



Decision Support

Analyzing online B2B exchange markets: Asymmetric cost and incomplete information

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ABSTRACT

This research applies the discriminating auction to analyze the online B2B exchange market in which a single buyer requests multiple items and several suppliers having equal capacity and asymmetric cost submit bids to compete for buyer demand. In the present model, we examine the impact of asymmetric cost and incomplete information on the participants in the market. Given the complete cost information, each supplier randomizes its price and the lower bound of the price range is determined by the highest marginal cost. In addition, the supplier with a lower marginal cost has a larger considered pricing space but ultimately has a smaller equilibrium one than others with higher marginal costs. When each supplier's marginal cost is private information, the lowest possible price is determined by the number of suppliers and the buyer's reservation price. Comparing these two market settings, we find whether IT is beneficial to buyers or suppliers depends on the scale of the bid process and the highest marginal cost. When the number of suppliers and the difference between the highest marginal cost and the buyer's reservation price are sufficiently large, each supplier can gain a higher profit if the marginal costs are private information. On the contrary, when the highest marginal cost approaches the buyer's reservation price, complete cost information benefits the suppliers.

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1. Introduction

In most real-world markets, information technology is used to analyze data and manipulate information. As the world's economy becomes increasingly competitive, more and more transactions are being processed by computers. With the growth of the Internet and commercial web-based applications, a B2B (Business to Business) exchange has been universally recognized as an online platform that creates a trading marketplace linked by the Internet and offers substantial cost savings to buyers (Zhu, 2004). Based on a recently released IDC report, it is estimated that business transactions on the internet will swell to 450 billion a day by 2020 (Johnston, 2010). Currently, there are many successful B2B exchange portals, such as BusyTrade.com, Global Sources, and IndiaMarkets. In these B2B exchange portals, a buyer can post his/her offer, and suppliers bid their selling prices to compete for demand.

The bidding process, also known as online procurement auction, usually composed of one buying firm and many suppliers, is one of the popular exchange mechanisms helping business buyers procure better contract terms and prices for the goods and services they purchase. In online procurement auctions, buyers host the on-

line auction and invite potential suppliers to bid on announced request-for-quotations (RFQs) (Parente et al., 2001). Empirical evidences have revealed the importance of online procurement auctions on B2B exchange markets. For example, software maker Ariba helps a major fast food company set up an online auction for suppliers to bid, reducing the total cost to \$30 million (Yang, 2009). Before adopting the auction, the company was spending \$40 million on 13 different suppliers with about 30 different contracts. Recently, many public departments have also adopted the same approach to achieve competitive pricing for public projects and outsourcing contracts, such as street maintenance projects and power purchase agreements (Louisville, 2009; Isensee, 2009).

Bandyopadhyay et al. (2005) study a buy-centric market, which is common in online B2B exchanges, in which suppliers are bidding for the business of large buyers. By examining IndiaMarkets, a B2B exchange portal, Bandyopadhyay et al. (2005) confirm that the supplier submitting the lowest bid price will be first invited to cater to the demand, followed by the supplier with the second-lowest bid price, and so on, until the demand is satisfied. Through the mechanism of reverse auctions implemented on the B2B exchange portal, a buyer can select the best offer in a time bound manner, and suppliers can lower the cost for selling and marketing. A series of studies have been conducted by them to provide a theoretical basis for understanding the effect of costs and capacities on B2B exchanges (Bandyopadhyay et al., 2006, 2008). Procurement auctions usually involve a sealed bid due to difficult bid evaluation

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(Cramton, 1998). Within a buy-centric B2B exchange framework, they analyze the case of competition among sellers, each of which cannot fulfill the entire market's demand, and focus on the mechanism of a one-sided, sealed-bid reverse auction.

Since no supplier can fulfill the entire market's demand, in the two-supplier case where overall capacity is greater than demand, the low priced supplier can sell its entire capacity, whereas the high priced supplier has residual demand which is less than its capacity. Much recent economic research in residual demand is associated with the discriminating auction format, in which each winning supplier sells its capacity at the actual price bid, rather than a single price common to all suppliers (Fabra et al., 2006). For example, in the England and Wales electricity market, the discriminating auction is adopted as the solution for determining electricity supply prices. Before each period that the market is open, the generating companies, such as National Power and Nuclear Electric, submit their bids to the National Grid Company. When demand is sufficiently large, the generators have incentive to raise bids above their marginal costs, and thus prices cannot get driven down toward marginal costs (Fehr and Harbord, 1993).

1.1. Motivation and problems

Motivated by the research initiated by Bandyopadhyay et al. (2005, 2006, 2008), we explore the impact of cost information on participants in online B2B exchange markets, in which suppliers submit their bids under the consideration of residual demand. When each supplier's cost information is common knowledge, the discriminating auction is essentially a Bertrand-Edgeworth game (Deneckere and Kovenock, 1996). However, the problem is, as pointed out by Edgeworth, an equilibrium (in pure strategies) in such a game may not exist (Vives, 1999). Bandyopadhyay et al. (2008) model the competition between identical suppliers vying for the same business in the discriminating auction and find the suppliers may randomize their prices to form a mixed strategy Nash equilibrium. In their setting, each supplier has identical marginal cost, which is common knowledge among all participants in the auction.

In the present study, we consider each supplier can have asymmetric marginal cost and the information can be common knowledge or privately owned. In reality, each supplier in B2B marketplaces cannot know other potential competitors' cost structures in advance unless information transparency can be improved. Prior studies indicate that information systems can improve information transparency (Zhu, 2004; Singh et al., 2005; Granados et al., 2006). For example, each supplier in B2B marketplaces may infer other potential competitors' cost structures in advance by exploiting the technique of data mining and artificial reasoning, both of which rely on information systems such as data warehousing and expert systems. In addition, a buyer may have incentive to learn cost information and implicitly distribute it by inviting buyers with cost advantage to participate in the online procurement auction. The investment in IT could be expensive; as a result, it is critical for buyers and suppliers to better understand the value of IT in the auction. Therefore, the purpose of this study is to analyze the effect of cost information in a tractable model designed to capture some of the key features of an online bid process. In addition, we seek to provide valuable analytical insights into how the auction is executed in B2B marketplaces.

1.2. Findings and contributions

Comparing these two market settings, we have the following findings. First, given the complete cost information, each supplier would randomize its price and the lower bound of the price range is determined by the highest marginal cost. Thus, when the highest

marginal cost approaches the buyer's reservation price, each supplier would submit a sufficiently high bid which is close to the buyer's reservation price. In addition, the supplier with a lower marginal cost has a larger considered pricing space but ultimately a smaller equilibrium one than others with higher marginal costs. Second, given the incomplete cost information, each supplier bids a price only according to its individual marginal cost and the number of suppliers in the bid process would significantly affect their profits. When the number of suppliers in the bid process increases, it is possible each supplier submits a higher bid. The bids would converge to the buyer's reservation price when the number of suppliers is sufficiently large. Further, we find a supplier would have a higher profit when its capacity increases or its competitors' capacities decrease.

Comparing the suppliers' profits in the two different market settings, we find whether IT is beneficial to buyers or suppliers depends on the scale of the bid process and the highest marginal cost. When the number of suppliers and the difference between the highest marginal cost and the buyer's reservation price are sufficiently large, each supplier can gain a higher profit if the marginal costs are private information. On the contrary, when the highest marginal cost approaches the buyer's reservation price, complete cost information is beneficial for the suppliers. The rest of this paper is organized as follows. In the next section, we review prior research and highlight their contribution. In Section 3, we introduce our model. In Sections 4 and 5, we derive mixed strategy Nash Equilibrium and Bayesian Nash equilibrium, respectively. In Section 6, we analyze the impact of the cost information on participants in the online B2B exchange market. Finally, we conclude our research and address future study in Section 7.

2. Related work

2.1. B2B market

In the past few years, many neutral B2B exchanges have struggled to survive and many successful B2B markets are buyer-owned (Yoo et al., 2007). A successful business to business (B2B) electronic market has to attract enough participants. In a single buyer electronic market, lower price plays an important role as an order winning criterion (Lee et al., 2009). Assuming electronic marketplaces provide positive network effects, Yoo et al. (2007) examine the role of the ownership structure of electronic marketplaces on various participants in the marketplace. Their study shows a buyer-owned marketplace where some buyers jointly own the marketplace can provide greater benefits to participants than a neutral marketplace owned by an independent third party. Aron et al. (2008) point out the creation of a private electronic exchange by the large producer will result in significant welfare loss when upstream suppliers are highly efficient.

2.2. Online auctions and residual demand

For games in which the payoff functions are discontinuous, Dasgupta and Maskin (1986a) proved the existence of a mixed-strategy equilibrium in such games. The empirical evidence of the mixed-strategy equilibrium can be found in retail markets in which each store changes its price over time (Varian, 1980). Fabra et al. (2006) studied bidding behavior and market outcomes in the uniform-price and discriminating electricity auctions. In their basic duopoly model, two independent suppliers with asymmetric marginal costs and limited capacities must submit a single price offer for its entire capacity. If demand is low, there is a pure equilibrium in the two auction formats in which both suppliers submit offer prices equaling the cost of the inefficient supplier (i.e., the high

cost). However, only mixed-strategy equilibria exist in high-demand realizations, in which the higher-bidding supplier's capacity is then dispatched to serve the residual demand. As for online auction environments, policy makers should focus on the design and frequency of the auction or the amount of real-time information made available to market participations (Puller, 2007).

If a buyer decides to procure multiple items at one go in a single auction, there are many possibilities, such as competitive auctions (also known as uniform-price auctions), discriminating auctions (also known as pay-your-bid auctions), and Vickrey auctions. Harris and Raviv (1981) show the expected revenues generated from the discriminating and the competitive auctions are the same if the bidders are risk neutral. Moreover, some studies have pointed out collusion is less likely under discriminatory auctions than under uniform-price auctions (Klemperer, 2002; Fabra, 2003). In recent years there has been considerable interest in the multi-item auction in which different products or services are auctioned simultaneously (Jin et al., 2006; Teich et al., 2006; Mishra and Veeramani, 2007). Dasgupta and Maskin (1986b) illustrate an application of their research in which the firm quoting the lower price serves the entire market up to its capacity, and the residual demand is met by the other firm. The existence of equilibria for all combination of capacities is founded on a mixed-strategy. Bandyopadhyay et al. (2008) analyze the scenario in which several identical suppliers are vying for business from a single larger buyer in a B2B exchange framework. By simulating suppliers in a synthetic environment, they find the agents altering their pricing strategy over time do indeed converge toward the theoretical Nash equilibrium (Bandyopadhyay et al., 2006).

2.3. Unique feature comparing with existing literatures

In the present study, we analyze a buyer-owned B2B market in which a buyer endeavors to procure multiple identical items supplied by several firms with heterogeneous marginal costs, and each can submit a sealed bid in the buyer's procurement process. A unique feature of our approach is each supplier's cost information can be common knowledge or privately owned. Compared with the model studied by Bandyopadhyay et al. (2008), our model has the following different features. First, to measure the value of IT in online B2B exchange markets, we analyze the impact of sharing cost information on buyer's expected cost and supplier's expected profit. Second, because each supplier's marginal cost is not identical in the present study, the impact of asymmetric cost on each supplier's pricing strategy can be investigated. In addition, we also discuss the impact of heterogeneous capacity on suppliers' pricing strategies when each supplier has private cost information. Most economics literatures utilize the cost to the buyer and profit to the supplier to measure the performance of auction mechanisms (Teich et al., 2004). We also adopt the same criteria to measure the impact of cost information on buyer's expected cost and supplier's expected profit in the auction. Based on our derived results, we can further understand the value of information technology and the function of residual demand in online B2B exchange markets.

3. The model

We consider the online B2B exchange market in which there are one buyer and n suppliers vying for the cumulative demand Q . For the level of demand, some of prior studies associated with Bertrand-Edgeworth equilibrium consider it a function of price (Allen and Hellwig, 1986; Osborne and Pitchik, 1986; Deneckere and Kovenock, 1996), while some of them consider that it is independent of the market price (Bandyopadhyay et al., 2005; Fabra

et al., 2006; Sakellaris, 2010). One of the reasons to model price-inelastic demand can be attributed to the fact that observation in electricity markets shows that the price elasticity of demand is very low, which forms a common assumption in electricity market literature (Fehr and Harbord, 1993; Crampe and Creti, 2005; Fabra et al., 2006; Sakellaris, 2010). Another reason is that the assumption of inelastic demand has been verified by IndiaMarkets.com, India's largest exchange market. Bandyopadhyay et al. (2005) confirm that the demand for materials posted on the B2B portal will tend to be inelastic. In essence, the present study concentrates on online B2B exchanges, so we enhance their model and inherit the verified assumption on the level of demand.

For simplicity, the suppliers are denoted as S_i where $1 \leq i \leq n$. A summary of all the variables used in the present paper appears in Table 1. The supplier S_i 's individual marginal cost is denoted as c_i . The buyer's reservation price is denoted as r and the suppliers have equal capacities k . The assumption of equal capacities in the present model is the same as that studied by Bandyopadhyay et al. (2008). The buyer will not accept any bid higher than its reservation price.

Here, we consider the capacity constraint $(n-1)k < Q < nk$. That is, each supplier can sell its entire capacity other than the one submitting the highest bid supplies to the residual demand. Given the complete cost information, if $nk \leq Q$, each supplier can sell its entire capacity as long as its bid is not larger than the buyer's reservation price. As a result, each supplier bids the buyer's reservation price. In the case where $(m-1)k < Q < mk$ with $n > m$, $(n-m)$ suppliers do not supply anything in this bidding game. It is definitely a profitable move for a new supplier to join the bidding. Accordingly, the positive equilibrium profits for some suppliers are destroyed. However, in any equilibrium system marginal price cannot exceed the marginal cost of the $m+1$ st most efficient supplier if $n > m$ (Fehr and Harbord, 1993, p. 535). Since the case where only a fraction of the suppliers can sell is enough to result in a Bertrand competition (Bandyopadhyay et al., 2008), we are interested in the case (the supplier with the highest price is at least able to sell the residual) where pure equilibrium does not exist. For consistency and comparability, we adopt the capacity constraint throughout the present study to examine the impact of cost information on the buyer and suppliers.

Therefore, the supplier S_i bidding a price p_i can gain the profit $(p_i - c_i)(Q - (n-1)k)$ or $(p_i - c_i)k$, depending on whether its price is the highest. If the suppliers submit different bids, the lower-bidding supplier's capacity is dispatched first. If there are multiple suppliers submitting equal bids, we adopt the same approach as proposed by Fabra et al. (2006). That is, when the case arises, the service rank is determined by the supplier's marginal cost and the supplier with a lower marginal cost is ranked first. In addition, in most auction literature, when the suppliers bidding the same

Table 1
Notations.

Notation	Description
Q	Total quantity demanded
k	Capacity of each supplier
c_i	Constant marginal cost of each supplier i
r	The reservation price of the buyer
F	Supplier's cumulative density function of prices
f	Supplier's probability density function of prices
f_{v_j}	The j th order statistic
n	The number of suppliers
Φ^l, Φ^c	Buyer's expected cost
$\pi^*(\cdot), \bar{\pi}_i$	Supplier's expected profit
p_i	A specific threshold in which $G_i(p_i) = 1$, where $1 \leq i \leq n-1$
$p^*(\cdot)$	The symmetric equilibrium bidding function
F_{ij}	Supplier i 's cumulative density function of prices in $[p_{j-1}, p_j)$
$G_i(\cdot)$	Supplier i 's cumulative density function

price have the same marginal costs, a random draw will determine their service rank. In fact, because there are no point masses in the equilibrium bidding functions when the suppliers have the same marginal cost, we can omit the consideration.

4. An online B2B exchange market with public cost information

In the section, we consider the setting in which each supplier's marginal cost is common knowledge. To begin with, we demonstrate the simple case in which the two suppliers, S_H and S_L , compete for the cumulative demand Q . The idea of the example is helpful to build the mixed strategy Nash equilibrium in the B2B exchange market with n suppliers.

4.1. A two-supplier auction game

In the two-supplier auction game, both suppliers have equal capacity k and the inequality $k < Q < 2k$ holds. The winner can supply its total capacity k , whereas the loser can only sell the residual demand $Q - k$. The supplier S_H 's (S_L 's) bid and marginal cost are denoted as $p_H(p_L)$ and $c_H(c_L)$, respectively. The supplier S_H has the highest marginal cost; that is, $0 \leq c_L < c_H \leq r$. There can be no pure-strategy equilibrium for the two-supplier auction game if both suppliers have the same marginal costs, which has been proved by Bandyopadhyay et al. (2008). If both suppliers have asymmetric marginal costs in the two-supplier auction game, we can derive the same result by considering $\hat{p} \equiv c_H + (r - c_H)(Q - k)/k$.

Therefore, we seek the mixed strategy Nash equilibrium in the two-supplier auction game and then examine suppliers' pricing strategies. In the mixed-strategy equilibrium, the goal is to find cumulative distribution functions $F_H(\cdot)$ and $F_L(\cdot)$ so the suppliers, S_H and S_L , are able to gain indifferent expected profits by submitting their bids according to $F_H(\cdot)$ and $F_L(\cdot)$, respectively. We adopt the approach similar to that in Fabra et al. (2006) to construct the cumulative distribution functions as follows. For conciseness, given the supplier S_i , we use $F_{-i}(\cdot)$ to denote the cumulative distribution function of the other supplier. If the two suppliers bid according to $F_H(\cdot)$ and $F_L(\cdot)$, their expected profits are given by

$$\pi_i(p) = (p - c_i)(k(1 - F_{-i}(p)) + (Q - k)F_{-i}(p)), \quad \text{where } i \in \{L, H\}. \tag{4.1}$$

Lemma 1. *In the two-supplier auction game, $F_L(\hat{p}) = F_H(\hat{p}) = 0$ and $F_L(r) = F_H(r) = 1$ under the mixed-strategy equilibrium. All the proofs can be found in the Appendix.*

A necessary condition for the supplier S_H to be indifferent between any price in $[\hat{p}, r]$ is, for all $p \in [\hat{p}, r]$, $\pi_H(p) = \pi_H(r)$. Note, the supplier S_H would certainly supply the residual demand by setting the price at r . That is, $\pi_H(r) = (Q - k)(r - c_H)$. Using (4.1) and the necessary condition, we can obtain the expression for $F_L(p)$ as follows:

$$F_L(p) = \frac{(p - c_H)k - (r - c_H)(Q - k)}{(p - c_H)(2k - Q)}. \tag{4.2}$$

Note, $F_L(p)$ is a continuous function on $[\hat{p}, r]$. Indeed, the property of $F_L(p)$ fits our requirement given in Lemma 1. However, $\hat{p} \equiv c_L + (r - c_L)(Q - k)/k$ is the lower limit of p_L . Thus, a straightforward evaluation of $F_H(p)$ with $F_H(\hat{p}) = 0$ would imply $F_H(r) < 1$. To compensate for this, $\Pr(p_H = r)$ has a positive probability θ , i.e., $F_H(r^-) = 1 - \theta$. Now, using this value and the equation $\pi_L(p) = \lim_{p \rightarrow r} \pi_L(p)$, we have

$$F_H(p) = \begin{cases} \frac{(p - c_L)k - (r - c_L)(Q - k) - (2k - Q)(r - c_L)\theta}{(p - c_L)(2k - Q)}, & \hat{p} \leq p < r, \\ 1, & p = r, \end{cases} \tag{4.3}$$

where $\theta = \frac{c_H - c_L}{r - c_L}$ is derived by solving $F_H(\hat{p}) = 0$. According to the rules described in Section 3, the supplier S_L is dispatched first when both suppliers bid $p = r$. Therefore, the equilibrium profits become

$$\begin{aligned} \pi_H &= (r - c_H)(Q - k) = (\hat{p} - c_H)k, \\ \pi_L &= (r - c_L)\{\Pr(p_H < r)(Q - k) + \Pr(p_H = r)k\} = (\hat{p} - c_L)k. \end{aligned} \tag{4.4}$$

Proposition 1. *In the two-supplier auction game, the probability the supplier with the higher marginal cost bids $p_H = r$ increases with the difference between the two suppliers' marginal costs. Formally, $\partial\theta/\partial\Delta > 0$, where $\Delta \equiv c_H - c_L$.*

As in Bandyopadhyay et al. (2008), both suppliers would bid according to the same cumulative distribution function when they have the same marginal costs. Note, supplier S_L can bid a price lower than \hat{p} if $c_H > c_L$ holds. However, in this case, supplier S_H would always obtain the residual demand. That is, because of the disadvantage of the higher marginal cost, the supplier S_H has to bid $p_H = r$ with a positive probability to ensure the supplier S_L has no incentive to bid a price lower than \hat{p} . Thus, supplier S_H also has a chance of supplying its total capacity by randomizing its price on $[\hat{p}, r]$. As for the supplier S_L , it just simply randomizes its price on $[\hat{p}, r]$ to ensure the supplier S_H can receive indifferent expected profit for all $p_H \in [\hat{p}, r]$.

Because the supplier S_H 's profit under the mixed strategy Nash equilibrium is positively proportional to the residual demand, we have $\partial\pi_H/\partial k < 0$. However, examining $\partial\pi_L/\partial k$, we have the following interesting finding.

Proposition 2. *In the two-supplier auction game, the supplier S_L 's equilibrium profit may increase (decrease) with the given capacity when the supplier S_H 's marginal cost is higher (smaller) than a specific threshold. Formally, $\partial\pi_L/\partial k \geq 0$ if and only if $c_H \geq (c_L + r)/2$.*

In fact, the equilibrium profit of the supplier S_L can be rewritten as follows:

$$\pi_L = \pi_H + (c_H - c_L)k. \tag{4.5}$$

That is, the profit of the supplier S_L is the profit of the supplier S_H plus $(c_H - c_L)k$. As the given capacity increases, supplier S_L has to randomize its price within a pricing space containing smaller prices than before. In other words, $\partial\hat{p}/\partial k < 0$. Since the profit of supplier S_H is inversely proportional to the given capacity, supplier S_L 's profit may increase or decrease with the given capacity, depending on whether its cost advantage can dominate the effect of competition.

4.2. Extending the model to a n-supplier auction game ($n \geq 2$)

Suppose there are n suppliers competing for the cumulative demand Q . All suppliers have equal capacities k and the inequality $(n - 1)k < Q < nk$ holds. Supplier S_i 's marginal cost is given by c_i where $0 \leq c_1 < c_2 < \dots < c_n \leq r$. Based on the approach of the two-supplier auction game, we can determine the bottom of the range of the active bids for all suppliers as follows:

$$p_0 \equiv c_n + (r - c_n)(Q - (n - 1)k)/k. \tag{4.6}$$

Lemma 2. *Suppose each supplier bids a random price according to the cumulative density function $G_i(\cdot)$, forming a mixed strategy Nash equilibrium. Denoting P_i as the support of $G_i(\cdot)$, we have $P_i \subseteq [p_0, r]$ for $1 \leq i \leq n$.*

Because of Lemma 2, we can only put attention on the specific closed set $[p_0, r]$ to figure out the formulation of $G_i(\cdot)$. In the following, we first give the complete formulation of $G_i(\cdot)$, and then show the n -supplier auction game has a mixed strategy Nash equilibrium

if each supplier S_i bids a random price on $[p_0, p_i]$ according to the cumulative density function $G_i(p)$. The formal formulation of $G_i(\cdot)$ is given by

$$\forall i = 1, 2, \dots, n$$

$$G_i(p) = \begin{cases} F_{ij}(p), & p_{j-1} \leq p < p_j, \\ 1, & p_i \leq p, \end{cases} \quad (4.7)$$

where

$$1 \leq j \leq \min\{i, n - 1\}, \quad (4.8)$$

$$F_{ij}(p) = \left(\frac{k(p - p_0)(p - c_i)^{n-j-1}}{(nk - Q) \prod_{\delta \neq i, \delta \geq j} (p - c_\delta)} \right)^{\frac{1}{n-j}}, \quad (4.9)$$

$$F_{i,i}(p_i) = 1 \quad \text{for} \quad 1 \leq i \leq n - 1 \quad \text{and} \quad p_n = r. \quad (4.10)$$

Clearly, $G_i(\cdot)$ is a piecewise function composed of $F_{ij}(p)$, in which $1 \leq j \leq \min\{i, n - 1\}$. To examine the validity of $G_i(\cdot)$, we have to check whether the relation $p_0 < p_1 < \dots < p_{n-1}$ holds.

Lemma 3. $p_0 < p_1 < \dots < p_{n-1} = r$ if $F_{i,i}(p_i) = 1$ for $1 \leq i \leq n - 1$. Moreover, $F_{i,i}(p)$ is an increasing function.

The function $F_{ij}(p)$ is the key component of the mixed strategy Nash equilibrium in the n -supplier auction game. The basic idea of the piecewise functions is as follows. To find $G_i(\cdot)$ so the equilibrium profit of each supplier is indifferent among all prices higher than p_0 , we can follow the approach of the two-supplier auction game to derive each $G_i(\cdot)$. However, if $G_i(\cdot)$ is not a piecewise function, we can find p_1 which satisfies $G_1(p_1) = 1$ is less than the buyer's reservation price r when $n \geq 3$, implying $G_i(\cdot)$ must be a piecewise function. Thus, we have to adopt the piecewise approach to construct $G_i(\cdot)$. The difference between the n -supplier auction game ($n \geq 3$) and 2-supplier auction game is we only consider partial suppliers in each segment other than the first segment $[p_0, p_1]$. For example, we solve all suppliers in the first segment $[p_0, p_1]$, but solve the only two suppliers, S_{n-1} and S_n , in the final segment $[p_{n-2}, p_{n-1}]$. In the case of the final segment, because we have $\lim_{p \rightarrow r} F_{n,n-1}(p) < 1$, supplier S_n has to set its price at $p = r$ with a positive probability to compensate this. Because supplier S_n has the same pricing space as supplier S_{n-1} , we use the constraint (4.8) to treat with the concern.

Next, because p_0 is the bottom of the range of active bids for all suppliers and each supplier's profit has to be indifferent among the support of prices, each supplier's expected profit is given by

$$\bar{\pi}_i = k(p_0 - c_i). \quad (4.11)$$

Lemma 4. For the suppliers S_j, S_{j+1}, \dots , and S_n , if they are the only suppliers bidding on $[p_{j-1}, p_j]$ and each supplier S_i , where $j \leq i \leq n$, bids $p \in [p_{j-1}, p_j]$ according to the cumulative distribution function $F_{ij}(p)$, then each supplier's expected profit on $[p_{j-1}, p_j]$ is $\bar{\pi}_i$.

Relying on Lemma 4, the final task is to check whether each supplier has no incentive to deviate the mixed strategy Nash equilibrium derived from $G_i(p)$.

Proposition 3. In the n -supplier auction game, there is a mixed strategy Nash equilibrium if each supplier S_i bids a random price in $[p_0, p_i]$ according to the cumulative density function $G_i(p)$.

Compared with the case discussed by Bandyopadhyay et al. (2008), our results display an interesting finding. It is considered the supplier with cost advantage would bid as high as possible. However, for the case of $n \geq 3$, supplier S_i , where $i \leq n - 2$, with a cost advantage wouldn't bid a price higher than the specific threshold p_i . Actually, a price higher than the specific threshold p_i would increase the profit of supplier S_i when it is not the highest bidder; however, the price also results in a higher probability of

getting residual demand for supplier S_i . When supplier S_i 's bid is higher than p_i , the increase in the profit resulting from a higher price is dominated by the loss of getting residual demand. Therefore, the highest possible bid submitted by supplier S_i is less than that submitted by other suppliers whose marginal costs are higher than supplier S_i 's.

Note, if there are multiple suppliers having the same marginal costs, they would bid prices according to the same cumulative distribution functions. Recently, with two suppliers, the case of heterogeneity in both costs and capacities has been studied by Bandyopadhyay et al. (2005), in which the lowest support price depends on the relative rankings of indifference prices, where each supplier has its own indifference price, being indifferent between bidding the highest price to sell residual demand and the indifference price to sell all capacity. Moreover, Fabra et al. (2006) demonstrate the interesting case in which three suppliers differ in both costs and capacities. In this case, there exist multiple pure equilibria when uniform auction is adopted. Generally speaking, if all suppliers have unequal capacities and asymmetric costs, this complicates the analysis although it may introduce some sort of new phenomena. The analysis for the case becomes more complex and less tractable for more than the scenario of two suppliers (Bandyopadhyay et al., 2005); therefore, in this research we only treat the issue of heterogeneity in costs when extending to the n -supplier case. The extension to the n -supplier case of heterogeneity in both costs and capacities remains an open question in various domains. A numerical example of $G_i(p)$ is shown in Fig. 1 in which $c_i = i/10, n = 5, k = 5, r = 1$, and $Q = 21$. Subsequently, the buyer's expected cost is given by

$$\Phi^C = \sum_{i=1}^n \bar{\pi}_i + \sum_{i=1}^{n-1} \int_{p_0}^{p_i} c_i \left\{ \frac{p_0 - c_i}{p - c_i} \right\} kdG_i(p) + \lim_{p \rightarrow r} \int_{p_0}^p c_n \left\{ \frac{p_0 - c_n}{p - c_n} \right\} kdG_n(p) + \frac{c_n(c_n - c_{n-1})(Q - (n-1)k)}{r - c_{n-1}} \quad (4.12)$$

The first term of Φ^C is the sum of each supplier's expected profit, whereas the second term of Φ^C is the sum of each supplier's expected cost. Note, the supplier with the highest marginal cost would randomize its price in $[p_0, r)$ and bid r with a positive probability; therefore, we need the third and fourth terms to calculate S_n 's expected cost. On the other hand, because $G_i(p)$ is a piecewise function, there is no general closed form for the buyer's expected cost in the n -supplier auction game. Thus, by numerical integration, we can derive the suppliers' expected costs and then utilize the results to figure out the suppliers' expected revenues (i.e., the buyer's expected cost). For the suppliers ($S_i, 1 \leq i \leq n$), their expected costs are given by 0.4964, 0.9777, 1.3851, 1.5579, and

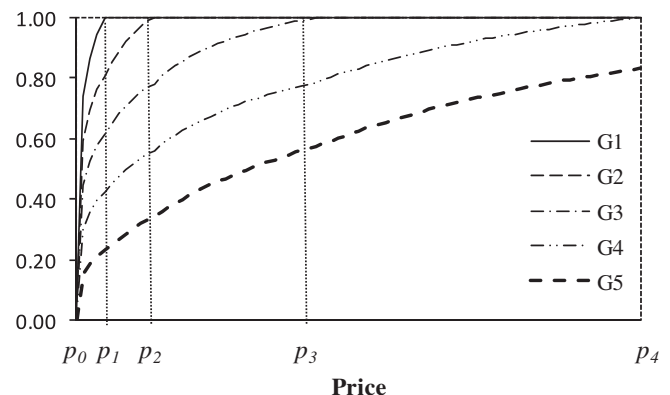


Fig. 1. Graph of $G_i(\cdot)$ $p_0 = 0.6, p_1 = 0.62, p_2 = 0.65, p_3 = 0.77, p_4 = 1$.

1.2637, respectively. Further, the buyer’s average expected cost (i.e., Φ^c/Q) is about 0.628 in the case. We can observe supplier S_5 ’s expected cost is less than S_4 ’s and S_3 ’s, although supplier S_5 ’s marginal cost is higher than S_4 ’s and S_3 ’s. The result stems from supplier S_5 having more chance than others to get residual demand; thus, we have this interesting finding.

5. An online B2B exchange market with private cost information

In reality, supplier’s cost information should be private and each supplier only knows the distribution of the supplier’s marginal cost. In the online auction, each supplier submits a sealed-bid during a specific period of time. Here, we emphasize the construction of a strictly increasing bidding function that is optimal for each supplier to employ, given all other suppliers also employ this bidding function. The symmetric equilibrium bidding function is denoted as $p(\cdot)$. Clearly, for every i , $c_i \leq p(c_i) \leq r$.

We assume the marginal costs of the suppliers are independent and identically distributed, drawn from the common distribution $F(c_i)$ with $F(0) = 0$, $F(r) = 1$ and $F(c_i)$ strictly increasing and differentiable over the interval $[0, r]$. The assumption was first presented by Vickrey, and has been frequently employed in the bidding literature. Based on the approach presented by Vickrey, the probability b_i submitted by the supplier S_i exceeds the bids of all other suppliers is given by $F^{n-1}(b_i)$. Thus, if the supplier S_i chooses b_i as its bid, the expected profit of the supplier S_i is given by

$$\pi_i = (p(b_i) - c_i)\{ (1 - F^{n-1}(b_i))k + F^{n-1}(b_i)(Q - (n - 1)k) \} = (p(b_i) - c_i)\{ k + F^{n-1}(b_i)(Q - nk) \}. \tag{5.1}$$

The Bayesian Nash equilibrium is given by solving $\partial\pi_i/\partial b_i = 0$ and setting $b_i = c_i$. Letting f denote the probability distribution function associated with the cumulative distribution function F . Then, we have the following equation:

$$\frac{\partial}{\partial c_i} p(c_i)\{ k + F^{n-1}(c_i)(Q - nk) \} = c_i(n - 1)F^{n-2}(c_i)f(c_i)(Q - nk). \tag{5.2}$$

Integrating both sides, we have

$$p(c_i)\{ k + F^{n-1}(c_i)(Q - nk) \} = (n - 1)(Q - nk) \times \int_0^{c_i} F^{n-2}(x)xf(x)dx + C. \tag{5.3}$$

Applying the condition $p^*(r) = r$, we can derive C as follows:

$$C = rk + (Q - nk)\left(r - \int_0^r x dF^{n-1}(x) \right). \tag{5.4}$$

Thus, the symmetric equilibrium bidding function is given by

$$p^*(c_i) = \frac{rk + (Q - nk)(r + \int_0^{c_i} x dF^{n-1}(x) - \int_0^r x dF^{n-1}(x))}{k + F^{n-1}(c_i)(Q - nk)}. \tag{5.5}$$

To derive the analytical results, we assume each supplier’s marginal cost is uniformly distributed on $[0, r]$. Consequently, the strictly increasing symmetric bidding function is

$$p^*(c_i) = \frac{r^n k + (Q - nk)\left(\frac{(n-1)c_i^n}{n} + \frac{r^n}{n}\right)}{r^{n-1}k + c_i^{n-1}(Q - nk)}. \tag{5.6}$$

Relying on the symmetric bidding function, the supplier S_i ’s expected profit is

$$\pi^*(c_i) = (r - c_i)k + (Q - nk)\left(\frac{r^n - c_i^n}{nr^{n-1}}\right). \tag{5.7}$$

Proposition 4. *In the auction in which each supplier has private cost information drawn from common uniform distribution, each supplier’s price and profit decrease with capacity k but increase with cumulative demand Q . Formally, $\partial p^*(c_i)/\partial k \leq 0$, $\partial \pi^*(c_i)/\partial k \leq 0$, $\partial p^*(c_i)/\partial Q \geq 0$, and $\partial \pi^*(c_i)/\partial Q \geq 0$.*

Under the inequality $(n - 1)k < Q < nk$, if capacity k increases, the amount of residual demand would decrease. Therefore, each supplier in the case would lower its price to prevent from becoming the highest bidder. Because each supplier adopts the same strategy, the supplier S_i ’s expected profit would decrease when capacity k increases. On the contrary, when the cumulative demand Q increases, we have the opposite results because the amount of residual demand increases. Given $r = 1$, $Q = 21$, $n = 5$, and $k = 5$, the numerical example of $p^*(c_i)$ and $\pi^*(c_i)$ are plotted in Figs. 2 and 3. Fig. 2 shows the bidding function $p^*(c_i)$ is a strictly increasing one, whereas Fig. 3 shows each supplier’s expected profit decreases with its marginal cost.

Although we only consider equal capacities in the present study, we can, in fact, relax the assumption of equal capacities when each supplier has private cost information, discussing its implications in the following. If each supplier has asymmetric capacity denoted as k_i , the symmetric equilibrium bidding function and the supplier S_i ’s expected profit are given by

$$p^*(c_i) = \left\{ r^n k_i + \left(Q - \sum_{j=1}^n k_j \right) \left(\frac{(n-1)c_i^n}{n} + \frac{r^n}{n} \right) \right\} / \left\{ r^{n-1} k_i + c_i^{n-1} \left(Q - \sum_{j=1}^n k_j \right) \right\}, \tag{5.8}$$

$$\pi^*(c_i) = (r - c_i)k_i + \left(Q - \sum_{j=1}^n k_j \right) \left(\frac{r^n - c_i^n}{nr^{n-1}} \right). \tag{5.9}$$

Proposition 5. *In the auction in which each supplier has asymmetric capacity and private cost information drawn from common uniform distribution, the impact of exogenous parameters on the symmetric equilibrium bidding function and each supplier’s expected profit is summarized in Table 2 and can be summarized as follows:*

1. Each supplier S_i ’s bid $p^*(c_i)$ decreases with its capacity, k_i , and the capacities of others, k_j , but increases with cumulative demand Q and its marginal cost, c_i .
2. Each supplier S_i ’s expected profit $\pi^*(c_i)$ decreases with its marginal cost, c_i , and the capacities of others, k_j , but increases with its capacity, k_i , and cumulative demand Q .

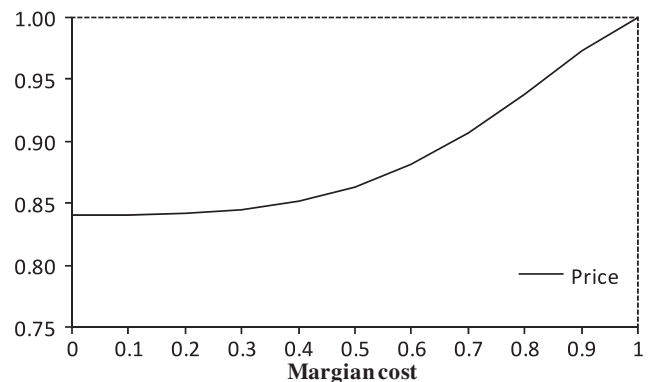


Fig. 2. Graph of $p^*(\cdot)$.

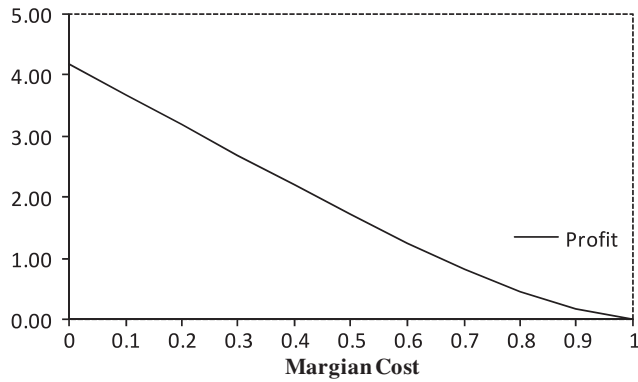


Fig. 3. Graph of $\pi^*(\cdot)$.

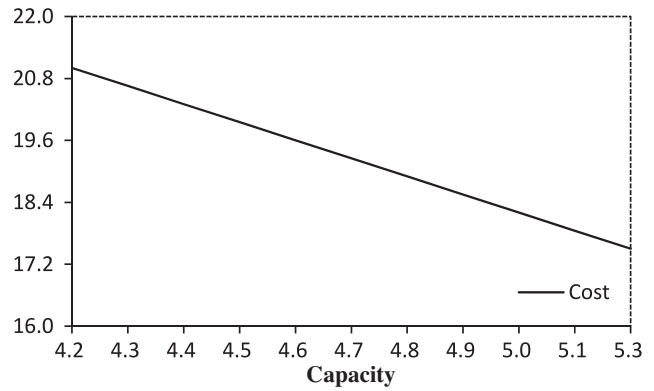


Fig. 4. Graph of Φ^l .

Table 2
Impact of exogenous parameters on bidding function and expected profit.

	k_i	k_j	Q	c_i
$p^*(c_i)$	–	–	+	+
$\pi^*(c_i)$	+	–	+	–

+ increase; – decrease.

When each supplier can have asymmetric capacity, most results in Proposition 5 remain the same as in Proposition 4; however, for supplier S_i , its profit increases with individual capacity but decreases with its competitor's. Formally, $\partial\pi^*(c_i)/\partial k_i \geq 0$ and $\partial\pi^*(c_i)/\partial k_j \leq 0$. The reason for the results is as follows. No matter which supplier's capacity increases, each supplier lowers its prices to respond. If one of the suppliers' capacities increases (e.g., k_j), supplier S_j 's profit, of course, would decrease. However, if only supplier S_i 's capacity increases, the positive effect of the increasing capacity dominates the negative effect of the decreasing price. Therefore, supplier S_i can gain a higher profit when only its capacity increases.

Subsequently, we want to know how much the buyer would pay in the online auction. From the order statistics, the buyer's expected total cost is given by

$$\begin{aligned} \Phi^l &= \int_0^r \left\{ \sum_{j=1}^{n-1} p^*(x) \cdot k \cdot f_{Y_j}(x) + p^*(x)(Q - (n-1)k)f_{Y_n}(x) \right\} dx \\ &= \int_0^r \left\{ k \sum_{j=1}^{n-1} f_{Y_j}(x) + (Q - (n-1)k)f_{Y_n}(x) \right\} p^*(x) dx, \end{aligned} \tag{5.10}$$

where $f_{Y_j}(x) = \frac{n!}{(n-j)!(j-1)!} F^{j-1}(x)(1-F(x))^{n-j} f(x)$.

The following formulation based on the binomial theorem is helpful to derive the explicit form of (5.10).

$$\sum_{j=1}^n f_{Y_j}(x) = \sum_{j=1}^n n \binom{n-1}{j-1} F^{j-1}(x)(1-F(x))^{n-j} f(x) = \frac{n}{r}. \tag{5.11}$$

Because of $f_{Y_n}(x) = \frac{n}{r} \left(\frac{x}{r}\right)^{n-1}$, the buyer's expected total cost can be rewritten as follows:

$$\begin{aligned} \Phi^l &= \int_0^r \left\{ k \frac{n}{r} \left(1 - \left(\frac{x}{r}\right)^{n-1}\right) + (Q - (n-1)k) \frac{nx^{n-1}}{r^n} \right\} p^*(x) dx \\ &= \frac{r \cdot n}{n+1} (2Q - k(n-1)). \end{aligned} \tag{5.12}$$

We find the buyer's expected total cost increases with cumulative demand Q .

When cumulative demand Q increases, each supplier would bid a higher price since each can sell a higher amount of residual units even if losing the auction. This leads to an increase in the buyer's expected total cost. Ideally, a small residual demand can serve as a strategy to make each supplier bid a higher price. The numerical example of Φ^l is shown in Fig. 4, in which the parameters are given by $r = 1$, $Q = 21$, $n = 5$, and $4.2 < k < 5.25$. Clearly, when the amount of each supplier's capacity sold increases (i.e., the residual demand decreases), the buyer's expected total cost decreases.

In the following, we consider the impact of the number of suppliers on the buyer's expected total cost. Intuitively, we can differentiate Φ^l with respect to the number of suppliers, n , to examine the impacts. However, because the capacity constraint $(n-1)k < Q < nk$ must hold for all $n \geq 2$, the value of capacity would vary with the number of suppliers to satisfy the capacity constraint. Thus, we have to adopt other approaches to examine the effect of the number of suppliers in the auction.

Proposition 6. *In the auction in which each supplier has private cost information, the buyer's expected cost decreases with capacity k . Moreover, if the buyer can choose capacity k and the number of suppliers so the capacity constraint $(n-1)k < Q < nk$ holds for all $n \geq 2$, the expected average cost per unit approaches the buyer's reservation price when the number of suppliers is sufficiently large.*

When the amount of capacity increases, each supplier hazards the decrease in the residual demand; therefore, the suppliers participating in the auction would lower their prices. From the buyer's point of view, the number of suppliers should be as small as possible to derive a competitive price, which is contrary to the findings of general competition study. For example, in Cournot competition, when the number of suppliers approaches infinity, the price would fall to the level of the suppliers' marginal costs. The reason for the difference is as follows. In the assumption of our model, only the highest bidder would obtain the residual demand. However, when the number of suppliers increases, the probability of becoming the highest bidder decreases; therefore, each supplier raises its bid to increase the buyer's expected total cost. Thus, the buyer should limit the number of suppliers and raise the amount of capacity supplied by each bidder.

6. Managerial implications: the design of online B2B exchange market

The goal of this study is to provide a theoretical basis for understanding the effect of asymmetric cost and incomplete information on B2B exchanges. In this research, in order to derive an explicit closed form of bidding function for further analytical analysis, we adopt the classic assumption of uniform distribution, which is

frequently used in auction literature, although other distribution of the unit cost may result in different comparative results. Examining the two settings which differ in cost information, we find that the buyer has an incentive to introduce as much competition as possible; however, our results also show the buyer's expected cost may increase with the number of suppliers when the assumption of capacity constraints must hold (i.e., $(n-1)k < Q < nk$). In addition, regardless of whether the supplier's cost information is known, the impact of residual demand on supplier's pricing strategy is the same. That is, each supplier would bid as high as possible when the highest bidder can supply nearly its entire capacity and lower the price when the residual demand is a small fraction of the suppliers' capacity. Currently, the private exchange, a privately built electronic market by a single buyer attracting many suppliers into the buyer's market, such as Wal-Mart, forms the largest part of e-commerce (Laudon and Traver, 2008; Lee et al., 2009). To strengthen the relationship between the buyer and suppliers, the function of residual demand can be considered the participation motivation of the suppliers. This research also points out residual demand can serve as a strategy to make each supplier bid a lower price in the online B2B exchange market.

For each supplier S_i , when the cost information is private, its profit is only associated with its individual marginal cost. However, if the cost information is known, its profit is not only associated with its individual marginal cost but also associated with the highest marginal cost. Therefore, comparing $\pi^*(c_i)$ with $\bar{\pi}_i$, we find $\bar{\pi}_i > \pi^*(c_i)$ when $n(r - c_n) < r - c_i(c_i/r)^{n-1}$ holds and $\bar{\pi}_i < \pi^*(c_i)$ when the opposite holds true. The result shows complete cost information benefits supplier S_i when the highest marginal cost approaches the buyer's reservation price. Because the lower bound of the bid range of each supplier depends on the highest marginal cost when the cost information is known, the buyer would pay more in that case.

On the contrary, when the number of suppliers is large enough and the highest marginal cost is less than the buyer's reservation price, sharing cost information cannot improve the supplier S_i 's expected profit. The reason for this result is as follows. When marginal cost is not common knowledge, each supplier bids according to its individual marginal cost; however, when the number of suppliers grows, the chance of getting the residual demand is reduced. As a result, as long as the number of suppliers is sufficiently large, each supplier can gain a high expected profit $\pi^*(c_i)$ approximating $(r - c_i)k$. Compared with $\bar{\pi}_i = (p_0 - c_i)k$, clearly, when the required condition is satisfied, $\bar{\pi}_i < \pi^*(c_i)$ holds.

Consequently, from the supplier's point of view, investing in IT to improve information transparency benefits them is uncertain because it depends on the scale of the auction and the highest marginal cost. On the other hand, from the buyer's point of view, it can collect a large amount of suppliers' data from the Internet in advance and then invite qualified suppliers with cost advantages to submit their bids. Our research also suggests restraining the number of suppliers is also helpful if the cost of IT investment is expensive, which is consistent with the traditional market where most raw materials or components buyers demand are supplied by a limited set of suppliers.

7. Conclusion

In the present study, we examine the function of residual demand and the value of IT in the online B2B exchange market in which the buyer requests multiple items and the suppliers have differing marginal costs. In the online B2B exchange market, each supplier can sell its entire capacity other than the one bidding the highest price can only supply the residual demand. To strengthen the relationship between the buyer and suppliers in

the supply chain, residual demand plays an important role in the motivation for participation of the suppliers because each supplier can make a profit. On the other hand, residual demand can serve as a strategy for deriving competitive prices from the suppliers because no supplier wants to be the loser supplying the residual demand. Our research points out the buyer should only keep a limited set of suppliers if it wants to both maintain a long-term partner relationship and procure materials at lower prices.

To understand the value of IT in online B2B exchange markets, we consider the suppliers' cost information may be common knowledge or private information. Whether IT in the online B2B exchange market benefits buyers or suppliers depends on the scale of the auction and the highest marginal cost. Public cost information can prevent suppliers from submitting high bids even if the number of participants in the online auction is sufficiently large; however, when the highest marginal cost is sufficiently high, the buyer would have a higher expected total cost because each supplier would bid high after they know there is a competitor having such a high marginal cost as to barely constitute a threat at all.

Some future directions of research are as follows. First, we only consider each supplier has equal capacity in the current research. Although we can relax the assumption when each supplier has private cost information, the general case in which the suppliers have varied capacity can be further studied. Second, although a buyer, indeed, has incentive to aggregate more suppliers to lead a Bertrand competition in prices, time cost can be a plausible reason so the buyer prefer receiving materials as soon as possible rather than waiting. This can form a future research that studies whether a buyer should wait (that is, the buyer set a longer deadline to add more bidders to the supplier pool). Third, some common knowledge given in the study, such as the buyer's reservation price and the number of suppliers, could be incomplete information in the future research. In addition, based on the interest of mathematics, it would be worthy to examine whether the mixed strategy Nash equilibrium given in the present study is unique. Fourth, a more realistic model should include the number of suppliers, supplier's capacity, and time variation to gather a different number of suppliers because these factors will certainly affect buyer's decision. In addition, better product quality and reputation, which improve product reliability and buyer's trust, are beneficial to suppliers; thus, future directions can include the analysis of product quality and supplier's reputation mechanism after auctions. Finally, although the mathematical result shows the less residual demand, the lower the procurement cost, whether suppliers have an incentive to participate in the bidding process remains to be solved. That is, the buyer has to ensure the motivation for participation of the suppliers by offering a sufficiently high residual demand. On the other hand, the lower residual demand can reduce the procurement costs. Thus, how to design an optimal residual demand is worthy of further study.

Appendix A. Proof of Lemma 1

Clearly, the suppliers cannot submit any bid higher than r , implying $F_L(r) = F_H(r) = 1$. For the supplier S_H , if the supplier S_L randomizes its price in $[\hat{p}, r]$, the pure strategy $p_H = r$ dominates the choices $p_H < \hat{p}$, implying $F_H(\hat{p}) = 0$. For the supplier S_L , if the supplier S_H only bids in $[\hat{p}, r]$, the pure strategy $p_L < \hat{p}$ is a strictly dominated strategy since its profit is strictly increasing in p_L up to \hat{p} , implying $F_L(\hat{p}) = 0$. \square

Appendix B. Proof of Propositions 1 and 2

By first-order condition, we complete the proof. \square

Appendix C. Proof of Lemma 2

Clearly, each supplier's bid cannot be larger than r . Moreover, the supplier with the highest marginal cost has no incentive to bid $p < p_0$ because the pure strategy $p = r$ dominates the choices $p < p_0$, implying $P_n \subseteq [p_0, r]$. Again, similar to Lemma 1, all suppliers whose marginal costs are less than c_n have no incentive to bid $p < p_0$; consequently, we have $P_i \subseteq [p_0, r]$. \square

Appendix D. Proof of Lemma 3

Because $F_{n-1,n-1}(r) = 1$ holds, we have $p_{n-1} = r$. Suppose we have found p_{n-j} so $F_{n-j,n-j}(p_{n-j}) = 1$. Then, we can easily verify $F_{n-j-1,n-j-1}(p_{n-j}) > 1$. Because of $F_{n-j-1,n-j-1}(p_0) = 0$, we can find p_{n-j-1} so $F_{n-j-1,n-j-1}(p_{n-j-1}) = 1$ holds, where $p_0 < p_{n-j-1} < p_{n-j}$. If there are multiple values satisfying the equation, we choose the smallest one. \square

D.1. The increasing property of $F_{ij}(p)$

Subsequently, it is easy to verify that $\partial F_{n,n-1}(p)/\partial p > 0$ and $\partial F_{n-1,n-1}(p)/\partial p > 0$; therefore, we consider the other cases as follows. Because of $F_{ij}(p) > 0$ for $p \in (p_0, r]$ and $F_{ij}(p_0) = 0$, we can ensure that $F_{ij}(p)$ increases with p when $p_0 < p < p_0 + \varepsilon$; otherwise, $F_{ij}(p) < 0$, which violates the fact that $F_{ij}(p) > 0$. Next, we examine the properties of $F_{ij}(p)$ where $p > p_0$ by the following approach:

$$\begin{aligned} \text{sign} \frac{\partial F_{ij}(p)}{\partial p} &= \text{sign} \frac{\partial (p - p_0)(p - c_i)^{n-j-1}}{\partial p \prod_{\delta \neq i, \delta \geq j} (p - c_\delta)} \\ &= \text{sign} \frac{\partial}{\partial p} \ln \frac{(p - p_0)(p - c_i)^{n-j-1}}{\prod_{\delta \neq i, \delta \geq j} (p - c_\delta)}. \end{aligned}$$

That is, we can examine the following function instead:

$$\Gamma_{ij}(p) \equiv \frac{1}{(p - p_0)} + (n - j - 1) \frac{1}{(p - c_i)} - \sum_{\delta \neq i, \delta \geq j} \frac{1}{(p - c_\delta)}.$$

Notice that $\lim_{p \rightarrow p_0} \Gamma_{ij}(p) > 0$ holds. Moreover, $\Gamma_{k,i}(p) \geq \Gamma_{i,i}(p)$ where $i \leq k \leq n$ and $i \in \{1, 2, \dots, n - 2\}$. Because $\frac{1}{(p - p_0)} + (n - j - 1) \frac{1}{(p - c_i)}$ and $\sum_{\delta \neq i, \delta \geq j} \frac{1}{(p - c_\delta)}$ are decreasing convex functions in p and $\lim_{p \rightarrow \infty} \frac{1}{p - x} = 0$, both functions can at most cross once. Accordingly, it is possible that $\Gamma_{ij}(p)$ is less than zero when p reaches a certain value; however, $\Gamma_{i,i}(p) < 0$ holds forever as long as p is higher than the value. Subsequently, we discuss the case that $\Gamma_{i,i}(p) < 0$ when $p < p_i$.

If $\Gamma_{i,i}(p) < 0$ when $p < p_i$, we have $F_{i,i}(p_i) < 1$; however, we have shown the result that there exists p_i such that $F_{i,i}(p_i) = 1$ in the previous part, which leads contradiction. That is, $\Gamma_{i,i}(p) < 0$ only holds when $p > p_i$. Therefore, $F_{i,i}(p)$ increases with p in the given interval. Moreover, $F_{ij}(p)$ increases with p in the given interval due to $\Gamma_{k,i}(p) \geq \Gamma_{i,i}(p) > 0$, which completes the proof. \square

Appendix E. Proof of Lemma 4

Given the range $p_{j-1} \leq p < p_j$, there are $n - j + 1$ suppliers bidding in the range. Therefore, if the supplier S_i , where $j \leq i \leq n$, bids $p \in [p_{j-1}, p_j)$ according to the cumulative distribution function $G_i(p)$, the supplier's profit is

$$\pi_i(p) = (p - c_i) \left\{ \left(1 - \prod_{m \neq i, m \geq j} G_m(p) \right) k + \prod_{m \neq i, m \geq j} G_m(p) (Q - (n - 1)k) \right\}.$$

To ensure the supplier S_i 's profit equals $\bar{\pi}_i$, we have the equation $(p - c_i) \{ k + \prod_{m \neq i, m \geq j} G_m(p) (Q - nk) \} = (p_0 - c_i) k$, which can be rewritten as follows: $\prod_{m \neq i, m \geq j} G_m(p) = \frac{k}{nk - Q} \left\{ \frac{p - p_0}{p - c_i} \right\}$. Because of

$$\prod_{j \leq i \leq n} \prod_{m \neq i, m \geq j} G_m(p) = \left(\prod_{m \geq j} G_m(p) \right)^{n-j} \quad \text{and} \quad G_i(p) = \prod_{m \geq j} G_m(p) / \prod_{m \neq i, m \geq j} G_m(p),$$

Appendix F. Proof of Proposition 3

By Lemma 2–4, we can confirm each supplier S_i can gain the expected profit $\bar{\pi}_i$ by bidding $p \in [p_0, p_i]$. In addition, the supplier with the highest marginal cost can also gain the expected profit $\bar{\pi}_n$ by bidding $p \in [p_0, r]$. However, because of $F_{n,n-1}(p) < 1$, to compensate for this, the supplier n bids the price r with a positive probability $(c_n - c_{n-1}) / (r - c_{n-1})$. Examining $G_i(p)$, we find they are continuous functions other than $G_n(p)$ having a jump at $p = r$. For the case of $n = 2$, we have completed the proof. For the case of $n \geq 3$, due to $D_i \subseteq [p_0, r]$, we only need to check whether each supplier S_i , other than supplier S_n and S_{n-1} , can gain more profit by bidding any price in $[p_i, r]$. For supplier S_i bidding in $[p_{j-1}, p_j)$ in which $p_i \leq p_{j-1} < p_j \leq r$, its profit is given by

$$\pi_i(p) = (p - c_i) \left\{ k + \prod_{m \geq j} G_m(p) (Q - nk) \right\}.$$

Suppose the profit is larger than $\bar{\pi}_i$, because term $(Q - nk)$ is negative, the following inequality must be satisfied.

$$\prod_{m \geq j} G_m(p) \leq \frac{k}{nk - Q} \left\{ \frac{p - p_0}{p - c_i} \right\}.$$

However, by the formulation of $G_i(p)$ given in (4.7)–(4.10), we can easily find

$$\prod_{m \geq j} G_m(p) = \left(\left(\frac{k}{nk - Q} \right)^{n-j+1} \prod_{j \leq m \leq n} \frac{p - p_0}{p - c_m} \right)^{\frac{1}{n-j}} > \frac{k}{nk - Q} \left\{ \frac{p - p_0}{p - c_i} \right\},$$

which is a contradiction. \square

Appendix G. Proof of Proposition 4 and 5

By first-order conditions, we have the results. \square

Appendix H. Proof of Proposition 6

By first-order conditions, we have $\partial \Phi^l / \partial k < 0$. Moreover, as n increases, the capacity k decreases. Otherwise, the capacity constraint cannot hold. Consider $L < Q < R$. Consider $L = Q - \alpha k$ and $R = Q + \beta k$ where $\alpha + \beta = 1$, $\alpha \geq 0$, and $\beta \geq 0$. Clearly, for any $n \geq 2$, we can find α , β , and k to satisfy the capacity constraint, $(n - 1)k < Q < nk$. Since the value of β is bounded and the capacity k decreases with the number of suppliers, R approaches Q when n is sufficiently large. This implies each supplier would bid a price close to the buyer's reservation price as long as the number of suppliers is sufficiently large.

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