In general, introducing dither reduces the dynamic range available from an SDM, because the addition of dither will both increase the total noise power within the loop and reduce modulator stability [4]. Given that 1 bit dithering of SDM ADCs is advantageous for the reasons outlined above, a series of simulations was performed to determine whether such an approach carries an additional SNR penalty compared to multilevel dither.

Simulations: 64-times oversampled modulators with noise-shaping orders between 2 and 5 and with a range of dither amplitudes, were individually optimised to yield maximum dynamic range, using the approach described in [4]. The noise-shaping poles of each modulator were arranged in a Butterworth highpass configuration, and noise-shaping zeros were set to yield maximum baseband SNR. Optimisation was achieved by controlling the cutoff frequency of the Butterworth poles. Dithered systems individually optimised for maximum dynamic range were simulated to determine the presence of unwanted baseband errors. A modulator was deemed 'linear' if two conditions were satisfied:

(i) no idle tones were visible in the baseband noise floor power spectrum across a range of DC input signals; the noise floor was examined using a 4096 point FFT

(ii) naseband noise modulation was <1dB for sinusoidal excitation across the dynamic range of the modulator.

Three dither amplitude distributions were investigated: single-bit quantised, i.e. bipolar probability distribution (BPD), rectangular probability distribution (RPD), and triangular probability distribution (TPD). For each distribution, the minimum dither amplitude that successfully linearised the modulators was determined, and the associated SNR penalty relative to the undithered modulator noted.

Table 1: SNR penalties for dithered sigma-delta modulators

Order	Undithered SNR_{pk} [dB]	Dithered SNR penalty [dB]		
		BPD	RPD	TPD
2	75.8	9.2	4.7	5.4
3	93.6	3.3	4.5	4.7
4	109.3	4.9	5.3	5.6
5	121.6	5.0	4.8	6.4

Results: The results of the simulations are shown in Table 1. It is seen that the SNR penalty for linearising SDMs using dither remains approximately constant with changes in modulator order, with an average value of 5.5dB. For higher-order systems (order >2), single-bit (BPD) dither can successfully linearise sigma-delta modulators with no significant additional SNR penalty compared to RPD-dithered systems. However, for second-order systems, there appears to be an additional SNR penalty to pay for quantising the dither to one bit. We speculate that use of dither signals with many levels introduces an added degree of randomness to the dithering process, which can be of benefit when linearising simple (low-order) systems. The results also show that TPD dither carries a slightly higher SNR penalty compared to RPD dither.

Fig. 2*b* shows a noise-floor plot for a fourth-order modulator linearised using single-bit dither of amplitude ±0.14 (determined by the optimisation process). No idle tones are visible, the power spectral density of the noise floor essentially being invariant with input signal characteristics.

Conclusions: We have demonstrated that sigma-delta modulators can be efficiently linearised using dither that has been quantised to one bit, a technique that is relatively straightforward to implement in sigma-delta ADCs. For higher-order systems, the SNR penalty for linearisation with single-bit dither is no greater than that paid when using multilevel dither.

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Adaptive IIR blind algorithms

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Indexing terms: Adaptive equalisers, Adaptive filters, Equalisers

Infinite impulse response (IIR) filtering, when compared with finite impulse response (FIR) filtering, can result in a substantial computational saving and small mean-squared error (MSE). Two IIR blind algorithms based on the second and fourth order cumulants are presented. Simulation results indicate that the proposed IIR blind algorithms not only have faster convergence rates but also lower MSEs than their FIR counterparts.

Introduction: The purpose of blind equalisation is to recover the intersymbol-interference and noise-corrupted signal from the received signal without the help of a training signal. Earlier investigators of Godard [2], Benvensite and Goursat [1] used different FIR-type algorithms to deal with this problem.

The application of IIR adaptive filtering has recently drawn the interest of many researchers, because of its potential advantages of achieving better performance and saving computational load, when compared with the FIR filtering technique. We present two blind IIR algorithms based on higher-order statistics. These two algorithms have decision-feedback and parallel-form structures, respectively. We describe these two blind IIR algorithms and provide numerical simulation results concerning the performance of the proposed algorithms and its FIR counterpart.

Adaptive IIR blind algorithms: Let $\mathbf{h} = [...h_{-1}\ h_0\ h_1\ ...]$ represent the system (channel) impulse response, a_i be the system (channel) input sequence, consisting of zero-mean i.i.d. real random variables with an arbitrary discrete probability distribution and y_i represent the system (channel) output. We want to select a filter $C = [...c_{-1}\ c_0\ c_1\ ...]$ such that the filter output z_i is identical to the input a_i up to a constant delay, i.e. the overall impulse response

$$s_i = h_i \circ c_i = \sum_l h_{i-l} c_l \tag{1}$$

where '$\circ$' denotes the convolution operator, must be of the following form:

$$\mathbf{s} \triangleq [\cdots s_{-1} s_0 s_1 \cdots] = e^{j\theta}(0...1...0) \tag{2}$$

Consider the relationship between a_i and z_i: $z_i = a_i \circ s_i = \Sigma_l\, a_{i-l} s_l$, and hence the following two relations exist [3]:

$$E[z_i^2] = E[a_i^2] \sum_l |s_l|^2 \tag{3}$$

$$K[z_i] = K[a_i] \sum_l |s_l|^4 \tag{4}$$

where E denotes the expectation operator and $K[z_i]$ is the kurtosis associated with z_i. That is,

$$K[z_i] = E[z_i^4] - 3E^2[z_i^2] \tag{5}$$

The inequality

$$\sum_l |s_l|^4 \le \left(\sum_l |s_l|^2\right)^2 \qquad (6)$$

where equality holds if and only if **s** has at most one nonzero component, implies that to achieve perfect equalisation, the following two relations must hold instantaneously:

$$\sum_l |s_l|^2 = 1 \qquad (7)$$

$$\sum_l |s_l|^4 = 1 \qquad (8)$$

This is equivalent to minimising the cost function

$$J = (E[z_i^2] - E[a_i^2])^2 + (E[z_i^4] - E[a_i^4])^2 \qquad (9)$$

Decision-feedback IIR blind algorithm: Consider an equaliser **C** with an IIR structure that satisfies the input-output relationship

$$z_k = \sum_{n=0}^{L} d_n y_{k-n} + \sum_{n=1}^{L} b_n \hat{z}_{k-n} = \mathbf{W}_k^T \mathbf{U}_k \qquad (10)$$

where $\hat{z}_k$ is the estimate of z_k; the time-varying weight vector $\mathbf{W}_k$ and the new signal vector $\mathbf{U}_k$ are defined as

$$\mathbf{W}_k = [d_{0k}, d_{1k}, \cdots d_{Lk}, b_{1k}, \cdots b_{Lk}]^T \qquad (11)$$

$$\mathbf{U}_k = [y_k, y_{k-1}, \cdots y_{k-L}, \hat{z}_{k-1}, \cdots \hat{z}_{k-L}]^T \qquad (12)$$

By defining $J_1 = E[(\hat{z}_k - z_k)^2]$, applying the gradient-descent method to the cost function $J_{tot} = J + J_1$ and making the approximations

$$\frac{\partial \hat{z}_i}{\partial d_n} \simeq \frac{\partial z_i}{\partial d_n} \qquad (13)$$

$$\frac{\partial \hat{z}_i}{\partial b_n} \simeq \frac{\partial z_i}{\partial b_n} \qquad (14)$$

we obtain

$$\mathbf{W}_{k+1} = \mathbf{W}_k - \mu[4(E[z_k^2] - E[a_k^2])z_k + 8(E[z_k^4] - E[a_k^4])z_k^3 + 2(\hat{z}_k - z_k)]\Theta_k \qquad (15)$$

where $\Theta_k = [\alpha_{0k}, \ldots, \alpha_{Lk}, \beta_{1k}, \ldots, \beta_{Lk}]^T$ and

$$\alpha_{nk} \doteq \frac{\partial z_k}{\partial d_n} = y_{k-n} + \sum_{l=1}^{L} b_l \alpha_{n,k-l} \qquad (16)$$

$$\beta_{nk} \doteq \frac{\partial z_k}{\partial b_n} = \hat{z}_{k-n} + \sum_{l=1}^{L} b_l \beta_{n,k-l} \qquad (17)$$

Parallel form IIR blind algorithm: The IIR equaliser C has the input-output relationship

$$z_k = \sum_{i=1}^{L} z_k^{(i)} \qquad (18)$$

where

$$z_k^{(i)} = b^{(i)} z_{k-1}^{(i)} + \sum_{n=0}^{1} d_n^{(i)} y_{k-n} \qquad (19)$$

Defining $\Upsilon_k = [b_k^{(1)}\ d_{0k}^{(1)}\ d_{1k}^{(1)} \ldots b_k^{(L)}\ d_{0k}^{(L)}\ d_{1k}^{(L)}]$, $\Psi_k = [z_{k-1}^{(L)}\ y_k\ y_{k-1} \ldots z_{k-1}^{(L)}\ y_k\ y_{k-1}]$, employing a cost function J and followed by a procedure similar to that described in the preceding Section, we then arrive at

$$\Upsilon_{k+1} = \Upsilon_k - \mu[4(E[z_k^2] - E[a_k^2])z_k + 8(E[z_k^4] - E[a_k^4])z_k^3]\Sigma_k \qquad (20)$$

where $\Sigma_k = [\theta_k^{(1)}\ \eta_{0k}^{(1)}\ \eta_{1k}^{(1)} \ldots \theta_k^{(L)}\ \eta_{0k}^{(L)}\ \eta_{1k}^{(L)}]$ and

$$\theta_k^{(i)} = y_k + b_k^{(i)} \theta_{k-1}^{(i)} \qquad (21)$$

$$\eta_{nk}^{(i)} = z_{k-n}^{(i)} + b_k^{(i)} \eta_{n,k-1}^{(i)} \qquad (22)$$

Simulation results: We performed Monte-Carlo simulations of the proposed blind algorithms. Binary PSK data were transmitted. The two channels defined below were used in the simulations.

$$\text{channel } 1: y_k = y_{k-1} + 0.25 y_{k-2} + a_k + 0.8 a_{k-1} + 0.6 a_{k-2}$$

$$\text{channel } 2: y_k = a_k + 0.9 a_{k-1}$$

The step sise μ is chosen to be 10^{-4}, the length of the IIR equaliser is $L = 4$. The number of taps for the FIR equaliser under comparison is $M = 20$.

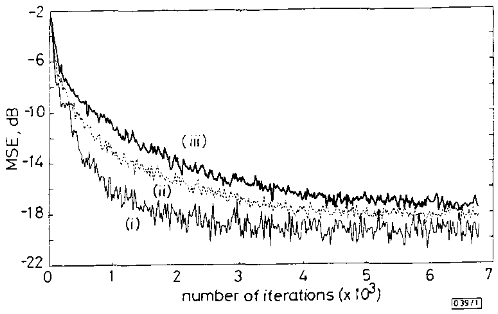

Fig. 1 *Channel 1*

(i) Decision-feedback IIR algorithm
(ii) Parallel form IIR algorithm
(iii) FIR algorithm

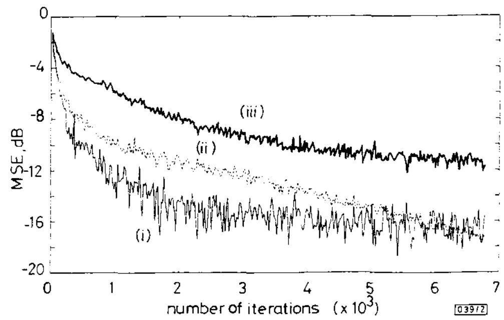

Fig. 2 *Channel 2*

(i) Decision-feedback IIR algorithm
(ii) Parallel form IIR algorithm
(iii) FIR algorithm

Shown in Figs 1 and 2 are the learning curves for these two IIR algorithms and their FIR counterpart in channels 1 and 2, respectively. These curves indicated that the proposed algorithms not only have faster convergence speeds but also yield smaller steady state MSEs.

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