A method for measuring the rotatory power of a chiral medium

Der-Chin Su [*a], Chen-Chih Hsu [a], Jiun-You Lin [a], and Ju-Yi Lee [*b]

[a] Institute of Electro-Optical Engineering, National Chiao Tung University, 1001 Ta-Hsueh Road, Hsin-Chu, Taiwan, R.O.C.

[b] Center for Measurement Standars, Industrial Technology Research Institute, Bldg. 16, 321 Kuang Fu Road, Sec 2, Hsin-Chu, Taiwan, R.O.C.

ABSTRACT

A light beam coming from a circularly polarized heterodyne light source passes through a chiral medium, its rotation angle is just half of the phase difference between p- and s- polarization components. And this phase difference can be measured accurately with heterodyne interferometric technique. The rotatory power is obtained by dividing the estimated rotation angle with its path length. This method has many advantages, such as, high stability, high resolution, easy operation, and real-time measurement.

Keywords: chiral medium, rotatory power, optical rotation angle, heterodyne interferometry.

1. INTRODUCTION

Rotatory power or specific rotation is an important characteristic constant of a chiral medium[1]. Although there are some techniques that have been proposed for measuring rotatory power, almost all of them are related to the light intensity variations[2-4]. However, the stability of light source, the scattering light, the internal reflection, and other factors influence the accuracy of measurements and decrease the resolution of results. In order to overcome these drawbacks, some improved methods using heterodyne interferometry are proposed[5-8]. Their heterodyne light sources are achieved by either introducing an acoustic-optic modulator to each arm of Mach-Zehnder interferometer or rotating polarization optical elements. Owing to their uncommon path configurations or the mechanical vibration, they are easily influenced by the surrounding vibration and air turbulence.

Further author information-

[*] D.C.S. (correspondence): Email: t7503@cc.nctu.edu.tw; Telephone: 886-3-5731951; Fax: 886-3-5716631

[*] J.Y.L.: Email: 880713@itri.org.tw; Telephone: 886-3-5743747; Fax: 886-3-5726445

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In this paper, an improved method for measuring the rotatory power of a chiral medium is presented. A circularly polarized heterodyne light source consisting of a linearly polarized laser source, an electro-optic modulator driven by a voltage signal generator, and a quarter-wave plate is used. The light beam coming from this light source passes through the tested chiral medium, its phase difference between p- and s- polarization components is twice of the rotation angle. And it can be measured accurately with heterodyne interferometric technique. Then, the rotatory power can be estimated by dividing the estimated rotation angle with the concentration and the the path length of the tested chiral solution[9]. Because of its common-path heterodyne interferometric configuration, this method has many advantages, such as, high stability, easy operation, and real-time measurement. And its feasibility is demonstrated and it has 0.015 degree resolution

2. PRINCIPLE

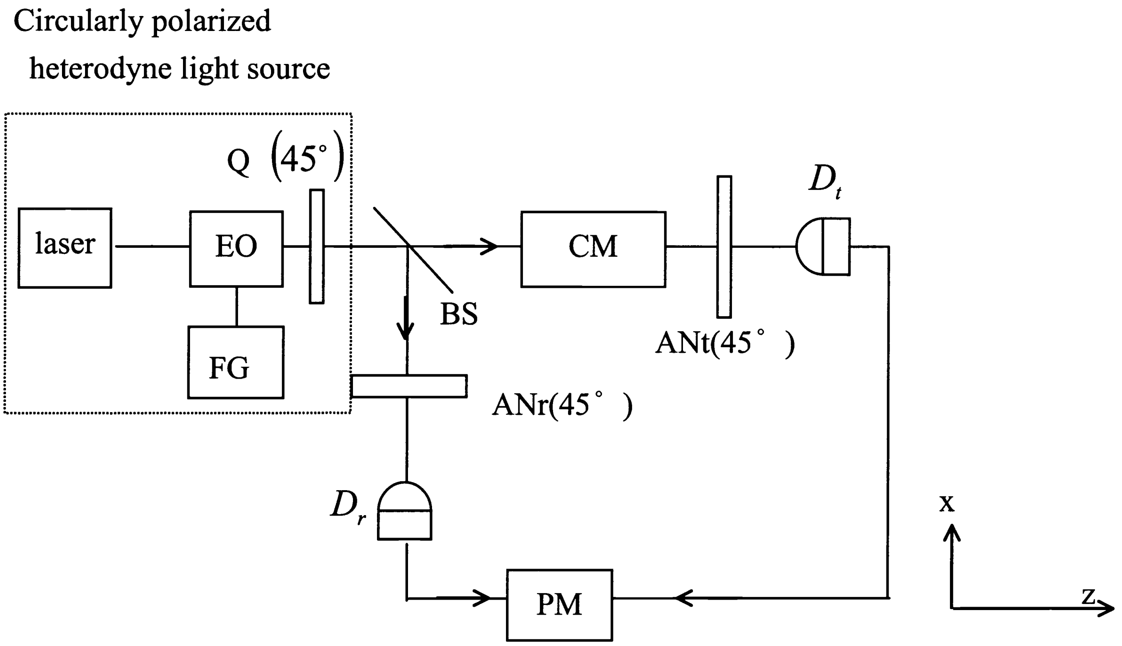

Fig. 1: Schematic diagram for measuring the phase difference. EO: electro-optic modulator; FG: function generator; Q: quarter-wave plate; BS: beam splitter; CM: chiral medium; AN: analyzer; D: photodetector; PM: phasemeter.

The schematic diagram of this method is shown in Fig. 1. For convenience, the +z axis is chosen along the propagation direction and the y-axis is along the vertical direction. If the light beam coming from a laser source is linearly polarized at 45° with respective to the x-axis, then its Jones vector can be written as

$$E = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}. \qquad (1)$$

And it passes through an electro-optic modulator EO and a quarter-wave plate Q with the fast axis being at 45° with respective to the x-axis. If the fast axis of EO is along the x-axis, and an external sawtooth voltage signal from a function generator FG with angular frequency ω and amplitude $V_{\lambda/2}$, the half-wave voltage of EO, is applied to EO, then the retardation produced by EO can be expressed as ωt, and the Jones vector of the light can be written as

$$\begin{aligned}
E' &= Q(45^\circ)EO(\omega t)E \\
&= \frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}\begin{pmatrix}e^{i\frac{\omega t}{2}} & 0\\ 0 & e^{-i\frac{\omega t}{2}}\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \\
&= \frac{1}{2}\begin{pmatrix}1\\i\end{pmatrix}e^{i\frac{\omega t}{2}} + \frac{1}{2}\begin{pmatrix}1\\-i\end{pmatrix}e^{-i\frac{wt}{2}+i\frac{\pi}{2}}\,. \qquad (2)
\end{aligned}$$

From Eq.(2), it is obvious that left- and right-circular polarizations have an angular frequency difference ω. The complete set-up for performing the operation Eq. (2) consists of a laser, an electro-optic modulator EO which is driven by a function generator FG and a quarter-wave plate Q. This set-up acts as a circularly polarized heterodyne light source. Then, the light is incident on the beam-splitter BS and is divided into two parts by BS : the reflected light and the transmitted light. The reflected light passes through an analyzer ANr with the transmission axis being at 45° with respective to the x-axis and enters into a photodetector Dr. Consequently, the Jones vector of the light becomes

$$\begin{aligned}
E_r &= AN_r(45^\circ)\,E' \\
&= \frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1 & i\\ i & 1\end{pmatrix}\begin{pmatrix}e^{i\frac{\omega t}{2}} & 0\\ 0 & e^{-i\frac{\omega t}{2}}\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \\
&= \frac{1}{2}(1+i)\cos\left(\frac{\omega t}{2}\right)\begin{pmatrix}1\\1\end{pmatrix}. \qquad (3)
\end{aligned}$$

Hence the intensity measured by D_r is

$$I_r = |E_r|^2 = \frac{1}{2}(1+\cos\omega t)\,. \qquad (4)$$

Here, I_r is the reference signal. On the other hand, the transmitted light enters the solution of a chiral medium contained in a cylindrical glass tube. Then, it passes through an analyzer AN_t with the transmission axis being at 45° with respective to the x-axis and is detected by another photodetector D_t. And the Jones vector of the light becomes

$$\begin{aligned} E_t &= AN_t(45^\circ)CM(\theta)E' \\ &= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} e^{i\frac{\omega t}{2}} & 0 \\ 0 & e^{-i\frac{\omega t}{2}} \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ &= \frac{1}{2}(1+i)\cos(\frac{\omega t}{2}+\theta)\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \end{aligned} \tag{5}$$

where CM(θ) is the Jones matrices of the chiral medium[6] and θ is the optical rotation angle. Therefore, the intensity measured by D_t is

$$I_t = |E_t|^2 = \frac{1}{2}[1+\cos(\omega t+2\theta)] \quad . \tag{6}$$

Here, I_t is the test signal. From Eq.(4) and Eq.(6), it is obvious that both the reference signal and the test signal are sinusoidal with angular frequency ω. These two sinusoidal signals are sent to the phase meter PM as shown in Fig. 1. The phase difference between the reference signal and test signal

$$\phi = 2\theta \quad , \tag{7}$$

can be obtained. From Eq. (7), it is seen that the phase difference ϕ is just twice of the rotation angle θ. From the relation $\theta=\phi/2$, the rotation angle θ can be estimated. Finally, the rotatory power or the specific rotation

$$[\alpha]_{\lambda,ph}^{T} = \frac{\theta}{C \cdot L} \quad , \tag{8}$$

can be calculated under the measurements of the rotation angle θ, the concentration C of the solution of the chiral medium, and the path length L of the cylindrical glass tube. $[\alpha]_{\lambda,ph}^{T}$ is expressed in degree/(dm g/l) or degree/(dm g/g)[9]. C and L are expressed in g/l or g/g and decimeters (dm), respectively. The symbols λ, pH and T means that the rotatory power α depends on the wavelength λ, pH and the temperature of the solution of the chiral medium.

3. EXPERIMENTS AND RESULTS

A He-Ne laser with a 632.8 nm wavelength and an electro-optic modulator (Model PC200/2, manufactured by England Electro-Optics Developments Ltd.) with a half-wave voltage of 170 V were used[10]. The frequency of the sawtooth signal applied to the electro-optic modulator was 2kHz. At first, to show the validity of this technique, a half-wave plate was introduced to replace the solution of the chiral medium. Fig. 2 shows the relation curve of the measured phase difference ϕ versus the optical rotation angle θ introduced by the half-wave plate. This relation curve is a straight line with a slope of 2. Hence, it is clear that our derivation shows good validity.

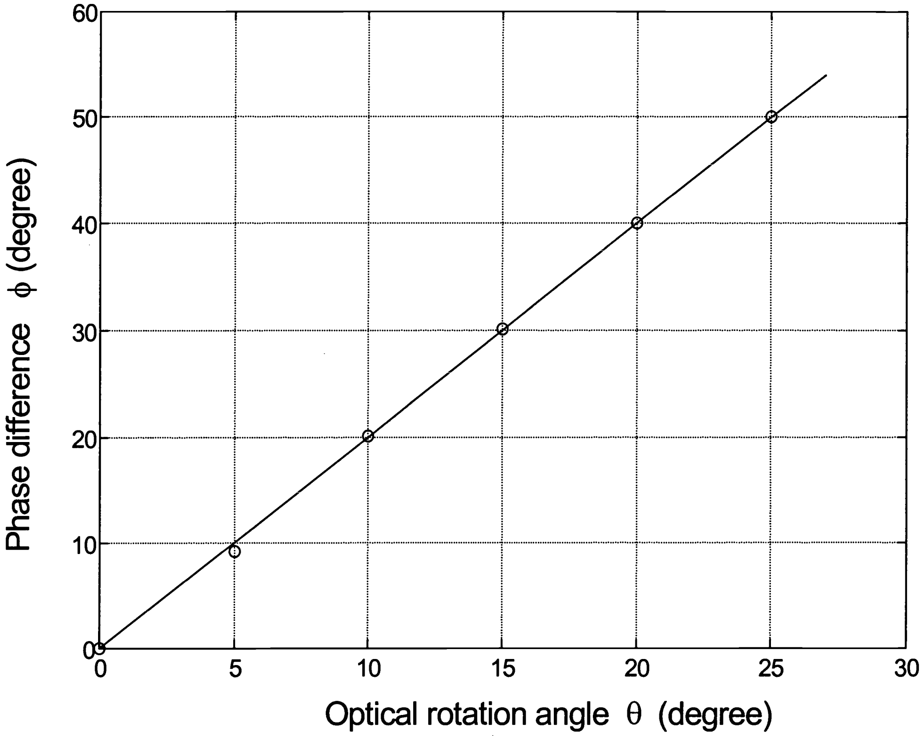

Fig. 2 : The relation curve of the phase difference ϕ versus the optical rotation angle θ for a half-wave plate.

Secondly, several sucrose solutions with different concentrations were measured. A cylindrical glass tube with a 2 decimeter length and 20 ml volume was used to contain the sucrose solution. The theoretical and experimental curves of the rotation angle θ versus the concentration C (in g/100g) are shown in Fig. 3. In this figure, the solid curve represent the theoretical values which are obtained by introducing the reference rotatory power[9] into Eq. (8). It is clear that this curve shows good correspondence except at high concentration. From the experimental results, the rotatory power of the sucrose solution is 56.25(g/100g $)^{-1}$(dm)$^{-1}$ at 632.8nm and the reference value is 56.51(g/100g)$^{-1}$(dm)$^{-1}$ at 636.2 nm.[9]

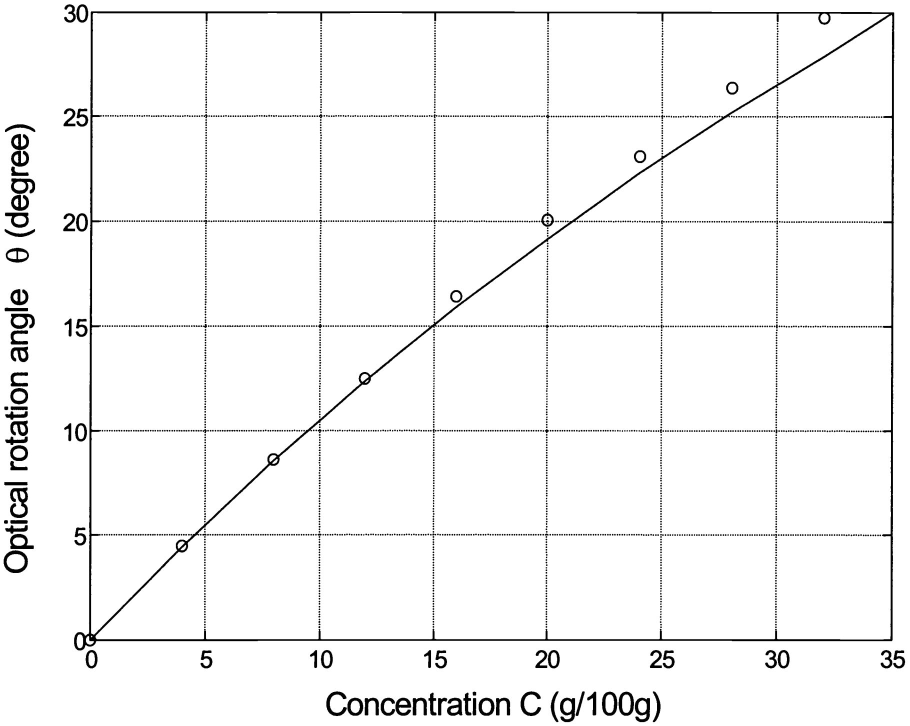

Fig. 3. The theorectical and experimental curves of the optical rotation angle θ versus the concentration C for sucrose solution.

4. DISCUSSION

Because a circularly polarized heterodyne light source is used in this method, the intensities of the signals are independent on the azimuth angles of the transmission axes of analyzers. But, when the transmission axis of the analyzer fixed at 45°, the phase difference error coming from the extinction ratio of analyzer will be minimum. The resolution depends on the resolution of the phasemeter and the optical configuration. From Eq. (7), we can estimate the error in optical rotation angle θ. It can be written as

$$\Delta\theta = \frac{\Delta\phi}{2} \quad . \tag{9}$$

where $\Delta\theta$ is the error in the phase difference. Angular resolution, second harmonic errors and polarization-mixing error may influence the errors in the phase difference in this method[10]. So the total phase difference error can decrease to 0.03° in our experiments. Substituting this data into Eq. (9), we have $\Delta\phi = 0.015^\circ$.

Eq. (8) is valid only for the chiral liquid and the solution of the chiral medium. As for a crystal, this method is still valid. But the rotatory power is defined as the measured rotation angle divided by its thickness (in mm unit).

This method is not related to the measurement of light intensity variations, it is free from the stability of a light source. In addition, because of its common-path interferometric structure, it is very stable and has a high resolution.

5. CONCLUSION

A light beam coming from a circularly polarized heterodyne light source passes through a chiral medium, its rotation angle is just half of the phase difference between p- and s- polarization components. And this phase difference can be measured accurately with heterodyne interferometric technique. Based on these facts, an improved method for measuring the rotatory power of a chiral medium is presented in this paper. It is very stable and has a high resolution. And its feasibility is demonstrated and its resolution is 0.015 degree.

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