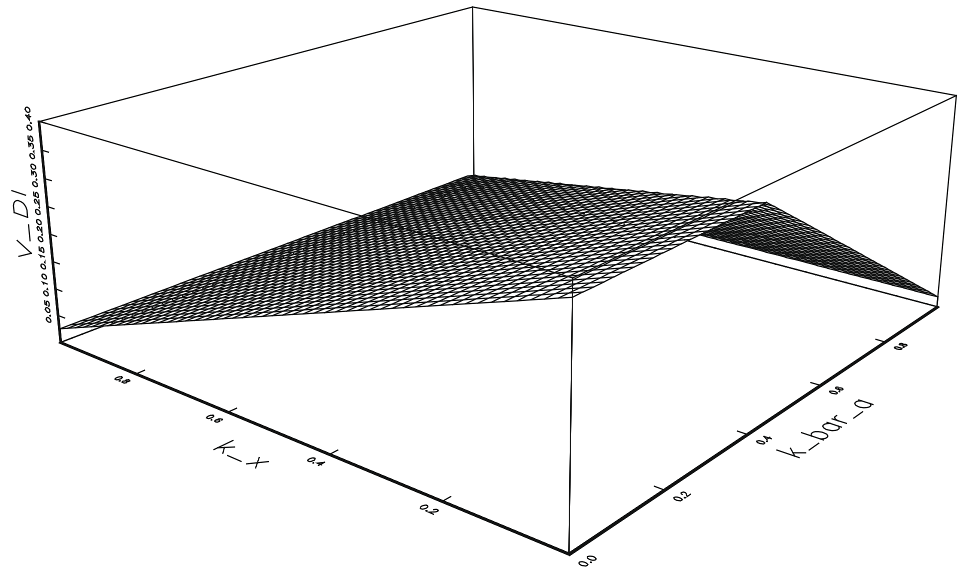

Fig. 3. The net value of deposit insurance with bankruptcy costs. This figure plots the net values of deposit insurance, $i_{bc}(a,\infty;1)$, with different bankruptcy costs. The parameters are set with the following values: $r = 0.0649$ and $\sigma = 0.0963$, which are taken from Ibbotson et al. (1985). The exogenously determined regulatory closure rule, $\bar{a} = 0.97$, is taken from Ronn and Verma (1986).

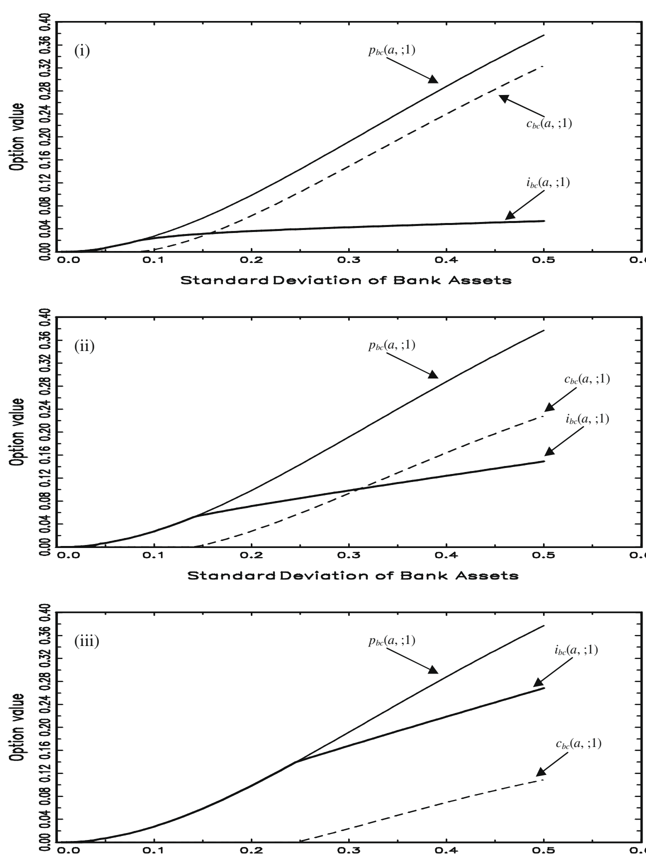

Fig. 4. The value of deposit insurance with risk-based bankruptcy costs and with fixed $k_x = 1$. This figure plots the values of the deposit insurance premiums with different types of bank risk. The thick solid line shows $p_{bc}(a,\infty;1)$ and the light solid line shows $i_{bc}(a,\infty;1)$, while the dashed line is $c_{bc}(a,\infty;1)$. The parameter $r = 0.0649$, which is taken from Ibbotson et al. (1985). The exogenously determined regulatory closure rule, $\bar{a} = 0.97$, is taken from Ronn and Verma (1986). The setup for each graph is (i) $k_{\bar{a}}(\sigma) = 1 - 0.05\sigma$; (ii) $k_{\bar{a}}(\sigma) = 1 - 0.25\sigma$; and (iii) $k_{\bar{a}}(\sigma) = 1 - 0.5\sigma$, respectively.

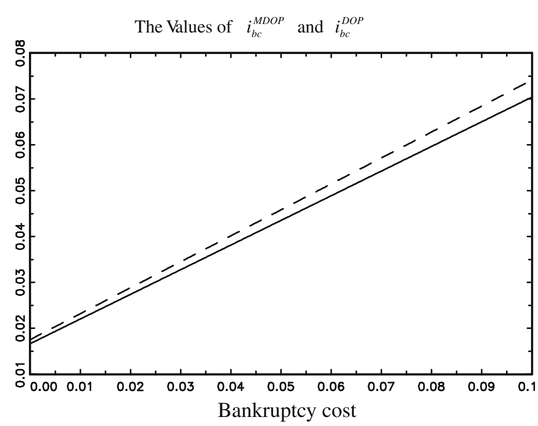

Fig. 5. The value of deposit insurance premium evaluated by MDOP and DOP with different bankruptcy costs. This figure plots the values of the deposit insurance premiums with different bankruptcy costs. The solid line shows $i_{bc}^{\mathrm{MDOP}} =$ and the dashed line shows i_{bc}^{DOP}. The parameters are set with the following values: $r = 0.0649$ and $\sigma = 0.0963$, which are taken from Ibbotson et al. (1985). The exogenously determined regulatory closure rule, $\bar{a} = 0.97$, is taken from Ronn and Verma (1986), and $T = 1$.

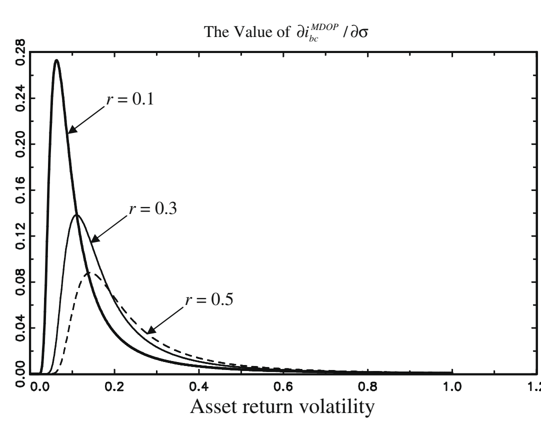

Fig. 6. The value of $\partial i_{bc}^{\mathrm{MDOP}}/\partial\sigma$. This figure plots the values of $\partial i_{bc}^{\mathrm{MDOP}}/\partial\sigma$ with different risk-free rates and standard deviations. The exogenously determined regulatory closure rule, $\bar{a} = 0.97$, is taken from Ronn and Verma (1986), $k_{\bar{a}} = 1$ and $T = 1$.

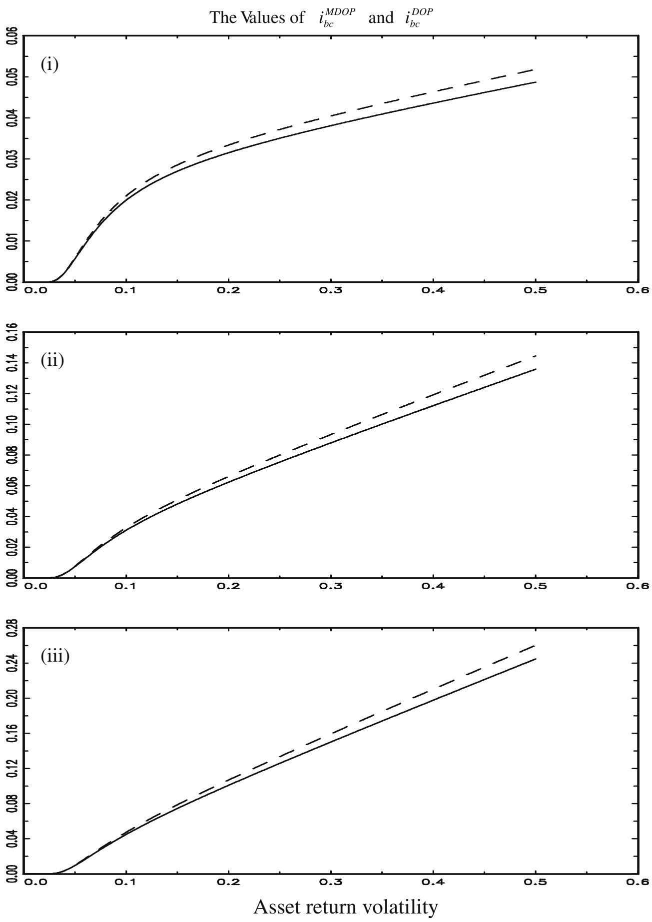

Fig. 7. The value of the deposit insurance premium evaluated by MDOP and DOP with different functional forms of asset return volatility. This figure plots the values of the deposit insurance premiums with different asset return volatilities. The solid line shows i_{bc}^{MDOP} and the dashed line shows i_{bc}^{DOP}. $r = 0.0649$, which is taken from Ibbotson et al. (1985). The exogenously determined regulatory closure rule, $\bar{a} = 0.97$, is taken from Ronn and Verma (1986), and $T = 1$. The setup for each graph is (i) $k_a(\sigma) = 1 - 0.05\sigma$; (ii) $k_a(\sigma) = 1 - 0.25\sigma$; and (iii) $k_a(\sigma) = 1 - 0.5\sigma$, respectively.

5. Conclusion

Facing the unprecedented financial turmoil since August 2007, many regulators decide to impose the explicit deposit insurance system with full coverage of deposits to all depositors. Under this protection banks are even more encouraged to take excessive risks. Hence fair pricing of deposit insurance is particularly a crucial issue in alleviating the moral hazard problem under such an environment.

In this paper, we have studied the pricing issue of deposit insurance with explicit bankruptcy costs and more reasonable closure rules that regulators have to consider, especially during the crisis period. Firstly, we extended the study of Allen and Saunders (1993), but incorporated the bankruptcy cost to evaluate deposit insurance as a callable perpetual American put. Applying the isomorphic relationship between deposit insurance and put option, we derived a closed-form solution for the pricing model of deposit insurance embedding bankruptcy costs and closure policies. Our model shows that the value of deposit insurance, with risk-based bankruptcy costs, increases monotonically with the increase in the bank asset risk. This is in contrast to the result in Allen and Saunders (1993) in which the riskier bank may be charged with a lower premium beyond a certain level. Furthermore, the value of deposit insurance, with risk-based bankruptcy costs, increases monotonically with the higher bankruptcy costs. These properties are supported by the numerical simulations and conform to the spirit of risk-based deposit insurance premium. In sum, we have solved the inconsistencies in Allen and Saunders (1993) by incorporating the "risk-based bankruptcy cost" into the deposit insurance pricing, and the properties of the new model are in line with normal expectation.

Secondly, we have applied the barrier option concept to price the deposit insurance with which not only bankruptcy costs but more realistic closure policy and regulatory forbearance are incorporated. Although it is difficult to quantify the bankruptcy cost, we set it as a function of the asset return volatility. The values of the deposit insurance, calculated by both MDOP and DOP approaches, increase monotonically with the increase in the bank asset risk. This conforms to the result from the perpetual put model with regards to the risk-based premium. We therefore suggest that bankruptcy costs and closure policies should be considered to improve the valuation of deposit insurance. The bankruptcy cost may be determined by the logistic regression so the proxy of the asset quality or market data can be used in the pricing model.

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Appendix A

A.1. Proof of Lemma 1

The general solution to (4) is

$$p_{bc} = a_1 a + a_2 a^{-\gamma},$$

where $\gamma = 2r/\sigma^2$

From condition (5) we can get $a_1 = 0$. And from condition (6) with $a_1 = 0$ we have $a_2 = (1-k_x x)x^{\gamma}$. Substitute a_1 and a_2 in p_{bc}, we obtain

$$p_{bc} = (1 - k_x x)\left(\frac{a}{x}\right)^{-\gamma}. \quad \square$$

A.2. Proof of Lemma 2

$$p_{bc} = a^{-\gamma}(x^{\gamma} - k_x x^{1+\gamma})$$

$$\frac{\partial p_{bc}}{\partial x} = a^{-\gamma}[\gamma x^{\gamma-1} - (1+\gamma)k_x x^{\gamma}] = 0$$

$$\gamma x^{\gamma-1} - (1+\gamma)k_x x^{\gamma} = 0$$

$$\frac{x^{\gamma}}{x^{\gamma-1}} = \frac{\gamma}{(1+\gamma)k_x}$$

$$x = \frac{\gamma}{(1+\gamma)k_x}. \quad \square$$

A.3. Proof of Lemma 3

$$p_{bc} = \left(1 - k_x \frac{\gamma}{(1+\gamma)k_x}\right)\left(\frac{a}{\gamma/[(1+\gamma)k_x]}\right)^{-\gamma}$$

$$= \left(\frac{1}{1+\gamma}\right)\left(\frac{(1+\gamma)k_x a}{\gamma}\right)^{-\gamma}. \quad \square$$

A.4. Proof of Lemma 6

To prove (22a) and (22b), we first prove the special case where X_t is a $(0, 1)$ Brownian motion. We can use the reflection principle to get

$$\Pr(X_T \geqslant X, m_T \leqslant B) = \Phi\left(\frac{-X+2B}{\sqrt{t}}\right). \tag{A.1}$$

Let hitting time, τ, be the first time, t, at which X_t=B. Then

$$\begin{aligned}\Pr(X_T \geqslant X, m_T \leqslant B) &= \Pr(X_T \geqslant X, \tau \leqslant T) \\ &= \Pr(\tau \leqslant T, X_T \leqslant -X+2B) \\ &= \Pr(X_T \leqslant -X+2B) \\ &= \Phi\left(\frac{-X+2B}{\sqrt{t}}\right).\end{aligned} \tag{A.2}$$

Therefore,

$$\begin{aligned}\Pr(X_T \geqslant X, m_T \geqslant B) &= \Pr(X_T \geqslant X) - \Pr(X_T \geqslant X, m_T \leqslant B) \\ &= \Phi\left(\frac{-X}{\sqrt{t}}\right) - \Phi\left(\frac{-X+2B}{\sqrt{t}}\right).\end{aligned} \tag{A.3}$$

Since τ is the first t at which $X_t = B$, obviously $\tau \geqslant T$ if and only if $m_T \geqslant B$. Assuming $X = B$ in (A.3) gives

$$\begin{aligned}\Pr(\tau \geqslant T) &= \Pr(m_T \geqslant B) \\ &= \Pr(X_T \geqslant B, m_T \geqslant B) \\ &= \Phi\left(\frac{-B}{\sqrt{t}}\right) - \Phi\left(\frac{B}{\sqrt{t}}\right).\end{aligned} \tag{A.4}$$

Therefore,

$$\begin{aligned}\Pr(X_T \leqslant X, m_T \geqslant B) &= \Pr(m_T \geqslant B) - \Pr(X_T \geqslant X, m_T \geqslant B) \\ &= \Phi\left(\frac{-B}{\sqrt{t}}\right) - \Phi\left(\frac{B}{\sqrt{t}}\right) - \Phi\left(\frac{-X}{\sqrt{t}}\right) + \Phi\left(\frac{-X+2B}{\sqrt{t}}\right) \\ &= \left[\Phi\left(\frac{-B}{\sqrt{t}}\right) - \Phi\left(\frac{-X}{\sqrt{t}}\right)\right] - \left[\Phi\left(\frac{B}{\sqrt{t}}\right) - \Phi\left(\frac{-X+2B}{\sqrt{t}}\right)\right].\end{aligned} \tag{A.5}$$

Then (A.5) and (A.3) are the special cases of (22a) and (22b), respectively. For the general case, we can use the "change of measure theorem" by Harrison (1985). For the proof of (22c), see Rubinstein and Reiner (1991). □

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