Laboratory cranking wave functions and ground-state moments of inertia of heavy deformed nuclei

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Nuclear ground-state moments of inertia for heavy deformed nuclei are calculated using a rotating intrinsic wave function in the laboratory system. Numerical calculations are reported for rare-earth nuclei. The results show that the agreements between the theoretical values and the experimental observations have been improved.

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I. INTRODUCTION

Consider the motion of nuclear rotation as a many-body problem, nearly all of the fully microscopic theories are based on or related to some version of the cranking model of Inglis [1]. A very large amount of progress has been made along the way, see, for instance, [2–9]. Denote the Hamiltonian in the lab system by H_0, the semiclassical equation of motion for the nuclear intrinsic state $|n\rangle$ in the rotating frame is given as

$$(H_0 - \omega j_x)\Phi_n = E_n \Phi_n \ . \tag{1}$$

The same form of Eq. (1) can also be obtained by a constraint variational method where $H_0 - \omega j_x$ is the auxiliary constrained Hamiltonian. Φ_n is then a solution of Eq. (1) with the condition $\langle \Phi_n | j_x | \Phi_n \rangle$ =const.

Usually, when a Hartree-Bogoliubov method without any special technique is applied to solve the equation

$$H_0 \Psi_n = \varepsilon_n \Psi_n \ , \tag{2}$$

we obtain a set of independent particle wave functions localized in the lab system. In order to study the rotational effects of the system on the individual particles, the cranking idea can be used, and the problem is reduced to solve Eq. (1) in the rotating frame. However, the physical experiments can only be performed in the lab system. Furthermore, when we are observing the nuclear intrinsic system which is rotating in the lab system, there is no Coriolois force acting on the particles in the nucleus. Therefore, it should be more appropriate if we could describe the nuclear intrinsic system in our lab system. In a recent paper, the author [10] has suggested a method of finding a simplified Hartree-Bogoliubov type of wave function for Eq. (2) to represent a rotating nuclear intrinsic state in the lab system where experiments are performed. The purpose of this work is to apply this wave function to calculate the nuclear ground-state moments of inertia for rare-earth nuclei.

With self-consistent Hartree-Fock wave functions, the cranking model gives results in a rigid-body value for the nuclear moments of inertia which is two to three times larger than that of the experimental observation. It was pointed out by Bohr and Mottelson [11] that residual two-body forces, not included in the one-body self-consistent field, would lower the moment, and that correlations due to pairing would be the most important. Subsequently, Belyaev [12] showed that residual interactions of the pairing type indeed lower the moment of inertia from the rigid-body value through an increased energy denominator and a reduction of the j_x matrix element in the numerator by a factor $(uv' - vu')$, where u and v are coefficients of the Bogoliubov transformation [13]. Numerical calculations were performed by Griffin and Rich [14] and by Nilsson and Prior [15]. With the "best" choice of parameter values, they obtained remarkable agreements with experiments for both the rare-earth and the actinide nuclei. However, the theoretical values for the moments of inertia were systematically still about 20–30 % too small on the average. Corrections due to residual interactions were derived by Migdal [16] and Belyaev [17]. Numerical calculations by Mayer *et al.* [18] showed that the effects of the residual particle-hole and particle-particle interactions on the moments of inertia of all rare-earth nuclei nearly cancelled each other and left the simple cranking value approximately unchanged. Calculations by Birbrair and Nikolaev [19] and by Kammuri and Kusuno [20] on the same subject did not take into account the effects of rotation.

Mottelson and Valatin [21] observed that, in a rotating reference system, the Coriolis forces act in opposite directions on particles forming a time-reversed pair and tend to decouple the pairing correlations. The authors showed that, in the second-order approximation, the reduction of the pairing interaction energy is proportional to the square of the rotational frequency ω. This is reflected in a decrease in the pairing-gap parameter Δ and, consequently, an increase in the moment of inertia of the nucleus. As we shall see below, this Mottelson-Valatin type of Coriolis antipairing effect is indeed included in the rotating solution of Eq. (2), which represents the intrinsic state of the nucleus observed in the lab system. For the completeness of this present work, we shall give a brief derivation of the relevant formalism in the following.

II. MOTTELSON-VALATIN EFFECT

To study the Mottelson-Valatin Coriolis antipairing effect, we shall follow an approach derived by Lin and Faessler [22] for studying strongly deformed nuclei in

which state-dependent antipairing forces were treated. In the constrained system, we take a Hamiltonian consisting of an axially symmetric Nilsson potential plus a pairing force, and a constraint Coriolis term, neglecting terms connecting $m = \pm\frac{1}{2}$ states in the matrix elements of j_x. Numerical calculations show that the contribution of these terms to the moments of inertia should be no more than 5%. This would not change the final results significantly. This approximation reduces the sizes of matrices to be diagonalized by a factor of 2. Furthermore, it introduces a twofold degeneracy which allows a separation of the single-particle space into a space $|\alpha\rangle$, with positive projection $m_\alpha > 0$, and a space $|\bar{\alpha}>$ with $m_{\bar{\alpha}} < 0$. In this approximation, the constrained Hamiltonian is given as

$$H = \sum_{\alpha>0}(\epsilon_\alpha - \lambda)(C^\dagger_\alpha C_\alpha + C^\dagger_{\bar{\alpha}} C_{\bar{\alpha}}) - \frac{G}{4}\sum_{\alpha,\beta} C^\dagger_\alpha C^\dagger_{\bar{\alpha}} C^\dagger_{\bar{\beta}} C_\beta - \hbar\omega \sum_{\alpha,\beta>0} \langle\alpha|j_x|\beta\rangle (C^\dagger_\alpha C_\beta - C^\dagger_{\bar{\alpha}} C_{\bar{\beta}}) \ . \tag{3}$$

Here α and $\bar{\alpha}$ are the Nilsson state and its time-reversal conjugate, ϵ_α the single-particle energy, λ the Fermi energy, G the pairing strength, and ω the angular frequency. We use Greek subscripts for states in the Nilsson representation.

In order to take into account the Coriolis force directly, we first diagonalize the Nilsson part and the Coriolis force of the Hamiltonian. Let the transformation be

$$C_\alpha = \sum_i B_{\alpha i} b_i, \quad C_{\bar{\alpha}} = \sum_i B_{\bar{\alpha}\bar{i}} b_{\bar{i}} \ . \tag{4}$$

We designate states in the b representation by Roman subscripts. It can be shown by simple matrix algebra that the eigenvalues ϵ_i and $\epsilon_{\bar{i}}$ are equal and

$$\begin{aligned} &B_{\alpha k} = B_{\bar{\alpha}\bar{k}} \quad \text{for } \alpha = k \ , \\ &B_{\alpha k} = -B_{\bar{\alpha}\bar{k}} \quad \text{for } \alpha \neq k \ , \\ &B^2_{\alpha k} = B^2_{\bar{\alpha}\bar{k}} \ . \end{aligned} \tag{5}$$

This greatly simplifies the formalism and the numerical calculation. It enables us to write the Hamiltonian in the b-representation as

$$H = \sum_{k>0}(\epsilon_k - \lambda)(b^\dagger_k b_k + b^\dagger_{\bar{k}} b_{\bar{k}}) - G \sum_{ijkl>0} R_{i\bar{j}} R_{\bar{k}l} b^\dagger_i b^\dagger_{\bar{j}} b_{\bar{k}} b_l \ , \tag{6}$$

where

$$\begin{aligned} R_{i\bar{j}} &= \sum_{\alpha>0} B_{\alpha i} B_{\bar{\alpha}\bar{j}} \ , \\ R_{\bar{i}j} &= \sum_{\alpha>0} B_{\bar{\alpha}\bar{i}} B_{\alpha j} \ . \end{aligned} \tag{7}$$

The R_{ij}'s have been computed as a function of ω. The result indicates that R_{ij} is strongly state dependent, and that $R^2_{i\bar{i}} \gg R^2_{i\bar{j}}$ by an order of magnitude for rotational frequency up to the backbending region corresponding to $\hbar\omega \simeq 0.3$ MeV. There is no guarantee that $R_{k\bar{k}}$'s are all positive definite, but this can be simply accomplished by redefining the phase of the single-particle states in the b representation.

When the rotation frequency ω is zero, the transformation (4) is a unity transformation with $B_{\alpha k} = \delta_{\alpha k}$ for all α. For small ω, the $R_{\alpha k}$'s are likewise small for $\alpha \neq k$. From the orthonormalization conditions for the transformation (4) and the relation (5), we have

$$\begin{aligned} R_k &\equiv R_{k\bar{k}} \\ &= 1 - 2\sum_{\alpha\neq k} B^2_{\alpha k} \ . \end{aligned} \tag{8}$$

Therefore, as long as ω remains sufficiently small, all the R_k's are positive.

From the above consideration, we can neglect all small terms in the Hamiltonian (6) so that it can be reduced to a standard form in the constrained system as

$$H = \sum_{k>0}(\epsilon_k - \lambda)(b^\dagger_k b_k + b^\dagger_{\bar{k}} b_{\bar{k}}) - G \sum_{i,k>0} R_i R_k b^\dagger_i b^\dagger_{\bar{i}} b_{\bar{k}} b_k \ . \tag{9}$$

The neglected terms can be included in the residual interaction which can be taken into account when we are improving the Hamiltonian. For small ω, the transformation coefficients $B_{\alpha i}$ of Eq. (4) can be expanded in terms of ω by perturbation method as

$$\begin{aligned} B_{ik} &= \frac{\langle i|j_x|k\rangle\omega}{\epsilon_i - \epsilon_k} \ , \\ B_{\bar{i}\bar{k}} &= \frac{\langle \bar{i}|j_x|\bar{k}\rangle\omega}{\epsilon_i - \epsilon_k} \ . \end{aligned} \tag{10}$$

Using Eq. (8), we have

$$R_k = 1 - 2\sum_{i\neq k}\left[\frac{\langle i|j_x|k\rangle\omega}{\epsilon_i - \epsilon_k}\right]^2 \ . \tag{11}$$

With Eq. (11), take the average over states for $R_i R_k$, and sum up terms to the second order in ω, the Hamiltonian (9) is then given as

$$H = \sum(\epsilon_k - \lambda)(b^\dagger_k b_k + b^\dagger_{\bar{k}} b_{\bar{k}}) - \bar{G}\sum_{i,k>0} b^\dagger_i b^\dagger_{\bar{i}} b_{\bar{k}} b_k \ , \tag{12}$$

where

$$\bar{G} = G\left\{1 - 4(\hbar\omega)^2\left[\sum_k\left(\frac{\langle i|j_x|k\rangle}{\epsilon_i - \epsilon_k}\right)^2_{\text{av}}\right]\right\} \ . \tag{13}$$

This is the well-known Mottelson-Valatin Coriolis antipairing effect for a rotating system.

It is standard procedure to diagonalize the Hamiltonian (12) by the quasiparticle approximation. The results are the following:

$$H = U_0 + \sum_{k>0} E_k(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) \ , \tag{14}$$

$$U_0 = \sum_{k>0}[(\eta_k + \tfrac{1}{2}\mu)v_k^2 - \tfrac{1}{2}u_k v_k \Delta], \quad E_k = \sqrt{\eta_k + \Delta^2} \ ,$$

$$b_k = u_k a_k + v_k a_{\bar{k}}^\dagger, \quad b_{\bar{k}} = u_k a_{\bar{k}} - v_k a_k^\dagger \ ,$$

$$\eta_k = (E_k - \mu - \lambda) = (u_k^2 - v_k^2)E_k \ ,$$

$$\Delta = \bar{G}\sum_{k>0} u_k v_k = 2u_k v_k E_k \ , \tag{15}$$

$$\mu = \bar{G}\sum_{k>0} v_k^2, \quad \mathcal{N} = \sum_{k>0} 2v_k^2 \ ,$$

$$u_k^2 = \frac{1}{2}\left[1 + \frac{\eta_k}{E_k}\right], \quad v_k^2 = \frac{1}{2}\left[1 - \frac{\eta_k}{E_k}\right] \ ,$$

where $\mathcal{N}$ is the total number of particles. The ground-state wave function is given as

$$\Psi(\omega) = \prod_{k>0}(u_k + v_k b_k^\dagger b_{\bar{k}}^\dagger)|0\rangle \ , \tag{16}$$

where $|0\rangle$ is the particle ground state.

III. ROTATING INTRINSIC STATES IN THE LAB SYSTEM

In Sec. II, we have obtained a complete set of rotational single-particle wave functions. This set of functions can be used as a set of basis vectors to express the wave function of any single particle in a rotating nucleus. In this representation, the Hamiltonian H_0 in the lab system is given as

$$\begin{aligned} H_0 = {} & U_0 + \omega \cdot J_x + \sum_{k>0} E_k(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) \\ & + \omega \sum_{i,k>0} [R_{ik} a_i^\dagger a_k \\ & + S_{ik} a_i^\dagger a_{\bar{k}} + T_{ik}(a_i^\dagger a_{\bar{k}}^\dagger - a_i a_{\bar{k}})] \ , \end{aligned} \tag{17}$$

with

$$j_{ik} = \langle i|j_x|k\rangle, \quad J_x = \sum_{k>0} v_k^2(j_{kk} + j_{\bar{k}\bar{k}}) \ , \tag{18}$$

$$R_{ik} = u_i u_k j_{ik} - v_i v_k j_{\bar{i}\bar{k}} \ , \tag{19}$$

$$S_{ik} = u_i u_k j_{\bar{i}\bar{k}} - v_i v_k j_{ik} \ , \tag{20}$$

$$T_{ik} = u_i v_k j_{ik} + v_i u_k j_{\bar{i}\bar{k}} \ . \tag{21}$$

Note that the states $|i\rangle$ and $|k\rangle$ are single-particle states in the constrained frame and, to first order in ω, we have

$$\langle i|j_x|k\rangle = \langle \bar{i}|j_x|\bar{k}\rangle \ \text{ for } i = k \ , \tag{22}$$

$$\langle i|j_x|k\rangle = -\langle \bar{i}|j_x|\bar{k}\rangle \ \text{ for } i \neq k \ . \tag{23}$$

In the constrained frame, j_x has nonvanishing diagonal matrix elements.

To diagonalize the Hamilton (17), we shall follow a method used by [23] and write the equations of motion for a_i and $a_i^\dagger$ ($i = 1, 2, , \ldots, N$, where N is the dimensionality of the single-particle space $|i\rangle$),

$$[a_i, H_0] = E_i a_i + \omega \sum_{k>0}(R_{ik} a_k + T_{ik} a_{\bar{k}}^\dagger) \ , \tag{24a}$$

$$[a_{\bar{i}}^\dagger, H_0] = -E_i a_{\bar{i}}^\dagger + \omega \sum_{k>0}(T_{ki} a_k - S_{ik} a_{\bar{k}}^\dagger) \ . \tag{24b}$$

When we use a real representation for j_x, Eq. (24) is a symmetrical matrix equation for the vector $(a_1, \ldots, a_N, a_{\bar{1}}^\dagger, \ldots, a_{\bar{N}}^\dagger)$. It can thus be diagonalized by an orthonormal transformation. Let the normal modes be written as

$$\beta_i = \sum_{k>0}(f_{ik} a_k + g_{ik} a_{\bar{k}}^\dagger) \ , \tag{25a}$$

$$\beta_{\bar{i}}^\dagger = \sum_{k>0}(f_{i\bar{k}} a_k^\dagger + g_{i\bar{k}} a_k) \ . \tag{25b}$$

The coefficients, f's and g's, can be determined by a simple diagonalization program for a real symmetric matrix. The Hamiltonian H_0 can be written in the β representation as

$$H_0 = W_0 + \sum_{k>0} W_k(\beta_k^\dagger \beta_k + \beta_{\bar{k}}^\dagger \beta_{\bar{k}}) \ , \tag{26}$$

where W_k is the eigenvalue of the kth normal mode of the matrix in Eq. (24), and W_0 is a constant of the new ground-state energy in the laboratory system which is

$$\begin{aligned} W_0 = {} & U_0 + \omega \cdot J_x + \omega \cdot \sum_{i,k>0} \Big[E_i(g_{ik}^2 + g_{\bar{i}\bar{k}}^2 + g_{\bar{i}k}^2) \\ & + T_{ik} \sum_{j>0}(g_{\bar{j}i} f_{j\bar{k}} - f_{ji} g_{j\bar{k}}) \Big] \ . \end{aligned} \tag{27}$$

The ground state $|\Psi_0\rangle$ of H_0 (for an even-mass system) is defined as

$$\beta_k|\Psi_0\rangle = 0 \ \text{ for any } k \ , \tag{28}$$

and accordingly, we have

$$|\Psi_0\rangle = A\prod_k(\beta_k \beta_{\bar{k}})|0\rangle \ , \tag{29}$$

where A is the normalization constant. When $\omega = 0$, $|\Psi_0\rangle$ is reduced to the Nilsson-BCS ground state (without rotation), and $W_0 = U_0(\omega = 0)$, the Nilsson-BCS ground-state energy. $|\Psi_0\rangle$ can be expressed in terms of the basis states of the constrained system as

$$|\Psi_0\rangle = |\Phi_0\rangle + \omega \cdot (2 \text{ qp states}) + \omega^2 \cdot (4 \text{ qp states}) + \cdots . \tag{30}$$

IV. NUCLEAR GROUND-STATE CRANKING MOMENT OF INERTIA

Suppose the nuclear Hamiltonian $\mathcal{H}$ can be transformed into an intrinsic part H_0, a rotational part H_r, and coupling term H_c

$$\mathcal{H} = \frac{R^2}{2\mathcal{F}} + H_0 + H_c , \tag{31}$$

where $\mathcal{F}$ is an operator involving the intrinsic coordinates of the various particles and H_0 depends only on the intrinsic particles. We are now considering the nuclear structure near the ground state, the coupling term H_c can be neglected. To find the connection between the intrinsic wave functions Ψ_n and the moment of inertia, we follow a derivation of de Shalit and Feshback [24], introduce an angle variable ϕ depending on the coordinates of all the particles and let ϕ satisfy the following assumptions.

(a) The angle ϕ describes a collective orientation of the nucleus in the sense that it commutes with all intrinsic coordinates

$$[\mathcal{F}, \phi] = [H_0, \phi] = 0 . \tag{32}$$

(b) ϕ is the conjugate variable to the collective angular momentum R in the sense that

$$[R, f(\phi)] = -if'(\phi) . \tag{33}$$

(c)

$$[R, \mathcal{F}] = 0, \quad [H_0, \mathcal{F}] = 0 . \tag{34}$$

With these assumptions, de Shalit and Feshback [24] have shown that the moment of inertia of the ground state $|\Psi_0\rangle$ is given as

$$\langle\Psi_0|\mathcal{F}|\Psi_0\rangle = 2\hbar^2 \sum_{W_n \neq W_0} \frac{|\langle\Psi_0|R|\Psi_n\rangle|^2}{W_n - W_0} , \tag{35}$$

where R is j_x because it is a rotation around the direction perpendicular to the nuclear symmetry axis. Substitute Eq. (29) and $|\Psi_n\rangle = \beta_i^+\beta_k^+|\Psi_0\rangle$ into (35) we have

$$\langle\Psi_0|\mathcal{F}|\Psi_0\rangle = 2\hbar^2 \sum_{i,k>0} \frac{|\langle\Psi_0|j_x\beta_i^\dagger\beta_k^\dagger|\Psi_0\rangle|^2}{W_i + W_k} . \tag{36}$$

For small ω, all the quantities can be expressed in terms

TABLE I. Theoretical and experimental values of the ground-state moments of inertia for rare-earth nuclei.

Nuclide					$\omega = 0$				$\omega \neq 0$		
	A	$\frac{2}{\hbar^2}\theta_{\text{exp}}$ $(\text{MeV})^{-1}$	$G_n \times A$ (MeV)	$G_p \times A$ (MeV)	Δ_n (MeV)	Δ_p (MeV)	$\frac{2}{\hbar^2}\theta_{\text{th}}$ $(\text{MeV})^{-1}$	$\hbar w$ (MeV)	Δ_n (MeV)	Δ_p (MeV)	$\frac{2}{\hbar^2}\theta_{\text{th}}$ $(\text{MeV})^{-1}$
Sm	152	49.2	16.24	19.63	1.108	1.083	40.3	0.026	0.982	1.017	48.9
	154	73.2	16.22	19.83	1.019	1.015	53.0	0.034	0.929	0.921	60.5
Gd	154	48.8	16.46	19.59	1.131	1.082	37.0	0.026	0.983	0.986	45.5
	156	67.4	16.45	19.79	1.051	0.021	49.4	0.031	0.941	0.924	56.6
	158	75.5	16.43	19.98	0.978	1.004	55.3	0.034	0.883	0.902	63.2
	160	79.7	16.42	20.18	0.903	1.009	58.7	0.039	0.787	0.882	68.9
Dy	160	69.0	16.65	19.94	1.011	1.029	47.7	0.031	0.772	0.914	62.9
	162	74.4	16.64	20.14	0.922	1.024	54.2	0.035	0.764	0.893	66.9
	164	81.8	16.63	20.33	0.839	1.027	59.2	0.042	0.652	0.856	75.3
Er	164	66.7	16.86	20.10	0.954	1.046	50.9	0.034	0.726	0.918	67.0
	166	74.5	16.84	20.29	0.870	1.050	56.8	0.042	0.632	0.879	76.7
	168	75.2	16.83	20.48	0.847	1.062	56.9	0.044	0.605	0.869	76.6
	170	75.6	16.82	20.67	0.844	1.079	56.6	0.044	0.625	0.872	76.5
Yb	170	71.2	17.05	20.44	0.892	1.111	52.6	0.037	0.715	0.965	66.8
	172	76.2	17.03	20.63	0.870	1.121	54.4	0.041	0.656	0.948	72.0
	174	78.5	17.02	20.82	0.860	1.130	54.5	0.043	0.641	0.945	73.6
	176	73.1	17.01	21.00	0.898	1.139	50.8	0.039	0.725	0.973	64.6
Hf	176	67.9	17.23	20.78	0.910	1.156	48.2	0.038	0.710	0.981	64.9
	178	64.4	17.22	20.97	0.927	1.155	44.5	0.038	0.682	0.954	62.2
	180	64.3	17.21	21.15	0.967	1.158	40.5	0.044	0.536	0.873	70.3
W	182	60.0	17.42	21.12	1.010	1.130	36.5	0.041	0.602	0.834	62.4
	184	54.1	17.41	21.30	1.041	1.126	32.4	0.036	0.776	0.855	49.7
	186	49.0	17.40	21.48	1.059	1.129	28.8	0.035	0.806	0.844	44.6

of functions of ω [23]. It can be shown in a straightforward manner that by taking only the lowest-order terms of (36) we have

$$\langle\Psi_0|\mathcal{F}|\Psi_0\rangle = 2\hbar^2 \sum_{i,k>0} \frac{|u_i v_k j_{ik} + v_i u_k j_{\bar{i}\bar{k}}|^2}{E_i + E_k} . \quad (37)$$

V. NUMERICAL CALCULATIONS, RESULTS, AND DISCUSSION

In performing numerical calculations, we take an axially symmetric Nilsson model from [25] and neglect the P_4 force. The major shells included are $N = 3$, 4, 5, and 6 for protons and $N = 4$, 5, 6, and 7 for neutrons. This larger dimensionality is important for the calculation since otherwise the single-particle basis states may not be complete enough. Especially for rare-earth nuclei with higher mass number, the occupation of single-particle orbitals may easily go up to $j_{15/2}$. For the deformation parameter δ we take from [15]. For the pairing strength parameter G we use the prescription of [25] as

$$G \times A = g_0 \pm g_1 \frac{N-Z}{A} , \quad (38)$$

where the plus sign holds for protons and the minus sign for neutrons. In [25] the authors also emphasize the surface dependence of the pairing strength G as the following. In order to reproduce the indicated large energy gap at the fission saddle point [26] one must assume [27] an increase of G with surface area. For a small deviation in G from a "normal" value G_0, they put

$$G = G_0 + \delta G \cong G_0 + G_0 \frac{\delta S}{S} , \quad (39)$$

where S is the surface area of the nucleus. If the nature of Eq. (39) exists for the G in a nucleus, there is no reason that it would not exist for nuclei with different mass number A, even the effect is small. We shall suggest that this small variation of G should be included in Eq. (38) with $S \sim A^{2/3}$.

The general form of this dependence of G on S may be complicated which may depend on the shell structure. However, we are not only interested in making an estimated correction of this dependence within a region of nuclei which the shell structure of those nuclei are similar. The simplest way is to directly take Eq. (39) and obtain

$$G \times A = \left[g_0 \pm g_1 \frac{N-Z}{A}\right]\left(1 + g_2 \frac{A^{2/3} - A_0^{2/3}}{A_0^{2/3}}\right) . \quad (40)$$

It was found that, with our single-particle basis space, we could reproduce reasonably well the empirical odd-even mass differences by using Eq. (40) from $A = 152$ to 189. The values of $G \times A$, Δ_N and Δ_p for rare-earth nuclei when $\omega = 0$ are given in Table I, using $g_0 = 18.7$ MeV, $g_1 = 9.6$ MeV, and $g_2 = 0.6$.

The experimental values of nuclear moments of inertia Θ are extracted according to

$$2\Theta/\hbar^2 = \frac{\Delta[J(J+1)]}{\Delta E(J)} . \quad (41)$$

Since only integral values of J can exist and $\Delta J = 2$, Eq. (41) for ground-state moments of inertia becomes

$$2\Theta/\hbar^2 = \frac{2(2+1)}{E(J=2) - E(J=0)} . \quad (42)$$

For theoretical calculation, we cannot solve the complete Hamiltonian $\mathcal{H}$ of Eq. (31) to calculate the moment of inertia for Eq. (42). We have to use Eq. (37) through the use of the intrinsic wave function Ψ_0. For $\omega = 0$, we have $\langle j_x\rangle = 0$, there is no rotation and it is meaningless to talk about moment of inertia. Therefore we calculate (37) by using the ground state with the lowest J, which is $J = 2$, to find the corresponding Θ for Eq. (42). That is, we have to find the value of ω such that we have the

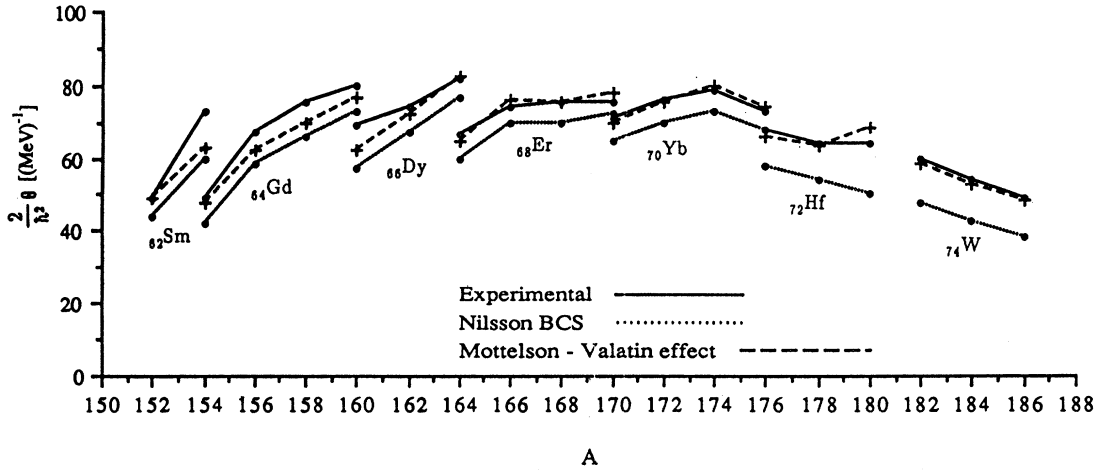

FIG. 1. Comparison of the calculated ground-state cranking moments of inertia and the experimental observations for rare-earth nuclei.

ground state $\Psi(\omega, J = 2)$ of the intrinsic system. To do this, we construct a $\Psi(\omega)$ and calculate $\langle\Psi|j_x|\Psi\rangle$. Then we vary ω until $\langle\Psi|j_x|\Psi\rangle = 2$ [if the magnitude of the angular moment is $\sqrt{J(J+1)}$, its maximum projection in one direction is J].

The calculated values of the Δ's and Θ's for rare-earth nuclei at $\omega = 0$ and $\omega = \omega(J = 2)$, together with the experimental values for Θ are shown in Table I. Those values for $\Theta(\omega)$, $\Theta(0)$, and $\Theta(\text{exp})$ are shown in Fig. 1. In Table I, we can see, as it is expected, that the values of Δ's are decreased as a result of the Mottelson-Valatn effect due to rotation. We can also see that the overall behavior of the calculated values of ground-state moments of inertia for rare-earth nuclei have actually been improved. If one wishes to improve the calculation further, solutions of $\mathcal{H}$ with good angular momenta and energies should be obtained by using angular momentum projection and Θ should be calculated according to Eq. (41). In this way, the results should be, in principle, better and quantitatively more accurate. However, all of the quantities would be computed numerically and the simple analytical form of the cranking formula would no longer exist. Nevertheless, a treatment like the present work can be considered as an approximation method which has more analytical aspects of a problem such that we can use it easier to study the physical qualitative features of a system.

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