QRD-based Precoder Selection for Maximum-likelihood MIMO Detection

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Abstract—**Precoding is an effective method to improve the transmission quality in multiple-input multiple-output (MIMO) systems. In a real-world system, the precoder is selected from a codebook, and its index is fed back to the transmitter. For a maximum-likelihood (ML) receiver, the criterion for precoder selection is equivalent to maximizing the minimum distance of the received signal constellation. The derivation of the optimum solution, however, may be of high computational complexity due to the requirement of the exhaustive search. To reduce the computational complexity, a suboptimum solution based on singular value decomposition (SVD) has been proposed in literature. In this paper, we propose using a QR decomposition (QRD) based method for precoder selection. To further improve the system performance, we also propose an enhanced QRD-based selection method. With Givens rotations, the computational complexity of the enhanced QRD-based method can be effectively reduced. Finally, we combine precoding with receive antenna selection, and use the proposed QRD-based methods to solve this joint optimization problem. Simulation results show that the proposed approaches can significantly improve the system performance.**

I. Introduction

Spatial multiplexing is a promising method to achieve high spectra efficiencies in multiple-input multiple-output (MIMO) systems [1]. The drawback of the spatial multiplexing scheme is that the error rate performance is greatly affected by channel fading [2]. One way to alleviate the performance loss is to adopt the precoding technique at the transmitter, where the transmit symbol vector is multiplied by a precoding matrix before signal transmission. The main problem for precoding is that the precoding matrix must be fed back to the transmitter, and it is not possible to use infinite precision for the matrix. In practice, a finite-set codebook, which is pre-developed and available at both the transmitter and the receiver, is used for conducting precoding. For one MIMO channel, a precoder is selected from the codebook at the receiver, and then the index of the selected precoder is fed back to the transmitter. This is a simple yet effective approach in real-world precoding [3]. Thus, how to choose the precoder from a codebook becomes an important issue.

It is well-known that in precoding, different receiver structures may require different selection criteria. For linear receivers, several precoder selection methods have been proposed in [3], including post signal-to-noise ratio (SNR) maximization and mean-square-error (MSE) minimization. In this paper, we focus on the maximum-likelihood (ML) receiver. Note that, under high SNR, the error rate performance of an ML receiver strongly depends on the minimum distance of the received signal constellation, referred to as free distance. Therefore, we can choose the precoder that reshapes the MIMO channel to have the largest free distance. However, it is difficult to evaluate the free distance of a MIMO channel. This is because an exhaustive search is usually required, and the computational complexity can be very high. Thus, a suboptimum solution based on singular value decomposition (SVD) was then proposed in [3]. Instead of maximizing the free distance itself, the SVD-based method maximizes the lower bound of the free distance. Recently, another lower bound for the free distance via QR decomposition (QRD) was developed in [4]. It has been theoretically proved [5] that the QRD-based lower bound is tighter than the SVD-based one. In this paper, we propose using the QRD-based lower bound as a precoder selection criterion. To further improve the system performance, we also propose a method, referred to as enhanced QRD-based method, to tighten the lower bound. With Givens rotations, the computational complexity of the enhanced QRD-based method can be effectively reduced.

Except for precoding, antenna selection is also a common approach to improving the transmission quality in MIMO systems [6], [7]. It is simple to conduct antenna selection, and the computational complexity is very low. Antenna selection can be combined with precoding. With this scheme, the performance can be improved while the additional complexity is limited. For ML receivers, the objective in either precoding or antenna selection is to maximize the free distance. Thus, we can perform these two schemes jointly and optimize the joint selection with our proposed QRD-based methods. Note that antenna selection can be conducted at either the transmitter or the receiver. In this paper, we only consider receive antenna selection [8] since the overhead of required feedback bits will not be increased, or the system can achieve the target performance with less feedback bits. Simulation results show that the proposed methods outperform the conventional SVD-based method and the joint precoder/antenna selection scheme improve the system performance even further.

The remainder of this paper is organized as follows. Section II outlines the system and signal model we use. Section III gives the SVD-based and proposed QRD-based selection criteria, and Section IV describes the joint optimization for precoder and antenna selection. Section V provides simulation

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results evaluating the performance of the proposed algorithms. Finally, we draw conclusions in Section VI.

II. System and Signal Models

Consider a wireless MIMO system with N_t transmit antennas and N_r receive antennas, as described in Fig. 1. Let $\mathbf{H}$ denote an $N_r \times N_t$ $(N_t \geq N_r)$ channel matrix. We assume that $\mathbf{H}$ is available to the receiver, but not to the transmitter. For precoder selection, the receiver first chooses a precoder $\mathbf{F}_p$ from the codebook $\mathcal{F}$. Note that the codebook consists of a finite set of precoding matrices, i.e., $\mathcal{F} = \{\mathbf{F}_1, \mathbf{F}_2, \cdots, \mathbf{F}_B\}$, where B is the size of the codebook, and $p \in \{1, \cdots, B\}$. Assume this codebook is known at both the transmitter and the receiver. Via a feedback channel, the receiver sends this index p to the transmitter. Finally, the transmitter uses the corresponding precoder $\mathbf{F}_p$ for precoding. We assume each precoding matrix in $\mathcal{F}$ has unit-norm columns so that the total transmit power is constrained. For spatial multiplexing, the input symbols are multiplexed into an $M \times 1$ symbol vector $\mathbf{s}$, and then multiplied by an $N_t \times M$ $(N_t \geq N_r \geq M)$ precoder $\mathbf{F}_p$ before transmission. Thus, the received signal vector can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{F}_p\mathbf{s} + \mathbf{n} \tag{1}$$

where $\mathbf{n}$ is the $N_r \times 1$ Gaussian noise vector with the covariance $\sigma^2\mathbf{I}_{N_r}$. The ML estimate of the transmit vector $\mathbf{s}$ can be expressed as

$$\hat{\mathbf{s}} = \min_{\mathbf{s}_i \in S} ||\mathbf{y} - \mathbf{H}\mathbf{F}_p\mathbf{s}_i\,||^2 \tag{2}$$

where S is the set of all possible transmitted symbol vectors. Note that when a colored noise is considered in the system model, we have to conduct the whitening process at the receiver so that the minimum distance criterion in (2) can be used for signal detection.

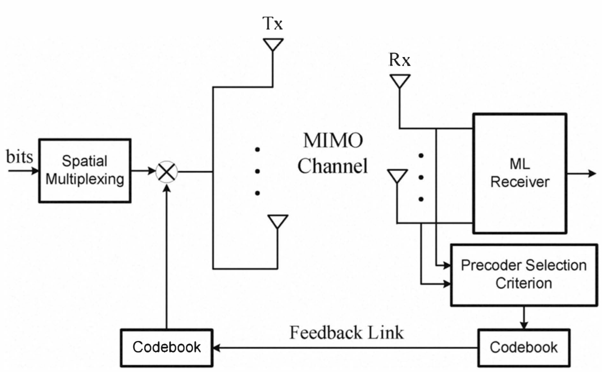

Fig. 1. System model for a limited-feedback precoding MIMO system.

III. Precoder Selection criteria

The free distance, which dominates the error rate performance of ML detection for high SNR regimes, is defined as

$$\hat{d}^2_{min} = \min_{\mathbf{s}_i, \mathbf{s}_j \in S, \mathbf{s}_i \neq \mathbf{s}_j} ||\mathbf{H}\mathbf{F}_p(\mathbf{s}_i - \mathbf{s}_j)\,||^2\,. \tag{3}$$

Let $(\mathbf{s}_i - \mathbf{s}_j)$ denote the difference vector, where $i \neq j$. Thus, for a given channel realization $\mathbf{H}$, the optimum precoder selection criterion is to choose $\mathbf{F}_p \in \mathcal{F}$ such that the free distance is maximized. From (3), we observe that the optimum solution is found by the exhaustive search over all possible difference vectors. Such numerical search, however, may be prohibitive when a large number of transmit bit-streams and a high-order QAM are adopted. In practice, we consider a suboptimum solution in which a lower bound of the free distance is maximized.

A. SVD-based selection criterion

Assume that $\mathbf{H}\mathbf{F}_p$ is an $N_r \times M$ full column rank matrix, and its SVD is given as $\mathbf{H}\mathbf{F}_p = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^*$, where $*$ represent the operation of Hermitian transpose, $\mathbf{U}$ is an $N_r \times N_r$ unitary matrix, $\mathbf{V}$ is an $M \times M$ unitary matrix, and $\mathbf{\Lambda} = \left[\text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_M)\ \mathbf{0}_{M\times(N_r - M)}\right]^*$ is an $N_r \times M$ matrix. The non-zero entries of $\mathbf{\Lambda}$ are the singular values of $\mathbf{H}\mathbf{F}_p$. Based on the Rayleigh-Rits theorem, a lower bound for the free distance using SVD was derived in [6]. It is shown that

$$\hat{d}^2_{min,\mathbf{H}\mathbf{F}_p} \geq \lambda_M^2 \hat{d}^2_{min} \tag{4}$$

where λ_M is the minimum singular value of the matrix $\mathbf{H}\mathbf{F}_p$, and d^2_{min} is the minimum distance between any two distinct transmit symbol vectors. Note that $\hat{d}^2_{min}$ is a deterministic value for a fixed QAM modulation size. Therefore, the lower bound only depends on the minimum singular value of $\mathbf{H}\mathbf{F}_p$. In [3], Heath *et al.* proposed the use of (4) to solve the precoder selection problem. The SVD-based precoder selection method can then be described as follows. With a given channel matrix $\mathbf{H}$, conduct SVD for each $\mathbf{H}\mathbf{F}_p$. Choose the precoder $\mathbf{F}_p \in \mathcal{F}$ whose $\mathbf{H}\mathbf{F}_p$ provides the largest λ_M.

With this criterion, only computing the minimum singular value λ_M of each $\mathbf{H}\mathbf{F}_p$ is required, and the computational complexity can be reduced dramatically. However, the problem for the SVD-based method is that the lower bound (4) may not be tight enough for evaluating the free distance.

B. QRD-based selection criterion

With QRD, we can factorize the matrix $\mathbf{H}\mathbf{F}_p$ in the form of $\mathbf{H}\mathbf{F}_p = \mathbf{Q}\mathbf{R}$, where $\mathbf{Q}$ is an $N_r \times M$ column-wise orthonormal matrix and $\mathbf{R}$ is an $M \times M$ upper triangular matrix with positive real-valued diagonal entries as

$$\mathbf{R} = \begin{pmatrix} R_{1,1} & R_{1,2} & \ldots & R_{1,M} \\ 0 & R_{2,2} & \ldots & R_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & R_{M,M} \end{pmatrix}.$$

Via this decomposition, we can have another lower bound for the free distance as

$$\hat{d}^2_{min,\mathbf{H}\mathbf{F}_p} \geq [\mathbf{R}]^2_{min} \hat{d}^2_{min} \tag{5}$$

where $[\mathbf{R}]_{min}$ is the minimum diagonal value of $\mathbf{R}$. Thus, we can have the QRD-based selection criterion described as follows. With a given channel matrix $\mathbf{H}$, conduct QRD for each $\mathbf{HF}_p$. Choose the precoder $\mathbf{F}_p \in \mathcal{F}$ whose $\mathbf{HF}_p$ provides the largest $[\mathbf{R}]_{min}$.

With the Cholesky factorization, it has been shown [5] that the lower bound achieved with QRD is tighter than that with SVD; that is

$$[\mathbf{R}]_{min} \geq \lambda_M. \tag{6}$$

In other words, (5) will be more accurate when evaluating the free distance. We hereby provide another proof for (6). The main idea is treating the QRD as a special case of the generalized triangular decomposition (GTD) [9] and use the corresponding properties.

Definition 1: Let $\underline{a} = (a_1, a_2, \ldots, a_m)$ and $\underline{b} = (b_1, b_2, \ldots, b_m)$ be two positive, real-valued sequences satisfying

$$a_1 \geq a_2 \geq \ldots \geq a_m$$

and

$$b_1 \geq b_2 \geq \ldots \geq b_m.$$

We say that $\underline{a}$ majorizes $\underline{b}$ in the product sense [10], [11] if

$$\prod_{k=1}^{l} a_k \geq \prod_{k=1}^{l} b_k$$

for all $l = 1, 2, \ldots, m$, and with equality when $l = m$.

Proposition 1: The inequality in (6) holds for any $M \times M$ full rank matrix $\mathbf{HF}_p$.

Proof: With QRD $\mathbf{HF}_p = \mathbf{QR}$, we can express $\mathbf{R}$ as

$$\mathbf{R} = \mathbf{Q}^* \mathbf{HF}_p. \tag{7}$$

Since the singular values $\underline{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_M)$ of $\mathbf{HF}_p$ are invariant under the unitary transformation, it follows that $\mathbf{HF}_p$ and $\mathbf{R}$ provide the same singular values. Note that $\mathbf{R}$ is an upper triangular matrix, which means its diagonal elements $\underline{r} = (r_1, r_2, \ldots, r_M)$ are exactly the eigenvalues of $\mathbf{R}$. Arrange both sequences $\underline{\lambda}$ and $\underline{r}$ in a decreasing order. By a theorem in [12], we can have that $\underline{\lambda}$ majorizes $\underline{r}$, which is equivalent to the following inequality

$$\prod_{k=1}^{l} \lambda_k \geq \prod_{k=1}^{l} r_k \tag{8}$$

for all $l = 1, 2, \ldots, M$, and with equality when $l = M$. Thus, we can conclude that $[\mathbf{R}]_{min} \geq \lambda_M$, which completes the proof.

C. Enhanced QRD-based selection criterion

The tightness of the QRD-based lower bound in (5) may degrade for large M even though (6) still holds. In this subsection, we propose a simple method to enlarge $[\mathbf{R}]_{min}$ for each $\mathbf{HF}_p$ so that the bound provided by QRD is even tighter. Assume that the same QAM modulation is adopted for each transmit bit-stream. Under this assumption, the ML detection criterion in (2) can be written as

$$\begin{aligned} \hat{\mathbf{s}} &= \min_{\mathbf{s}_i \in S} \|\mathbf{y} - \mathbf{HF}_p \mathbf{PP}^* \mathbf{s}_i \|^2 \\ &= \min_{\mathbf{s}'_i \in S} \|\mathbf{y} - \mathbf{H}' \mathbf{s}'_i \|^2 \end{aligned} \tag{9}$$

where $\mathbf{P}$ is a permutation matrix, $\mathbf{H}' = \mathbf{HF}_p\mathbf{P}$, and $\mathbf{s}'_i = \mathbf{P}^*\mathbf{s}_i$. The key observation that leads to the enhanced QRD-based method is that the solution of the ML detection in (9) remains the same since $\mathbf{s}'_i$ is a vector obtained with an element ordering of $\mathbf{s}_i$. Note that $\mathbf{H}'$ is a matrix obtained with a column ordering of $\mathbf{HF}_p$, and the QRD of $\mathbf{H}'$ will give a different $[\mathbf{R}]_{min}$. This fact motivates us to adopt the permutation method to further tighten the QRD-based lower bound. Also note that the permutation method proposed here cannot be adopted in the SVD-based scheme since the singular values of $\mathbf{HF}_p$ are independent of columns permutation. The details for the enhanced-QRD method can be summarized as follows. For a given $\mathbf{HF}_p$, we can obtain $M!$ matrices with column permutations denoted as $\mathbf{H}_1 = \mathbf{HF}_p\mathbf{P}_1, \mathbf{H}_2 = \mathbf{HF}_p\mathbf{P}_2, \ldots, \mathbf{H}_{M!} = \mathbf{HF}_p\mathbf{P}_{M!}$, where $\mathbf{P}_n$ is a permutation matrix corresponding to a specific permutation pattern n. Let their QRDs be expressed as

$$\mathbf{H}_n = \mathbf{Q}_n \mathbf{R}_n \tag{10}$$

where $n = 1, \ldots, M!$. From the above permutation method, $M!$ different minimum diagonal entries for a given $\mathbf{HF}_p$ can be obtained, and we can choose the maximum one, denoted by $[\mathbf{R}]_{min,max}$, as the minimum diagonal entry of $\mathbf{HF}_p$. Therefore, the enhanced QRD-based method is given as follows. With a given channel matrix $\mathbf{H}$, use the permutation method to compute the $[\mathbf{R}]_{min,max}$ for each $\mathbf{HF}_p$. Then, choose the precoder $\mathbf{F}_p \in \mathcal{F}$ whose $\mathbf{HF}_p$ provides the largest $[\mathbf{R}]_{min,max}$.

The permutation method we propose can tighten the lower bound in (5), but the computational complexity will be increased due to the extra $(M-1)$ QRD operations. To reduce the complexity, we can use Givens rotations [10], [13] for computing the QRD of each $\mathbf{H}_n$. First, we assume that $\mathbf{H}_1 = \mathbf{Q}_1\mathbf{R}_1$ is available via a complete QRD, and $\mathbf{H}_2$ is another matrix different from $\mathbf{H}_1$ by exchanging two neighbor columns. We then seek to obtain $\mathbf{R}_2$ of $\mathbf{H}_2$ without another complete QRD. Denote $\bar{\mathbf{P}}$ as a permutation matrix that exchanges two specific neighbor columns of $\mathbf{H}_1$. We then have

$$\mathbf{H}_2 = \mathbf{Q}_1 \mathbf{R}_1 \bar{\mathbf{P}} = \mathbf{Q}_1 \bar{\mathbf{R}}_1 \tag{11}$$

where $\bar{\mathbf{R}}_1$ is a near upper triangular matrix. Now, all we have to do is to transfer $\bar{\mathbf{R}}_1$ into an upper triangular matrix. Since $\bar{\mathbf{P}}$ only exchanges two neighbor columns of $\mathbf{R}_1$, we can upper-trianglize $\bar{\mathbf{R}}_1$ by applying a simple Givens rotation matrix $\mathbf{G}_1$, that is, $\mathbf{G}_1\bar{\mathbf{R}}_1 = \mathbf{T}$, where $\mathbf{T}$ is an upper triangular matrix. Thus we can rewrite (11) as

$$\mathbf{H}_2 = \mathbf{Q}_1 \mathbf{G}_1^* \mathbf{G}_1 \bar{\mathbf{R}}_1 = \bar{\mathbf{Q}}_2 \mathbf{T} \tag{12}$$

where $\bar{\mathbf{Q}}_2 = \mathbf{Q}_1\mathbf{G}_1^*$ is a unitary matrix. Since the QRD of a full column rank matrix is unique [10], we know that $\bar{\mathbf{Q}}_2\mathbf{T}$ in (12) is the QRD of $\mathbf{H}_2$, and $\mathbf{T}$ is equal to $\mathbf{R}_2$. In other words, we obtain $\mathbf{R}_2$ by simply left-multiplying a Givens rotation matrix on $\bar{\mathbf{R}}_1$ rather than by performing a complete QRD on $\mathbf{H}_2$. Therefore, we can dramatically reduce the computational complexity of the enhanced QRD-based scheme. Fig. 2 illustrates (for $M = 3$) how each $\mathbf{R}_n$ can be derived with Givens rotations.

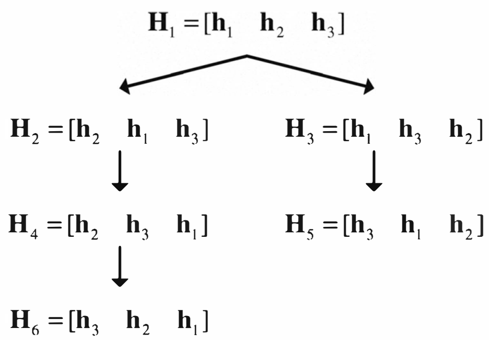

Fig. 2. The ordering of computing each $\mathbf{R}_n$ of $\mathbf{HF}_p$, $M = 3$

D. Capacity-based selection criterion

The criterion of capacity maximization is also widely considered in the precoding problem [3]. With a given equivalent channel matrix $\mathbf{HF}_p$, the capacity can be expressed as

$$C = \log_2 \det(\mathbf{I}_M + \frac{\rho}{M}\mathbf{F}_p^*\mathbf{H}^*\mathbf{HF}_p) \tag{13}$$

where ρ is the average SNR per receive antenna, det() denotes the determinant, and $\mathbf{I}_M$ is an $M \times M$ identity matrix. Therefore, the capacity-based method can be given as follows. Compute channel capacity using (13) for each $\mathbf{HF}_p$. Choose the precoder $\mathbf{F}_p \in \mathcal{F}$ whose $\mathbf{HF}_p$ provides the largest C.

The capacity-based selection criterion is derived from a general capacity formula, which is independent of the receiver structure. Thus, it may not provide the guaranteed performance improvement for some channel realizations. Also, the permutation method in our enhanced QRD-based scheme cannot be adopted in the capacity-based method since the channel capacity C is invariant under the column permutation of the matrix $\mathbf{HF}_p$.

E. Complexity comparisons

One way to quantify the complexity of the matrix computation is to count the number of floating operations (FLOPS). Several efficient algorithms for conducting QRD and SVD are given in [13]. In general, SVD requires more FLOPS than QRD does. As a result, the QRD-based selection scheme not only has better performance, but also requires lower computational complexity. For the enhanced QRD-based scheme, the computational complexity of performing QRD on all $\mathbf{H}_n$ (for a given $\mathbf{HF}_p$) is $O(M!M^3)$. As mentioned, we can reduce the complexity via Givens rotations, in which only one complete QRD and $(M! - 1)$ upper-triangulization operations are required. Each upper-triangulization operation only needs $O(3\times 4^2)$ FLOPS. Thus, the overall computational complexity is reduced from $O(M!M^3)$ to $O(M^3 + 48(M! - 1)) \simeq O(M^3 + M!)$. As for the capacity-based method, the computational complexity is $O(M^3)$, which mainly arises from computing the determinant and the matrix multiplication $\mathbf{F}_p^*\mathbf{H}^*\mathbf{HF}_p$ in (13). Note that there is an additional overhead for the capacity-based method since the variance of the channel noise is required.

IV. Joint Precoder and Antenna Selection

Antenna selection is a simple yet effective method to enhance the diversity gain in a wireless MIMO system. It has been shown that with ML detection, the optimum antenna subset is the one giving the largest free distance. We hereby propose a scheme that combines transmit precoding with antenna selection. Note that antenna selection can be conducted at either the transmitter or the receiver side. In this paper, we only consider the receive antenna selection. The advantage of receive antenna selection is that the feedback overhead will not be increased. The system model for joint precoder and receive antenna selection is shown in Fig. 3.

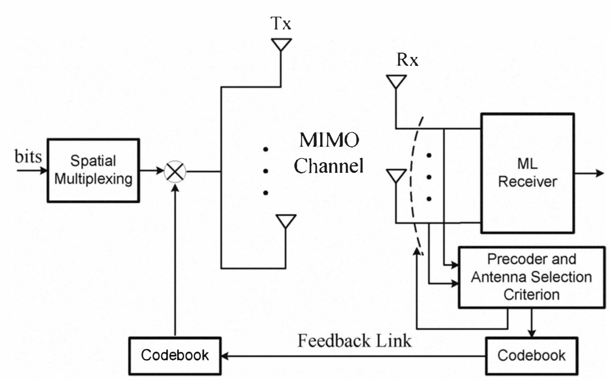

Fig. 3. System model for joint precoder and receive antenna selection in a MIMO system.

Assume that $N_t \geq N_r > M$. It means we have $\binom{N_r}{M}$ receive antenna subsets to choose. According to some criterion, the receiver jointly determines the optimum precoder $\mathbf{F}_p \in \mathcal{F}$, and the receive antenna subset indicated by the index q. Note that, via the feedback channel, only the index p will be sent back to the transmitter since the antenna selection is performed at the receiver side. The received signal in (1) can be rewritten as

$$\mathbf{y} = \mathbf{H}_q\mathbf{F}_p\mathbf{s} + \mathbf{n} \tag{14}$$

where $\mathbf{H}_q$ is the channel matrix corresponding to the selected receive antenna subset. The ML detection can then be rewritten as

$$\hat{d}^2_{min}(\mathbf{H}_q\mathbf{F}_p) = \min_{\mathbf{s}_i,\mathbf{s}_j \in S, \mathbf{s}_i \neq \mathbf{s}_j} ||\mathbf{H}_q\mathbf{F}_p(\mathbf{s}_i - \mathbf{s}_j)\,||^2 . \tag{15}$$

Thus, among $B\binom{N_r}{M}$ possible combinations, we choose a pair of $\mathbf{H}_q\mathbf{F}_p$ that can give the largest free distance. The joint precoder and receive antenna selection problem can be viewed as a two-dimensional optimization problem as shown in Fig. 4. As mentioned, the optimum solution needs an exhaustive search over all possible difference vectors in (15), which requires high computational complexity. Thus, we can use the suboptimum solution described in Section III for this joint optimization problem. The inequalities (4) and (5) can be rewritten as

$$\hat{d}^2_{min,\mathbf{H}_q\mathbf{F}_p} \geq \lambda_M^2 \hat{d}^2_{min} \tag{16}$$

and

$$\hat{d}^2_{min,\mathbf{H}_q\mathbf{F}_p} \geq [\mathbf{R}]^2_{min} \hat{d}^2_{min} \tag{17}$$

respectively. Since $[\mathbf{R}]_{min} \geq \lambda_M$, we expect that (17) will be a better criterion for the optimization problem. Furthermore, the enhanced QRD-based method can also be used for further performance improvement. Similarly, the computational complexity of the enhanced QRD-based method can be reduced by applying Givens rotations.

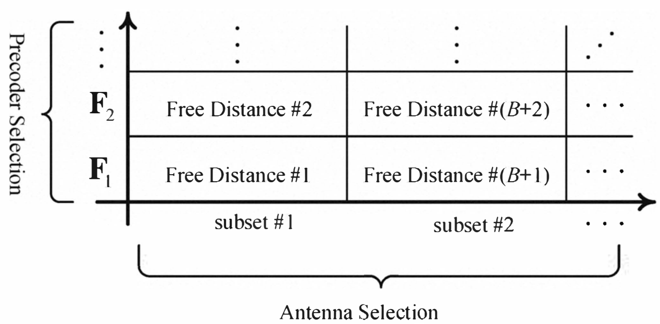

Fig. 4. Two-dimensional search for the optimum $\mathbf{H}_q\mathbf{F}_p$ in joint optimization problems.

V. Simulation Results

In this section, we report simulation results demonstrating the effectiveness of the proposed algorithms. In simulations, we consider a flat-fading MIMO channel, of which the entries are assumed to be i.i.d complex Gaussian random variables with zero mean and unit variance. The QPSK modulation is assumed at the transmitter while the ML detection is conducted at the receiver. The codebooks we use in the simulations are obtained from [14].

Fig. 5 shows the bit error rate (BER) performance of precoding. Here, $N_t = 6, N_r = M = 3$, and $B = 64$. As we can see, the scheme without precoding (3×3) suffers from the performance loss in channel fading. For precoding, the QRD-based method indeed outperforms the SVD-based method, and the enhanced QRD-based method achieves the best performance, about 2 dB better than the SVD-based method. Besides, the performance of capacity-based method is slightly better the SVD-based method but worse than the proposed methods. Note that the computational complexity of the capacity-based method is higher since the receiver needs to estimate the variance of channel noise.

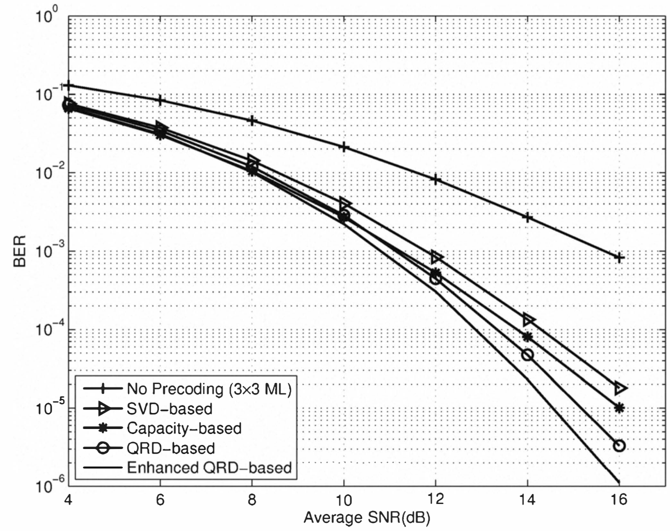

Fig. 5. BER performance comparison for precoder selection with $N_t = 6, N_r = 3, M = 3$, and $B = 64$.

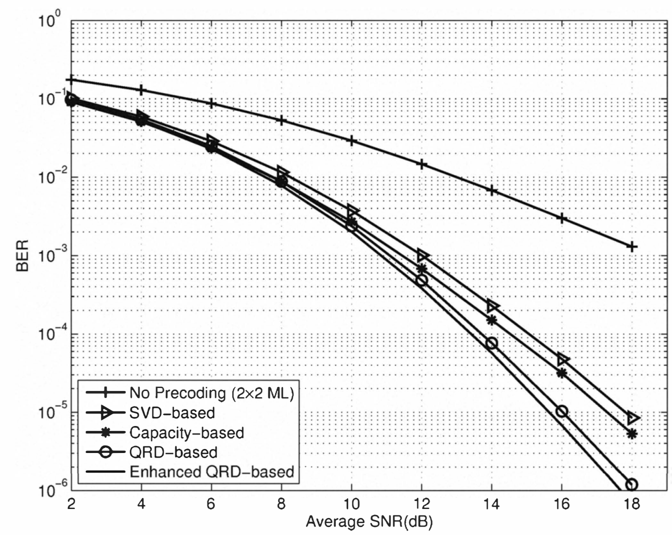

Fig. 6. BER performance comparison for precoder selection with $N_t = 4, N_r = 2, M = 2$, and $B = 64$.

Fig. 6 compares the BER performance of precoding for the case with $N_t = 4, N_r = M = 2$, and $B = 64$. Similarly, the proposed selection methods outperform the SVD-based and capacity-based methods in high SNR regimes. In this set of simulations, the gap between the QRD-based and the enhanced QRD-based method is not obvious since we only have two permutation patterns for $M = 2$.

Fig. 7 shows the performance improvement for joint precoding and receive antenna selection. In this case, we let $N_t = 4, N_r = 3, M = 2$, and $B = 16$, which means we have 3 receive antenna subsets to choose. As we can see, the method is very effective for performance improvement. The proposed joint selection methods outperform other selection methods. Compared to the result in Fig. 6, the capacity-based method exhibits some performance loss for high SNR. This

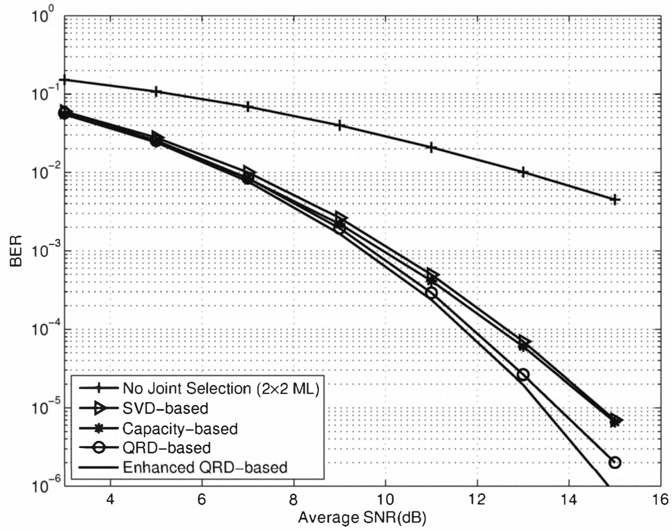

Fig. 7. BER performance comparison for joint precoder and receive antenna selection with $N_t = 4, N_r = 3, M = 2$, and $B = 16$.

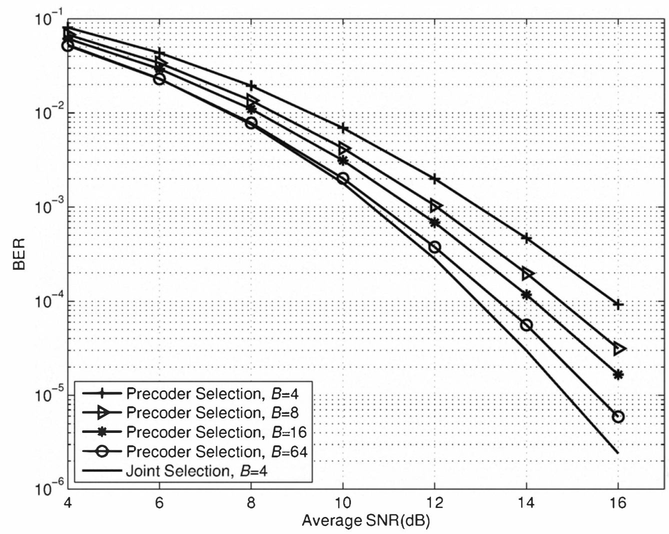

Fig. 8. BER performance comparison between joint selection ($N_t = 4, N_r = 3$, and $M = 2$) and precoder selection ($N_t = 4, N_r = 2$, and $M = 2$).

can be explained by the fact that it maximizes the channel capacity, not the free distance. Thus, its performance may degrade for some channel conditions. Besides, we observe that the gap between the QRD-based and SVD-based method is reduced somewhat since we only have $\binom{3}{2}2^4 = 48$ candidate matrices in this case. Fig. 8 shows the reduction of the required feedback bits when the joint selection scheme is considered. Here, all results are obtained with the enhanced QRD-based method. We observe that at least $\log_2^{64} - \log_2^{4} = 4$ bits can be saved when an extra antenna is used at the receiver. Note that increasing the receive antenna may not be always possible for some applications due to the size constraint at the receiver. Thus, the joint selection method can be viewed as a tradeoff between the feedback bits and the number of receive antennas.

VI. Conclusions

In this paper, we propose a QRD-based precoder selection method for ML receivers. Theoretical and simulation results indicate that the QRD-based method is not only better than the conventional SVD-based method, but also has lower computational complexity. To further improve the performance, we also propose the enhanced QRD-based method that can provide a more accurate estimate of the free distance. Using Givens rotations, the computational complexity of the enhanced QRD-based method can be reduced effectively. Besides, we combine the precoding with receive antenna selection, and solve the selection problem using the proposed methods. Simulations show that the proposed approaches can provide the significant performance improvement. Moreover, the proposed QRD-based approaches will exhibit a significant advantage when sphere-decoding (SD) [15], an efficient algorithm for the ML detection, is used at the receiver. Note that the QRD is also required in the SD algorithm, which implies that the same QRD unit can be shared by proposed selection methods and the SD algorithm. Based on the above reasons, we conclude that the QRD-based selection algorithms will be much more efficient in real-world applications.

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