Joint MMSE Transceiver Design in Amplify-and-Forward MIMO Relay Systems with Tomlinson-Harashima Source Precoding

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***Abstract*—Existing precoding schemes in amplify-and-forward (AF) multiple-input-multiple-output (MIMO) relay systems use linear precoders. In this paper, we consider a precoding scheme in which a Tomlinson-Harashima (TH) precoder (THP) is used at the source and a linear precoder at the relay. With a minimum-mean-squared-error (MMSE) receiver at destination, we propose a new joint precoders design method. Since two precoders are involved, the transceiver design, formulated as an optimization problem, is difficult to solve. To overcome the problem, we propose cascading an additional unitary precoder with the TH precoder. The unitary precoder can not only simplify the optimization problem but also improve the MMSE performance. With the specially designed unitary precoder at the source, we can then adopt the primal decomposition method to solve this problem. With the method, the original optimization problem can first be decomposed into a master and a subproblem optimization problems, and then transferred to a relay precoder optimization problem. However, the optimization is not a convex problem and the solution is not obtainable. We then propose a method being able to transfer it to a convex optimization problem. A closed-form solution can then be obtained by the Karuch-Kuhn-Tucker (KKT) conditions. Simulations show that the proposed transceiver can significantly outperform existing linear transceivers.**

***Index Terms* – Amplify-and-forward (AF), multiple-input multiple-output (MIMO), cooperative communication, precoder, Tomlinson-Harashima precoding (THP), minimum-mean-squared-error (MMSE), primal decomposition approach, Karush-Kuhn-Tucker (KKT).**

I. Introduction

Current research on amplify-and-forward (AF) multiple-input multiple-output (MIMO) cooperative networks, namely MIMO relay system, mainly focuses on linear transceiver designs, either for boosting capacity [1]-[2], or for improving link reliability [3]-[5]. Most of these proposals, however, only consider relay precoders [1]-[4]. Some of them even neglect the direct (or, source-to-destination) link so as to simplify the design [1], [3], [4]. Recently, a joint source/relay precoders design method was proposed in [5], [6] and it is shown that the system performance can be significantly improved. However, the precoders considered in previous works are all linear. In this paper, we consider a nonlinear precoding scheme in which a Tomlinson-Harashima (TH) precoder (THP) is used at the source and a linear precoder at the relay, and a minimum-mean-squared-error (MMSE) receiver is applied at the destination. THP is a well known precoding scheme and has been shown to have a better performance than the linear ones in point-to-point MIMO systems [7], [8].

Since the MMSE is a complicated function of the source and relay precoders, the design is difficult. In addition, the problem is non-convex. To facilitate the optimization, we propose cascading an unitary precoder after the TH precoded signal. In this manner, as we will see, it can greatly simplify the optimization via the primal decomposition [9]. The primal decomposition decomposes the original optimization into a master and a subproblem optimizations. The source precoder is first derived in the subproblem and it can be expressed as a function of the relay precoder which is optimized in the master problem. By the additional unitary precoder $\mathbf{F}_S$, we can not only minimize the MSE but also facilitate the optimizations. With the proposed approach, the two-precoder nonconvex design problem can be finally transferred into a convex relay-precoder-only optimization problem. As a result, the closed-form solution of the relay precoder can be obtained by the Karush-Kuhn-Tucker (KKT) conditions. Simulations show that the proposed method can significantly outperform existing non-precoded and precoded systems, either in terms of MSE or bit-error-rate (BER).

This paper is organized as follows. The proposed transceiver structure and the precoders optimization problem are introduced in Section II. In Section III, a new method to solve the optimization is proposed. The performance is then evaluated in Section VI. Finally, conclusions are drawn in Section V.

II. System Model

A. System Model

We consider a three-node AF MIMO relay precoding system in which N, R, and M antennas are placed at the source, the relay and the destination, respectively, as shown in Fig. 1.

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As we can see, two precoders are included – a TH source precoder and a linear relay precoder $\mathbf{F}_R$. Also, a linear MMSE receiver, **G,** is applied at the destination. Here, we consider the general two-phase transmission protocol [1]-[6]. In the first phase, the source signal $\mathbf{s} \in \mathbb{C}^{N\times 1}$ is first fed into the nonlinear THP in which a successive cancellation operation characterized by a backward squared matrix **B** and a modulo operation MOD$_m$(•). The source signals $\mathbf{s} = [s_1, \cdots, s_N]^T$ are modulated by *m*-QAM where the real and image parts of s_k are drawn from the set $\{\pm 1, \cdots, \pm(\sqrt{m}-1)\}$. The feedback matrix **B** has a lower triangular structure and the diagonal elements are all zeros. The modulo operation acts over the real and imaginary parts of the inputs, respectively, expressed as follows:

$$\mathrm{MOD}_m(x) = x - 2\sqrt{m} \cdot \left\lfloor \frac{x+\sqrt{m}}{2\sqrt{m}} \right\rfloor. \quad (1)$$

It is clear that the transmitted signal **x** is bounded between $-\sqrt{m}$ and $\sqrt{m}$. With **B** and the operation in (1), the elements of ***x*** can be recursively expressed as [7], [8]

$$\mathbf{x}_k = \mathbf{s}_k - \sum_{l=1}^{k-1} \mathbf{B}(k,l)\mathbf{x}_l + \mathbf{e}_k, \quad (2)$$

where $\mathbf{x}_k$ is the *k*th elements of vector **x** and $\mathbf{B}(k,l)$ is the (*k*,*l*) element of matrix **B**; $\mathbf{e} = [\mathbf{e}_1, \cdots, \mathbf{e}_N]^T$ denotes the errors of the modulo operation (the difference of the input and the output). From (2), we can reformulate the transmitted signal **x** after THP with the following matrix form

$$\mathbf{x} = \mathbf{C}^{-1}\mathbf{v}, \quad (3)$$

where $\mathbf{C} = \mathbf{B} + \mathbf{I}_N$ is a lower triangular with ones in its diagonal, and $\mathbf{v} = \mathbf{s} + \mathbf{e}$. The THP precoded **x** is then passed through a unitary matrix $\mathbf{F}_S$ and subsequently sent to the relay and the destination simultaneously.

In the second phase, the received signal at the relay is multiplied with the relay precoder and then is transmitted to the destination. Therefore, the signal received at the destination after the two consecutive phases can be expressed as a vector form as

$$\mathbf{y}_D := \underbrace{\begin{bmatrix} \mathbf{H}_{SD} \\ \mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR} \end{bmatrix}}_{:=\mathbf{H}} \mathbf{F}_S\mathbf{x} + \underbrace{\begin{bmatrix} \mathbf{n}_{D,1} \\ \mathbf{H}_{RD}\mathbf{F}_R\mathbf{n}_R + \mathbf{n}_{D,2} \end{bmatrix}}_{:=\mathbf{w}}, \quad (4)$$

where **H** and **w** denote the equivalent channel matrix and the equivalent noise vector, respectively. In (4), $\mathbf{x} \in \mathbb{C}^{N\times 1}$ is the TH precoded signal vector (3); $\mathbf{y}_D \in \mathbb{C}^{2M\times 1}$ is the received signal vector at the destination; $\mathbf{H}_{SR} \in \mathbb{C}^{R\times N}$, $\mathbf{H}_{SD} \in \mathbb{C}^{M\times N}$ and $\mathbf{H}_{RD} \in \mathbb{C}^{M\times R}$ are the channel matrices of the source-to-relay, the source-to-destination, and the relay-to-destination links, respectively; $\mathbf{n}_{D,1} \in \mathbb{C}^{M\times 1}$, $\mathbf{n}_R \in \mathbb{C}^{R\times 1}$, and $\mathbf{n}_{D,2} \in \mathbb{C}^{M\times 1}$ are the received noise vectors at the destination and at the relay in the first-phase, and at the destination in the second-phase. Here, we assume that $N \le M$ to provide sufficient degree of freedom for signal transmission.

Note that if **v** can be estimated at the destination, **s** can then be recovered by the modulo operation in (1). Thus, the optimum $\mathbf{G} \in \mathbb{C}^{2M\times N}$ can be found by minimizing the MSE defined as

$$J = E\left\{\left\|\mathbf{G}\mathbf{y}_D - \mathbf{v}\right\|^2\right\}. \quad (5)$$

To solve the problem in (5), we assume that the precoded signal $\mathbf{x}_k$'s are statistically independent and they have the zero-mean and the same variance. Let the variance of each element in **s** be denoted as σ_s^2. We then have $E\left[\mathbf{x}\mathbf{x}^H\right] = \sigma_s^2\mathbf{I}_N$ and $E\left[\mathbf{v}\mathbf{v}^H\right] = \sigma_s^2\mathbf{C}\mathbf{C}^H$. It is noted that the assumption is valid when the QAM size is large ($m \ge 16$) [8]. Then, the optimum solution of (5) can be obtained as [10]

$$\mathbf{G}_{opt} = \sigma_s^2\mathbf{C}\mathbf{F}_S^H\mathbf{H}^H\left(\sigma_s^2\mathbf{H}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}^H + \mathbf{R}_w\right)^{-1}, \quad (6)$$

where $\mathbf{G}_{opt}$ is the optimum **G**, $\mathbf{R}_w = E\left[\mathbf{w}\mathbf{w}^H\right]$ is the covariance matrix of the equivalent noise vector **w**. Note here that **w** is not white. Denote the variance of the noise components at the destination as $\sigma_{n,d}^2$, and that at the relay as $\sigma_{n,r}^2$. Substituting (6) in (5), we can have the MSE matrix

$$\begin{aligned} \mathbf{E} &= \mathbf{C}\left(\sigma_s^{-2}\mathbf{I}_N + \mathbf{F}_S^H\mathbf{H}^H\mathbf{R}_w^{-1}\mathbf{H}\mathbf{F}_S\right)^{-1}\mathbf{C}^H \\ &= \mathbf{C}\left(\sigma_s^{-2}\mathbf{I}_N + \mathbf{F}_S^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{F}_S\right)^{-1}\mathbf{C}^H \end{aligned} \quad (7)$$

and

$$J_{\min} = tr\{\mathbf{E}\}, \quad (8)$$

where $J_{\min}$ is the minimum MSE and

$$\begin{aligned} \tilde{\mathbf{H}} &= \mathbf{R}_w^{-1/2}\mathbf{H} \\ &= \begin{bmatrix} \sigma_{n,d}^{-1}\mathbf{H}_{SD} \\ \left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1/2}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}_{SR} \end{bmatrix} \end{aligned} \quad (9)$$

is defined as the equivalent channel matrix after noise whitening. Note that the MSE is contributed by both the direct and relay links. By ignoring the direct link and adopting a single precoder at the relay, the problem is reduced to that considered in [3] and [4]. Here, we incorporate the THP as the source precoder and take the direct link into consideration. A significant performance enhancement can then be expected.

B. Problem Formulation

With the MMSE criterion in (7)-(8), we now can formulate our joint design problem as:

$$\min_{\mathbf{C},\mathbf{F}_S,\mathbf{F}_R} tr\left\{\underbrace{\mathbf{C}\left(\sigma_s^{-2}\mathbf{I}_N+\mathbf{F}_S^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{F}_S\right)^{-1}\mathbf{C}^H}_{:=\mathbf{E}}\right\}$$
$$s.t.$$
$$\mathbf{F}_S=\alpha\mathbf{U}_S$$
$$tr\left\{E\left[\mathbf{F}_S\mathbf{x}\mathbf{x}^H\mathbf{F}_S^H\right]\right\}=\sigma_s^2 tr\left\{\mathbf{F}_S\mathbf{F}_S^H\right\}\le P_{S,T},$$
$$tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R+\sigma_s^2\mathbf{H}_{SR}\mathbf{F}_S\mathbf{F}_S^H\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\}\le P_{R,T} \quad (10)$$

where the inequalities in (10) indicates the transmitted power constraints at the source and the relay (the maximal available power is $P_{S,T}$ and $P_{R,T}$, respectively). Here we let $\mathbf{F}_S=\alpha\mathbf{U}_S$ where α is a scalar and $\mathbf{U}_S$ is an unitary matrix. As we will see in the next section, the unitary structure can greatly facilitate the optimization. Taking a close look at (10), we can observe that the cost function is a nonlinear function of $\mathbf{F}_S$ and $\mathbf{F}_R$. Moreover, (10) is not a convex optimization problem. As a result, it is difficult to solve the problem, directly. In the next section, we propose a new method to overcome the problem.

III. Joint Source/Relay Precoder Design

A. Proposed Method

Since simultaneously finding the optimum $\mathbf{F}_S$ and $\mathbf{F}_R$ in (10) is very difficult, we resort to the primal decomposition method [9] translating (10) into a subproblem and a master problem. The subproblem is first solved to obtain the source precoder, and subsequently the master problem solves the relay precoder. To proceed, we reformulate (10) as

$$\min_{\mathbf{C},\mathbf{F}_S,\mathbf{F}_R} tr\{\mathbf{E}\}=\min_{\mathbf{F}_R}\min_{\mathbf{C},\mathbf{F}_S} tr\{\mathbf{E}\}$$
$$s.t.$$
$$\mathbf{E}=\mathbf{C}\left(\sigma_s^{-2}\mathbf{I}_N+\mathbf{F}_S^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{F}_S\right)^{-1}\mathbf{C}^H$$
$$\mathbf{F}_S=\alpha\mathbf{U}_S$$
$$N\sigma_s^2\alpha^2\le P_{S,T}$$
$$tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R+\sigma_s^2\alpha^2\mathbf{H}_{SR}\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\}\le P_{R,T}. \quad (11)$$

B. Proposed Subproblem Optimization

In the subproblem, the relay precoder $\mathbf{F}_R$ is assumed to be given. The optimal $\mathbf{C}$ and $\mathbf{F}_S$ can be first derived as a function of $\mathbf{F}_R$. Therefore, the joint precoders design can be re-formulated to the master problem in which the only unknown is the relay precoder. The unitary precoder $\mathbf{F}_S$ included here has two reasons: *(i)* It can simply the formulation of the relay precoder. *(ii)* By a proper design of $\mathbf{U}_S$, the minimum MSE can have an amenable form, leading to a tractable optimization problem. The subproblem then becomes the optimization of α, $\mathbf{U}_S$ and $\mathbf{C}$, given as

$$\min_{\mathbf{C}(\mathbf{F}_R),\alpha,\mathbf{U}_S(\mathbf{F}_R)} tr\left(\mathbf{C}\left(\sigma_s^{-2}\mathbf{I}_N+\alpha^2\mathbf{U}_S^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{U}_S\right)^{-1}\mathbf{C}^H\right)$$
$$s.t.$$
$$N\sigma_s^2\alpha^2\le P_{S,T},$$
$$tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R+\alpha^2\sigma_s^2\mathbf{H}_{SR}\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\}\le P_{R,T}. \quad (12)$$

To find the solution of (12), we can first find the optimum α, denoted α_{opt}, with $\mathbf{U}_S$ and $\mathbf{C}$ fixed. The solution can be easily obtained as

$$\alpha_{opt}=\sqrt{\frac{P_{S,T}}{N\sigma_s^2}}. \quad (13)$$

The result is because the cost function is strictly decreasing function on α.

Substituting (13) in (12), we find that the resultant relay power constraint, expressed as

$$tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R+\frac{P_{S,T}}{N}\mathbf{H}_{SR}\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\}\le P_{R,T}. \quad (14)$$

It is clear that the constraint is not a function of the source precoder. As a result, we only have to consider it in the master problem. The subproblem thus becomes

$$\min_{\mathbf{C}(\mathbf{F}_R),\mathbf{U}_S(\mathbf{F}_R)} tr\left\{\mathbf{C}\left(\sigma_s^{-2}\mathbf{I}_N+\frac{P_{S,T}}{N\sigma_s^2}\mathbf{U}_S^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{U}_S\right)^{-1}\mathbf{C}^H\right\}. \quad (15)$$

With a known relay precoder, the problem in (15) is similar to that of the THP scheme in conventional point-to-point MIMO systems, and the optimum solution **C**, denoted as $\mathbf{C}_{opt}$, has been solved as [7]

$$\mathbf{C}_{opt}=\mathbf{D}\mathbf{L}^{-1}, \quad (16)$$

where

$$\mathbf{L}\mathbf{L}^H=\left(\sigma_s^{-2}\mathbf{I}_N+\frac{P_{S,T}}{N\sigma_s^2}\mathbf{U}_S^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{U}_S\right)^{-1}, \quad (17)$$

is the Cholesky factorization of

$$\left(\sigma_s^{-2}\mathbf{I}_N+\frac{P_{S,T}}{N\sigma_s^2}\mathbf{U}_S^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{U}_S\right)^{-1};$$

D is a diagonal matrix scaling the diagonal elements of **C** to unity. Substituting (16) in (15), we then have

$$J_{\min}=tr\left\{\mathbf{C}\left(\sigma_s^{-2}\mathbf{I}_N+\frac{P_{S,T}}{N\sigma_s^2}\mathbf{U}_S^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{U}_S\right)^{-1}\mathbf{C}^H\right\}$$
$$=\sum_{k=1}^{N}\mathbf{L}(k,k)^2\ge N\left(\prod_{k=1}^{N}\mathbf{L}(k,k)\right)^{2/N} \quad (18)$$

which is a function of $\mathbf{U}_S$. The inequality in (18) is obtained from arithmetic-geometric inequality (AGI) and the equality holds when $\mathbf{L}(i,i)=\mathbf{L}(j,j)$, $i \neq j$. The next step we have to do is finding $\mathbf{U}_S$ so that that lower bound in (18) is achieved. To start with, we first decompose $\mathbf{U}_S$ as

$$\mathbf{U}_S = \mathbf{V}_{\tilde{\mathbf{H}}}\mathbf{U}'_S, \tag{19}$$

where $\mathbf{V}_{\tilde{\mathbf{H}}} \in \mathbb{C}^{N\times N}$ is the left singular matrices of $\tilde{\mathbf{H}}$ and $\mathbf{U}'_S \in \mathbb{C}^{N\times N}$ is another unitary matrix. Note that this decomposition is always possible for any unitary matrix. Substituting (19) into (17), we then have

$$\mathbf{L}\mathbf{L}^H = \mathbf{U}'^H_S \underbrace{\left(\sigma_s^{-2}\mathbf{I}_N + \frac{P_{S,T}}{N\sigma_s^2}\Lambda\right)^{-1}}_{:=\tilde{\mathbf{D}}} \mathbf{U}'_S \tag{20}$$

where $\Lambda = \mathrm{diag}\left\{\lambda_{\tilde{\mathbf{H}},1},\cdots,\lambda_{\tilde{\mathbf{H}},N}\right\}$ is the eigenvalues of $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$. It is simple to see that $\tilde{\mathbf{D}}$ is a diagonal matrix. Applying the geometric mean decomposition (GMD) [12] on $\tilde{\mathbf{D}}^{1/2}$, we have

$$\tilde{\mathbf{D}}^{1/2} = \mathbf{Q}\mathbf{R}\mathbf{P}^H, \tag{21}$$

where $\mathbf{Q}$ and $\mathbf{P}$ are some unitary matrices, and $\mathbf{R}$ is an upper triangular matrix with equal diagonal elements. Substituting (21) into (20), we then have

$$\mathbf{L}\mathbf{L}^H = \mathbf{U}'^H_S\mathbf{P}\mathbf{R}^H\mathbf{R}\mathbf{P}^H\mathbf{U}'_S = \mathbf{R}^H\mathbf{R}. \tag{22}$$

In (22), we let $\mathbf{U}'_S = \mathbf{P}$. Thus lower bound (18) is achieved due to $\mathbf{L}(i,i)=\mathbf{L}(j,j)$, $i \neq j$. The optimal $\mathbf{F}_S$, denoted as $\mathbf{F}_{S,opt}$, can then be expressed as

$$\mathbf{F}_{S,opt} = \sqrt{\frac{P_{S,T}}{N\sigma_s^2}}\mathbf{V}_{\tilde{\mathbf{H}}}\mathbf{P}. \tag{23}$$

From (22), the resultant MSE is then

$$J_{\min} = \sum_{k=1}^{N}\mathbf{R}(k,k)^2 = N\prod_{k=1}^{N}\left(\frac{P_{S,T}}{N\sigma_s^2}\lambda_{\tilde{\mathbf{H}},k} + \sigma_s^{-2}\right)^{-1/N}. \tag{24}$$

Now, the problem becomes the minimization of (24) in the master problem.

C. Proposed Master Optimization

Consider the following equation.

$$\prod_{k=1}^{N}\left(\frac{P_{S,T}}{N\sigma_s^2}\lambda_{\tilde{\mathbf{H}},k} + \sigma_s^{-2}\right) = \det\left(\sigma_s^{-2}\mathbf{I}_N + \frac{P_{S,T}}{N\sigma_s^2}\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right). \tag{25}$$

Substituting (25) into (24), we can then reformulate the master problem as

$$\begin{aligned} &\max_{\mathbf{F}_R}\ \det\left(\frac{N}{P_{S,T}}\mathbf{I}_N + \tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right) \\ &s.t. \\ &tr\left\{\mathbf{F}_R\left(\sigma_{n,r}^2\mathbf{I}_R + \frac{P_{S,T}}{N}\mathbf{H}_{SR}\mathbf{H}_{SR}^H\right)\mathbf{F}_R^H\right\} \le P_{R,T}. \end{aligned} \tag{26}$$

Solving (26) is very difficult since the cost function is a complicated function of $\mathbf{F}_R$ and problem is not convex. To overcome the difficulty, we consider the following lemma.

Lemma: For any positive definite matrix $\mathbf{M} \in \mathbb{C}^{N\times N}$, we have [11]

$$\det(\mathbf{M}) \le \prod_{i=1}^{N}\mathbf{M}(i,i), \tag{27}$$

where $\mathbf{M}(i,i)$ denotes the ith diagonal element of $\mathbf{M}$ and the equality holds when $\mathbf{M}$ is diagonal matrix. Therefore, to maximize the determinant, we can try to design the relay recoder so that the matrix of the determinant is diagonal. However, $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$ in (10) consists of two matrices and one is not related to $\mathbf{F}_R$. The diagonalization is till difficult. To overcome the problem, we use the following equivalence [6]

$$\begin{aligned} &\arg\min_{\mathbf{F}_R}\det\left(\frac{N}{P_{S,T}}\mathbf{I}_N + \tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right) = \arg\max_{\mathbf{F}_R}\det\Big(\mathbf{I}_N + \sigma_{n,d}^2\mathbf{H}'^H_{SR} \\ &\mathbf{F}_R^H\mathbf{H}_{RD}^H\left(\sigma_{n,r}^2\mathbf{H}_{RD}\mathbf{F}_R\mathbf{F}_R^H\mathbf{H}_{RD}^H + \sigma_{n,d}^2\mathbf{I}_M\right)^{-1}\mathbf{H}_{RD}\mathbf{F}_R\mathbf{H}'_{SR}\Big) \end{aligned} \tag{28}$$

where $\mathbf{H}'_{SR} = \mathbf{H}_{SR}\left(N/P_{S,T}\mathbf{I}_N + \sigma_{n,d}^{-2}\mathbf{H}_{SD}^H\mathbf{H}_{SD}\right)^{-1/2}$. The right equation in (28) suggests a feasible way to diagonalize the cost function. To start with, we consider the singular value decomposition (SVD)

$$\mathbf{H}_{RD} = \mathbf{U}_{rd}\Sigma_{rd}\mathbf{V}_{rd}^H; \tag{29}$$

$$\mathbf{H}'_{SR} = \mathbf{U}'_{sr}\Sigma'_{sr}\mathbf{V}'^H_{sr}, \tag{30}$$

where $\mathbf{U}_{rd} \in \mathbb{C}^{M\times M}$ and $\mathbf{U}'_{sr} \in \mathbb{C}^{R\times R}$ are left singular matrices of $\mathbf{H}_{RD}$ and $\mathbf{H}'_{SR}$, respectively; $\Sigma_{rd} \in \mathbb{R}^{M\times R}$ and $\Sigma'_{sr} \in \mathbb{R}^{R\times N}$ are the diagonal singular value matrices of $\mathbf{H}_{RD}$ and $\mathbf{H}'_{SR}$, respectively; $\mathbf{V}_{rd}^H \in \mathbb{C}^{R\times R}$ and $\mathbf{V}'^H_{sr} \in \mathbb{C}^{N\times N}$ are the right singular matrices of $\mathbf{H}_{RD}$ and $\mathbf{H}'_{SR}$, respectively. We found that if the optimal $\mathbf{F}_R$ have the following structure, a full diagonalization of the matrix of the determinant in (28) can be achieved:

$$\mathbf{F}_{R,opt} = \mathbf{V}_{rd}\Sigma_r\mathbf{U}'^H_{sr}, \tag{31}$$

where Σ_r is a diagonal matrix with ith diagonal element $\sigma_{r,i}$, yet to be determined. Let $\sigma_{rd,i}$ and $\sigma'_{sr,i}$ be the ith diagonal element of Σ_{rd} and Σ'_{sr}, respectively. Substituting (29), (30) and (31) into (28) and taking the ln operation to the cost function, we can then rewrite (26) as

$$\begin{aligned} &\max_{p_{r,i},\ 1\le i\le N} \sum_{i=1}^{N} \ln\left(1+\frac{p_{r,i}\sigma_{n,d}^{2}\sigma_{rd,i}^{2}\sigma_{sr,i}^{\prime 2}}{p_{r,i}\sigma_{n,r}^{2}\sigma_{rd,i}^{2}+\sigma_{n,d}^{2}}\right) \\ &s.t. \\ &\sum_{i=1}^{N} p_{r,i}\left(P_{S,T}\sigma_{sr,i}^{\prime 2}\mathbf{D}_{sr}^{\prime}(i,i)+\sigma_{n,r}^{2}\right)\le P_{R,T}, \\ &p_{r,i}\ge 0, \end{aligned} \tag{32}$$

where $p_{r,i}=\sigma_{r,i}^{2}$ and $\mathbf{D}_{sr}^{\prime} = \mathbf{V}_{sr}^{\prime}\left(N/P_{ST}\mathbf{I}_N+\mathbf{H}_{SD}^{H}\mathbf{H}_{SD}\right)\mathbf{V}_{sr}^{\prime H}$ with $\mathbf{D}_{sr}^{\prime}(i,i)$ being the ith diagonal element of $\mathbf{D}_{sr}^{\prime}$. The cost function now is simplified to a function of scalar parameters. Since the cost function and the inequalities are all concave for $p_{r,i}\ge 0$ [9], (32) is a standard concave optimization problem. As a result, the optimal solutions $p_{r,i}$, $i=1,\cdots,N$, can be solved by means of KKT conditions and are given as equation (33) at the bottom of this page, where μ is chosen to satisfy the power constraint in (32) and $[y]^{+}=\max(0,y)$. Substituting (33) into (31), we can finally obtain the optimum relay precoder. With the relay precoder, $\tilde{\mathbf{H}}$ in (10) can be obtained. Subsequently, the $\mathbf{F}_S$ can be derived by substituting (21) into (23) and $\mathbf{C}$ can be obtained by (16) accordingly.

IV. SMULATIONS

We consider an AF MIMO relay system with $N=R=M=4$. The elements of each channel matrix are assumed to be independent and identically-distributed (i.i.d.) complex Gaussian random variables with zero-mean and unity variance. Let SNR_{sr}, SNR_{rd}, and SNR_{sd} denote, respectively, the SNR per receive antenna of the source-to-relay, the relay-to-destination, and the source-to-destination links. Here, we let $SNR_{sr}=15$ dB, $SNR_{rd}=10$ dB and vary SNR_{sd}. Also, we use 16-QAM for each transmitted symbols. Fig. 2 and Fig. 3 show the MSE and BER performances comparison, respectively, for (a) an un-precoded system with the MMSE receiver, (b) an MIMO relay system with the optimum relay precoder in [3], [4], (c) an MIMO relay system with the source/relay precoders in [5], and (d) an MIMO relay system with the proposed source/relay precoders. Note that the optimum relay precoder in [3], [4] only considers the relay link. For better performance, we further include the direct link when implementing the MMSE receiver. As we can see, the proposed method significantly outperforms other methods. Although two precoders are used in [5], the performance is limited. This is because both precoders are linear.

V. CONCLUSIONS

In this paper, we consider a precoding scheme in AF MIMO relay systems. In this scheme, a TH source precoder is used at the source, a linear relay precoder at the relay, and an MMSE receiver at the destination. Since MSE is a complicated function of the source and relay precoders, a direct minimization is difficult. To solve the problem we propose cascading an unitary precoder after the THP precoded signals. We have found that the unitary precoder can minimize the MSE, and, most importantly, it greatly facilitates the use of the primal decomposition. Using this approach, the original problem can be formulated as a relay-precoder design problem called the master problem, and a source-precoder design problem called the subproblem. A two-precoder optimization is then translated to a single relay-precoder optimization and the problem is convex. By the KKT conditions, we can obtain the closed-form solutions of the precoders. Simulations show that the proposed method significantly outperforms the existing un-precoded and precoded systems.

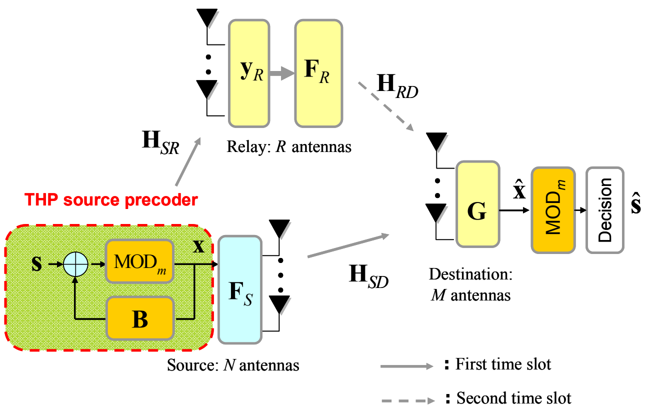

Figure 1. Three node AF MIMO relay system with TH source precoder and linear relay precoder.

$$p_{r,i}=\left[\sqrt{\frac{\mu}{\sigma_{rd,i}^{2}\left(\frac{P_{S,T}}{N}\sigma_{sr,i}^{\prime 2}\mathbf{D}_{sr}^{\prime}(i,i)+\sigma_{n,r}^{2}\right)\left(\sigma_{n,r}^{2}\sigma_{n,d}^{-2}\sigma_{sr,i}^{\prime -2}+1\right)}+\frac{\frac{1}{4}\frac{\sigma_{n,d}^{4}}{\sigma_{n,r}^{4}}}{\sigma_{rd,i}^{4}\left(\frac{\sigma_{n,r}^{2}}{\sigma_{rd,i}^{2}\sigma_{sr,i}^{\prime 2}}+1\right)^{2}}}-\frac{1+\frac{1}{2}\frac{\sigma_{n,d}^{2}\sigma_{sr,i}^{\prime 2}}{\sigma_{n,r}^{2}}}{\sigma_{rd,i}^{2}\left(\frac{\sigma_{n,r}^{2}}{\sigma_{n,d}^{2}}+\sigma_{sr,i}^{\prime 2}\right)}\right]^{+} \tag{33}$$

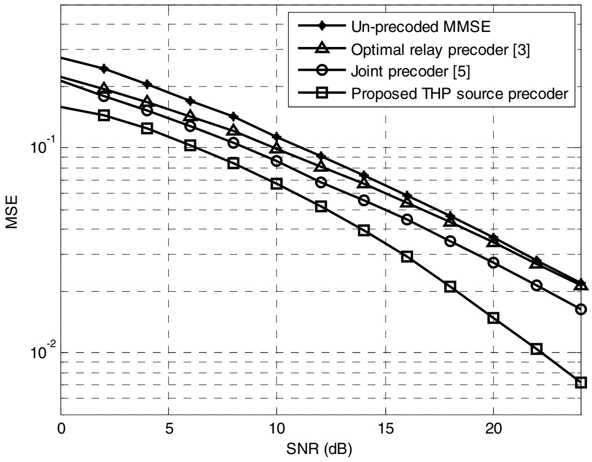

Figure 2. MSE performance comparison of the proposed precoders method and the other schemes.

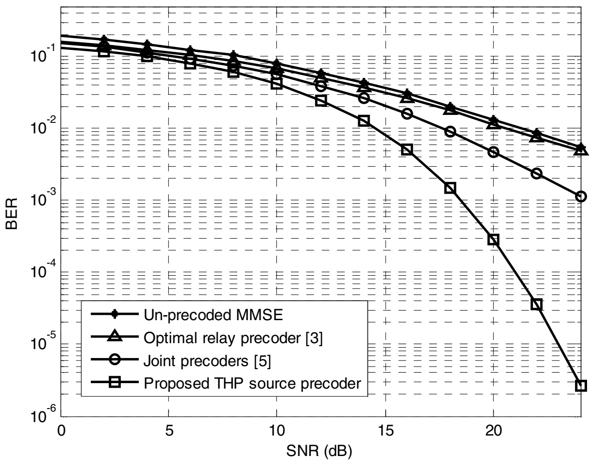

Figure 3. BER performance comparison of the proposed precoders method and the other schemes.

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