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Determining a common set of weights in a DEA problem using a separation vector

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ABSTRACT

A separation method to be used for locating a set of weights, also known as a common set of weights (CSW), in the Data Envelopment Analysis (DEA) is proposed in this work. To analyze the methods of finding the CSW, it is necessary to solve a particular form of a multiple objectives fractional linear programming problem (MOFP). One of the characteristics of this particular MOFP is that the decision variables can be separated into two parts; one part of variables present in the numerator and the other part of variables present in the denominator. Based on this characteristic, this research utilized an auxiliary vector to convert the MOFP to a single objective linear programming to obtain a CSW for calculating the DMU's efficiency ratio. Finally, the developed method is applied to analyze the data of the last Beijing Olympic Games.

1. Introduction

Let $N = \{1, 2, \dots, n\}$ and consider the following multiple objectives fractional programming problem:

$$\text{(MOFP)}\quad \underset{U,V}{\text{Max}} \left\{ Z^k(U,V) = \frac{UY^k}{VX^k}, k \in N \right\}$$
$$\text{st.}\quad \frac{UY^k}{VX^k} \le 1, \quad k \in N,$$
$$U, V \ge \varepsilon,$$

where Z is an $1 \times n$ row vector, U and V are $1 \times s$ and $1 \times r$ row vectors with nonnegative elements, respectively. X is a $r \times n$ matrix and Y is a $s \times n$ matrix; X^k and Y^k are kth column of matrices X and Y, respectively.

This formulation is derived from the problem of determining a set of weights for all decision making units (DMUs) in the Data Envelopment Analysis (DEA) [1]. In the problem setting, matrices X and Y represent the inputs and outputs of all DMUs, and the decision variables (U, V) represent a set of weights among inputs and outputs. This set of weights also is named as a common set of weights (CSW).

This formulation is derived from the problem of how to determinate a set of weight for all decision making units (DMUs) in the Data Envelopment Analysis (DEA) [1]. In the problem setting, matrices X and Y represent the inputs and outputs of all DMUs, and the decision variables (U, V) represent a set of weights among inputs and outputs. This set of weights also is named as a common set of weights (CSW).

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Using the concept of max–min, Chiang and Tzeng [1] presented the following multiple objectives model to determine CSW:

$$\begin{aligned} &\underset{U,V}{\text{Max}}\ \text{Min}\left\{Z^k(U,V)=\tfrac{UY^k}{VX^k},k\in N\right\}\\ &\text{st.}\quad \tfrac{UY^k}{VX^k}\le 1,\quad k\in N,\\ &\qquad U,V\ge\varepsilon. \end{aligned}$$

By introducing the nonnegative variable α, which represents the level of achievement, the problem can be converted to the following nonlinear problem:

$$\begin{aligned} &\underset{U,V}{\text{Max}}\ \alpha\\ &\text{st.}\quad UY^k-\alpha(VX^k)\ge 0,\quad k\in N,\\ &\qquad UY^k-VX^k\le 0,\quad k\in N,\\ &\qquad U,V\ge\varepsilon,\qquad \alpha\ge 0. \end{aligned}$$

Note that instead of solving n linear programming DEA models, only one nonlinear programming problem is solved, and the efficiency for all DMUs are obtained. Because all DMUs' efficiency scores are evaluated by the same set of weights, then the efficiency index of DMUs can be accepted for the purpose of ranking. Jahanshahloo et al. [2] also have presented a similar model with Chiang and Tzeng [1] for solving CSW.

Based on the concept of the compromise solution, Kao and Hung [3] proposed the following models to obtain the CSW. The authors first calculated the efficiency scores of DMUs from the standard DEA model, and regarded these scores as the ideal solution for the DMUs to achieve. Then a CSW closest to the ideal solution can be derived based on the generalized measure of distance.

$$\begin{aligned} &\underset{U,V}{\text{Min}}\ D_p=\|\theta-(UY/VX)\|_p\\ &\text{st.}\quad \tfrac{UY^k}{VX^k}\le 1,\quad k\in N,\\ &\qquad U,V\ge\varepsilon, \end{aligned}$$

where $\theta=(\theta_1,\theta_2,\ldots,\theta_n)$ is a vector of ideal points that calculated by CCR DEA model, and D_p is the distance from θ to (UY/VX) according to l_p norm, p represents a positive number.

Cook and Zhu [4] examined a set of power plants, with each containing a set of power units under a common plant management. They developed a goal programming model to derive a common-multiplier set.

$$\begin{aligned} &\underset{U,V}{\text{Min}}\ \gamma\\ &\text{st.}\quad UY^k-V(\theta_kX_k)+\gamma=0,\quad k\in N\\ &\qquad UY^k-VX^k\le 0,\quad k\in N\\ &\qquad U,V\ge\varepsilon. \end{aligned}$$

To rank the units on the DEA frontier with CSW, Liu and Peng [5] proposed the following goal programming problem:

$$\begin{aligned} &\underset{U,V}{\text{Min}}\ \sum_l(\Delta_O^l+\Delta_I^l)\\ &\text{st.}\quad UY^l-VX^l+\Delta_O^l+\Delta_I^l=0,\quad l\in E,\\ &\qquad U,V\ge\varepsilon,\qquad \Delta_O^l,\Delta_I^l\ge 0, \end{aligned}$$

where E is the set of all efficient DMU l.

The above-mentioned four methods of solving MOFP are either with nonlinear formulations or solution process consisting of many steps. The nonlinear model developed by Cook and Zhu [4] is almost in line with that of Chiang and Tzeng [1], except that the solution approaches are different: One is by goal programming and the other by the bisection method. No matter what technique is used, a nonlinear model is inherently more difficult to solve than a linear model. As for the rest of models, there are two main steps of the method proposed by Kao and Hung [3], thus the solution procedure takes lots of time and effort. Similarly, Liu and Peng's method also consists of two steps: First find the efficient DMUs by conventional DEA, and then a goal programming model is derived for obtaining CSW. In this paper, we aim at formulating a linear model with an easier procedure for solving MOFP.

As we can see in MOFP, there are two important characteristics: (i) one part of variables, U, is only present in the numerator while the other part of variables, V, is only present in the denominator; (ii) these two parts of variables have a relation: $UY^k\le VX^k$, $k\in N$. These two characteristics motivate the authors to think about the possibility of utilizing an auxiliary variable refers to a separation vector to convert the MOFP to two simpler problems that contain a decision variable, either U or V.

In this paper we propose two methods based on the characteristics of separation of variables to determine a CSW. The structure of this paper is organized as follows. In the next section, we first provide the main theorem for developing methods to solve MOFP. In Section 3, the proposed method is illustrated by using data from the 2008 Beijing Olympic Games to rank nations. The conclusions and remarks are presented in the last section.

2. Main theorem of methods

In this section, we provide a main theorem for converting the MOFP to an equivalent formulation in the form of multiple objective linear programming. Then we propose a method to solve the equivalent problem.

Theorem 1. *The formulation of MOFP can be transformed to the following multiple objective linear programming with a separation vector:*

$$\text{(MOLPSV)} \quad \underset{U,V,\Lambda}{\text{Min}} \; [VX^1 - UY^1, \ldots, VX^n - UY^n]$$
$$\text{st.} \quad \begin{aligned} & VX^k \geq \lambda_k, \quad k \in N, \\ & UY^k \leq \lambda_k, \quad k \in N, \\ & V \geq \varepsilon, \qquad U \geq \varepsilon. \end{aligned}$$

Proof. By the constraints of MOFP, the constraints $\frac{UY^k}{VX^k} \leq 1, k \in N$, can be rearranged as $UY^k \leq VX^k, k \in N$. It follows that there exists a positive separation vector $\Lambda = [\lambda_1, \lambda_2, \ldots, \lambda_n]$ between the point $[UY^1, UY^2, \ldots, UY^n]$ and the point $[VX^1, VX^2, \ldots, VX^n]$ such that $UY^k \leq \lambda_k \leq VX^k, k \in N$. In addition, the value of each objective function in MOFP is between 0 and 1, i.e. $0 \leq \frac{UY^k}{VX^k} \leq 1, k \in N$. Multiplying each part of the inequality with $-VX^k$, and then adding the term VX^k to each part of the inequality, we obtain the following conditions: $VX^k \geq VX^k - UY^k \geq 0$. These results suggest that the fractional objective function can be transformed to a linear objective function with the form of difference of VX^k and UY^k. In other words, maximizing the ratio of UY^k and VX^k with the constraints $\frac{UY^k}{VX^k} \leq 1, k \in N$ in MOFP is equivalent to minimizing the differences between VX^k and UY^k, i.e $\text{Min}_{U,V}\{Z^k(U, V) = VX^k - UY^k, k \in N\}$. Therefore we obtain a multiple objective linear programming with a separation vector, which is equivalent to MOFP. □

According to the Theorem 1, the multiple objective programming with fractional form has been transformed to a multiple objective programming with general linear form. Next we introduce a simple method to solve the transformed problem, MOLPSV.

Proposition 1. *Given X^k, Y^k, $k \in N$ are nonnegative column vectors. If an optimal solution of the following single objective programming exists, then this optimal solution will be an efficient solution of MOLPSV.*

$$\text{(SOLPSV)} \quad \underset{U,V,\Lambda}{\text{Min}} \; \sum_{k=1}^{n} VX^k - \sum_{k=1}^{n} UY^k$$
$$\text{st.} \quad \begin{aligned} & VX^k \geq \lambda_k, \quad k \in N, \\ & UY^k \leq \lambda_k, \quad k \in N, \\ & U, V \geq \varepsilon, \qquad \Lambda \geq 0. \end{aligned}$$

Proof. Let $W(U^*, V^*)$ be an optimal solution of (SOLPSV). Suppose that $W(U^*, V^*)$ is not an efficient solution of (MOLPSV) then there exists a vector V' such that $V^*X^k - U^*Y^k > V'X^k - U'Y^k$ for some k, and $V^*X^l - U^*Y^l \geq V'X^l - U'Y^l, l \in N \setminus k$. It follows that $\sum_{k=1}^{n} V^*X^k - \sum_{k=1}^{n} U^*Y^k > \sum_{k=1}^{n} V'X^k - \sum_{k=1}^{n} U'Y^k$. This contradicts that $W(U^*, V^*)$ is an optimal solution of (SOLPSV). □

3. Ranking for the Beijing Olympic Games

In this section, the proposed methods are illustrated using data from the 2008 Beijing Olympic Games. In the literature there are some studies for alternative rankings for Olympic Games. The Olympic Committee has never issued an official ranking, but the International Olympic Committee (IOC) presents the medal data by the gold first ranking system to suggest a ranking [6]. While the gold first ranking system has been used by the IOC, some medias publish medal tables ordered by the total number of medals won.

Another ranking system (demographic ranking) in use is the per-capita ranking, where the number of medals is divided by the population of the country. In addition, systematic rankings based upon a weighted point system with the most points awarded to a gold medal have also been devised. For example, a system awarding gold medals 5 points, silver medals 3 points, and bronze medals 1 point (5:3:1) was used. In addition to the system of (5:3:1), systems of (3:2:1) and (4:2:1) were also used in some places at some time but none of them have been adopted on a large scale.

There are already some approaches using DEA to establish Olympic rankings. For instance, Lozano et al. [7] and Lins et al. [8] concentrate on the technical issues of the DEA-based approach to measure the performance of nations at the

Table 1
Inputs and Outputs of different nations.

Nation	GDP (Billions)	Population (Millions)	Gold	Silver	Bronze	Total medals
United States (USA)	14204.3	304.1	36	38	36	110
China (CHN)	4326.2	1325.6	51	21	28	100
Russia (RUS)	1607.8	141.8	23	21	28	72
Great Britain (GBR)	2645.6	61.4	19	13	15	47
Australia (AUS)	1015.2	21.4	14	15	17	46
Germany (GER)	3652.8	82.1	16	10	15	41
France (FRA)	2583.1	62	7	16	18	41
South Korea (KOR)	929.1	48.6	13	10	8	31
Italy (ITA)	2293	59.9	8	9	10	27
Ukraine (UKR)	180.4	46.3	7	5	15	27
Japan (JAP)	4909.3	127.7	9	6	11	26
Cuba (CUB)	55.2	11.2	2	11	11	24
Spain (ESP)	1604.2	45.6	5	10	3	18
Canada (CAN)	1400.1	33.3	3	9	6	18
Belarus (BLR)	60.3	9.7	4	4	9	17
Netherlands (NED)	860.3	16.4	7	5	4	16
Brazil (BRA)	1612.5	192	3	4	8	15
Kenya (KEN)	34.5	38.5	6	4	4	14
Kazakhstan (KAZ)	132.2	15.7	2	4	7	13
Jamaica (JAM)	15.1	2.7	6	3	2	11
Hungary (HUN)	356.8	11.2	3	6	2	11
Poland (POL)	527	38.1	3	6	1	10
Norway (NOR)	450	4.8	3	5	1	9
New Zealand (NZL)	130.7	4.3	3	2	4	9
Romania (ROU)	200.1	21.5	4	1	3	8
Turkey (TUR)	794.2	73.9	1	4	3	8
Ethiopia (ETH)	26.5	80.7	4	1	2	7
Denmark (DEN)	342.7	5.5	2	2	3	7
Switzerland (SUI)	488.5	7.6	2	0	5	7
Azerbaijan (AZE)	46.3	8.7	1	2	4	7
Czech Republic (CZE)	216.5	10.4	3	3	0	6
Slovakia (SVK)	95	5.4	3	2	1	6
Georgia (GEO)	12.8	4.4	3	0	3	6
North Korea (PRK)	26.2	23.9	2	1	3	6
Argentina (ARG)	328.4	39.9	2	0	4	6
Uzbekistan (UZB)	27.9	27.3	1	2	3	6
Armenia (ARM)	11.9	3.1	0	0	6	6
Slovenia (SLO)	54.6	2	1	2	2	5
Bulgaria (BUL)	49.9	7.6	1	1	3	5
Indonesia (INA)	514.4	228.2	1	1	3	5
Sweden (SWE)	480	9.2	0	4	1	5
Croatia (CRO)	69.3	4.4	0	2	3	5
Lithuania (LTU)	47.3	3.4	0	2	3	5
Mongolia (MGL)	5.3	2.6	2	2	0	4
Thailand (THA)	260.7	67.4	2	2	0	4
Zimbabwe (ZIM)	3.4	12.5	1	3	0	4
Finland (FIN)	271.3	5.3	1	1	2	4
Greece (GRE)	356.8	11.2	0	2	2	4
Nigeria (NGR)	212.1	151.3	0	1	3	4
Chinese Taipei (TPE)	393.1	23.1	0	0	4	4
Mexico (MEX)	1086	106.4	2	0	1	3
Latvia (LAT)	33.8	2.3	1	1	1	3
India (IND)	1217.5	1140	1	0	2	3
Austria (AUT)	416.4	8.3	0	1	2	3
Ireland (IRL)	281.8	4.5	0	1	2	3
Serbia (SRB)	50.1	7.4	0	1	2	3
Belgium (BEL)	497.6	10.7	1	1	0	2
Dominican Republic (DOM)	45.8	9.8	1	1	0	2
Estonia (EST)	23.1	1.3	1	1	0	2
Portugal (POR)	242.7	10.6	1	1	0	2
Iran (IRI)	385.1	72	1	0	1	2
Trinidad and Tobago (TRI)	23.9	1.3	0	2	0	2
Algeria (ALG)	173.9	34.4	0	1	1	2
Bahamas (BAH)	6.9	0.3	0	1	1	2
Colombia (COL)	242.3	44.5	0	1	1	2
Kyrgyzstan (KGZ)	4.4	5.3	0	1	1	2
Morocco (MAR)	86.3	31.2	0	1	1	2
Tajikistan (TJK)	5.1	6.8	0	1	1	2
Cameroon (CMR)	23.4	18.9	1	0	0	1
Panama (PAN)	23.1	3.4	1	0	0	1

(*continued on next page*)

Table 1 (*continued*)

Nation	GDP (Billions)	Population (Millions)	Gold	Silver	Bronze	Total medals
Tunisia (TUN)	40.2	10.3	1	0	0	1
Chile (CHI)	169.5	16.8	0	1	0	1
Ecuador (ECU)	52.6	13.5	0	1	0	1
Iceland (ISL)	16.7	0.3	0	1	0	1
Malaysia (MAS)	194.9	27	0	1	0	1
Singapore (SIN)	181.9	4.8	0	1	0	1
South Africa (RSA)	276.8	48.7	0	1	0	1
Sudan (SUD)	58.4	41.3	0	1	0	1
Vietnam (VIE)	90.7	86.2	0	1	0	1
Afghanistan (AFG)	10.2	32.7	0	0	1	1
Egypt (EGY)	162.8	81.5	0	0	1	1
Israel (ISR)	199.5	7.3	0	0	1	1
Mauritius (MRI)	8.7	1.3	0	0	1	1
Moldova (MDA)	6	3.6	0	0	1	1
Togo (TOG)	2.8	6.5	0	0	1	1
Venezuela (VEN)	313.8	27.9	0	0	1	1

Table 2
The results of CSW.

	U_1	U_2	U_3	V_1	V_2
CSW	0.004	0.002	0.001	0.001	0.0063

Summer Olympics. Churilov and Flitman [9] combine the data-mining tools and DEA-based approach to present a study of the factors influencing countries' preferences towards a different ranking system. In the study of Lozano et al. [7], the authors used population and GNP as inputs and medals as outputs. However, Churilov and Flitman [9] use the "utility" of modal counts, not the numbers of medals as an output for the study.

In this paper, we use the medals as outputs, and population and Gross Domestic Product (GDP) as inputs, i.e. $s = 3$, $r = 2$. A high population gives a country more athletes to draw from, while GDP could be assumed to represent a country's prosperity, with a prosperous country more likely to spend money on sport. Table 1 presents the data of medals earned by each nations in the 2008 Beijing Olympic Games, and the corresponding population and GDP. These data were obtained from the IOC and World Bank.

To take into account the difference in importance among gold, silver, and bronze medals, we add constraints in SOLPSV to insure that the difference in importance between gold and silver medals is greater than the difference between silver and bronze medals. These constraints are as follows:

$$U_1 - U_2 \geq 0.001,$$
$$U_2 - U_3 \geq 0.001,$$
$$U_1 - 2U_2 + U_3 \geq 0.001.$$

The results of the common set of weights are shown as Table 2. Note that the weights for gold, silver, and bronze are 4: 2: 1, which is same as one of the weighted point system of (4:2:1).

Calculating the efficiency ratio for each nation and sorting nations' ratio we obtain the eighty-six national rankings of the 2008 Beijing Olympic Games. The efficiency ratio for each nation is shown as the third columns in Table 3. Comparing Tables 3 and 2, there are significant differences of national ranking. According to the efficiency ratio, Jamaica is the most efficient nation for wining medals at the 2008 Beijing Olympic Games. Its ranking rises from 20th to first place.

Table 4 shows twenty nations' differences between two rankings calculated by total medals and efficiency index respectively. As we can see, the USA drops dramatically from the first place ranked by total medals to 59th place in the efficiency ratio ranking system. Japan also has the same situation which drops from 11th to 66th place. Conversely, Cuba's ranking raises from 12th to 5th place. In fact, except for Jamaica and Cuba, the ranking for Belarus and Kenya both raise their rankings based on the efficiency ratio.

From the viewpoint of efficiency, the totals medals method utilized by the IOC is definitely not a suitable measure of a country, since it does not consider how a country gets medals by investing in various resources. It seems that the conventional DEA models and other modified DEA methods are adequate approaches for measuring the medal efficiency of nations; however, the weights for outputs, i.e. the degree of how a nation's people pay attention to gold, silver, and bronze medals, should not be variant among nations. In other words, no matter which nation, people always value gold medals more than silver and then bronze medals. Thus CSW is a reasonable approach to deal with the problem of a nation's efficiency in obtaining medals. According to the results shown as Table 4, many developing countries demonstrate high efficiency compared with developed countries.

Table 3
National rankings by efficiency ratio calculated by CSW.

Nation	Total medals	Efficiency ratio
Jamaica (JAM)	11	1.000000
Mongolia (MGL)	4	0.556223
Georgia (GEO)	6	0.371833
Bahamas (BAH)	2	0.341772
Cuba (CUB)	24	0.327205
Belarus (BLR)	17	0.272694
Estonia (EST)	2	0.192080
Armenia (ARM)	6	0.191671
Slovenia (SLO)	5	0.148990
Latvia (LAT)	3	0.145239
Slovakia (SVK)	6	0.131988
Kenya (KEN)	14	0.130680
New Zealand (NZL)	9	0.126892
Trinidad and Tobago (TRI)	2	0.124855
Zimbabwe (ZIM)	4	0.122488
Azerbaijan (AZE)	7	0.119100
Ukraine (UKR)	27	0.112717
Iceland (ISL)	1	0.107656
Lithuania (LTU)	5	0.102068
Kazakhstan (KAZ)	13	0.099796
Bulgaria (BUL)	5	0.092336
Panama (PAN)	1	0.090128
Australia (AUS)	46	0.089632
Kyrgyzstan (KGZ)	2	0.079842
North Korea (PRK)	6	0.073949
Croatia (CRO)	5	0.072284
Russia (RUS)	72	0.064920
South Korea (KOR)	31	0.064867
Czech Republic (CZE)	6	0.063921
Tajikistan (TJK)	2	0.062942
Romania (ROU)	8	0.062748
Hungary (HUN)	11	0.060904
Mauritius (MRI)	1	0.059393
Dominican Republic (DOM)	2	0.056001
Uzbekistan (UZB)	6	0.055338
Norway (NOR)	9	0.047912
Netherlands (NED)	16	0.043616
Serbia (SRB)	3	0.041486
Denmark (DEN)	7	0.039775
Great Britain (GBR)	47	0.038615
Tunisia (TUN)	1	0.038215
Ethiopia (ETH)	7	0.037621
Moldova (MDA)	1	0.035047
Poland (POL)	10	0.032659
Cameroon (CMR)	1	0.028229
Finland (FIN)	4	0.026275
Switzerland (SUI)	7	0.024251
France (FRA)	41	0.024065
Germany (GER)	41	0.023760
Togo (TOG)	1	0.022996
Spain (ESP)	18	0.022756
Italy (ITA)	27	0.022489
Canada (CAN)	18	0.022381
China (CHN)	100	0.021706
Argentina (ARG)	6	0.020756
Portugal (POR)	2	0.019414
Thailand (THA)	4	0.017581
Sweden (SWE)	5	0.016742
United States (USA)	110	0.015893
Ecuador (ECU)	1	0.014588
Greece (GRE)	4	0.014055
Ireland (IRL)	3	0.012905
Turkey (TUR)	8	0.011935
Morocco (MAR)	2	0.010654
Belgium (BEL)	2	0.010627
Japan (JAP)	26	0.010335
Brazil (BRA)	15	0.009949
Singapore (SIN)	1	0.009436
Austria (AUT)	3	0.008541
Algeria (ALG)	2	0.007708

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Table 3 (*continued*)

Nation	Total medals	Efficiency ratio
Chinese Taipei (TPE)	4	0.007439
Chile (CHI)	1	0.007282
Sudan (SUD)	1	0.006311
Iran (IRI)	2	0.005983
Colombia (COL)	2	0.005760
Malaysia (MAS)	1	0.005496
Mexico (MEX)	3	0.005137
Afghanistan (AFG)	1	0.004654
Indonesia (INA)	5	0.004633
Nigeria (NGR)	4	0.004314
Israel (ISR)	1	0.004078
South Africa (RSA)	1	0.003439
Vietnam (VIE)	1	0.003173
Venezuela (VEN)	1	0.002047
Egypt (EGY)	1	0.001486
India (IND)	3	0.000718

Table 4
Ranking differences among twenty nations.

Nation	Ranking by total medals	Ranking by CWS
United States (USA)	1	59
China (CHN)	2	54
Russia (RUS)	3	27
Great Britain (GBR)	4	40
Australia (AUS)	5	23
Germany (GER)	6	49
France (FRA)	6	48
South Korea (KOR)	8	28
Italy (ITA)	9	52
Ukraine (UKR)	9	17
Japan (JAP)	11	66
Cuba (CUB)	12	5
Spain (ESP)	13	51
Canada (CAN)	13	53
Belarus (BLR)	15	6
Netherlands (NED)	16	37
Brazil (BRA)	17	67
Kenya (KEN)	18	12
Kazakhstan (KAZ)	19	20
Jamaica (JAM)	20	1
Hungary (HUN)	20	32

4. Conclusions

In this paper, the authors proposed a linear model with a separation vector for obtaining CSW in a DEA problem; then for demonstrating its effectiveness, this model is utilized to evaluate nations' efficiency for winning medals at the 2008 Beijing Olympic Games. Unlike the nonlinear models developed by Chiang and Tzeng [1], and Cook and Zhu [4], the separation vector model is a simple linear model, which is easier to solve; besides, compared to Liu and Pengs' method, the proposed model utilized fewer auxiliary variables, making it more adequate when applied to a large scale problem.

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