# Two-optical-component method for designing zoom system

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**Abstract.** A new method is presented to solve a zoom system in which the lenses are divided into two combined units, and each combined unit is defined as an optical component. It is shown that most zoom systems can be considered as two-component systems and can be solved using principal plane techniques of optical component, and the first-order zoom system design is thus made easier. The theory with some examples are described.

*Subject terms: zoom lenses; zoom design.*

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## 1 Introduction

Most of the many published papers concerning zoom have concentrated on the first-order zoom design. A zoom system is generally considered to consist of three parts: the focusing, zooming, and fixed parts. The focusing part is placed in front of the zooming part, to adjust the object distance. The zooming part is literally used for zooming, and the fixed rear part serves to control the focal length or magnification and reduce the aberrations of the whole system. Some papers are focused on the paraxial design of the zooming part.[1–5] Yamaji[1] considered first-order designs for basic types of zoom systems, used the inverse Galilean system consisting of a negative front lens and a positive rear lens as the zooming part, and then divided each lens into two or three lenses for different types of zoom systems. Clark[2] provided an overview of the historical development of zoom lenses and discussed the various types of zoom lenses, including mechanically compensated and optically compensated systems. Tanaka[3] reported a paraxial method of mechanically compensated zoom lenses in terms of Gaussian brackets. Using Gaussian brackets, the expressions that define the displacement of components at zooming, the extremum of displacement, and the singular point of displacement are derived. Tao[4] used the varifocal differential equation to express the zoom process and discussed its solvable region. Oskotsky[5] described a grapho-analytical method for the first-order design of two-lens zoom systems and the canonic equation of the lens motion and discussed the region of Gaussian solution.

There are mainly two categories of zoom systems: one is optically compensated and the other is mechanically compensated. Because the various high-precision curves made with a computer numerical control machine are well developed, making a zoom cam is not difficult; therefore, the mechanically compensated zoom system is widely used today. In this paper, we will concentrate on this kind of zoom system.

In general, almost all zoom systems can be solved using only two lenses, except the two-conjugate zoom system[6] in which not only the distance from object to image but also the entrance and exit pupils are fixed during zooming; in this case, at least three lenses are required. The number of adjustable parameters of two-lens systems are not enough to correct image aberrations if the system has a larger zoom ratio and field of view, thus more than two lenses are needed.

In this paper, a two-optical-component method is presented to solve a zoom system where each component can be considered as a combined unit containing a number of lenses. In other words, a zoom system in which more than two lenses are used is still considered as a two-optical-component system if the lenses are divided into two combined units. We then solve each combined unit, find its related principal plane, and adjust the separation between the two combined units to get the solution of the two-component zoom system.

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## 2 Theory

### 2.1 *Sign and Notation*

To describe the formulas, we use the following sign and notation conventions (Fig. 1):

1. $u$ and $u'$ are the marginal ray slope angles in object and image spaces, respectively. The angle is positive if the ray is counterclockwise to the axis.
2. $\ell$ and $\ell'$ are the distances from the first principal plane to the object plane and the second principal plane to the image plane, respectively. For a thin lens, both the principal planes coincide with the lens. The distance to the right of a lens is positive; to the left, negative.
3. $h$ is the height of marginal ray at the lens. The height above the axis is positive; that below the axis, negative.
4. $n$ and $n'$ are the indices of object and image spaces, respectively.
5. $K$ and $F$ are the power and focal length of a lens, respectively ($F=1/K$).
6. $H$ and $H'$ are the first and second principal points of the combined unit (Fig. 4 in Sec. 2.3).

The lens equation (if $n=n'=1$) for a thin lens is

$$1/\ell' - 1/\ell = K \ ,$$

$$\text{or } u' - u = hK \ . \quad (1)$$

The transverse magnification $M$ is defined as

$$M = \ell'/\ell = u/u' \ , \quad (2)$$

where $\ell = (1/M - 1)F$ and $\ell' = (1-M)F$. The distance from object to image is

$$T = (2 - M - 1/M)F \ . \quad (3)$$

### 2.2 *Two-Lens System*

The two-lens system is the simplest case for a two-component system, in which each component contains only one lens and the first and second principal planes of each component coincide, and has been well described.[2,5,7] These two lenses move along a linear and a nonlinear locus, respectively. The two-optical-component method we describe here is simple to use in this case.

In Fig. 2, for an infinite-conjugate system, the total power of the two-lens system is

$$K = K_1 + K_2 - d_1 K_1 K_2 \ , \quad (4)$$

where $d_1$ is the separation between principal planes of lenses 1 and 2. In zooming, $d_1$ and $K$ are changed and we have

$$\ell_2' = (1 - K_1 d_1)/K \ , \quad (5)$$

where $\ell_2'$ is the distance from lens 2 to the image plane. Using Eq. (4), $d_1$ is calculated after zooming and substituted into Eq. (5), which gives $\ell_2'$.

If the system is finite-conjugate, as shown in Fig. 3, the distance from object to image remains constant during zooming and the related equations are obtained as

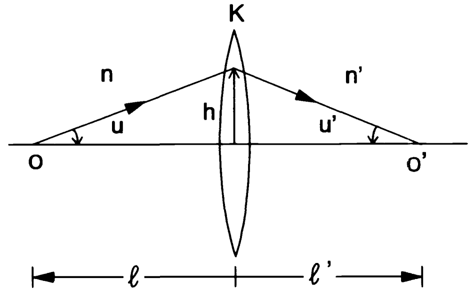


**Fig. 1** Signs and notations.

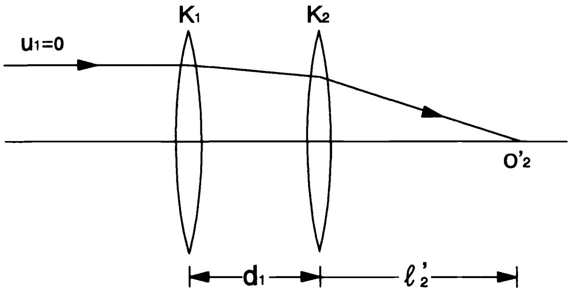


**Fig. 2** Two-lens infinite-conjugate system.

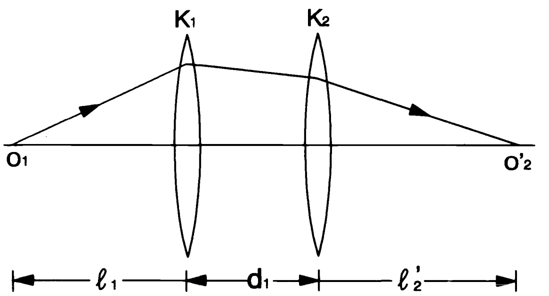


**Fig. 3** Two-lens finite-conjugate system.

$$T_{12} = (2 - M_1 - 1/M_1)F_1 + (2 - M_2 - 1/M_2)F_2 \ , \quad (6)$$

$$\ell_1 = (1/M_1 - 1)F_1 \ , \quad (7)$$

$$\ell_1' = (1 - M_1)F_1 \ , \quad (8)$$

$$\ell_2 = (1/M_2 - 1)F_2 \ , \quad (9)$$

$$\ell_2' = (1 - M_2)F_2 \ , \quad (10)$$

$$d_1 = \ell_1' - \ell_2 \ , \quad (11)$$

where $T_{12}$ is the object/image distance and $M_1$ and $M_2$ are the magnifications of lenses 1 and 2, respectively.

In zooming, adjust $\ell_1$ and then $M_1$ and $M_2$ are changed from Eqs. (6) and (7). By substituting the two $M$ values into Eqs. (8), (9), and (10), $d_1$ and $\ell_2'$ are solved.

### 2.3 *Three-Lens System*

The three-lens system is a two-component system in which one component contains one lens and the other contains two lenses. Three cases are discussed here.

#### 2.3.1 *First lens fixed*

In Fig. 4, for an infinite-conjugate system, the first lens is fixed and regarded as the first component, and the remaining two lenses are combined as the second component. The combined unit has magnification $M_{23}$, object/image distance $T_{23}$, focal length $F_{23}$ (power $K_{23}$), separation $\triangle$ between the two principal planes, object (image) distance $\ell_H(\ell'_H)$, and distance $\delta$ ($\delta'$) from the first (second) lens of the combined unit to the first (second) principal plane. We then have the following:

$$T_{23}=(2-M_2-1/M_2)F_2+(2-M_3-1/M_3)F_3$$
$$=(2-M_{23}-1/M_{23})F_{23}+\triangle \ , \qquad (12)$$

$$\delta=K_3d_2/K_{23} \ , \qquad (13)$$

$$\delta'=-K_2d_2/K_{23} \ , \qquad (14)$$

$$\triangle=d_2+\delta'-\delta=-K_2K_3d_2^2/K_{23} \ , \qquad (15)$$

$$D_1=d_1+\delta=\ell'_1-\ell_H \ , \qquad (16)$$

$$\ell_H=(1/M_{23}-1)F_{23}=\ell'_1-D_1 \ , \qquad (17)$$

$$\ell'_H=(1-M_{23})F_{23}=\ell'_3-\delta' \ , \qquad (18)$$

$$K=K_1+K_{23}-D_1K_1K_{23} \ , \qquad (19)$$

$$F=F_1M_{23} \ , \qquad (20)$$

where $T_{23}$ is a constant given by the initial condition. Substituting Eq. (15) into Eq. (12), we have

$$T_{23}=(2-M_{23}-1/M_{23})F_{23}-K_2K_3d_2^2/K_{23} \ , \qquad (21)$$

$$K_{23}=K_2+K_3-d_2K_2K_3 \ , \qquad (22)$$

where $M_{23}$ is changed during zooming and $d_2$ is obtained as

$$d_2=[-b\pm(b^2-4ac)^{1/2}]/2a \ , \qquad (23)$$

where

$$a=K_2K_3 \ ,$$

$$b=-T_{23}K_2K_3 \ ,$$

$$c=T_{23}(K_2+K_3)-(2-M_{23}-1/M_{23}) \ .$$

According to Eqs. (16) to (18), $D_1$, $d_1$, $\ell'_3$, and the other parameters of the zoom system are obtained. Figure 5 illustrates an example.

If the system is finite-conjugate, Eq. (20) should be replaced by the system magnification $M=M_1M_{23}$. Because the first lens is fixed during zooming, $M_1$ remains unchanged.

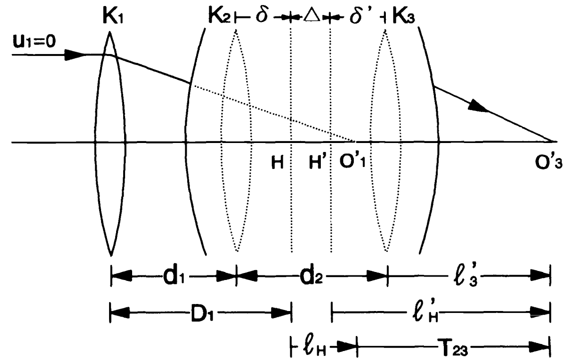


**Fig. 4** Three-lens infinite-conjugate system with the first lens fixed. The two solid curved lines represent the combined unit that contains two lenses.

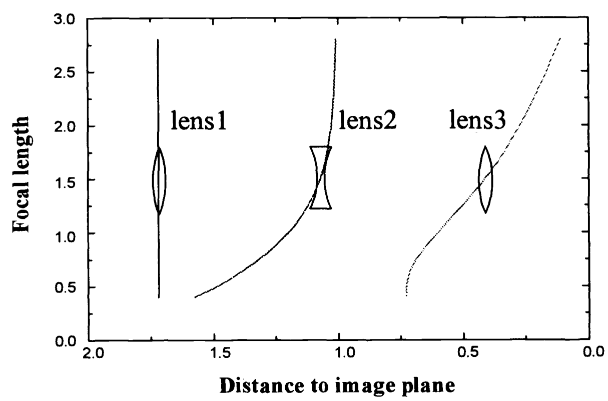


**Fig. 5** Loci of three-lens infinite-conjugate system with the first lens fixed for $F_1=1.0$, $F_2=-0.4$, $F_3=0.5$, and zoom ratio=7. The separation between the first lens and image is 1.719. The object/image distance $T_{23}$ is 0.719.

The value of $M$ can be calculated for various $M_{23}$, and $d_2$, $K_{23}$, and other parameters are also obtained from Eqs. (22) and (23) and the other related equations.

#### 2.3.2 *Middle lens fixed*

In this case, lens 2 is fixed, and for the two components, either lenses 1 and 2 are combined as the first component and lens 3 is the second component, or lens 1 is the first component and lenses 2 and 3 are combined as the second component.

*Combined unit consisting of the lenses 1 and 2.* In Fig. 6, for an infinite-conjugate system, lenses 1 and 2 are combined as a unit. The separation between lens 2 and image keeps constant. Similar to Sec. 2.3.1, we have

$$K=K_{12}+K_3-D_2K_{12}K_3 \ , \qquad (24)$$

$$D_2=d_2-\delta'=d_2+K_1d_1/K_{12} \ , \qquad (25)$$

$$K_{12}=K_1+K_2-d_1K_1K_2 \ , \qquad (26)$$

$$F_{12}=F_1M_2 \ , \qquad (27)$$

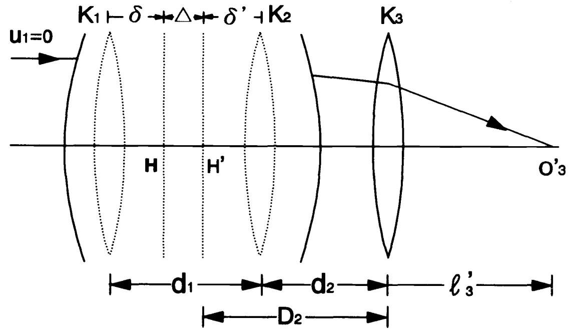


**Fig. 6** Three-lens infinite-conjugate system with the middle lens fixed. The first and second lenses are combined as a unit.

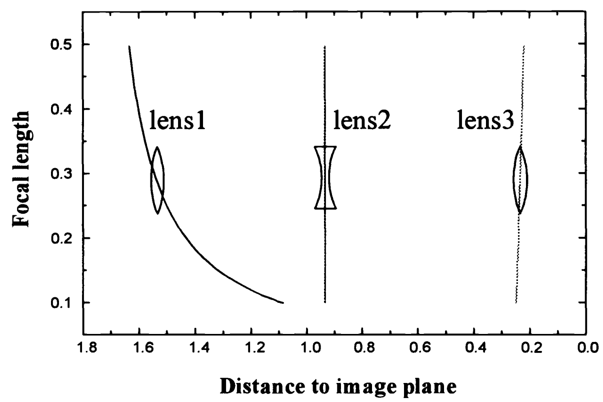


**Fig. 7** Loci of three-lens infinite-conjugate system with the middle lens fixed for $F_1=1.0$, $F_2=-0.25$, $F_3=0.2$, and zoom ratio=5. The separation between the middle lens and image is 0.934.

$$F=F_{12}M_3=F_1M_2M_3 \ , \tag{28}$$

$$\begin{aligned} d_2+\ell_3' &= d_2+(1-M_3)F_3 \\ &= \ell_2'-\ell_3+(1-M_3)\mathbf{F}_3 \\ &= (1-M_2)F_2+(2-M_3-1/M_3)F_3 \\ &= \text{constant} \ . \end{aligned} \tag{29}$$

In zooming, $d_1$ is changed, and $K_{12}$ and $M_2$ can be calculated by Eqs. (26) and (27). Substituting $M_2$ into Eq. (29), we get $M_3$ and $d_2$. From Eq.(25), $D_2$ is calculated. Finally, the power $K$ of the system is found from Eq. (24) or Eq. (28). Figure 7 shows an example.

If the system is finite-conjugate, the distance from object to image, the separation between object and lens 2, and the separation between lens 2 and image must remain constant during zooming. Related equations are derived as follows:

$$\begin{aligned} T &= T_{12}+T_3 \\ &= (2-M_{12}-1/M_{12})F_{12}-K_1K_2d_1^2/K_{12} \\ &\quad +(2-M_3-1/M_3)F_3 \ , \end{aligned} \tag{30}$$

where $T_{12}$ is distance between object and image of the combined unit. In this case, we have

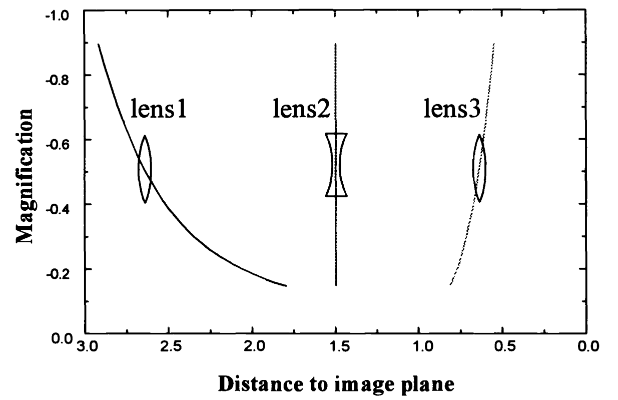


**Fig. 8** Loci of three-lens finite-conjugate system with the middle lens fixed for $F_1=1.0$, $F_2=-0.4$, $F_3=0.5$, and zoom ratio=6. The distance from object to image is 5.083. The separation between object and the middle lens is 3.586. The separation between the middle lens and image is 1.497.

$$K_{12}=K_1+K_2-d_1K_1K_2 \ , \tag{31}$$

$$\begin{aligned} -\ell_1+d_1 &= -(1/M_1-1)F_1+d_1 \\ &= (2-M_1-1/M_1)F_1-(1/M_2-1)F_2 \\ &= \text{constant} \ , \end{aligned} \tag{32}$$

$$\begin{aligned} d_2+\ell_3' &= d_2+(1-M_3)F_3 \\ &= \text{constant} \ . \end{aligned} \tag{33}$$

In zooming, $d_1$ is adjusted, $K_{12}$ is calculated from Eq. (31): $\ell_1$, $M_1$, and $M_2$ are given by Eq. (32); $M_3$ and $d_2$ are found from Eqs. (30) and (33); and the magnification $M$ of the system is then obtained. Figure 8 shows an example.

*Combined unit consisting of lenses 2 and 3.* In Fig. 9, for an infinite-conjugate system, lenses 2 and 3 are combined as a unit. Similar to section 2.3.1, we have

$$K=K_1+K_{23}-D_1K_1K_{23} \ , \tag{34}$$

$$\begin{aligned} D_1 &= d_1+\delta \\ &= F_1-\ell_2+\delta \\ &= F_1-\ell_2+K_3d_2/K_{23} \ , \end{aligned} \tag{35}$$

$$K_{23}=K_2+K_3-d_2K_2K_3 \ , \tag{36}$$

$$F=F_1M_{23} \ , \tag{37}$$

$$\begin{aligned} d_2+\ell_3' &= d_2+(1-M_3)F_3 \\ &= (1-M_2)F_2+(2-M_3-1/M_3)F_3 \\ &= \text{constant} \ . \end{aligned} \tag{38}$$

When $d_2$ is changed, $K_{23}$ is given by Eq. (36). Substituting $d_2$ into Eq. (38), we get $M_3$ and $M_2$. From Eq. (35), $D_1$ is found. Therefore, the power $K$ of the system can be obtained by using Eqs. (34) and (37). Figure 10 shows an example.

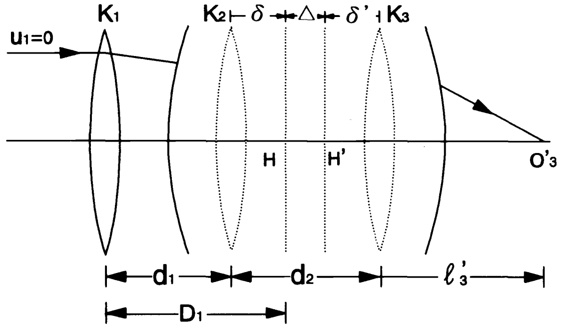


**Fig. 9** Three-lens infinite-conjugate system with the middle lens fixed. The second and third lenses are combined as a unit.

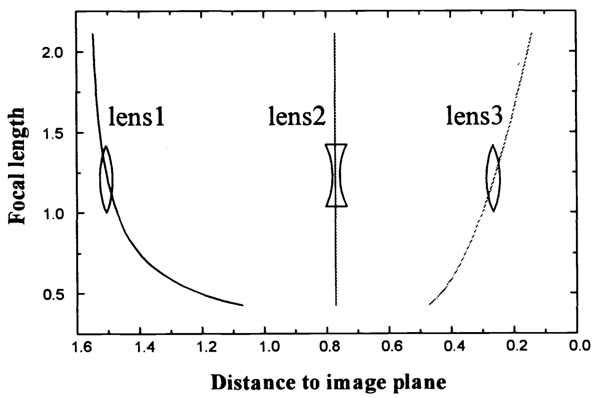


**Fig. 10** Loci of three-lens infinite-conjugate system with the middle lens fixed for $F_1=1.0$, $F_2=-0.3$, $F_3=0.3$, and zoom ratio=5. The separation between the middle lens and image is 0.771.

If the system is finite-conjugate, the related equations are similar to those for the case of a combined unit consisting of lenses 1 and 2, and we have

$$T=T_1+T_{23}$$
$$=(2-M_1-1/M_1)F_1$$
$$+(2-M_{23}-1/M_{23})F_{23}-K_2K_3d_2^2/K_{23} , \quad (39)$$

where $T_{23}$ is the object/image distance of the combined unit.

$$K_{23}=K_2+K_3-d_2K_2K_3 , \quad (40)$$

$$-\ell_1+d_1=(2-M_1-1/M_1)F_1-(1/M_2-1)F_2$$
$$=\text{constant} , \quad (41)$$

$$d_2+\ell_3'=d_2+(1-M_3)F_3$$
$$=(1-M_2)F_2+(2-M_3-1/M_3)F_3$$
$$=\text{constant} . \quad (42)$$

When $d_2$ is varied, $K_{23}$ is calculated from Eq. (40) and $M_2$ and $M_3$ are given from Eq. (42). Using $K_{23}$ in Eq. (39), we get $M_1$. From Eq. (41), $d_1$ is found. Finally, the magnification $M$ of the system is then obtained. Figure 11 shows an example.

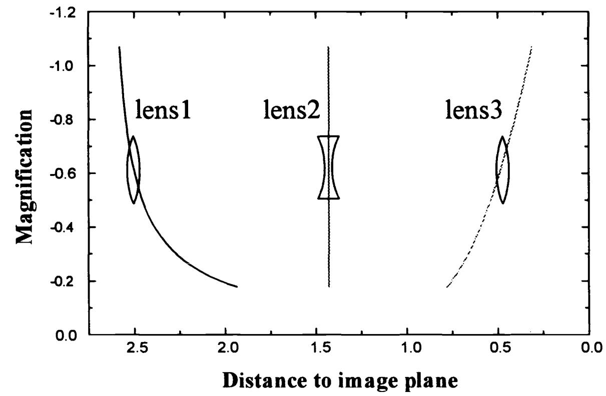


**Fig. 11** Loci of three-lens finite-conjugate system with the middle lens fixed for $F_1=1.0$, $F_2=-0.4$, $F_3=0.5$, and zoom ratio=6. The distance from object to image is 5.662. The separation between object and the middle lens is 4.233. The separation between the middle lens and image is 1.429.

### 2.3.3 *Third lens fixed*

In Fig. 12, for an infinite-conjugate system, the last lens is fixed and regarded as the second component, and the other two lenses are combined as the first component. Similar to Sec. 2.3.1, we have

$$K=K_{12}+K_3-D_2K_{12}K_3 , \quad (43)$$

$$D_2=d_2-\delta'$$
$$=d_2+K_1d_1/K_{12} , \quad (44)$$

$$K_{12}=K_1+K_2-d_1K_1K_2 , \quad (45)$$

$$F=F_{12}M_3 . \quad (46)$$

Because the third lens is fixed during zooming, $M_3$ is constant. Then $F_{12}$ is obtained from Eq. (46). Using Eqs. (43) to (45), we find $D_2$, $d_1$, and $d_2$. Figure 13 shows an example.

If the system is finite-conjugate, as shown in Fig. 14, the distance between object and image of the combined unit must remain constant during zooming. Related equations can be obtained as follows:

$$T_{12}=(2-M_1-1/M_1)F_1+(2-M_2-1/M_2)F_2$$
$$=(2-M_{12}-1/M_{12})F_{12}+\triangle , \quad (47)$$

$$\delta=K_2d_1/K_{12} , \quad (48)$$

$$\delta'=-K_1d_1/K_{12} , \quad (49)$$

$$\triangle=d_1+\delta'-\delta=-K_1K_2d_1^2/K_{12} , \quad (50)$$

$$M=M_{12}M_3 , \quad (51)$$

$$L_1=\ell_1+\delta=(1/M_{12}-1)F_{12} , \quad (52)$$

$$D_2=d_2-\delta' . \quad (53)$$

Here $T_{12}$ is a constant given by the initial condition. Rearranging Eq. (47), we have

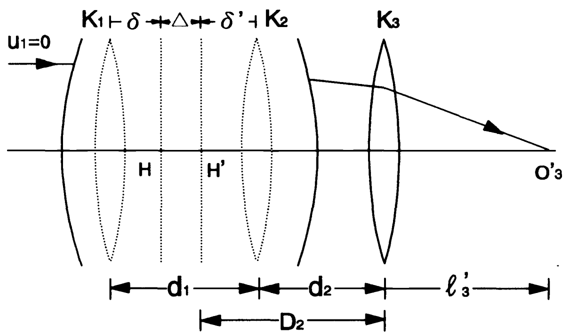


**Fig. 12** Three-lens infinite-conjugate system with the third lens fixed.

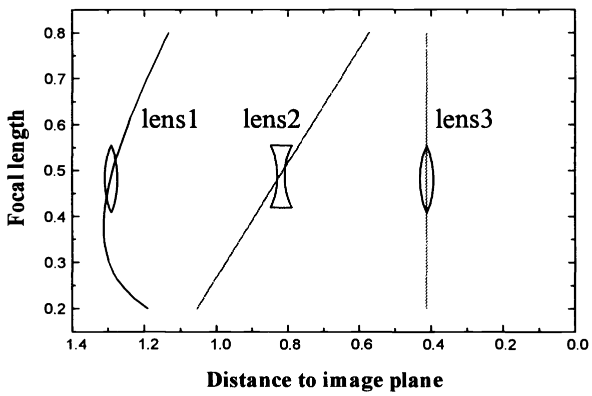


**Fig. 13** Loci of three-lens infinite-conjugate system with the third lens fixed for $F_1=1.0$, $F_2=-0.3$, $F_3=0.3$, and zoom ratio=4. The separation between the third lens and image is 0.413.

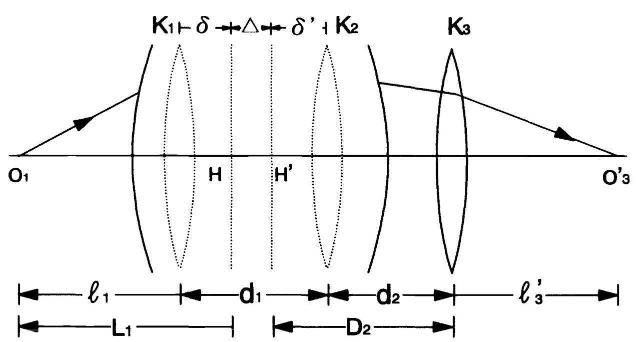


**Fig. 14** Three-lens finite-conjugate system with the third lens fixed.

$$T_{12}=(2-M_{12}-1/M_{12})F_{12}-K_1K_2d_1^2/K_{12} \ , \tag{54}$$

$$K_{12}=K_1+K_2-d_1K_1K_2 \ , \tag{55}$$

where $M_{12}$ is changed during zooming. Using the preceding equations, we have

$$d_1=[-b\pm(b^2-4ac)^{1/2}]/2a \ , \tag{56}$$

where

$$a=K_1K_2 \ ,$$

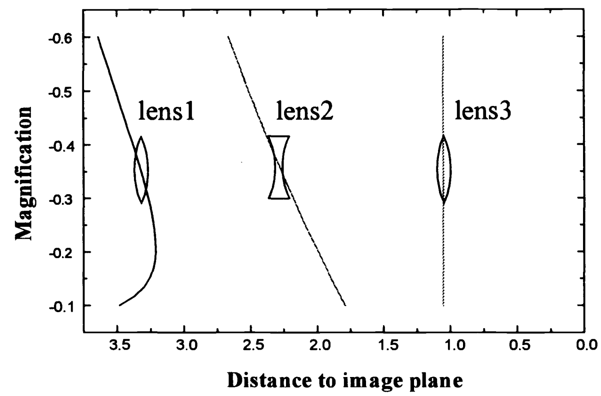


**Fig. 15** Loci of three-lens finite-conjugate system with the third lens fixed for $F_1=1.0$, $F_2=-0.4$, $F_3=0.5$, and zoom ratio=6. The distance from object to image is 9.052. The separation between the third lens and image is 1.052. The object/image distance $T_{12}$ is 7.047.

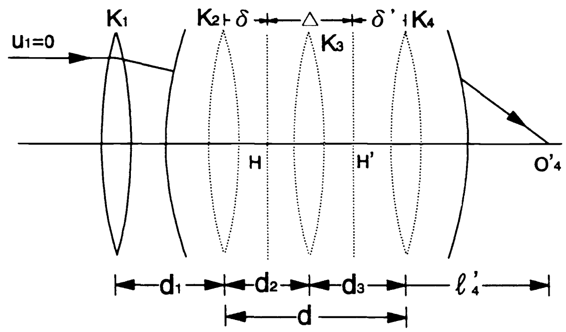


**Fig. 16** Four-lens system with the first lens fixed.

$$b=-T_{12}K_1K_2 \ ,$$

$$c=T_{12}(K_1+K_2)-(2-M_{12}-1/M_{12}) \ .$$

The parameters of $\ell_1$, $d_2$, and $\ell_3'$ can be found by using related equations. Figure 15 shows an example.

## 2.4 *Four-Lens System*

The four-lens system is a two-component system in which one component contains one lens and the other contains three lenses.

In Fig. 16, the first lens is assumed to be fixed and considered as the first component, and lenses 2 to 4 are combined as the second component. The combined unit has the magnification $M_{234}$, object/image distance $T_{234}$, focal length $F_{234}$ (power $K_{234}$), and the separation $\triangle$ between the two principal planes as follows:

$$K_{234}=K_2+K_3+K_4-dK_2K_4-(d_2K_2+d_3K_4)K_3 + d_2d_3K_2K_3K_4 \ , \tag{57}$$

$$T_{234}=(2-1/M_{234}-M_{234})F_{234}+\triangle \ , \tag{58}$$

$$\triangle=[-d^2K_2K_4-(d_2^2K_2+d_3^2K_4)K_3 + dd_2d_3K_2K_3K_4]/K_{234} \ , \tag{59}$$

$$F = F_1 M_{234} \ , \tag{60}$$

$$d = d_2 + d_3 \ , \tag{61}$$

$$K = K_1 + K_{234} - D_1 K_1 K_{234} \ , \tag{62}$$

where $M_{234}$ changes when lenses 2, 3, and 4 are moved. Because many solutions can be obtained using these equations, some constraints are needed in a practical design. Two special cases are discussed here.

### 2.4.1 *Four-lens system with the first and fourth lenses fixed*

If one lens of the combined unit is fixed during zooming, for example, lens 4 is fixed, the system can be simplified into a negatively compensated or an afocal zoom system and the combined unit is composed of lenses 2 and 3. The afocal zoom system is discussed here.

In Fig. 17, the marginal ray that exits from lens 3 is parallel to the optical axis and the second focal point of lens 1 coincides with the first focal point of the combined unit. For an infinite-conjugate system, the related equations are then given by

$$D_1 = F_1 + F_{23}$$

$$= d_1 + \delta \ , \tag{63}$$

$$\delta = K_3 d_2 / K_{23} \ , \tag{64}$$

$$K_{23} = K_2 + K_3 - d_2 K_2 K_3 \ , \tag{65}$$

$$M_{123} = -F_1 / F_{23} = h_1 / h_4 \ . \tag{66}$$

Solving Eqs. (63) to (66), we have

$$d_1 = F_1 + F_2 + F_1 F_2 / (F_3 M_{123}) \ , \tag{67}$$

$$d_2 = F_2 + F_3 + F_2 F_3 M_{123} / F_1 \ . \tag{68}$$

We choose $M_{123}$ and then obtain $d_1$ and $d_2$. Because $u_3' = 0$, $\ell_4'$ is equal to $F_4$. If the total length $L$ of the system is given, we will have

$$d_3 = L - d_1 - d_2 - \ell_4' \ . \tag{69}$$

The focal length of the system is then found by the following equation:

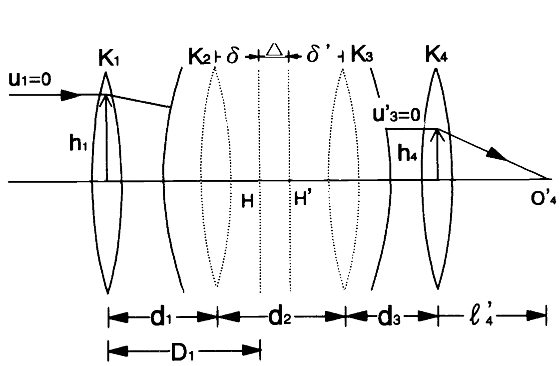


**Fig. 17** Four-lens afocal system with the first and fourth lenses fixed.

$$F = h_1 / u_4' = (h_1/h_4)(h_4/u_4') = M_{123} F_4 \ .$$

Figure 18 shows an example.

If the system if finite-conjugate and $u_3' = 0$, Eq. (63) is replaced by the following equation:

$$D_1 = \ell_1' + F_{23} = \ell_1 F_1 / (\ell_1 + F_1) + F_{23}$$

$$= d_1 + \delta$$

$$= d_1 + K_3 d_2 / K_{23} \ . \tag{70}$$

In zooming, $d_1$ is changed and $d_2$ is given by Eq. (70). If the total length $L$ of the system is known, $d_3$ is found from Eq. (69). The magnification of the system is then calculated with paraxial ray tracing and equal to $u_1/u_4'$.

### 2.4.2 *Four-lens system with the first lens fixed and the second and fourth lenses regarded as one moving group*

In Fig. 19, the second and fourth lenses are linked and regarded as one moving group during zooming. In other words, the separation between lenses 2 and 4 always remains constant. For an infinite-conjugate system, the related equations are

$$d = d_2 + d_3 = \text{constant} \ , \tag{71}$$

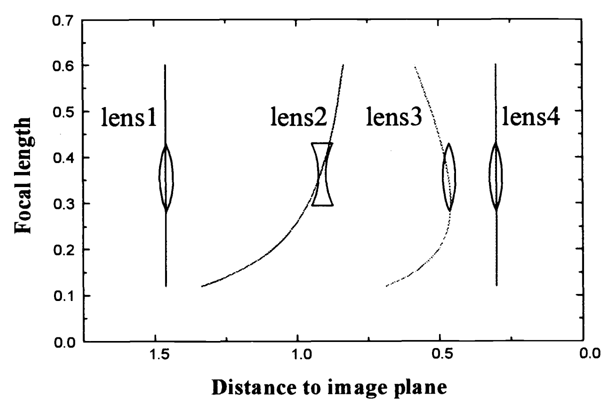


**Fig. 18** Loci of four-lens afocal system with the first and fourth lenses fixed for $F_1 = 1.0$, $F_2 = -0.25$, $F_3 = 1.0$, $F_4 = 0.3$, and zoom ratio = 5. The separation between the first lens and image is 1.460.

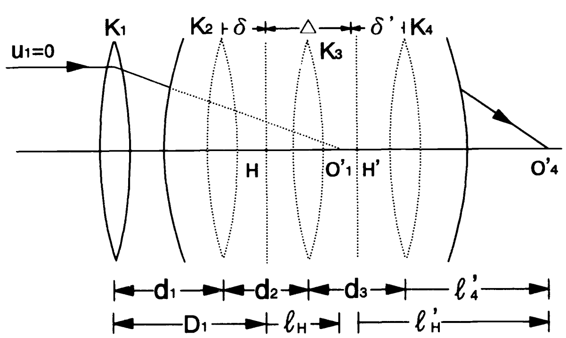


**Fig. 19** Four-lens infinite-conjugate system with the first lens fixed. The second and fourth lenses are linked and regarded as one moving group.

$$\ell_H = \ell_1' - D_1 = F_1 - (d_1 + \delta) = (1/M_{234} - 1)F_{234} \ , \qquad (72)$$

$$\ell_H' = \ell_4' - \delta' \ , \qquad (73)$$

$$\delta = \{d_3K_4 + d_2(K_3 + K_4 - D_3K_3K_4)]/K_{234} \ , \qquad (74)$$

$$\delta' = [-d_2K_2 - d_3(K_2 + K_3 - d_2K_2K_3)]/K_{234}. \qquad (75)$$

Here $T_{234}$ is a constant given by the initial condition. Rearranging Eqs. (57) to (61), $d_2$ can be expressed as follows:

$$d_2 = [-b \pm (b^2 - 4ac)^{1/2}]/2a \ , \qquad (76)$$

where

$$a = K_3K_4 + K_2K_3 - K_2K_3K_4(T_{234} - d) \ ,$$

$$b = -2dK_3K_4 + K_2K_3K_4d(T_{234} - d) - T_{234}(K_2K_3 - K_3K_4) \ ,$$

$$c = d^2(K_2K_4 + K_3K_4) + T_{234}(K_2 + K_3 + K_4 - K_2K_4 - K_3K_4) - (2 - 1/M_{234} - M_{234}) \ .$$

In zooming, $F$ is changed and $M_{234}$ is given by Eq. (60). Using $M_{234}$ in Eq. (76), $d_2$ is obtained. According to Eqs. (71) through (75), $d_1$, $d_3$, $\ell_4'$, and the other parameters of this zoom system can be found. Figure 20 illustrates an example.

If the system if finite-conjugate, Eq. (60) is replaced by the system magnification $M = M_1 M_{234}$. Because the first lens is fixed during zooming, $M_1$ remains unchanged. Therefore, $M, d_1, d_2, d_3, \ell_4'$, and other parameters are obtained as in the infinite case.

## 3 Discussion

From the preceding examples, we find that for a zoom system, there exist many solutions that depend on the initial parameters, such as the focal length of each lens. The first changeable parameter during zooming can be one of the separations between lenses, the focal length, or the magnification of the system, and the other parameters are then solved from related equations. If the loci of the lenses are not smooth or the zoom ratio of the system is too small, the related input parameters should be readjusted. In fact, the selection of the input parameters can be controlled by using an optimization method to find the loci.[8] This technique combined with the two-component method can thus solve all of the zoom systems. It is of interest to compare the two three-lens systems in which the middle lens is fixed in Sec. 2.3.2. We find that the loci of moving lenses are similar in these two cases, and the results will be identical if the given parameters are the same. Compared with other methods, the two-optical-component method presented here is simpler and easier to understand, especially for solving zoom systems that contain more than two lens cases.

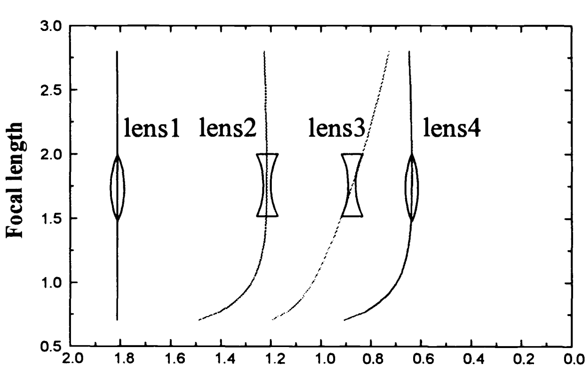


**Fig. 20** Loci of four-lens infinite-conjugate system with the first lens fixed for $F_1 = 1.0$, $F_2 = -0.7$, $F_3 = -0.3$, $F_4 = 0.36$, and zoom ratio$=4$. The second and fourth lenses are linked and regarded as one moving group. The distance from the first lens to image is 1.813. The separation between the second lens and the fourth lens is 0.580.

## 4 Conclusion

A proper initial layout will often give a satisfactory final optical design. We have shown that the two-optical-component method is a simple way to solve the zoom lens layout and several zoom lens systems, such as a stereoscopic microscope with zoom ratio 7:1 and fringe viewing system with zoom ratio 6.5:1 for Fizeau interferometer, have been designed by this method.

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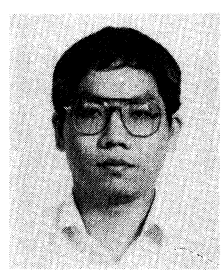

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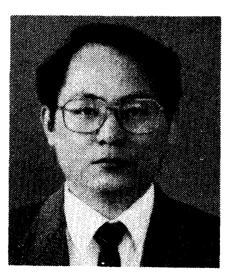

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