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Marketing-driven channel coordination with revenue-sharing contracts under price promotion to end-customers

Jiuh-Biing Sheu *

Institute of Traffic and Transportation, National Chiao Tung University, 4F, 118 Chung Hsiao W. Rd., Sec. 1, Taipei 10012, Taiwan

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ABSTRACT

This paper explores the equilibrium behavior of a basic supplier-retailer distribution channel with and without revenue-sharing contracts under price promotion to end-customers. Three types of promotional demand patterns characterized by different features of dynamic price sensitivity are considered to rationalize price promotional effects on end-customer demands. Under such a retail price promotion scheme, this work develops a basic model to investigate decentralized channel members' equilibrium decisions in pricing and logistics operations using a two-stage Stackelberg game approach. Extending from the basic model, this work further derives the equilibrium solutions of the dyadic members under channel coordination with revenue-sharing contracts. Analytical results show that under certain conditions both the supplier and retailer can gain more profits through revenue-sharing contracts by means of appropriate promotional pricing strategies. Moreover, the supplier should provide additional economic incentives to the retailer. Furthermore, a counter-profit revenue-sharing chain effect is found in the illustrative examples. Such a phenomenon infers that the more the retailer requests to share from a unit of sale the more it may lose under the revenue-sharing supply chain coordination scheme.

1. Introduction

Supply chain coordination under retail price promotion remains challenging in the areas of both marketing and operations management. In reality, such marketing-manufacturing interface issues were first discussed by Shapiro (1977) who pointed out that marketing and manufacturing decision makers are less coordinated. In practical areas, conflicting marketing-manufacturing problems, such as the incongruence between the retailer's promotion and manufacturer's production, can be easily found in the supplier-retailer channels of grocery and high-tech product industries. Whereas, dyadic channel members aim at assuring their own interests without adapting some opportunistic tasks to gain channel benefits (Iyer and Jain, 2003). Some researchers in management science also argue that price promotions may have a positive impact on manufacturer revenues. However, their effects on retailer revenues are mixed, depending on the promotional effort and commitment of the retailer (Srinivasan et al., 2004). Gerstner and Hess (1995) particularly stress that such diversion of total profits within a distribution channel may lead to channel conflict. Therein, the dyadic channel members can arbitrarily make self-interested operational decisions leading to a pernicious destruction of mutual profits.

In effect, channel coordination based on the concept of vertical integration which addresses double marginalization effects has been the mainstream in supply chain management (SCM) research. Double marginalization was first characterized in Spengler (1950), followed by a vast number of researchers devoted to channel coordination mechanisms from different perspectives such as economics (Machlup and Taber, 1960; Gal-Or, 1985; Klemperer and Meyer, 1986), marketing science (Jeuland and Shugan, 1983; Gerstner and Hess, 1995), and operations management (Cachon and Lariviere, 2005; Anand et al., 2008). (Jeuland and Shugan, 1983) discuss several mechanisms for channel coordination, where both profit sharing and quantity discounts are suggested as two feasible and complementary measures. Gerstner and Hess (1995) further suggest targeted pull pricing strategies in which discounts are offered directly to those price-conscious customers to gain more channel profits. Nevertheless, they also point out several drawbacks such as difficulties in targeting the aforementioned price-conscious customers and the costs in implementing the strategy.

In previous studies of operations management, coordinating contracts such as buy-back contracts (Pasternack, 1985), quantity-flexibility contracts (Tsay, 1999), price-discount contracts (Bernstein and Federgruen, 2005), and revenue-sharing contracts (Cachon and Lariviere, 2005; Koulamas, 2006) have continuously gained researchers' recognition as feasible mechanisms for supply chain coordination. The induced effects, however, vary with exogenous demand and operational conditions. For instance, Cachon

* Tel.: +886 2 2349 4963; fax: +886 2 2349 4953.
E-mail address: jbsheu@mail.nctu.edu.tw

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and Lariviere (2005) provide in-depth discussion of the strengths and limitations of revenue-sharing contracts. According to their analytical results, revenue-sharing mechanisms appear to be attractive to the dyadic members of supplier-retailer channels. However, these mechanisms offer mixed effects depending on retailers' actions (e.g., advertising, service quality, and store environments).

Motivated by the previous literature, this work aims to address the issue of channel coordination contingent on the use of revenue-sharing contracts as a coordinating mechanism in price promotion to end customers. A distinctive feature of the proposed model is the integration of promotional activities and operational management underlining marketing-oriented supply chain coordination. Based upon the existing theory of promotional consumer behavior, this work postulates three typical types of promotional demand patterns in Section 2. Then this work presents a Stackelberg game-based supplier-retailer channel model in Section 3 to investigate the dyadic members' decisions in pricing and logistics operations without revenue-sharing contracts. This is followed by Section 4 which describes the extension of the model for channel coordination through the revenue-sharing contract for qualitative analysis. Section 5 presents the analytical results of illustrative examples.

2. Promotional demand patterns

This section aims to specify promotional demand functions to rationalize price promotional effects on the investigated supplier-retailer channel behavior in the price promotional contexts. Based on previous research on price promotion (Boulding et al., 1994; Blattberg et al., 1995; Hanssens et al., 2000; Kurata and Liu, 2007) and economics (Foekens et al., 1999; Samuelson and Nordhaus, 2009), we formulate a promotional-demand function in a dynamic negative exponential form given by $D(p,t) = \lambda e^{-\beta(t)p}$, where p represents the promotional price; t represents a given time slice; λ denotes a positive parameter implying the potential market size faced by the retailer to sell the given product in t; and $\beta(t)$ represents the dynamic price sensitivity. Therein, the 1st and 2nd order derivatives of the above exponential form satisfy $\partial D(p,t)/\partial p < 0$ and $\partial^2 D(p,t)/\partial^2 p > 0$. These two conditions follow the empirical marketing generalizations that temporary price discounts may cause a significant short-term sales spike (Blattberg et al., 1995) and increased returns to scale (Hanssens et al., 2000). In addition, considering that promotional demand may also depend on factors such as potential market share (Kurata and Liu, 2007) and price sensitivity (Boulding et al., 1994), we further formulate $\beta(t)$ with a linear form as $\beta(t) = a + bt$, where a and b represent two non-negative parameters ($a \geqslant 0$ and $b \geqslant 0$) determining the shape of demand function. Such treatment follows the analytical results of Kopalle et al. (1999) which argue that the effect of dynamic price sensitivity is mixed and category-specific. Accordingly, the generalized form of the promotional demand function ($D(p,t)$) is finalized as

$$D(p,t) = \lambda e^{-(a+bt)p} \tag{1}$$

Based on the above promotional demand model, this work considers three types of promotional demand patterns characterizing three different types of consumer responses to the promotional price (i.e., price sensitivity) during the promotional period, as depicted in Fig. 1.

Type 1: Change rate of the promotional demand is constant (i.e., $a \geqslant 0$, $b = 0$)

The *Type-1* promotional demand pattern is a basic form of the price-promotional demand pattern, where consumer response to

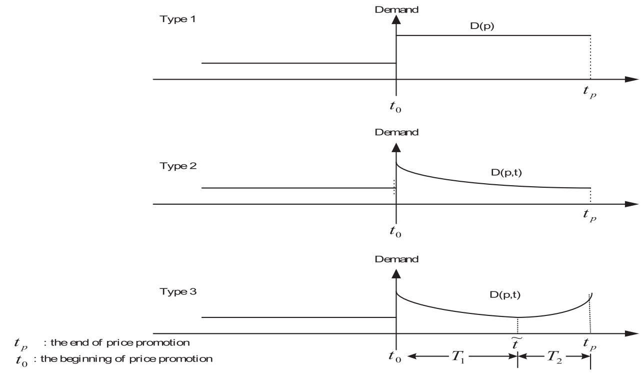

Fig. 1. Change patterns of time-varying promotional demands.

the promotional price is treated in a time-invariant form as $a \geqslant 0$, $b = 0 \Rightarrow \beta(t) = a$. In reality, such a treatment has been used by many previous studies to address the corresponding inventory and replenishment issues contingent on temporary price discounts (Tersine and Schwarzkopf, 1989; Goyal et al., 1991). The main advantage of *Type-1* promotional demand pattern is the simplicity of model formulation to facilitate the derivation of equilibrium solutions. Accordingly, the promotional demand function (Eq. (1)) can be expressed as $D(p,t) = \lambda e^{-ap} = D(p)$, which infers that the daily promotional demand rate may not change over time during the promotional period.

Type 2: Promotional demand steadily decreases over time (i.e., $a \geqslant 0$, $b > 0$)

By contrast, the *Type-2* promotional demand pattern is more realistic in practice since it follows previous studies (Neslin et al., 1985; Blattberg et al., 1995) claiming that the acceleration of consumers' purchases in response to price cuts may cause a post-promotion dip in subsequent weeks due to increasing inventories on the customer side. Similarly, Hanssens et al. (2000) point out that the magnitude of the long-term impact of price promotion may not be as high as its immediate effect given that the effects of most marketing activities tend to dissipate over time. Therefore, it is plausible that the promotional demand function can be characterized in a negative exponential form. This implies that promotional demands may increase sharply at the beginning of a promotional period, followed by continuous reduction till the end of the promotional period.

Type 3: Promotional demand changes in a two-regime concave-like curve

Compared to *Type 2*, the *Type 3*promotional demand pattern can be regarded as an extended form of Eq. (1) since it further considers the anticipation of consumers in terms of the upcoming termination of promotion, which may re-stimulate consumers' "secondary stockpiling" near the end of the promotional period. This may hold true particularly for those product categories consumed daily (e.g., milk and tissue paper) or without the concerns of freshness (e.g., paper towels), storage constraints (e.g., tuna fish). Actually, such a purchase acceleration effect can be further amplified by retailers' advertising strategies, e.g., "One Week Left for 50% off", etc. (Stiving and Winer, 1997). Accordingly, we postulate that such a two-regime concave-like promotional demand pattern may exist, and is given by

$$D(p,t) = \begin{cases} D_1(p,t) = \lambda e^{-(a+bt)p} & \text{for } t_0 \leqslant t < \tilde{t} \\ D_2(p,t) = \lambda e^{[\tilde{a}+\tilde{b}(t-\tilde{t})]p} & \text{for } \tilde{t} \leqslant t \leqslant t_p \end{cases} \tag{2}$$

where $a \geqslant 0,\ b > 0;\ \tilde{a} \geqslant 0,\ \tilde{b} > 0$; and $\tilde{t}$ represents the demand turning point during the promotional period; t_0 and t_p represent the start and end time points of the promotional period. Note that the above demand forms exhibited in these two regimes are not necessarily symmetric. In other words, the corresponding necessary conditions $a = \tilde{a}$ and $b = \tilde{b}$ are not absolutely required in this type of demand.

3. Baseline model: a decentralized channel system

Consider a basic dyadic channel composed of a supplier and a retailer selling a single product to end consumers. As a sales manager of the retailer, one may consider the use of price discounts to promote a product with a goal of maximizing the promotional profit during a given promotion period. Nevertheless, there are certain influential factors which may affect the performance of a price promotional plan. This work focuses on two external influential sources: (1) the variability of the induced promotional demands oriented from the demand side and (2) the wholesale and order pricing strategies of the supplier from the supply side. Fig. 2 depicts the above problem background, which also specifies the key decision variables, parameters, and their casual relationships exhibited in the retailer's joint promotional price-logistics decision scheme.

To facilitate model formulation within the specified scope of study, four assumptions are postulated as follows:

(1) Only the single retailer's promotion plan is considered, independent of other retailers' promotion plans. As an initial step we tend to limit the scope of the present study to a single distribution channel to facilitate the investigation of the target retailer's joint promotional price-logistics decision mechanism.
(2) The retailer is assumed to be the Stackelberg follower, and relatively, the supplier acts as the leader in the specified supplier-retailer distribution channel. This phenomenon may also be observed in manufacturer-retailer distribution channels in high-technology industries, where manufacturers typically have more power in dominating channel members' goals and decisions.
(3) The retailer adopts periodic reorder policy in this study. Although channel coordination in logistics control may also be a critical issue pointed out by some scholars (Weng, 1995; Anand et al., 2008), the present study scope focuses on channel members' pricing and procurement decisions given that the retailer undertakes periodic reorder policy.
(4) Only the short-term effect of price promotions is considered in the model formulation. Although the impact of a price promotion may also have a long-term effect on either the brand level or the sales level (Blattberg et al., 1995), this study merely considers the short-term effect of price promotions in model formulation and analyses.

Given these assumptions, this work formulates the decision schemes of the dyadic members using the two-stage Stackelberg game theory. According to the 2nd assumption, the supplier is the Stackelberg leader in the specified supplier-retailer distribution channel. In the two-stage game context, the supplier may speculate about the retailer's potential decisions to determine the unit procurement price (c_w) and order price (c_o) offered to the retailer. Herein, the unit procurement price is defined as the unit price for procurement of a unit product; order price refers to the price for each purchase order. The aforementioned supplier's decision process is thus defined as the first stage, followed by the retailer's joint promotional price-logistics decision process defined as the second stage in response to the supplier's decisions in c_w and c_o as well as the potential promotional demand patterns. The corresponding equilibrium solutions can then be backwardly approximated (termed backward induction in Kreps, 1990) from the echelon of the retailer to that of the supplier.

In the following subsections, we first introduce the retailer's joint promotional price-logistics model embedded in the 2nd stage, and then the supplier's demand-responsive model embedded in the 1st stage of the proposed two-stage Stackelberg game framework.

3.1. Retailer's joint promotional price-logistics model

Given the above three types of promotional demand patterns, we now consider the efficient reaction of the retailer to the induced variability of promotional demands in its promotional pricing and inbound logistics mechanisms during the promotional period. Therein, the equilibrium solutions in terms of the promotional price (p), reorder frequency (K), amount in each reorder (Q_{t_k}) should be determined. Let us assume that the retailer's order-up-to level ($\bar{S}$) at the beginning of the promotional period (t_0), the lead time (t_L), safety stock level (S_s), and the length of promotion period (T) are given.

3.2. Scenario 1-N: retailer's strategy responding to Type-1 promotional demand pattern

Facing the *Type-1* promotional demand pattern ($D(p,t) = \lambda e^{-ap}$), the retailer can carry out the periodic order policy with a constant reorder amount (Q) at each given reorder time t_k. This can maximize the promotional profits under which the corresponding reorder amount (Q_{t_k}) at the kth reorder time t_k would be

$$Q_{t_k} = Q = \bar{S} - S_s, \quad \text{for } k = 1, 2, \ldots, K \tag{3}$$

where K represents the order frequency of the retailer in the promotional period. Given the constant reorder amount (Q), we can derive both the reorder time (t_k) and frequency (K) by

$$t_k = \begin{cases} t_0 + \frac{Q}{\lambda e^{-ap}} - t_L, & k = 1 \\ t_{k-1} + \frac{Q}{\lambda e^{-ap}}, & k = 2, 3 \ldots K \end{cases} \tag{4}$$

$$K = \frac{T}{Q/\lambda e^{-ap}} \tag{5}$$

Based on the above derivations, we further infer that the retailer may attempt to maximize its promotional profits ($\pi(p,Q)$), which considers the induced revenue, purchase cost, inventory holding cost and order cost given by

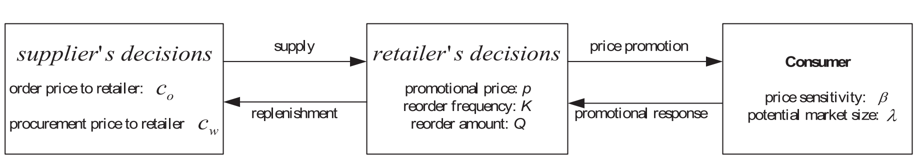

Fig. 2. Channel members' promotional price-logistics decision scheme (without channel coordination).

$$\max \pi(p,Q) = (p - c_w)(\lambda e^{-ap} \times T) - c_{inv} \times \left(S_s + \frac{Q}{2}\right) \times T - c_o \times \frac{T}{Q/\lambda e^{-ap}} \tag{6}$$

where c_{inv} represents the retailer's unit inventory holding cost. Using Eq. (6) we can approximate the tentative equilibrium solution of promotional price (p^*) given by

$$p^* = \frac{1 + ac_w + a\sigma}{a - \frac{a^2}{2}\sigma}, \quad \text{s.t.} \quad 1 - \frac{a\sigma}{2} > 0 \tag{7}$$

where $\sigma = \sqrt{\frac{c_{inv}c_o}{2\lambda}}$. Using Eq. (7) and the result of $\frac{\partial \pi(p^*,Q)}{\partial Q}$, Q^* can then be determined by

$$Q^* = \sqrt{\frac{2c_o \lambda e^{-ap^*}}{c_{inv}}} \tag{8}$$

Accordingly, the retailer can carry out the periodic order policy by approximating p^* and $Q^*_{t_k}$ using Eqs. (7) and (8). Then, the optimal reorder time (t^*_k) and frequency (K^*) can be determined according to Eqs. (4) and (5) to maximize the promotional profit.

3.2.1. Scenario 2-N: retailer's strategy in response to the Type-2 promotional demand pattern

Now, we consider the potential promotional pricing and logistics strategies of the retailer in response to the Type-2 promotional demand pattern (i.e., $D(p,t) = \lambda e^{-(a+bt)p}$). Similarly, given the retailer's ideal inventory level (S_{t_0}) at the beginning of the promotional period (t_0), lead time (t_L), and safety stock level (S_s), one might speculate that first-time replenishment must be accomplished at time point (t_1), before the operating level of inventory runs out. Thus the condition $S_{t_0} - \int_{t_0}^{t_1} D(p,t)dt \geqslant S_s$ must hold. Following the above postulation, we can infer that the retailer's ideal inventory level ($S_{\bar{t}_k}$) at each replenishment time point $\bar{t}_k$ (referring to the time at which the replenishment is practically completed) can be $S_{\bar{t}_k} = S_s + \int_{\bar{t}_k}^{\bar{t}_{k+1}} D(p,t)dt$ during the promotional period. Thus, we have $Q_{t_k} = S_{\bar{t}_k} - S_s = \int_{\bar{t}_k}^{\bar{t}_{k+1}} D(p,t)dt$, where $\bar{t}_k = t_k + t_L$. By taking the same form of profit maximization shown in Eq. (6), we can derive the retailer's objective function given by

$$\begin{aligned}\max \pi(p,Q) = &\sum_{k=0}^{K-1}(p - c_w)\int_{\bar{t}_k}^{\bar{t}_{k+1}} D(p,t)dt \\ &- c_{inv}\int_{\bar{t}_k}^{\bar{t}_{k+1}}\left[S_{\bar{t}_k} - \int_{\bar{t}_k}^{t'} D(p,t)dt\right]dt' - c_o(K-1) \\ &+ (p - c_w)\int_{\bar{t}_K}^{t_p} D(p,t)dt \\ &- c_{inv}\int_{\bar{t}_K}^{t_p}\left[S_{\bar{t}_K} - \int_{\bar{t}_K}^{t'} D(p,t)dt\right]dt' - c_o\end{aligned} \tag{9}$$

In Eq. (9), we also consider the possibility that the time point at which the last replenishment elapses may not be consistent with the end of the promotional period, and thus the terms associated with the last replenishment are separately specified. Additionally, we have $S_{\bar{t}_k} = S_s + Q_{\bar{t}_k} = S_s + \int_{\bar{t}_k}^{\bar{t}_{k+1}} D(p,t)dt$. Therefore, Eq. (9) can be rewritten as

$$\begin{aligned}\max \pi(p,Q) = &(p - c_w)\int_{t_0}^{t_p} D(p,t)dt - c_{inv} \times S_s \times T - c_o \times K \\ &- c_{inv}\left[\int_{\bar{t}_K}^{t_p}\int_{t'}^{t_p} D(p,t)dtdt' + \sum_{k=0}^{K-1}\int_{\bar{t}_k}^{\bar{t}_{k+1}}\int_{t'}^{\bar{t}_{k+1}} D(p,t)dtdt'\right]\end{aligned} \tag{10}$$

To facilitate finding the equilibrium solutions of Eq. (10), we can postulate that the periodic order policy with constant reorder cycles is also used in this scenario and $t_0 = 0$, which may lead Eq. (10) to

$$\max \pi(p,K) = \left(p - c_w - \frac{c_{inv} \times T}{2K}\right)\int_0^T D(p,t)dt - c_{inv} \times S_s \times T - c_o \times K \tag{11}$$

Similarly, using Eq. (11), we can approximate the tentative equilibrium solution of promotional price (p^*) given by

$$p^* = c_w + \sigma + \frac{1}{2a + bT} \tag{12}$$

By taking $\frac{\partial \pi(p^*,K)}{\partial K}$, we can determine K^* as

$$K^* = \sqrt{\frac{c_{inv}T[\lambda e^{-ap^*} - \lambda e^{-(a+bT)p^*}]}{2c_o bp^*}} \tag{13}$$

Once p^* and K^* are approximated, the reorder amount ($Q^*_{t_k}$) can then be readily determined by

$$Q^*_{t_k} = \int_{\frac{(k-1)T}{K^*}}^{\frac{kT}{K^*}} D(p^*,t)dt \tag{14}$$

3.2.2. Scenario 3-N: retailer's strategy in response to the Type-3 promotional demand pattern

The retailer's response in *Scenario 3-N* can be regarded as an extension of the model derived in *Scenario 2-N* due to the similarity of the promotional demand pattern exhibited in the first regime. However, it must be adapted from the beginning of the second regime to respond to the potential purchase re-acceleration driven by consumers' perception of the upcoming promotion termination. Accordingly, we can reconstruct the retailer's objective function by modifying Eq. (11) such that the aggregation of the promotional profits of these two respective regimes is maximized, i.e., max$\pi(p,K) = \pi(p,K_1) + \pi(p,K_2)$, where

$$\begin{aligned}\pi(p,K_1) = &\left(p - c_w - \frac{c_{inv} \times T_1}{2K_1}\right)\int_0^{T_1} D_1(p,t)dt - c_{inv} \times S_s \\ &\times T_1 - c_o \times K_1\end{aligned} \tag{15}$$

$$\begin{aligned}\pi(p,K_2) = &\left(p - c_w - \frac{c_{inv} \times T_2}{2K_2}\right)\int_0^{T_2} D_2(p,t)dt - c_{inv} \times S_s \\ &\times T_2 - c_o \times K_2\end{aligned} \tag{16}$$

Combining Eqs. (15) and (16), we have $\pi(p,K)$ given by

$$\begin{aligned}\pi(p,K) = &\left(p - c_w - \frac{c_{inv}T_1}{2K_1}\right)\int_0^{T_1} \lambda e^{-(a+bt)p}dt \\ &+ \left(p - c_w - \frac{c_{inv}T_2}{2K_2}\right)\int_0^{T_2} \lambda e^{(\tilde{a}+\tilde{b}t)p}dt - c_{inv}S_sT \\ &- c_o(K_1 + K_2)\end{aligned} \tag{17}$$

Similarly, we can derive the tentative equilibrium solution of p^* from the first-order derivative of Eq. (17) as

$$p^* = c_w + \sigma + \frac{T}{2aT_1 + bT_1^2 - 2\tilde{a}T_2 - \tilde{b}T_2^2}, \quad \text{s.t.} \quad 2aT_1 + bT_1^2 > 2\tilde{a}T_2 + \tilde{b}T_2^2 \tag{18}$$

Combining Eqs. (17) and (18), and then taking $\frac{\partial \pi(p^*,K_1,K_2)}{\partial K_1} = 0$ and $\frac{\partial \pi(p^*,K_1,K_2)}{\partial K_2} = 0$, we can further determine K^*_1 and K^*_2 given, respectively, by

$$K^*_1 = \sqrt{\frac{c_{inv}T_1\lambda(e^{-ap^*} - e^{-(a+bT_1)p^*})}{2c_o bp^*}} \tag{19}$$

$$K^*_2 = \sqrt{\frac{c_{inv}T_2\lambda\left(e^{(\tilde{a}+\tilde{b}T_2)p^*} - e^{\tilde{a}p^*}\right)}{2c_o\tilde{b}p^*}} \tag{20}$$

Once p^*, K_1^* and K_2^* are approximated, the reorder amount $\left(Q^*_{t_k}\right)$ can then be derived by

$$Q^*_{t_k} = \begin{cases} \int_{\frac{(k-1)T_1}{K_1^*}}^{\frac{k_1 T_1}{K_1^*}} D_1(p^*, t)dt, & \text{for } t_0 \leqslant t < \tilde{t} \\ \int_{\frac{(k_2-1)T_2}{K_2^*}}^{\frac{k_2 T_2}{K_2^*}} D_2(p^*, t)dt, & \text{for } \tilde{t} \leqslant t < t_p \end{cases}$$
$$= \begin{cases} \frac{\lambda\left[e^{-(a+\frac{b(k_1-1)T_1}{K_1})p^*} - e^{-(a+\frac{bk_1T_1}{K_1})p^*}\right]}{bp^*}, & \text{for } t_0 \leqslant t < \tilde{t} \\ \frac{\lambda\left[e^{(\tilde{a}+\frac{\tilde{b}k_2T_2}{K_2})p^*} - e^{(\tilde{a}+\frac{\tilde{b}(k_2-1)T_2}{K_2})p^*}\right]}{\tilde{b}p^*}, & \text{for } \tilde{t} \leqslant t < t_p \end{cases} \tag{21}$$

3.3. Supplier's demand-responsive model

Given the above three promotional demand scenarios, the following subsection derives the supplier's equilibrium solutions for the determination of c_w and c_o provided to the retailer in the promotional period. Our main purpose here is to investigate the supplier's potential profits without revenue-sharing contracts so as to compare with the case of channel coordination investigated in the next section. The derived equilibrium solutions of the retailer's decisions (i.e., p^*, K^*, and Q^*) are fed back to this stage so as to derive the equilibrium solutions for the supplier's decisions in c_w and c_o.

Given the Type-1 promotional demand pattern, the supplier may aim to maximize its own profits $\left(\psi^{1\text{st}}\right)$, conditional on the retailer's decisions on p^*, K^*, and Q^*, in response to the retailer's periodic order strategy. Therefore, using Eq. (6), we have the supplier's objective function given by

$$\psi^{1^{st}} = \lambda T e^{-ap^*}\left(c_w - c_g + \frac{c_o}{Q^*}\right) \tag{22}$$

where $p^* = \frac{1+ac_w+a\sigma}{a-\frac{a^2}{2}\sigma}$, $Q^* = \sqrt{\frac{2c_o\lambda e^{-ap^*}}{c_{inv}}}$; and c_g represents the unit production cost of the promoted product that includes the unit costs in manufacturing and inventory. Using backward induction, we can derive the equilibrium solutions c_o^* and c_w^* by taking the 1st-order derivatives of Eq. (22) with respect to c_o and c_w, respectively. In this study, c_o^* and c_w^* are interdependent, and their relationship is given by

$$c_o^* = \frac{2\lambda}{a^2 c_{inv}} \times e^{-(1+ac_w^*)} \tag{23}$$

Similarly, using Eqs. (11) and (17) we have the supplier's objective functions ($\psi^{2\text{nd}}$ and $\psi^{3\text{rd}}$) of *Scenarios 2-N and 3-N* given, respectively, by

$$\psi^{2\text{nd}} = (c_w - c_g)\frac{\lambda}{bp^*}\left[e^{-ap^*} - e^{-(a+bT)p^*}\right] + c_o K^* \tag{24}$$

where $p^* = c_w + \sigma + \frac{1}{2a+bT}$, and $K^* = \sqrt{\frac{c_{inv}T\left[\lambda e^{-ap^*} - \lambda e^{-(a+bT)p^*}\right]}{2c_o bp^*}}$.

$$\psi^{3\text{rd}} = (c_w - c_g)\frac{\lambda}{bp^*}\left[e^{-ap^*} - e^{-(a+bT_1)p^*}\right] + (c_w - c_g) \times \frac{\lambda}{\tilde{b}p^*}\left[e^{(\tilde{a}+\tilde{b}T_2)p^*} - e^{\tilde{a}p^*}\right] + c_o\left(K_1^* + K_2^*\right) \tag{25}$$

where $P^* = c_w + \sigma + \frac{T}{2aT_1+bT_1^2-2\tilde{a}T_2-\tilde{b}T_2^2}$, $K_1^* = \sqrt{\frac{c_{inv}T_1\lambda\left(e^{-ap^*} - e^{-(a+bT_1)p^*}\right)}{2c_o bp^*}}$, and $K_2^* = \sqrt{\frac{c_{inv}T_2\lambda\left(e^{(\tilde{a}+\tilde{b}T_2)p^*} - e^{\tilde{a}p^*}\right)}{2c_o\tilde{b}p^*}}$. Using Eqs. (24) and (25), this study derives c_o^* and c_w^* for the supplier's decisions for *Scenarios 2-N and 3-N* by taking the 1st-order derivatives of these equations with respect to c_o and c_w, respectively.

Once the equilibrium solutions (i.e., c_o^* and c_w^*) of the supplier's decisions are determined, this study feeds back these two solutions to the retailer's model to obtain the equilibrium solutions for the retailer's decisions (i.e., p^*, $Q^*_{t_k}$, and K^*) in the promotional pricing-logistics scheme. All the equilibrium solutions and induced promotional profits of channel members undertaken without channel coordination are summarized in *Appendix A*.

4. Extended model for channel coordination

By extending the aforementioned baseline model, this section investigates the dyadic channel members' joint promotional pricing-logistics decisions under channel coordination. In light of Cachon and Lariviere (2005), this study incorporates two decisions variables ω and γ (referring to the unit wholesale price and revenue-sharing percentage, respectively) into the extended model, subject to the condition $\omega < c_w$. This implies that the supplier may offer the retailer a lower procurement price in exchange for a proportion, $1-\gamma$, of p.

Herein, we propose a three-stage game-based channel coordination model to rationalize the decision process of the coordinated channel members, contingent on the revenue-sharing contract, as presented in Fig. 3. At stage 1, both the supplier and retailer may speculate about the contents of the revenue-sharing contract, particularly in γ and ω. This is followed by the supplier's decision in c_o at stage 2, and the retailer's decisions on the joint promotional pricing-logistics strategies at stage 3.

Backward induction (Kreps, 1990) remains used to derive the equilibrium solutions of coordinated channel members. This work derives the tentative equilibrium solution of promotional price p^* first, and then the supplier's decision in order price (c_o^*), followed by the equilibrium solutions of the contract parameters γ^* and ω^*. Once the equilibrium solutions (γ^* and ω^*) of stage 1 are determined, they are input to stage 2 to finalize the equilibrium solution of c_o^*, and then to stage 3 to finalize the equilibrium solutions of p^* and the other logistics-related variables in the retailer-decision layer. The computational procedures associated with the three promotional demand patterns are presented as follows:

4.1. Scenario 1-R: channel coordination in response to the Type-1 promotional demand pattern

This scenario investigates the equilibrium solutions of the dyadic members under the coordination of the revenue-sharing contract contingent on γ and ω in response to the *Type-1* promotional demand pattern. Hence, by Eq. (6), we can rewrite the retailer's objective function ($\pi(p(\gamma,\omega))$) as

$$\pi(p(\gamma,\omega)) = \lambda T e^{-ap}(\gamma p - \omega) - c_{inv}T\left(S_s + \frac{Q}{2}\right) - \frac{c_o T\lambda e^{-ap}}{Q} \tag{26}$$

Take the first-order condition of Eq. (25) with respect to p. We have the tentative equilibrium solution p^* given by

$$p^* = \frac{a\omega + \gamma + a\sigma}{a\gamma - \frac{a^2}{2}\sigma}, \quad \text{s.t.} \quad \gamma - \frac{a\sigma}{2} > 0 \tag{27}$$

Note that combining Eqs. (27), (5) and (8), we can further approximate the tentative equilibrium solutions of K^* and Q^*.

The next step is to derive the tentative equilibrium solution (i.e., c_o^*) for the supplier's decision. Based on Eqs. (22) and (27), we can rewrite the supplier's objective function $\psi^{1^{st}}(\gamma,\omega)$ as

$$\psi^{1^{st}}(\gamma,\omega) = \lambda T e^{-ap^*}\left[(1-\gamma)p^* + \omega - c_g + \frac{c_o}{Q^*}\right] \tag{28}$$

where $p^* = \frac{a\omega+\gamma+a\sigma}{a\gamma-\frac{a^2}{2}\sigma}$ and $Q^* = \sqrt{\frac{2c_o\lambda e^{-ap^*}}{c_{inv}}}$. Then, c_o^* can be approximated by $\frac{\partial\psi^{1^{st}}(\gamma,\omega)}{\partial c_o} = 0$, and thus, we have

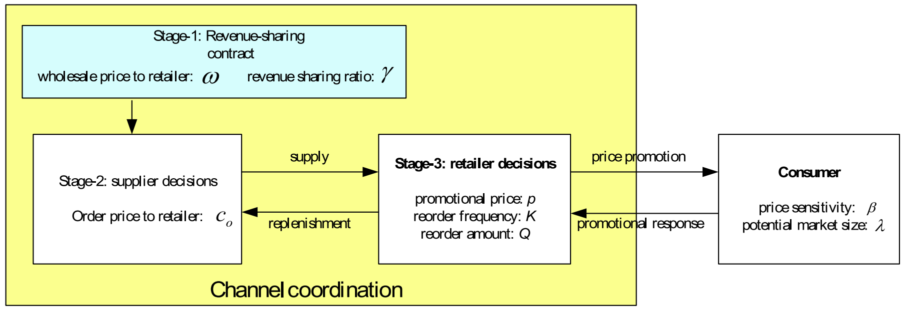

Fig. 3. Supply chain coordination (with a revenue-sharing contract).

$$c_o^* = \frac{2\lambda}{c_{inv}}\left[\frac{-(2a\omega+\gamma+3+ac_g)+\sqrt{(2a\omega+\gamma+3+ac_g)^2-20a(\omega-\gamma c_g)}}{5a}\right]^2 \tag{29}$$

Using the tentative equilibrium solutions p^* (stage 3) and c_o^* (stage 2) obtained previously, the equilibrium solutions of contact parameters γ^* and ω^* can then be derived (stage 1) as

$$\omega^* = -\frac{\gamma^*}{2a}+\frac{\tilde{c}_1}{\gamma^*} \tag{30}$$

where $\tilde{c}_1$ is a constant subject to $\frac{(\gamma^*)^2}{2a} < \tilde{c}_1 < \gamma^* c_w + \frac{(\gamma^*)^2}{2a}$ $(\because 0 < \omega^* < c_w)$.

4.2. Scenario 2-R: channel coordination in response to the Type-2 promotional demand pattern

Similarly, using Eq. (11), we can rewrite the retailer's objective function $(\pi(p(\gamma,\omega)))$ under channel coordination in response to the *Type-2* promotional demand pattern in this scenario. Therefore, we have

$$\pi(p(\gamma,\omega)) = \left(\gamma p - \omega - \frac{c_{inv}\times T}{2K}\right)\left[\frac{\lambda\left(e^{-ap}-e^{-(a+bT)p}\right)}{bp}\right] - c_{inv}\times S_s\times T - c_o\times K \tag{31}$$

By taking the 1st-order condition of Eq. (31) with respect to p, we obtain p^* given by

$$p^* = \frac{\omega+\sigma}{\gamma}+\frac{1}{2a+bT} \tag{32}$$

The next step is to derive the tentative equilibrium solution (i.e., c_o^*) for the supplier's decision in c_o. Combining Eqs. (24) and (32), the supplier's objective function $(\psi^{2nd}(\gamma,\omega))$ of this scenario can be rewritten as

$$\psi^{2nd}(\gamma,\omega) = \left[(1-\gamma)p^*+\omega-c_g\right]\frac{\lambda}{bp^*}\left(e^{-ap^*}-e^{-(a+bT)p^*}\right)+c_oK^* \tag{33}$$

where $p^* = \frac{\omega+\sigma}{\gamma}+\frac{1}{2a+bT}$ and $K^* = \sqrt{\frac{c_{inv}T[\lambda e^{-ap^*}-\lambda e^{-(a+bT)p^*}]}{2c_o bp^*}}$. Then, c_o^* can be derived by $\frac{\partial\psi^{2nd}(\gamma,\omega)}{\partial c_o} = 0$. Therefore, we have

$$c_o^* = \frac{2\lambda}{c_{inv}}\left(\frac{\gamma^2}{(1-\gamma)(2a+bT)}+\frac{\gamma c_g-\omega}{1-\gamma}\right)^2 \tag{34}$$

Following the same backward induction procedures, the equilibrium solutions of the contractual parameters γ^* and ω^* can then be derived as

$$\omega^* = -\frac{\gamma^*}{2(2a+bT)}+\frac{\tilde{c}_2}{\gamma^*} \tag{35}$$

where $\tilde{c}_2$ is a constant subject to $\frac{(\gamma^*)^2}{2(2a+bT)} < \tilde{c}_2 < \gamma^* c_w + \frac{(\gamma^*)^2}{2(2a+bT)}$ $(\because 0 < \omega^* < c_w)$.

4.3. Scenario 3-R: channel coordination in response to the Type-3 promotional demand pattern

Similar to the above computational procedures, we first derive the tentative equilibrium solution of promotional price p^* in response to the *Type-3* promotional demand pattern in this scenario. Using Eqs. (15) and (16), we can rewrite the retailer's objective function $(\pi(p(\gamma,\omega)))$ as

$$\begin{aligned}\pi(p(\gamma,\omega)) = {} & \left(\gamma p-\omega-\frac{c_{inv}T_1}{2K_1}\right)\frac{\lambda}{b}\left[\frac{\left(e^{-ap}-e^{-(a+bT_1)p}\right)}{p}\right] \\ & + \left(\gamma p-\omega-\frac{c_{inv}T_2}{2K_2}\right)\frac{\lambda}{\tilde{b}}\left[\frac{e^{(\tilde{a}+\tilde{b}T_2)p}-e^{\tilde{a}p}}{p}\right] \\ & - c_{inv}S_sT - c_o(K_1+K_2)\end{aligned} \tag{36}$$

By taking the first-order conditions of Eq. (36) with respect to p, we can have the equilibrium solution p^* given by

$$p^* = \frac{\omega+\sigma}{\gamma}+\frac{T}{(2a+bT_1)T_1-(2\tilde{a}+\tilde{b}T_2)T_2} \tag{37}$$

Combining Eqs. (25) and (37) the supplier's objective function $(\psi^{3rd}(\gamma,\omega))$ of this scenario can be rewritten as

$$\begin{aligned}\psi^{3rd}(\gamma,\omega) = {} & \left[(1-\gamma)p^*+\omega-c_g\right]\frac{\lambda}{bp^*}\left[e^{-ap^*}-e^{-(a+bT_1)p^*}\right] \\ & + \left[(1-\gamma)p^*+\omega-c_g\right]\frac{\lambda}{\tilde{b}p^*}\left[e^{(\tilde{a}+\tilde{b}T_2)p^*}-e^{\tilde{a}p^*}\right] \\ & + c_o\left(K_1^*+K_2^*\right)\end{aligned} \tag{38}$$

where $p^* = \frac{\omega+\sigma}{\gamma}+\frac{T}{(2a+bT_1)T_1-(2\tilde{a}+\tilde{b}T_2)T_2}$, $K_1^* = \sqrt{\frac{c_{inv}T_1\lambda\left(e^{-ap^*}-e^{-(a+bT_1)p^*}\right)}{2c_obp^*}}$, and $K_2^* = \sqrt{\frac{c_{inv}T_2\lambda\left(e^{(\tilde{a}+\tilde{b}T_2)p^*}-e^{\tilde{a}p^*}\right)}{2c_o\tilde{b}p^*}}$. Then, c_o^* can be derived as

$$c_o^* = \frac{2\lambda}{c_{inv}}\left[\frac{\gamma^2T}{(1-\gamma)\left((2a+bT_1)T_1-(2\tilde{a}+\tilde{b}T_2)T_2\right)}+\frac{\gamma c_g-\omega}{1-\gamma}\right]^2 \tag{39}$$

Following the same backward induction procedures, the equilibrium solutions of the contractual parameters γ^* and ω^* of *Scenario 3-R* can then be derived as

$$\omega^* = -\frac{\gamma^*T}{2[(2a+bT_1)T_1-(2\tilde{a}+\tilde{b}T_2)T_2]}+\frac{\tilde{c}_3}{\gamma^*} \tag{40}$$

where $\tilde{c}_3$ is a constant subject to

$$\begin{aligned}& \frac{(\gamma^*)^2T}{2[(2a+bT_1)T_1-(2\tilde{a}+\tilde{b}T_2)T_2]} < \tilde{c}_3 \\ & < \gamma^*c_w + \frac{(\gamma^*)^2T}{2[(2a+bT_1)T_1-(2\tilde{a}+\tilde{b}T_2)T_2]}\ (\because 0 < \omega^* < c_w)\end{aligned} \tag{41}$$

All the equilibrium solutions and induced promotional profits under channel coordination with the revenue-sharing contract are summarized in *Appendix A*.

By comparing the equilibrium solutions derived with and without channel coordination, one may be interested in the relative advantage of channel coordination in the induced promotional profit. In the following, p_R^* and p_N^* represent the equilibrium solutions of promotional prices determined for the cases with and without revenue-sharing contracts; $\pi_R(p(\gamma^*,\omega^*))$ and $\pi_N(p^*,Q^*)$ denote the retailer's profits obtained with and without revenue-sharing contracts under equilibrium conditions; $\psi_R(\gamma^*,\omega^*)$ and $\psi_N(c_o^*,c_w^*)$ represent the supplier's profits obtained with and without revenue-sharing contracts under equilibrium conditions; Ω_R^* and Ω_N^* represent the corresponding channel profits. Our findings observed from the derived results are presented below.

Corollary 1. *Given c_g, c_{inv} and c_o^* , under equilibrium conditions in any one of the three promotional-demand scenarios $p_R^* > p_N^*$ holds if $\omega^* \geqslant \gamma^* c_w^*$.*

In reality, Corollary 1 also infers that under channel coordination with revenue-sharing contracts the promotional price offered by the retailer to end-customers is allowed to be greater than that offered under contract-free conditions.

Corollary 2. *Given c_g, c_{inv} and $c_o^*, \Omega_R^* > \Omega_N^*$ holds when $\omega^* < \gamma^* c_w^*$ in Type-1 promotional-demand scenario; and however, $\omega^* \geqslant \gamma^* c_w^*$ in the other two promotional-demand scenarios.*

Corollary 2 definitely encourages the scheme of channel coordination with revenue-sharing contracts under price promotion to end-customers as it guarantees the increases in channel profits.

If Corollaries 1 and 2 are combined, it is apparent that through channel coordination with revenue-sharing contracts the retailer is allowed to offer a greater promotional price to end-customers without the concern of double marginalization effects on channel performance (Spengler, 1950). Such an inference is consistent with (Jeuland and Shugan, 1983) who claim that the hybrid use of profit sharing and quantity discount measures may achieve the desirable incentive structure of channel coordination.

Nevertheless, the dyadic channel members, particularly the retailer who is supposed to perform the follower in the study, may be further curious about whether such channel coordination is more profitable than the strategy without channel coordination under the price promotion scheme. The following generalizations may provide more insights on the dyadic members' performances.

Corollary 3-1. *Given c_g,c_{inv} and c_o^*, under the equilibrium conditions in the Type-1 promotional-demand scenario, (a) $\pi_R(p(\gamma^*,\omega^*)) > \pi_N(p^*,Q^*)$ if $p_R^* - p_N^* < \frac{\ln(\gamma^* p_R^* - \omega^*) - \ln(p_N^* - c_w^*)}{a}$; (b) $\psi_R(\gamma^*,\omega^*) > \psi_N(c_o^*,c_w^*)$ if $p_R^* - p_N^* < \ln \frac{\left((1-\gamma^*)p_R^* + \omega^* + \frac{\sigma^*}{\sqrt{e^{-ap_R^*}}}\right) - \ln\left(c_w^* + \frac{\sigma^*}{\sqrt{e^{-ap_N^*}}}\right)}{a}$.*

Corollary 3-1 reveals the respective conditions for the dyadic members to yield greater profits through a revenue-sharing contract relative to the case without revenue-sharing contracts.

Based on Corollary 3-1, let us further consider a case in which the supplier (i.e., the leader) would like to offer more incentives such as waiving reorder charges ($c_o^* = 0$) and quick response to the retailer's reorder to greatly reduce the retailer's unit inventory holding cost ($c_{inv} \approx 0$). Then, we have.

Corollary 3-2. *Given c_g, either $c_{inv} \approx 0$ or $c_o^* = 0$, under the equilibrium conditions in the Type-1 promotional-demand scenario, the win–win outcome (i.e., $\pi_R(p(\gamma^*,\omega^*)) > \pi_N(p^*,Q^*)$ and $\psi_R(\gamma^*,\omega^*) > \psi_N(c_o^*,c_w^*)$) is guaranteed if $p_R^* - p_N^* < \frac{\ln(\gamma^* p_R^* - \omega^*) - \ln(p_N^* - c_w^*)}{a}$.*

Corollary 3-2 also infers the importance of the retailer's profitability to the fulfillment of a win–win outcome in the studied supplier-retailer joint promotional program. Therein, a win–win outcome can be easily achieved once the retailer is guaranteed to yield profit greater than that obtained without channel coordination. Such a generalization is important to the dyadic members, particularly to the retailer who acts the follower in this study case. In reality, it is not surprising that the supplier (the leader) is always better off by revenue-sharing contracts as it possesses more bargaining power and resources. By contrast, the retailer should rely on appropriate promotional pricing strategies as well as economic and logistics incentives provided by the supplier to increase profits such that the win–win outcome can be achieved.

Corollary 3-3. *Given c_g, c_{inv} and c_o^*, under equilibrium conditions in either the Type-2 or Type-3 promotional-demand scenario, $\pi_R(p(\gamma^*,\omega^*)) < \pi_N(p^*,Q^*)$; however, $\psi_R(\gamma^*,\omega^*) > \psi_N(c_o^*,c_w^*)$.*

Corollary 3-3, in reality, characterizes the main purpose of revenue-sharing contracts, i.e., maximizing the channel-based aggregate profits rather than the dyadic members' disaggregate profits. Under channel coordination with revenue-sharing contracts, the retailer needs to sacrifice some revenues for the supplier's commitment to lower procurement prices and reliable supply, compared to the contract-free case. In contrast, the supplier may gain more profit from the revenue-sharing contract, despite the loss caused by offering the lower procurement price to the retailer.

Based on the above analytical results in channel profitability, we can conclude that channel coordination with revenue-sharing contracts guarantees the increases in the channel's and leader's (i.e., the supplier in this study case) profits. By contrast, the increases in the follower's (i.e., the retailer's) profits must hinge on a constrained promotional price strategy (Corollarys 3-1 and 3-2) and the prerequisite which a consumer's promotional price sensitivity is time-invariant. More importantly, acting as the leader the supplier should provide more incentives to the retailer to facilitate the retailer's relationship commitment in channel coordination such that a win–win outcome can be fulfilled in the joint promotional program.

In addition to channel profitability, the following generalizations may provide some insights on channel coordination in logistics operations under price promotion. Therein, Propositions 1 to 3 characterize the retailer's strategic procurement and induced inventory costs with and without revenue-sharing contracts.

Proposition 1. *Given the Type-2 promotional demand pattern, under equilibrium conditions either with or without revenue-sharing contracts the retailer's reorder amounts $Q_{t_k}^*$, $k = 1,2,\ldots,K^*$ may decrease over time following $Q_{t_1}^* > Q_{t_2}^* > \cdots > Q_{t_{K^*}}^*$ such that the induced average inventory cost ($S_{t_k}^*$) for each reorder cycle can also be reduced over the promotional period, i.e.,$S_{t_1}^* > S_{t_2}^* > \cdots > S_{t_{K^*}}^*$.*

The proof of Proposition 1 is quite straightforward. Note that the time-varying promotional demand may strictly decrease under the condition of the *Type-2* promotional demand pattern as $\frac{\partial \lambda e^{-(a+bt)p^*}}{\partial t} < 0$. Using Eq. (14) and $S_{t_k}^* = c_{inv}(S_s + Q_{t_k}^* - E[D(p^*,t)]) \times \frac{T}{2K^*}$, Proposition 1 can then be proved.

Proposition 2. *Given the Type-3 promotional demand pattern, under equilibrium conditions either with or without revenue-sharing contracts the retailer's reorder amounts ($Q_{t_{k_1}}^*$, $k_1 = 1,2,\ldots,K_1^*$) induced before the demand turning point $\tilde{t}$ decrease over time (i.e., $Q_{t_1}^* > Q_{t_2}^* > \cdots > Q_{t_{K_1^*}}^*$), followed by the increases of $Q_{t_{k_2}}^*$, $k_2 = 1,2,\ldots,K_2^*$ over time after$\tilde{t}$ until the end of the promotional period (i.e., $Q_{t_1}^* < Q_{t_2}^* < \cdots < Q_{t_{K_2^*}}^*$).*

Proposition 2 shows that under equilibrium conditions, two respective periodic order policies can be carried out in response to the different change patterns of the promotional demands exhibited before and after $\tilde{t}$, i.e., $\frac{\partial D_1(p^*,t)}{\partial t} < 0$ and $\frac{\partial D_2(p^*,t)}{\partial t} > 0$, respectively. This contributes to the decrements of reorder amounts before $\tilde{t}$, followed by increases after $\tilde{t}$ until the end of the promotional period. Using Eq. (14), we can further derive the decrements $(\Delta Q_{t_k,t_{k+1}})$ of reorder amounts between two successive reorder time points (t_{k_1} and) before $\tilde{t}$ as $\frac{\lambda}{bp^*}\left(2e^{-(a+\frac{bk_1T_1}{K_1^*})p^*} - e^{-(a+\frac{b(k_1-1)T_1}{K_1^*})p^*} - e^{-(a+\frac{b(k_1+1)T_1}{K_1^*})p^*}\right)$. Similarly the increments of reorder amounts between two successive reorder time points (t_{k_2} and t_{k_2+1}) after $\tilde{t}$ can be derived as $\frac{\lambda}{\tilde{b}p^*}\left(e^{(\tilde{a}+\frac{\tilde{b}(k_2-1)T_2}{K_2^*})p^*} + e^{-(\tilde{a}+\frac{\tilde{b}(k_2+1)T_2}{K_2^*})p^*} - 2e^{-(\tilde{a}+\frac{\tilde{b}k_2T_2}{K_2^*})p^*}\right)$.

Proposition 3. *Given the Type-3 promotional demand pattern, under equilibrium conditions either with or without revenue-sharing contracts the retailer's periodic inventory cost ($S^*_{t_{k_1}}$, $k_1 = 1, 2, \ldots, K_1^*$) induced before the demand turning point $\tilde{t}$ decreases over time (i.e., $S^*_{t_1} > S^*_{t_2} > \cdots > S^*_{t_{K_1^*}}$), followed by the increases of the cost ($S^*_{t_{k_2}}$, $k_2 = 1, 2, \ldots, K_2^*$) after $\tilde{t}$ until the end of the promotional period (i.e., $S^*_{t_1} < S^*_{t_2} < \cdots < S^*_{t_{K_2^*}}$).*

Similarly, Proposition 3 can be straightforwardly proved using the features of Proposition 2 coupled with the inventory cost function $S^*_{t_k} = c_{inv}(S_s + Q^*_{t_k} - E[D(p^*, t)]) \times \frac{T}{2K^*}$ which indicates that $S^*_{t_k}$ may vary in parallel with $Q^*_{t_k}$ across the promotional period.

5. Illustrative Examples

To gain more managerial insights, this section presents a numerical study which illustrates "bread", "notebooks", and "tissue paper" (termed product categories 1, 2, and 3, respectively) as three promotion cases associated with the *Type-1*, *Type-2*, and *Type-3* promotional demand patterns, respectively. Based on our preliminary analysis, we preset the corresponding key parameters needed in the model, as presented in Table 1.

Now we turn our attention to the impact of the retailer's revenue-sharing rate (γ^*) on the equilibrium solutions of channel members' decisions and performance which are derived using the proposed model. Fig. 4 illustrates the impact of γ^* on the supplier's decision in c_o^* and channel performance, where the x-axis represents the value of γ^* bounded within the feasible range 0 and 0.72. Note that although γ^* and ω^* are two key contractual variables, they are correlated. In the illustrative examples, we also preset appropriate values of $\tilde{c}_1$ such that the equilibrium solution of ω^* exists in the study cases. Thus, only the effects of γ^* are illustrated. According to Fig. 4, some interesting findings are observed. First, the effects of γ^* revealed in the case of product category-1 (i.e., bread) are significantly different from the others. In the case of product category-1, c_o^* shows straightforward increases as γ^* increases. In contrast, the (γ^*, c_o^*) curve is wavelike and bends upward remarkably when $\gamma^* > 0.56$ in both the cases of product categories 2 and 3. In addition, it is revealed that the supplier (i.e., the leader of the cooperative channel) remains dominant under such marketing-driven supply chain coordination, which is contingent upon revenue-sharing contractual mechanisms. No matter how γ^* changes, the supplier's profits are still greater than the retailer's profits except when $\gamma^* > 0.56$ under the *Type-1* promotional demand pattern. In contrast, the impact of γ^* on the retailer's profits is mixed. In the case of Type-1 promotional demand pattern, the retailer's profits increase with γ^*, and remains positive. Conversely, the retailer's profit curve is concave in both the Type-2 and Type-3 promotional demand cases and bends downward significantly when $\gamma^* > 0.56$, thus leading to the following paradox, as presented in **Remark 1**.

Remark 1. Given the existence of either the Type-2 or Type-3 promotional demand pattern, the more the retailer shares from a unit sale the more it loses under the revenue-sharing supply chain coordination scheme.

Actually, such a phenomenon is not surprising due to the $\gamma^*_\uparrow \Rightarrow c^*_{o\uparrow} \Rightarrow p^*_\uparrow$ chain effect on $\pi_R(p(\gamma^*, \omega^*))$. As can be seen in Fig. 4, a high value of γ^* (e.g., $\gamma^* > 0.56$) may cause striking increases in c_o^* and p^*, leading to a negative effect on π_R $(p(\gamma^*, \omega^*))$. That is, the retailer's anomalous request to raise γ^* in the revenue-sharing contract may incur the supplier's decision to increase c_o^*, followed by the retailer's response of raising the promotional price p^* to end customers. Consequently, the retailer becomes the final loser in the collaborative game as its profit decreases. Nevertheless, it is also noted that theoretically, the aforementioned $\gamma^*_\uparrow \Rightarrow c^*_{o\uparrow} \Rightarrow p^*_\uparrow$ counter-profit chain effect exists when $\gamma^* > 0.56$; however it may not often occur in practical cases, particularly when the retailer is a follower with relatively less bargaining power, as addressed in this study.

$c_o^* c_o^* c_o^*$

Drawing from Corollaries 2, 3-2 and our interest in the win–win outcome to the coordinated channel members in this study case, this work further aims at product category 1 (*Type-1* demand pattern) to conduct sensitivity analysis with respect to γ^* specified in the revenue-sharing contract. Then, this work compares the profits obtained with and without the revenue-sharing contract for both the supplier and retailer. Therein, the parameters shown in Table 1 remain used except for c_o^* and c_{inv}^* which are set to be zero, mimicking that the supplier is willing to provide more economic and logistics incentives to the retailer (by Corollary 3-2) for win–win solutions. The analytical results are shown in Figs. 5–7. Some managerial insights observed from Figs. 5–7 are provided as follows.

Overall, the results of sensitivity analysis are consistent with Corollaries 2, 3-1, and 3–2, indicating that the win–win outcome (i.e., $\pi_R(p(\gamma^*, \omega^*)) > \pi_N(p^*, Q^*)$ and $\psi_R(\gamma^*, \omega^*) > \psi_N(c_o^*, c_w^*)$) does exist as $\gamma^* \geqslant 0.4$. Therein, both the retailer's profit and channel profit increase as the value of γ^* increases. As a channel leader, the supplier needs to seek for an optimal solution of γ^* (e.g., $\gamma^* = 0.5$ is suggested in this study case) to maximize the increased profit. In contrast with the $\gamma^*_\uparrow \Rightarrow c^*_{o\uparrow} \Rightarrow p^*_\uparrow$ counter-profit chain effect revealed in Remark 1, Fig. 5 indicates that the decrease in the retailer's revenue-sharing ratio (γ^*) does not guarantee the increase in the sup-

Table 1
Preset parameters.

Parameter demand pattern	a	b	$\tilde{a}$	$\tilde{b}$	c_g	c_{inv}	λ	S_s	T	T_1	T_2
Type-1 (bread)	1	0			0.2	0.05	5000	0.1	30		
Type-2 (notebook)	0.0001	0.0001			100	5	500	1.0	30		
Type-3 (tissue paper)	0.002	0.01	0.07	0.007	0.8	0.1	900	0.1	30	23	7

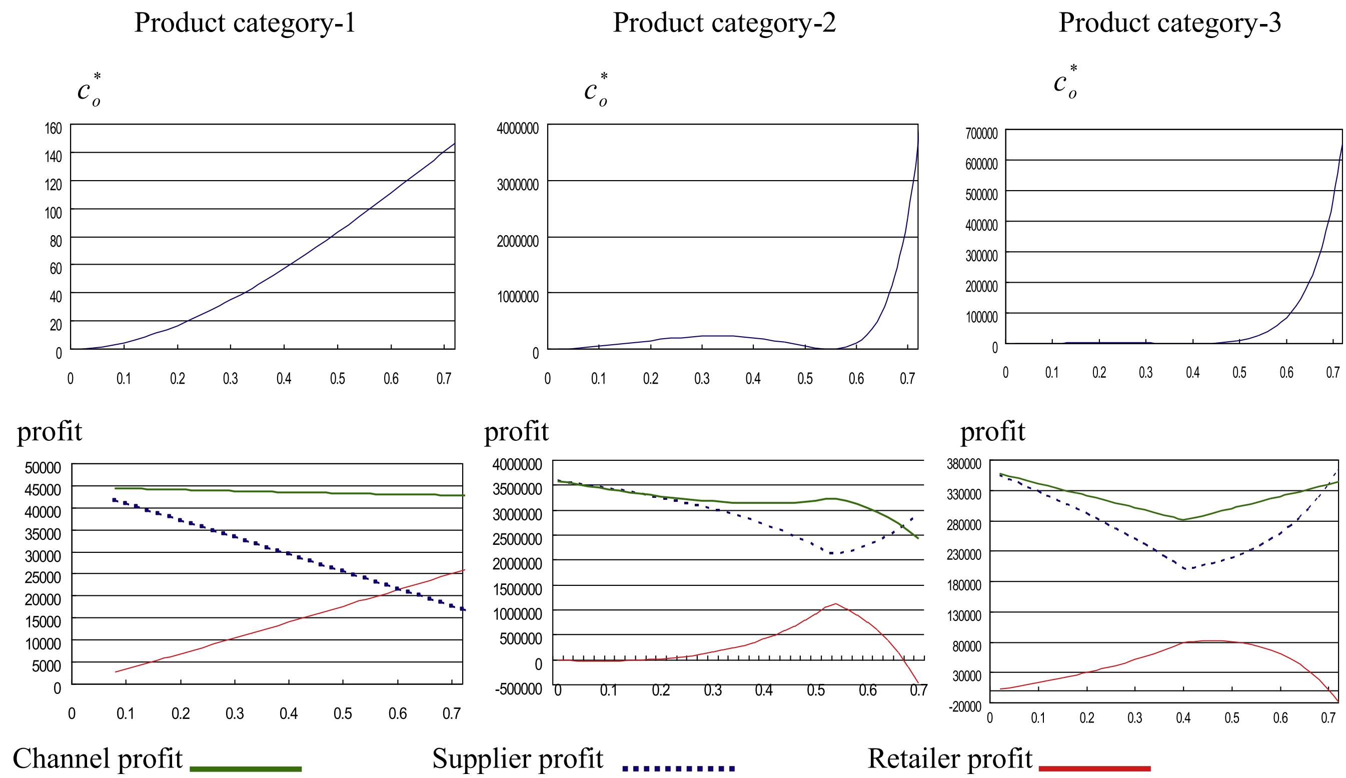

Fig. 4. Effects of γ^* on coordinated channel behavior.

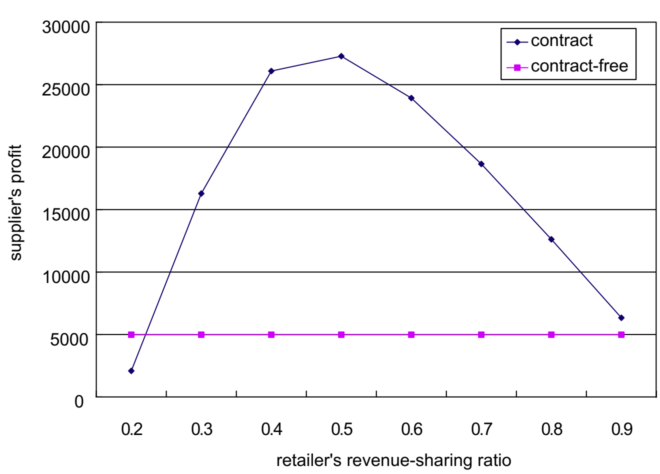

Fig. 5. Comparison of supplier profits (with and without revenue-sharing contract).

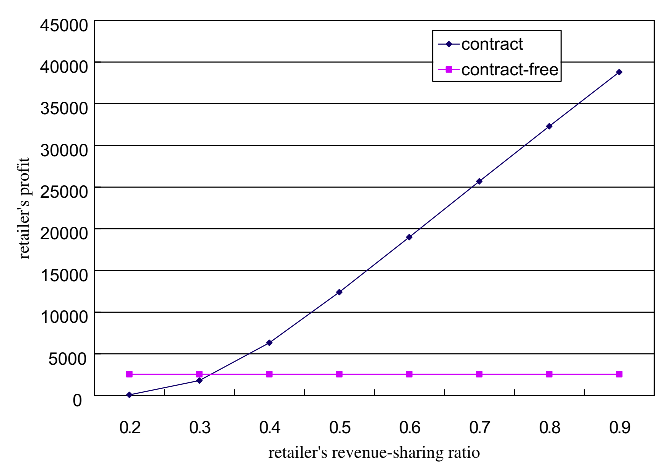

Fig. 6. Comparison of retailer profits (with and without revenue-sharing contract).

plier's profit under channel coordination. Instead, both the supplier's and retailer's profits increase as γ^* increases when $\gamma^* \leqslant 0.5$, followed by the decrease in the supplier's profit when $\gamma^* > 0.5$. Moreover, we conclude the following remark (Remark 2) from our observation in the sensitivity analyses.

Remark 2. Given c_g, either $c_{inv} \approx 0$ or $c_o^* = 0$, under the equilibrium conditions in the Type-1 promotional-demand scenario, the win–win outcome coexists with the conditions $p_R^* < p_N^*$ and $\omega^* < \gamma^* c_w^*$.

6. Managerial implications for future research

Based on analytical results, we can conclude that under equilibrium conditions, the resulting channel-based aggregate profit with revenue-sharing contracts is greater than that without contracts no matter which promotional demand pattern exists. Nevertheless, the corresponding price discounts offered to end customers under channel coordination with revenue-sharing contracts should be greater than the discounts offered without contracts to achieve the above goal. Furthermore, the supplier's profit induced by a revenue-sharing contract is proved to be greater than that obtained without a contract. Revenue-sharing contracts may not assure the retailer's profitable advantage, but they do reduce the periodic reorder amounts and inventory costs during the promotional period.

From the retailer's viewpoint, channel coordination with revenue-sharing contracts may not be necessary for the accomplishment of promotional profit maximization. However, this study deals with the case in which the retailer is a follower, and thus,

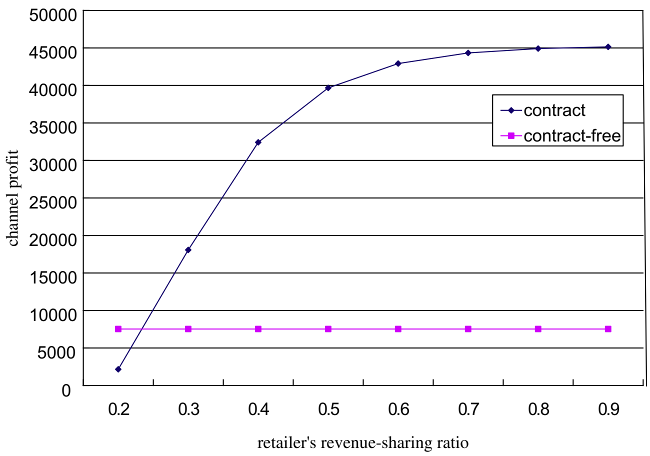

Fig. 7. Comparison of channel profits (with and without revenue-sharing contracts).

needs to appeal to revenue-sharing contracts to reciprocally gain reliable procurement and lower wholesaling prices from the supplier during a promotional period. As such, channel coordination through revenue-sharing contracts may alleviate the retailer's concerns about stock-imbalance risks and variant procurement costs in its joint promotional pricing-logistics planning scheme.

In addition, several suggestions for future research are given. First, model extension from either the demand side or supply side may warrant more research efforts. On the demand side, incorporation of other forms of promotional demand patterns characterized with either stochastic or fuzzy properties can be considered. Moreover, more investigation is warranted on the effects of other promotional programs, such as coupons, new product introductions, and celebrity endorsements. On the supply side, the phenomenon presented in Remark 1 coupled with the $\gamma^*_\uparrow \Rightarrow c^*_{o\uparrow} \Rightarrow p^*_\uparrow$ chain effect warrants more investigations. In addition, other channel collaborative measures not limited to contracts may also be noteworthy. For instance, the hybrid use of economic and non-economic channel power sources and the induced effects on the equilibrium solutions of the dyadic members' decisions in joint promotional pricing-logistics schemes may warrant more investigation. The use of "bargaining game theory" (e.g., Nash bargaining game and Rubinstein model) to characterize the bargaining power of the cooperative dyadic members and the resulting effect on negotiation and solutions can also be noteworthy for future research.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.ejor.2011.04.031.

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