Electron-atom scattering in an intense radiation field

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The electron-atom scattering in the presence of an intense radiation field is investigated by solving the Schrödinger equation in momentum space, which facilitates the extraction of the rapidly varying part of the wave function. It can be shown that the first-order Born approximation is only a limiting situation of the general approach. The angular distributions of the scattering probability are calculated for different field strengths and some interesting points regarding the multiphoton process are also discussed.

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I. INTRODUCTION

Charged-particle–atom scattering in the presence of a radiation field is a fundamental process in many physical systems such as plasma heating by electromagnetic radiation, gas breakdown, etc. During recent years, the availability of increasingly more powerful lasers in a wide range of frequencies has stimulated considerable interest in the study of the multiphoton phenomena in such a process. Many of the theoretical investigations of the laser-assisted electron-atom scattering are based on the lowest-order perturbative method [1–4]. It is obvious, however, that this method is not adequate and should be replaced either by a high-order perturbative calculation [5,6] or by some nonperturbative methods when the intensity of the laser field is strong. Starting with the well-know Kroll-Watson work [7,8] on the soft-photon approximation, there exists only a few nonperturbative treatments for this problem. Gavrila and Kaminski [9] proposed a method based on the Kramers-Henneberger transformation and suggested that this transformation might be particularly useful in the case of intense high-frequency fields. Several calculations and applications [10,11] were then performed following this theory. Shakeshaft [12] formulated a method of coupled integral equations to calculate the differential cross sections for stimulated photon absorption and a successful application for the case of a separable potential was obtained. Recently, an efficient method of solving the time-dependent Schrödinger equation for a system undergoing multiphoton processes has been introduced [13–15]. An important feature of this method is that the Schrödinger equation is solved in momentum space, which facilitates the extraction of the rapidly varying part of the wave function. Another advantage of this method is that, whereas in configuration space artificial boundaries must be introduced to absorb the electron as it moves far away, no absorbing boundaries need be introduced in momentum space. A preliminary calculation [14] using this method for the case of one-dimensional scattering of electron from a potential in the laser field was performed. The result is interesting in that the multiphoton absorption process is manifested with many peaks in the transition probability function. In this paper, we shall extend this method to the three-dimensional system and show that the dynamics of the mulitphoton process of electron-atom scattering in an intense field can be well understood in this formulation. A detailed calculation of the scattering probability from a hydrogen atom for different laser intensities will be performed and the dependence on the laser frequencies will also be discussed.

II. THEORY

Consider an electron moving in the radiation field of a vector potential $\mathbf{A}(t)$ and scattering by the atomic potential V; the time-dependent Schrödinger equation is

$$i\hbar\frac{d}{dt}|\Psi(t)\rangle=\left[\frac{p^2}{2m}+V+H_I(t)\right]|\Psi(t)\rangle \ , \tag{1}$$

where $H_I(t)=-(e/mc)\mathbf{A}\cdot\mathbf{p}$ is the interaction with the radiation field. The $A^2(t)$ term is removed by a trivial contact transformation [7].

Initially, at time $t\rightarrow-\infty$, the electron is far from the atom, i.e., free of the potential $V=0$, which is just the problem for the free electron moving in the radiation field. The solution to the time-dependent Schrödinger equation was originally derived by Volkov [16] and can be written in the form

$$H_0|\chi_\mathbf{k}(t)\rangle=-i\hbar\frac{\partial}{\partial t}|\chi_\mathbf{k}(t)\rangle \tag{2}$$

with

$$H_0=\frac{p^2}{2m}-\frac{e}{mc}\mathbf{A}\cdot\mathbf{p} \ , \tag{3}$$

$$|\chi_\mathbf{k}(t)\rangle=\exp\{-i[E_i t/\hbar+\theta_\mathbf{k}(t)]\}|\mathbf{k}\rangle \ , \tag{4}$$

where E_i is the initial energy, $|\mathbf{k}\rangle$ is the eigenvector of $\mathbf{p}$ with momentum eigenvalue $\hbar\mathbf{k}$ normalized as

$$\langle\mathbf{r}|\mathbf{k}\rangle=(2\pi)^{-3/2}e^{i\mathbf{k}\cdot\mathbf{r}} \ , \tag{5}$$

and the real phase $\theta_\mathbf{k}(t)$ is given by

$$\theta_\mathbf{k}(t)=\frac{1}{\hbar}\int_0^t dt'\left[\frac{\hbar^2k^2}{2m}-E_i+H_I(t')\right] . \tag{6}$$

For a monochromatic, linearly polarized field $\mathbf{A}=\mathbf{A}_0\cos wt$, we have

$$|\chi_{\mathbf{k}}(t)\rangle=(2\pi)^{-3/2}\exp[-i(\mathbf{k}\cdot\mathbf{r}-\mathbf{k}\cdot\boldsymbol{\alpha}\sin wt-E_i t/\hbar)] , \tag{7}$$

$$\theta_{\mathbf{k}}(t)=\frac{1}{\hbar}(E_k-E_i)t-(\mathbf{k}\cdot\boldsymbol{\alpha})\sin wt , \tag{8}$$

where $E_k=\hbar^2k^2/2m$ and $\boldsymbol{\alpha}=(e/mcw)\mathbf{A}_0$.

We shall use the wave function of the "unperturbed" system H_0 as our basis. Initially, the incident electron with incoming momentum $\hbar\mathbf{k}_i$ is in the state $|\chi_{\mathbf{k}_i}(t)\rangle$; the solution of Eq. (1) can be expressed as

$$|\Psi(t)\rangle=|\chi_{\mathbf{k}_i}(t)\rangle+|\phi(t)\rangle . \tag{9}$$

Expand $|\phi(t)\rangle$ in terms of the bases states

$$|\phi(t)\rangle=\int d\mathbf{k}a_{\mathbf{k}}(t)|\chi_{\mathbf{k}}(t)\rangle \tag{10}$$

with the boundary condition $|\Psi(t)\rangle\to|\chi_{\mathbf{k}_i}(t)\rangle$ as $t\to-\infty$, i.e., $|\phi(-\infty)\rangle=0$. Substituting Eqs. (9) and (10) into Eq. (1), an inhomogeneous integro-differential equation can be obtained for the coefficient $a_{\mathbf{k}}(t)$:

$$i\hbar\frac{d}{dt}a_{\mathbf{k}}(t)=e^{i\theta_{\mathbf{k}}(t)}[e^{-i\theta_{\mathbf{k}_i}(t)}\langle\mathbf{k}|V|\mathbf{k}_i\rangle+b_{\mathbf{k}}(t)] , \tag{11}$$

where

$$b_{\mathbf{k}}(t)=\int d\mathbf{k}'e^{-i\theta_{\mathbf{k}'}(t)}a_{\mathbf{k}'}(t)\langle\mathbf{k}|V|\mathbf{k}'\rangle \tag{12}$$

with the boundary condition $a_{\mathbf{k}}(-\infty)=b_{\mathbf{k}}(-\infty)=0$. It has been shown [13,15] that, because of the phase factor $\exp[i\theta_{\mathbf{k}}(t)]$ on the right-hand side of Eq. (11), the function $a_{\mathbf{k}}(t)$ varies rapidly with both $\mathbf{k}$ and t. On the other hand, $b_{\mathbf{k}}(t)$ varies relatively slowly with $\mathbf{k}$ and t. Consequently, we can interpolate $b_{\mathbf{k}}(t)$. Let us discuss several interesting points.

(i) Since $b_{\mathbf{k}}(t)$ is a slowly varying function of $\mathbf{k}$ and t, we first temporarily ignore its effect; Eq. (11) becomes

$$i\hbar\frac{d}{dt}a_{\mathbf{k}}(t)=e^{i\theta_{\mathbf{k}}(t)}e^{-i\theta_{\mathbf{k}_i}(t)}\langle\mathbf{k}|V|\mathbf{k}_i\rangle . \tag{13}$$

Substituting Eq. (8) into Eq. (13) we have

$$i\hbar\frac{d}{dt}a_{\mathbf{k}}(t)=\exp\left[\frac{i}{\hbar}(E_k-E_i)t\right]\times\exp[-i(\mathbf{k}-\mathbf{k}_i)\cdot\boldsymbol{\alpha}\sin wt]\langle\mathbf{k}|V|\mathbf{k}_i\rangle . \tag{14}$$

Using the Fourier-Bessel expansion

$$\exp(-i\xi_k\sin wt)=\sum_n J_n(\xi_k)\exp(-inwt) \tag{15}$$

and integrating Eq. (14) from $t=-\infty$ to $t=+\infty$, we obtain

$$a_{\mathbf{k}}(\infty)=\sum_n J_n(\xi_k)\langle\mathbf{k}|V|\mathbf{k}_i\rangle i\pi\delta[E_n(k)] , \tag{16}$$

where

$$\xi_k=(\mathbf{k}-\mathbf{k}_i)\cdot\boldsymbol{\alpha} , \tag{17}$$

$$E_n(k)=E_k-E_i-n\hbar w . \tag{18}$$

Clearly the delta function $\delta[E_n(k)]$ on the right-hand side of Eq. (16) expresses the conservation of energy such that the outgoing electron with momentum $\hbar\mathbf{k}$ would have energy

$$E_k=\frac{\hbar^2k^2}{2m}=E_i+n\hbar w . \tag{19}$$

It is interesting to note that by substituting Eq. (4) into Eq. (13), we find

$$i\hbar\frac{d}{dt}a_{\mathbf{k}}(t)=\langle\chi_{\mathbf{k}}(t)|V|\chi_{\mathbf{k}_i}(t)\rangle . \tag{20}$$

This result reduces to that of the first Born approximation (FBA) used in previous works [1,6]. Therefore, FBA is only the limiting case of ours by setting the function $b_{\mathbf{k}}(t)=0$. This is, of course, an oversimplification. The correct treatment must also take into account the effect of $b_{\mathbf{k}}(t)$, which we discuss in the following.

(ii) Equation (11) can be rewritten as

$$i\hbar\frac{d}{dt}a_{\mathbf{k}}(t)=e^{i\theta_{\mathbf{k}}(t)-i\theta_{\mathbf{k}_i}(t)}[\langle\mathbf{k}|V|\mathbf{k}_i\rangle+b_{\mathbf{k}}(t)e^{i\theta_{\mathbf{k}_i}(t)}] .$$

Using Eq. (12) for $b_{\mathbf{k}}(t)$, we have

$$i\hbar\frac{d}{dt}a_{\mathbf{k}}(t)=e^{i\theta_{\mathbf{k}}(t)-i\theta_{\mathbf{k}_i}(t)}\langle\mathbf{k}|V|\mathbf{k}_i\rangle+\int d\mathbf{k}'e^{i\theta_{\mathbf{k}}(t)-i\theta_{\mathbf{k}'}(t)}a_{\mathbf{k}'}(t)\langle\mathbf{k}|V|\mathbf{k}'\rangle .$$

Substituting Eqs. (7) and (15) into the preceding equation and integrating over t, we obtain

$$a_{\mathbf{k}}(t)=-\frac{i}{\hbar}\int_{-\infty}^{t}dt'\left\{\sum_n J_n(\xi_k)e^{(i/\hbar)(E_k-E_i-n\hbar w)t'}\langle\mathbf{k}|V|\mathbf{k}_i\rangle+\int d\mathbf{k}'\sum_m\sum_n J_n(\xi_k)e^{(i/\hbar)[E_k-E_{k'}-(n-m)\hbar w]t'}J_m(\xi_{k'})a_{\mathbf{k}'}(t)\langle\mathbf{k}|V|\mathbf{k}'\rangle\right\} . \tag{21}$$

Equation (21) gives the general expression for the scattering amplitude $a_{\mathbf{k}}(t)$ of the electron scattered from the atom at time t. The first term on the right-hand side of Eq. (21) is just the result described in point (i). The important effect on the scattering process comes from the second term on the right-hand side of Eq. (21). One sees that $a_{\mathbf{k}'}(t')$ represents the scattering amplitude of the electron with momentum $\hbar\mathbf{k}'$, at time t', having absorbed

m photons for which $J_m(\xi_{k'})$ is the amplitude for this process and, by conservation of energy, the electron energy is given by $E_{k'}=E_i+m\hbar w$. Then it reabsorbs $(n-m)$ photons (due to the propagator $\exp\{(i/\hbar)[E_k-E_{k'}-(n-m)\hbar w]\}t$) with amplitude $J_n(\xi_k)$. The net result is that the electron has absorbed n photons at time t and is represented by the scattering amplitude $a_{\mathbf{k}}(t)$. Therefore, Eq. (21) gives a general formulation to describe the electron-atom scattering in an intense radiation field. The dynamics of the multiphoton process during the scattering is clearly manifested in this theory.

(iii) We have pointed out that $b_{\mathbf{k}}(t)$ is a slowly varying function of $\mathbf{k}$ and t. Explicit and implicit methods [13,15] have been proposed for treating this problem. In the explicit method, we extrapolate $b_{\mathbf{k}}(t)$ using the Taylor-series expansion

$$b_{\mathbf{k}}(t)=b_{\mathbf{k}}(0)+\dot{b}_{\mathbf{k}}(0)t+\cdots \quad (22)$$

The time derivative of $b_{\mathbf{k}}(t)$ can be obtained from Eq. (12),

$$\dot{b}_{\mathbf{k}}(t)=\int d\mathbf{k}'\left\{-\frac{i}{\hbar}\dot{F}_{\mathbf{k}'}(t)\exp\left[-\frac{i}{\hbar}F_{\mathbf{k}'}(t)\right]a_{\mathbf{k}'}(t)+\exp\left[-\frac{i}{\hbar}F_{\mathbf{k}'}(t)\right]\dot{a}_{\mathbf{k}'}(t)\right\}\langle\mathbf{k}|V|\mathbf{k}'\rangle , \quad (23)$$

where

$$F_{\mathbf{k}}(t)=\left[\frac{\hbar^2k^2}{2m}-E_i\right]t-\hbar\mathbf{k}\cdot\boldsymbol{\alpha}\sin wt . \quad (24)$$

Using Eq. (11) for $\dot{a}_{\mathbf{k}}(t)$, we have

$$\dot{b}_{\mathbf{k}}(t)=-\frac{i}{\hbar}\int d\mathbf{k}'\left\{\dot{F}_{\mathbf{k}'}(t)\exp\left[-\frac{i}{\hbar}F_{\mathbf{k}'}(t)\right]a_{\mathbf{k}'}(t)+\exp\left[-\frac{i}{\hbar}F_{\mathbf{k}_i}(t)\right]\langle\mathbf{k}|V|\mathbf{k}_i\rangle+b_{\mathbf{k}'}(t)\right\}\langle\mathbf{k}|V|\mathbf{k}'\rangle . \quad (25)$$

Substituting Eq. (25) into Eq. (22) and to a first-order approximation, we find

$$b_{\mathbf{k}}(t)=b_{\mathbf{k}}(0)\exp(-iQt/\hbar) , \quad (26)$$

where

$$Q=\frac{1}{b_{\mathbf{k}}(0)}\int d\mathbf{k}'[\dot{F}_{\mathbf{k}'}(0)a_{\mathbf{k}'}(0)+\langle\mathbf{k}|V|\mathbf{k}_i\rangle+b_{\mathbf{k}'}(0)]\times\langle\mathbf{k}|V|\mathbf{k}'\rangle . \quad (27)$$

The convergence of the expansion series in Eq. (22) has been checked in a preliminary calculation in a previous work [14], where the one-dimensional electron-potential scattering was studied by also including the t^2 term in the expansion series and it was found that the convergence is reasonably good. Therefore, we use the first-order approximation in Eq. (26) so that the computational difficulty involved in the three-dimensional case studied here can be avoided. Substituting Eq. (26) into (11) and using Eq. (15), we obtain the scattering amplitude

$$a_{\mathbf{k}}(t)=a^B_{\mathbf{k}}(t)+\Delta a_{\mathbf{k}}(t) , \quad (28)$$

where $a^B_{\mathbf{k}}(t)$ is the amplitude for the first Born approximation

$$a^B_{\mathbf{k}}(t)=-\frac{i}{\hbar}\int_{-\infty}^{t}\langle\chi_{\mathbf{k}}(t)|V|\chi_{\mathbf{k}_i}(t)\rangle \quad (29)$$

and $\Delta a_{\mathbf{k}}(t)$ is the correction due to the effect of the function $b_{\mathbf{k}}(t)$,

$$\Delta a_{\mathbf{k}}(t)=-\frac{i}{\hbar}b_{\mathbf{k}}(0)\sum_n J_n(\xi_k)\times\int_{-\infty}^{t}dt' e^{(i/\hbar)(E_k-E_i-Q-n\hbar w)t'} . \quad (30)$$

III. RESULTS AND DISCUSSION

In this section we present the numerical calculation for the scattering of an electron with a hydrogen atom in the presence of a laser field. We have calculated the scatter-

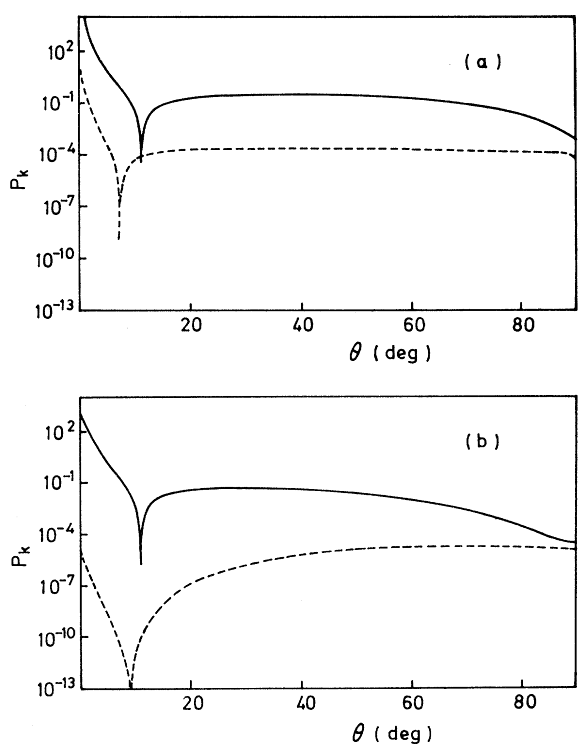

FIG. 1. Scattering probability density $P_k=|a_k(t\to\infty)|^2$ as a function of the scattering angle θ for the incident electron energy $E_i=3.67$ a.u. with the absorption of (a) one photon and (b) two photons. The field strength is $E_0=0.003$ a.u. and the laser frequency is $w=0.07$ a.u. Dashed curve, results for the FBA; full curve, full calculation using Eq. (28).

ing probability density $|a_k(\infty)|^2$ as a function of the scattering angle θ for different field strengths. We take the geometry such that the electric field $\mathbf{E}_0$ is parallel to the incident electron wave vector $\mathbf{k}_i$. Atomic units are used in the calculation [the atomic units of the electric field, frequency, and energy (hartree) have the following standard equivalents: 5.14×10^9 V/cm, 4.13×10^{16} sec^{-1}, and 27.21 eV]. We first calculate the angular distribution for the case that neglects the function $b_{\mathbf{k}}$, which is just the result of the FBA, as pointed out in the preceding section. Then the effect of the function $b_{\mathbf{k}}$ is included in the calculation according to Eq. (28). In order to calculate $\Delta a_{\mathbf{k}}$ in Eq. (28), we have to know the values of $a_{\mathbf{k}'}(0)$, which appear in the expression of Q in Eq. (27). The exact values of $a_{\mathbf{k}'}(t)$ at $t=0$ are not known; however, a reasonable approximation for $a_{\mathbf{k}'}(0)$ is obtained in the following. Please note that the results in Eqs. (26) and (27) are obtained by taking the first-order approximation and it can be seen from Eq. (26) that the term involving Q is already in first order; therefore the quantities in Q can be taken as a zeroth-order approximation. Thus we may use the first Born result $a^B_{\mathbf{k}'}(0)$ for $a_{\mathbf{k}'}(0)$ in the expression of Q, that is, $a_{\mathbf{k}'}(0)\approx a^B_{\mathbf{k}'}(0)$. Following the standard adiabatic approach in scattering theory, we replace the potential V by $Ve^{-\epsilon|t|}$, where ϵ is positive but very small, and $a_{\mathbf{k}'}(0)$ can then be easily obtained from Eq. (29) by integrating from $-\infty$ to 0. Figure 1 shows the result for the field strength $E_0=0.003$ and frequency $w=0.07$ at the incident electron energy $E_i=3.67$ with different numbers of photons absorbed. The results for a higher incident electron energy $E_i=20$ is shown in Fig. 2. It can be seen that, in general, the effect of the function $b_{\mathbf{k}}$ gives an enhancement relative to the FBA. Thus the function $b_{\mathbf{k}}$ gives an important contribution to the scattering process that should not be neglected, as assumed in previous calculations. One observes that the minima appearing in the FBA occur at angles such that the scalar product $\boldsymbol{\Delta}\cdot\boldsymbol{\varepsilon}=0$, where the momentum transfer $\boldsymbol{\Delta}=\mathbf{k}_i-\mathbf{k}_f$ and $\boldsymbol{\varepsilon}$ is the polarization vector. We see that the minimum shifts to a different angle as the effect of $b_{\mathbf{k}}$ is included. This is because there are two terms in the scattering amplitude, as given in Eq. (28). The minimum occurs when these two terms cancel each other. Therefore, it will occur at a value of scattering angle different from that of the FBA. Trombetta and Ferrante [6] performed a calculation with the second-order Born approximation for the charged-particle scattering in the presence of a strong field. They obtained a similar enhancement relative to the FBA and the minima are also shifted to larger angles. Therefore, our results are consistent with theirs. Figures 3 and 4 show the results for a higher field strength $E_0=0.02$. Now the oscillations are enhanced due to the larger argument of the Bessel functions. We see that the effect of the function $b_{\mathbf{k}}$ is also quite important for all cases of photons absorbed. The minima are again shifted to different angles. It is worth mentioning that Trombetta and Ferrante [6] provided a criterion of validity of the

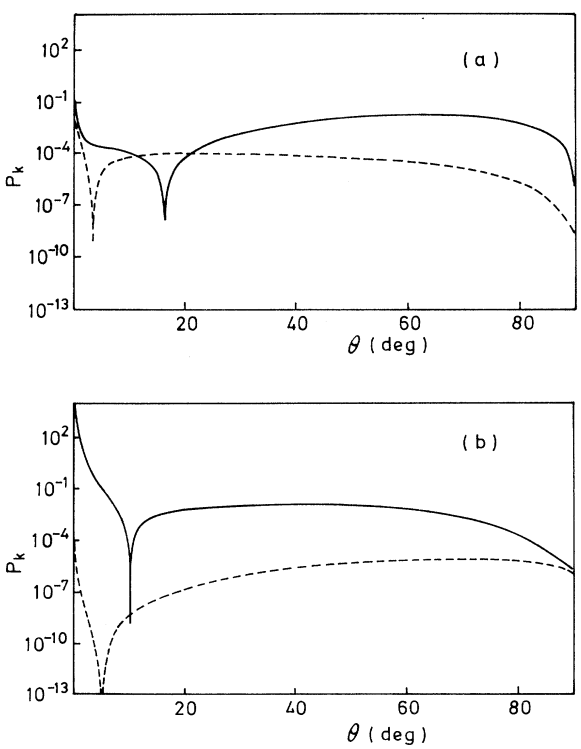

FIG. 2. Same as Fig. 1, but for an incident electron energy $E_i=20$ a.u.

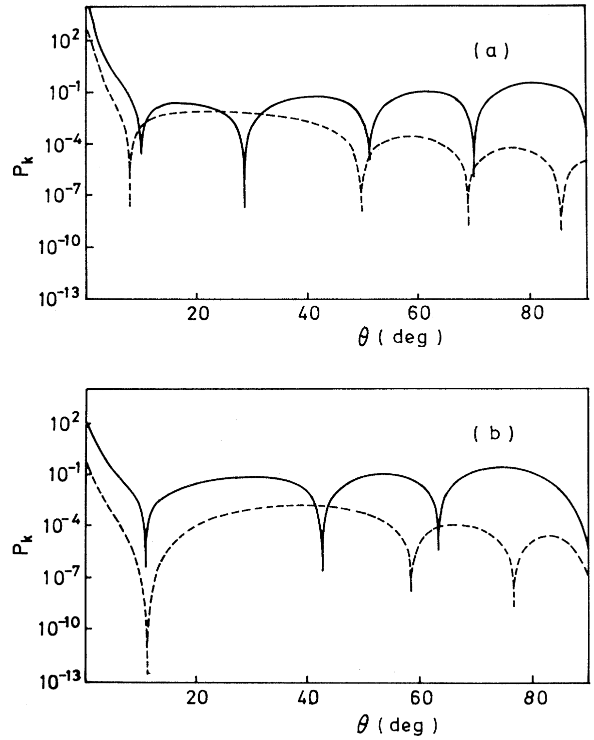

FIG. 3. Same as Fig. 1, but for a field strength $E_0=0.02$ a.u.

first Born approximation for the electron-atom scattering in the presence of a radiation field. In their calculations they found that the FBA holds better for stronger fields. Comparing our strong-field results in Figs. 3 and 4 with the weak-field results in Figs. 1 and 2, it can be seen that the agreement between the FBA and the more accurate calculation is much better for the case of the strong field, especially in small scattering angles. Therefore, our results seem to be very reasonable. It is also interesting to note from Eq. (28) that the effect of $b_{\mathbf{k}}$ gives not only a correction to the angular distribution of the scattering amplitude but also a different distribution to the outgoing electron energy. It can be seen in Eq. (30) that the energy of the scattered electron is shifted by an amount Q, which comes from the $\mathbf{A}\cdot\mathbf{p}$ interaction, which will have different values for different energy states. Further investigation about this interesting point for the energy spectra will be performed in a future study.

We have also studied the frequency dependence of the scattering probability function. Figure 5 gives the variation of the scattering probability in terms of the laser frequency at a fixed scattering angle $\theta = 10°$ and the incident electron energy $E_i = 3.67$ for the field strength $E_0 = 0.003$. It can be seen that as the frequency increases, so does the momentum transfer and one observes that the FBA decreases steadily, goes to zero when the condition $\boldsymbol{\Delta}\cdot\boldsymbol{\varepsilon} = 0$ is fulfilled, and then increases again toward an almost constant value. The situation becomes more complicated as the effect of $b_{\mathbf{k}}$ is included in the calculation. In general, several oscillations presented in

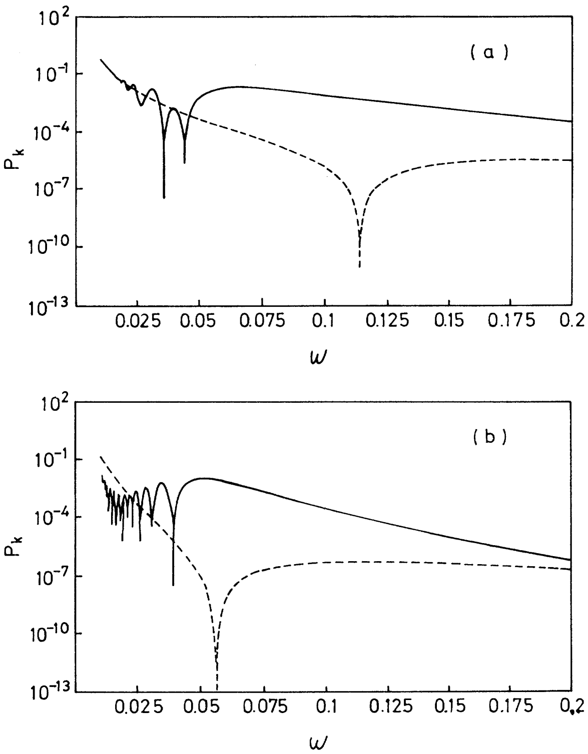

FIG. 5. Scattering probability density P_k as a function of laser frequency w at a fixed scattering angle $\theta = 10°$ with the absorption of (a) one photon and (b) two photons. The incident electron energy $E_i = 3.67$ a.u. and the field strength $E_0 = 0.003$ a.u. Dashed curve, results for the FBA; full curve, full calculation using Eq. (28).

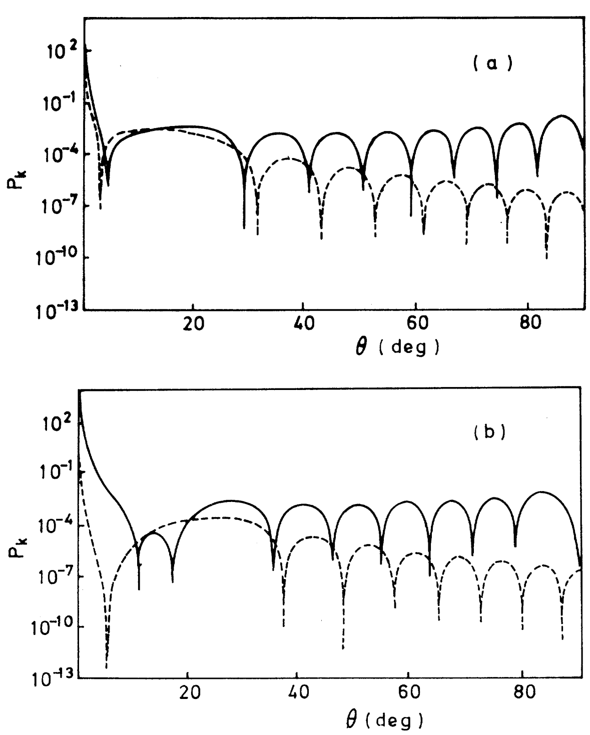

FIG. 4. Same as Fig. 2, but for a field strength $E_0 = 0.02$ a.u.

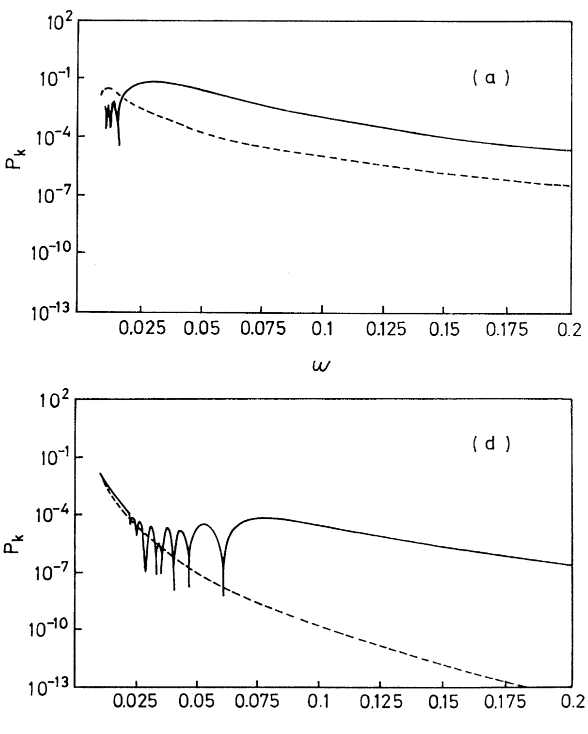

FIG. 6. Same as Fig. 5, but for an incident electron energy $E_i = 20$ a.u.

the scattering probability appear because of the interference effect of the correction term Δa_k with the FBA, as shown in the expression of the amplitude a_k. This behavior is more prominent for the case of a higher incident electron energy, as shown in Fig. 6 for $E_i=20$. In this case, the condition $\mathbf{\Delta}\cdot\boldsymbol{\varepsilon}=0$ cannot be met in the frequency range displayed here (the minimum would occur at $w=0.63$); therefore the curve for the FBA is monotonic decreasing. However, as the effect of $b_{\mathbf{k}}$ is included, the curve is again modified with some oscillations.

IV. CONCLUSION

Based on the method of solving the Schrödinger equation in momentum space, we have shown that the dynamics of the multiphoton process during the electron-atom scattering in an intense radiation field can be clearly manifested in this formulation. It was found that the first-order Born approximation is only a limiting situation of the general approach. We have calculated the angular distribution of the scattering probability for electron-hydrogen scattering with different field strenghts. We found that the correction due to the function $b_{\mathbf{k}}$ is quite important and, in general, gives an enhancement relative to the FBA. The minima occurring in the distribution are shifted to different angles. We also studied the frequency dependence of the scattering probability. It was found that the effect of the function $b_{\mathbf{k}}$ also gives an important modification due to the interference effect of the correction term with the FBA.

ACKNOWLEDGMENT

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[1] J. I. Gersten and M. H. Mittleman, Phys. Rev. A **13**, 123 (1976).
[2] M. H. Mittleman, Phys. Rev. A **19**, 134 (1978).
[3] G. Ferrante, C. Leone, and L. Lo. Caseio, J. Phys. B **12**, 2319 (1979).
[4] L. Rosenberg, Phys. Rev. A **23**, 2283 (1981); **34**, 4567 (1986).
[5] M. Gavrila, A. Maquet, and V. Véniard, Phys. Rev. A **42**, 236 (1990).
[6] F. Trombetta and G. Ferrante, J. Phys. B **22**, 3881 (1989).
[7] N. M. Kroll and K. M. Watson, Phys. Rev. A **8**, 804 (1973).
[8] B. Wallbank and J. K. Holmes, Phys. Rev. A **48**, R2515 (1993).
[9] M. Gavrila and J. Z. Kaminski, Phys. Rev. Lett. **52**, 613 (1984).
[10] R. Bhatt, B. Piraux, and K. Burnett, Phys. Rev. A **37**, 98 (1988).
[11] G. Csanak and Lee A. Collins, Phys. Rev. A **47**, 3240 (1993).
[12] R. Shakeshaft, Phys. Rev. A **28**, 667 (1983).
[13] R. Shakeshaft and M. Dörr, Z. Phys. D **8**, 255 (1988).
[14] R. Shakeshaft and C. S. Han, Phys. Rev. A **38**, 2163 (1988).
[15] C. S. Han, J. Phys. B **23**, L495 (1990).
[16] D. M. Volkov, Z. Phys. **94**, 250 (1935).