

Reconstruction of 3D Geometric Models of Pulmonary Arteries from Cardiac CT Images

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ABSTRACT

We present a method to construct the geometric model of the pulmonary arteries from a set of cardiac CT scan images. It is desired that the model is in rectangular meshes. The main difficulties in this work are insufficient resolution along z-direction, the requirement of the rectangular meshes, and the geometric shape of the pulmonary arteries. We present a method that is based on estimation the medial axis and the radii of the vessel along the axis. We evaluate the proposed method using a phantom data set. The proposed method can achieve good reconstructed result for the phantom data set.

Key words: pulmonary arteries, 3D geometric model, spline curve, CT scan.

1. INTRODUCTION

Cardiac CT scan is one of the most important tools for heart disease diagnosis. It has the advantage that the resolution of an image is the best among all of the modalities. Due to this reason, this work attempts to construct the geometric model of the pulmonary arteries from a set of CT images. The desired geometric model meets the following criterion.

- The meshes are rectangular.
- The direction of the rectangular meshes should be consistent with the medial axis of the vessel.

These requirements make the task a more difficult task comparing to simply reconstruct the geometric model using triangular patches. Another difficulty is due to the relative low resolution along z-direction.

In this paper, we present a method that reconstructs the geometric model to meet the requirements stated above. We present the method in the following section.

2. METHOD

The pulmonary artery (PA) is considered consisting of two tubes. One is LPA and PT and the other is RPA and PT. These two tubes are denoted LPA/PT and RPA/PT. Our approach is to first construct the medial axes of LPA/PT and RPA/PT. We then calculate the medial axes for the pulmonary arteries by merging the two medial axes. A point on the medial axes is associated with a radius. The geometric model of the pulmonary arteries is obtained from these information.

The medial axis of a tube is derived from the contours in consecutive slices of the CT images. The contours of the pulmonary arteries in an image can be easily calculated since pulmonary arteries are enhanced by contrast agent. A contour may contain the boundaries of both the LPA and RPA, In this case, the contours for LPA and RPA are obtained by separating the contour using user assists.

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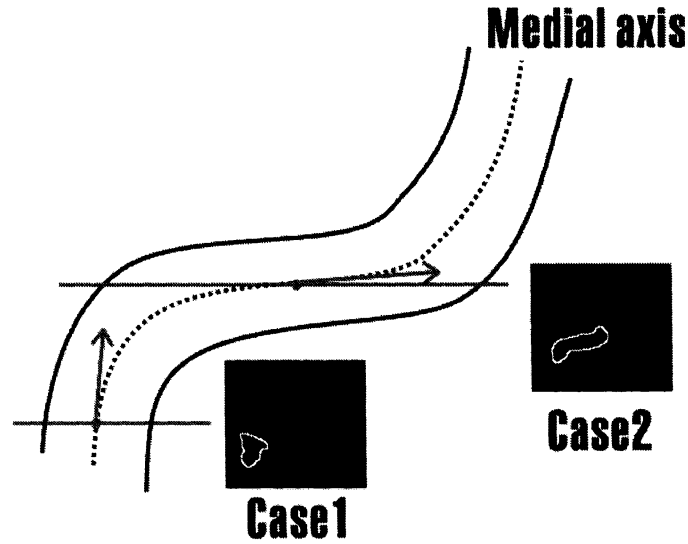


Figure 1: There are two cases, depending on the angle between the plane and the medial axis, we consider.

2.1. Compute the 3D Medial Axes

Observe that, given a tube and a plane, if the tube and the plane are perpendicular to each other, the intersection between the plane and the tube is a circle. The disk enclosed by the circle is the cross section of the tube. If the angle between the plane and the medial axis is close to $\pi/2$, the intersection between the tube and the plane is an ellipse. Furthermore, the center point of the ellipse is the center of the cross section, and the length of the short axis is equal to the radius of the cross section. If we have sufficiently dense cross sections along the medial axis of the tube, the tube can be easily reconstructed. The proposed method is based on the observation stated above.

The ellipse-specific direct least-square fitting is applied to process the ellipse fitting.³ The ellipse fitting can be separated into two major approaches. One is geometric method and the other is the algebraic method. The major difference between these two approaches is the measuring of the error. The geometric method⁵ uses Euclidean distance and the algebraic method uses the algebraic distance. The algebraic distance is briefly described in the follows. A general conic can be written in the form

$$F(\mathbf{a}, \mathbf{x}) = \mathbf{a}\mathbf{x} = ax^2 + bxy + xy^2 + dx + ey + f$$

where $\mathbf{a} = [a, b, c, d, e]$ and $\mathbf{x} = [x^2, xy, y^2, x, y, 1]$. We call the "algebraic distance" of a point \mathbf{x}_i to the conic $F(\mathbf{a}, \mathbf{x}) = 0$ to be

$$F(\mathbf{a}, \mathbf{x}_i) = d.$$

We solve the parameter \mathbf{a} by minimizing the sum of the distances from all points to the ellipse. The constraint that the conic function is an ellipse is as shown in the following equation.

$$b^2 - 4ac = \mathbf{a}^T \begin{bmatrix} 0 & 0 & -2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{a} = \mathbf{a}^T C \mathbf{a} < 0.$$

To construct the medial axis, we consider two cases(see Figure 1).

Case 1 The angle between the plane and the medial axis is close to $\pi/2$. In this case, the ellipse fitting is applied directly to the contour.

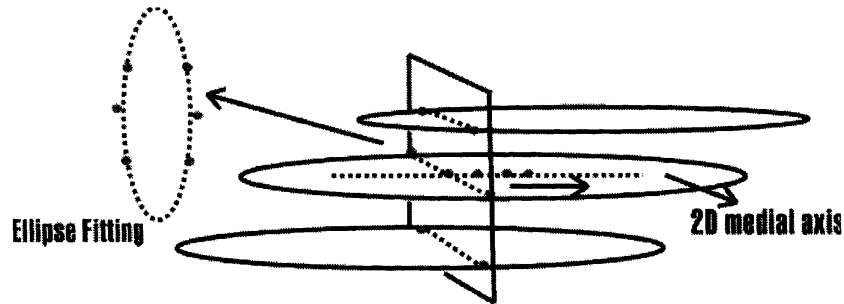


Figure 2. A plane perpendicular to the 2D medial axis is almost perpendicular to the 3D medial axis. And the intersection between the plane and contours is close to an ellipse.

Case 2 The plane is almost parallel to medial axis. In this case, we need to determine a plane that is almost perpendicular to the medial axis. To determine the plane, we first calculate the 2D medial axis of the contour in a slice. A plane perpendicular to the 2D medial axis of a contour can be considered to be almost perpendicular to the 3D medial axis of the tube. And the shape of the intersections between the plane and the contours in the consecutive slices is close to an ellipse (Figure 2). We can then apply the ellipse fitting to the newly obtained contour.

The set, C , of the center points of the ellipses are considered a set of points on the medial axis of a tube. The approximated medial axis is obtained by a spline curve fitting of the point set C . We used cubic and non-uniform B-spline for the spline curve fitting. A B-spline curve needs a set of control points. These control points are obtained from a polygonal path that approximates C . We use a recursive algorithm to establish the initial polygonal path. Let $C = \{p_i | i = 1, \dots, n\}$. Suppose that p_1 and p_n are respectively the first and the last points of the medial axis. We find p_j , $1 < j < n$, that is the farthest point from $\overline{p_1, p_n}$. If the distance between p_j and $\overline{p_1, p_n}$ is greater than a given threshold, we then split the $\overline{p_1, p_n}$ to two line segments $\overline{p_1, p_j}$ and $\overline{p_j, p_n}$. We recursively apply the method to these two line segments. By using the points on the polygonal path as the control points, we are able to design a spline curve that approximates the medial axis. Finally, we map each vertex on the polygonal path to a point u on the B-spline curve. Each point u is called a characteristic point and is described by a parameter in the parametric representation of the spline. The radius of the characteristic point on the B-spline curve is obtained by averaging the distances from u to its neighboring points.

2.2. Merging two Medial Axes

The two tubes have to be merged to establish the pulmonary artery. In the first merge step, we determine the “junction point”. The two medial axes meet at the junction point and become a three-branch medial axes. Each branch corresponds to PT, LPA, and RPA. In the second step, we design the “junction” of the tubes. The junction divides the pulmonary artery into four parts, i.e., the junction, and three tubes. Again, the three tubes are respectively PT, LPA, and RPA. The surface of the pulmonary artery is then obtained by union of the three tubes and the junction.

To combine the two B-spline curves, we determine the position of the junction point fp . The method to determine fp is stated in the follows. We examine all the characteristic points on the medial axis of RPA/PT in the order from the RPA end to the PT end. We find a characteristic point p that the distance from p to the medial axis of LPA/PT is small. It is not easy to calculate the distance between a point and a spline curve. We simplify the computation to calculate the distances between characteristic points on both medial axes. Another problem is to determine the p to be fp . Note that, since both tubes share PT. The distance between the characteristic points on PT of both tubes should be small. Furthermore, the junction point, that is the junction of LPA, RPA, and PT, is close to the RPA end. We choose the first p that the distance between p to the medial axis of LPA/PT is smaller than a given threshold. The closest characteristic point on LPA/PT to p is denoted q .

Finally, the three-branch medial axis is formed by using the following rule.

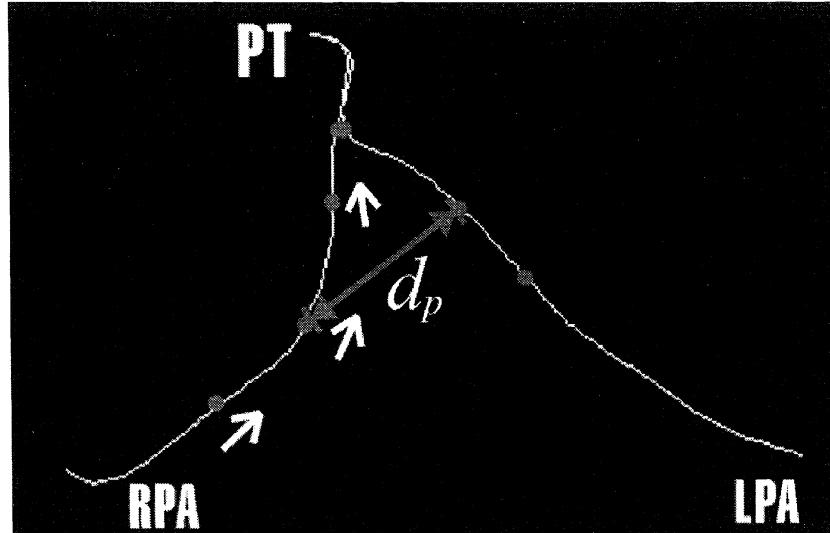


Figure 3: The junction point and the medial axes.

- The two branches are from medial axis of RPA/PT by dividing at fp .
- We partition the LPA/PT from the point q . We then connect q to fp to form the third branch.

The merged medial axes and the junction points are shown in Figure 3.

2.3. Surface Construction

The construction of the surface mesh of the model can be separated into two parts. One is the simple connection of the two consecutive circles and the other is to build the junction connection. To design the junction connection is the difficult part in this step. The junction part of the model consists of three cross sections of LPA, RPA, and PT and the inter-medium (Figure 4). In the first step, we determine the three cross sections. The three cross sections cannot intersect to each other and they should be close to the junction point.

We have three sequences of points on the medial axes of RPA, LPA and PT respectively. We start at fp and march along the three axes toward their ends. Each time, we advance a step along each sequence. We then test if the three cross sections could intersect to each other. If there is intersection, we advance to the next point. Otherwise we have three cross sections that do not intersect to each other.

The next step is to find the inter-medium. The inter-medium of the junction is to connection the three circles and its shape is showed in Figure 4. The inter-medium consists of three half ellipses. The short axes of three half ellipses are all equal to the radius associated with fp . As shown in Figure 5, the half ellipse between LAP and RPA is called EllipseLR, the half ellipse between LAP and PT is called EllipseLP and the half ellipse between RAP and PT is called EllipseRP. The plane that is made of the Fork, RPA and LPA intersects the circle associated with RPA at p_1 and p_2 . It also intersects the circle associated with LPA at p_3 and p_4 . We can form four pairs of points, (p_1, p_3) , (p_2, p_3) , (p_3, p_4) and (p_5, p_6) . We then select the pair that the distance between them is the least between them. The length of the long axis of the EllipseLR is the distance between the Fork and the mid-point between the pair. The length of the long axes of EllipseLP and EllipseRP are obtained in the similar way. We can then construct the junction. Then we connect the junction and the three circles by rectangular meshes. The result is showed in Figure 6.

The final geometric model is obtained by connecting the consecutive circles along PT, LPA, and RPA. The result is shown in Figure 10.

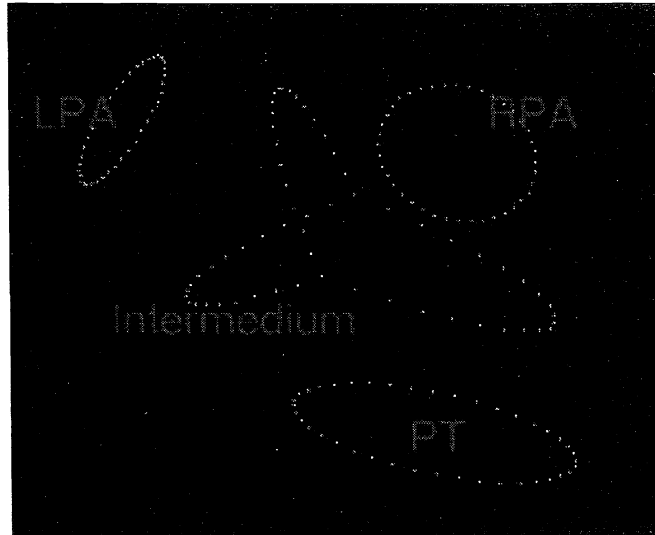


Figure 4: The designed junction of PT, LPA, and RPA.

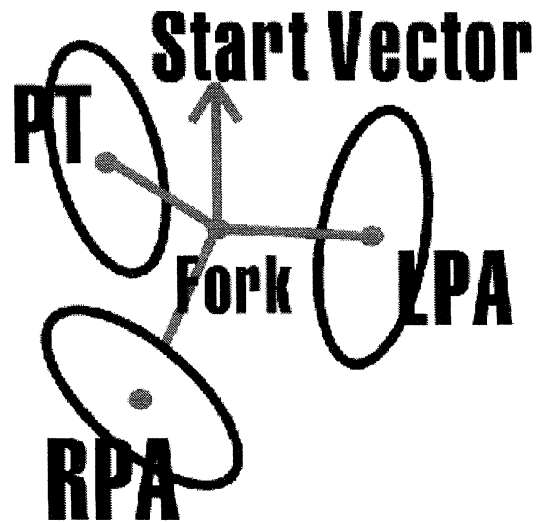


Figure 5: The geometric structure connects PT, LPA, and RPA.

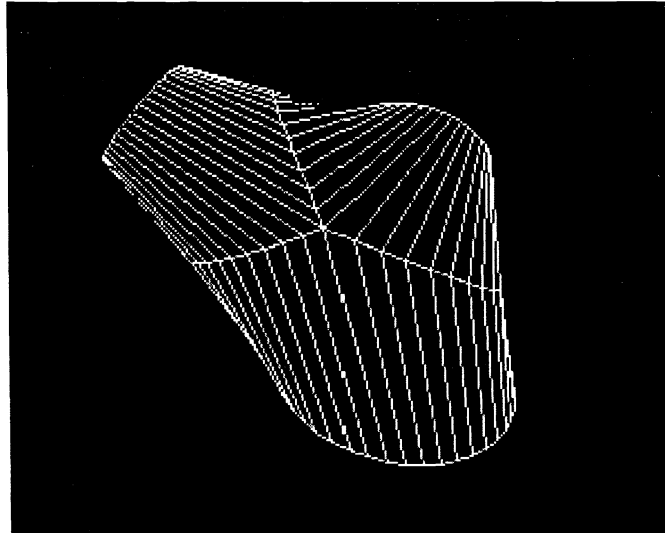


Figure 6: The geometric structure connects PT, LPA, and RPA.

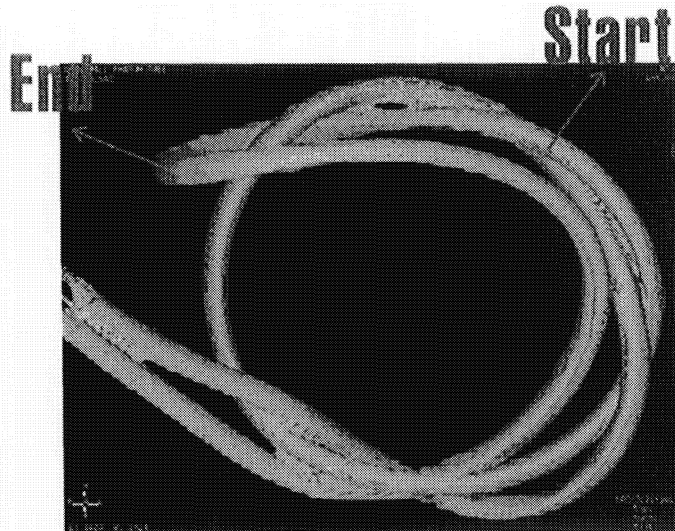


Figure 7: CT scan of a hose filled with contrast agent. The portion used in the experiment is shown in the Figure.

3. EXPERIMENTS AND RESULTS

In this section, we present the results obtained using the proposed method. We have done experiments on a phantom data and a cardiac CT scan of heart.

Phantom data set

The phantom data set was a hose filled with contrast agent. We took CT scan of the hose and tried to construct a portion of the hose that has medial axis almost parallel to the CT section plane. The hose had radius 12.19 pixels. The reconstructed hose had radius within 10% error. The phantom data set obtained by volume rendering is shown in Figure 7. The portion of the hose in the reconstructed experiment is also shown in the figure. The reconstructed result is shown in Figure 8. The error along the medial axis is shown in Figure 9. Note that, we choose the portion that has medial axis almost parallel to the CT section plane. This is the most difficult case. Our result show that we can still obtain good result.

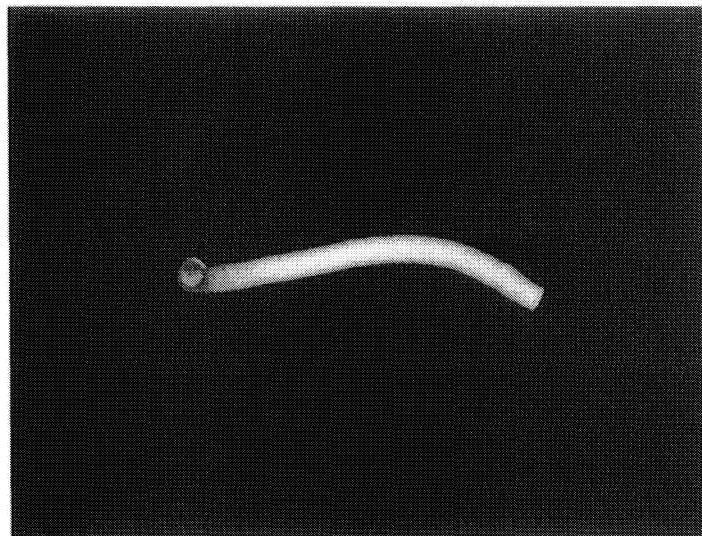


Figure 8: The reconstructed hose.

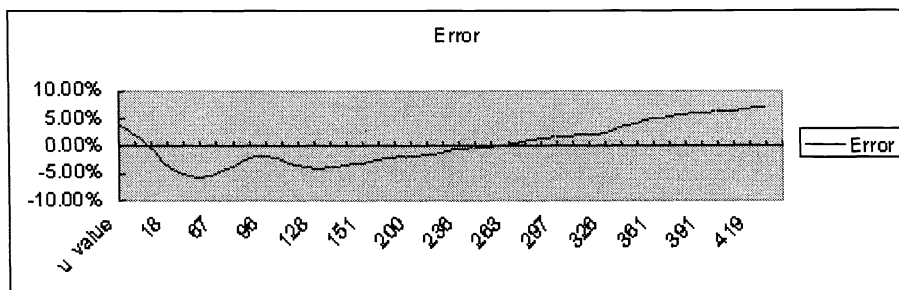


Figure 9: The error of the reconstructed result along the medial axis.

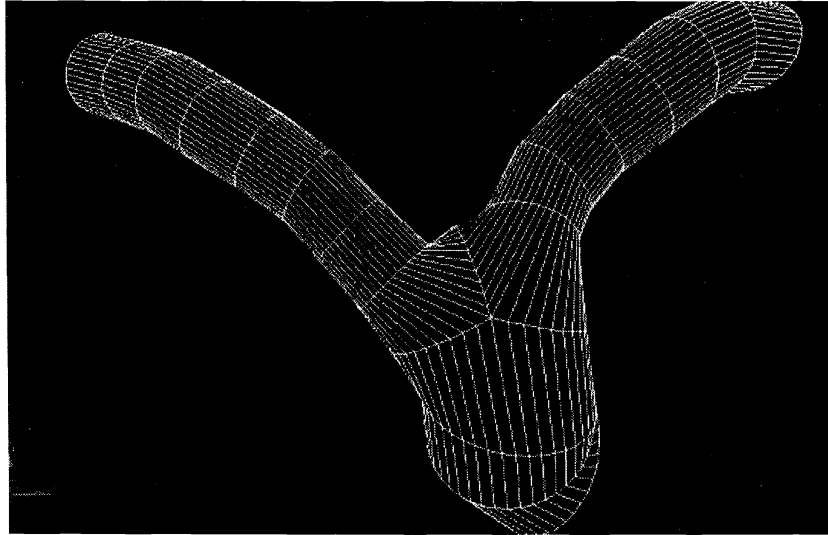


Figure 10: The reconstructed pulmonary arteries.

Cardiac CT Scan We also reconstructed a pulmonary artery from a set of CT scan of heart using the presented method. There were 10 slices in the original CT images. The pixel size was 0.496094 mm and the interval between two consecutive slices was 3mm. The reconstructed result is shown in Figure 10.

4. CONCLUSION

We present a method to reconstruct the pulmonary arteries from a set of CT scan of heart. The proposed method can well reconstruct a tube even the medial axis of the tube is almost parallel to the section plane. However, it is hard to evaluate the accuracy of the reconstructed junction of the pulmonary arteries. Since the meshes of the geometric model is rectangular and the longer side is almost parallel to the medial axis, it is possible to apply the result fluid analysis of the pulmonary arteries.

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