Photoelastic modulation polarimetry and its measurement of Twisted Nematic Liquid crystal

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ABSTRACT

For *in situ*/real time measurement, a retarder is substituted by a photoelastic modulator (PEM) in a polarimetry. The azimuthal position of the strain axis of PEM is directly determined with respect to the orientation of the transmission axis of polarizer and analyzer. The Mueller matrix of a twisted nematic liquid crystal (TN-LCD) is derived analytically. The phase retardation and the twisted angle of a TN-LCD are numerically obtained through two successive measurements of the Mueller polarimeter.

Keyword: polarimetry, photoelastic modulation, circular dichroism, liquid crystal

INTRODUCTION

Polarimetry has been well established[1] for measuring the anisotropic bulk medium, which includes the optical rotation (OR); circular dichroism (CD); linear birefringence (LB) and linear dichroism (LD). For measuring the circular anisotropic properties of the medium, one has to introduce a wave plate into the polarimetric system in addition to the analyzer and polarizer. Recently, the phase modulator has been used to substitute the phase retarder in the polarimetry, and speed up the polarization measurement process.[2-5] Basically, the polarimetric technique measures the intensity under varies polarization conditions, this may be why the Jones calculus has been supplanted by Stokes-Mueller matrices,[6,7] recently. Although, the Mueller matrix is more complicated than the Jones matrix, one can put the circular and linear eigenpolarization in the same Mueller matrix but not in Jones matrix. In this paper, we derived a generalized Mueller matrix from Jones matrix through the symbolic program of Mathematica by using the Pauli Spin matrices.[8] This effort drastically reduces the complication of deriving the intensity under a special polarization condition. For measuring the TN-LCD, we derived its Mueller matrix and found one element in the matrix is insensitive to its twisted angle. This element provides us the information of phase retardation. Use this information we deduce the twisted angle of the TN-LCD by successively measuring another element of the matrix.

BACKGROUND

1. The calibration of PEM

The most essential problem in polarization measurements is to calibrate the intrinsic polarization parameters in the polarimetry. From Fig. 1, the in-situ/real time polarimetry consists the polarizer (P), photoelastic modulator (PEM), sample (S) and analyzer (A). Light (L) is transmitted through the system, measured by a detector (D) and recorded in PC for analyzing. Before inserting the sample and PEM, we align the azimuths of polarizer and analyzer by the three-intensity technique[9], which is developed previously. The azimuth and the static retardation of PEM are calibrated as follows; as P = 45°, the measured intensity is

$$I(A) = I_0\{1 + \cos 2(A - C)\cdot \sin 2C + \cos\Delta \cdot \cos 2C \cdot \sin 2(A - C)\} \quad (1)$$

where A and C are the azimuths of analyzer and the strain axis of PEM, respectively. The phase of PEM is modulated as

$$\Delta = \Delta_0 \cdot \cos wt + \Delta_i \, .$$

Under this modulation, the intensity can be expanded in the following Bessel-function series

$$\sin\Delta = 2\sum J_{2k+1}(\Delta_0)\cdot \sin((2k+1)\cdot wt.$$

$$\cos\Delta = J_0(\Delta_0) + 2\sum J_{2k}(\Delta_0)\cdot \cos(2kwt)\,.$$

The DC component of intensity is used to calibrate the azimuth position of the strain axis of PEM. Since

$$I_{dc}(A) = I_o\{1 + \cos 2(A - C)\sin 2C + J_o(\Delta_o)\cos\Delta_i \cos 2C \sin 2(A - C)\}$$

we can use the three-intensity technique[9] for extracting the azimuth angle and $J_o(\Delta)\cos\Delta_\iota$ of PEM. Let

$$I_o = [I_{dc}(0) + I_{dc}(60) + I_{dc}(120)]/3$$
$$I_+ = [I_{dc}(60) + I_{dc}(120)]/I_o$$

$$I_- = [I_{dc}(60) - I_{dc}(120)] / I_o$$

we can prove

$$tan\,2C = -\frac{I_+ - 2}{1 - I_- / \sqrt{3}} \tag{2}$$

$$J_o(\Delta_o)\cos\Delta_i = (\frac{I_-}{\sqrt{3}})(1 + tan^2\,2C) - tan^2\,2C \tag{3}$$

By varying the amplitude of phase modulation, we are able to locate the zero point of the zero order Bessel function through equation (3).

2. The PEM Mueller Polarimetry

Let the Mueller matrix of a medium be

$$G = \begin{bmatrix} M_{00} & M_{01} & M_{02} & M_{03} \\ M_{10} & M_{11} & M_{12} & M_{13} \\ M_{20} & M_{21} & M_{22} & M_{23} \\ M_{30} & M_{31} & M_{32} & M_{33} \end{bmatrix}$$

In a P-PEM-Sample setup, while P = 45° and C = 0°, the measured intensities of 1f and 2f are

$$\frac{I_{1f}}{I_{dc}} = \frac{2J_1(\Delta_0)\cdot M_{03}/M_{00}}{1 + J_0(\Delta_0)\cdot M_{02}/M_{00}} \quad \text{and} \quad \frac{I_{2f}}{I_{dc}} = \frac{-2J_2(\Delta_0)\cdot M_{02}/M_{00}}{1 + J_0(\Delta_0)\cdot M_{02}/M_{00}},$$

respectively. For simplicity, the measured intensities are always taken at the zero-point of zero order Bessel function. This configuration can measure the circular and linear dichroism of the medium. By adding an analyzer, one can prove that the intensity distribution under varies azimuth angle of analyzer is

$$I(A) = I_o\{M_{00} + M_{10}\cos 2A + M_{20}\sin 2A + \cos\Delta[M_{02} + M_{12}\cos 2A + M_{22}\sin 2A] + \sin\Delta[M_{03} + M_{13}\cos 2A + M_{23}\sin 2A]\}$$

when A= 0°, we have

$$\frac{I_{1f}}{I_{dc}} = \frac{2J_1(\Delta_0)\cdot M_{13}/M_{00}}{1 + J_0(\Delta_0)\cdot M_{22}/M_{00}}; \quad and \quad \frac{I_{2f}}{I_{dc}} = \frac{-2J_2(\Delta_0)\cdot M_{12}/M_{00}}{1 + J_0(\Delta_0)\cdot M_{22}/M_{00}}. \tag{4}$$

While A= 45°, we have

$$\frac{I_{1f}}{I_{dc}} = \frac{2J_1(\Delta_0)\cdot M_{23}/M_{00}}{1+J_0(\Delta_0)\cdot M_{22}/M_{00}}; \quad and \qquad \frac{I_{2f}}{I_{dc}} = \frac{-2J_2(\Delta_0)\cdot M_{22}/M_{00}}{1+J_0(\Delta_0)\cdot M_{22}/M_{00}}. \tag{5}$$

From above analysis, one can obtain the 16 elements by rotating the sample. It is our interests to measure and compare an elliptical retarder to a linear retarder in this paper. The Jones matrix of a TN-LCD is expressed as[10]

$$M = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix} \cdot \begin{bmatrix} \cos\chi - i\frac{\Gamma\sin\chi}{2\chi} & \phi\frac{\sin\chi}{\chi} \\ -\phi\frac{\sin\chi}{\chi} & \cos\chi + i\frac{\Gamma\sin\chi}{2\chi} \end{bmatrix} \tag{6}$$

where the twisted angle (ϕ) and phase retardation (Γ) are related by χ as

$$\chi = \sqrt{\phi^2 + (\frac{\Gamma}{2})^2}.$$

When $\phi = 90^o$, the Jones matrix can be converted to a Mueller matrix through Pauli-Spin matrices and it is expressed as follows

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1-2(\frac{\Gamma}{2}\frac{\sin x}{x})^2 & -\frac{\pi\sin 2x}{2x} & \frac{\pi\Gamma\sin^2 x}{2x^2} \\ 0 & \frac{\pi\sin 2x}{2x} & -\cos 2x & \frac{\Gamma\sin 2x}{2x} \\ 0 & -\frac{\pi\Gamma\sin^2 x}{2x^2} & \frac{\Gamma\sin 2x}{2x} & 1-2(\frac{\Gamma}{2}\frac{\sin x}{x})^2 \end{bmatrix},$$

which is very similar to an elliptical retarder. The twisted angle and retardation can be obtained by measuring any two elements. For accurately measure the twisted angle and phase retardation, we need to calculate the analytical solution of Mueller matrix for eq. (4) and (5). Only the significant terms are listed as follows

$$m_{13} = \frac{(x\cdot\sin 2\phi\cdot\cos x - \phi\cdot\sin x\cdot\cos 2\phi)\cdot\Gamma\cdot\sin x}{x^2},$$

$$m_{23} = \frac{-(x \cdot \cos 2\phi \cdot \cos x + \phi \cdot \sin x \cdot \sin 2\phi) \cdot \Gamma \cdot \sin x}{x^2}.$$

By numerically analyzing the terms around $\phi = 90^o$, we find the m_{23} is lease sensitive to the twisted angle, such as shown in Fig. 2. Because the phase retardation periodically intersects with the measured value, the phase retardation needs to be estimated prior to an accurate determination.

EXPERIMENTAL SETUP

Fig. 1 shows the experimental setup. A He-Ne laser is used as the light source for the P-PEM-S or P-PEM-S-A systems. The azimuths of polarizer and analyzer are aligned by the three-intensity technique, then set P = 45°. The PEM (Hinds PEM-90) is inserted and aligned. Its strain axis is set at 0° and the zero-point of zero order Bessel function is located by varying the amplitude of modulation from 0.535 λ to 0.413λ by a step 0.005λ. A quartz quarter wave plate and a TN-LCD (twisted angle is about 90°, 6μm thick, 99% E7 and 1% CB5) are measured successively by setting A= 0 and 45°. Two lock-in amplifiers are used for measuring dc, 1f and 2f intensities simultaneously in order to obtain a real time measurement.

RESULTS

According to eqs. (2) and (3), we calculate the position of strain axis (2.73 ± 0.06^o) and $J_o(\Delta)\cos\Delta_i$ by the dc measurements. The distribution of zero-order Bessel function under varies amplitude modulation is plotted in Fig. 3. In the experiment, we did observe the temperature effect on the zero-point of zero order Bessel function of this PEM. For this experiment, the zero point is at 0.388λ instead of 0.383λ. According to the measurements of a quartz quarter wave plate, we obtain its phase retardation to be 83.40° instead of 86.75°[3]. This discrepancy can be improved by considering the intrinsic phase retardation (3.35 ± 0.05^o) of the PEM. Since the elements of Mueller matrix of TN-LCD is more complicated than the linear retarder, we graphically locate the phase retardation and twisted angle by zooming in the plots, such as shown in Fig. 4. From the intersection of measured value and simulated value, one can determine the twisted angle to be 89.2 ± 0.1^o; the phase retardation is 12.585 ± 0.002.

CONCLUSIONS

The Mueller polarimetry can measure all kinds of media, which include the circular and linear birefringence anisotropy. If we can use more than one detecting system[11], the PEM polarimeter can measure every parameter in the Mueller matrix in real time. It is our future interest to observe the switching property of the TN-LCD. Using the Pauli-spin matrices in the symbolic program of mathematica benefits us greatly. We hope to explore more by this powerful tool.

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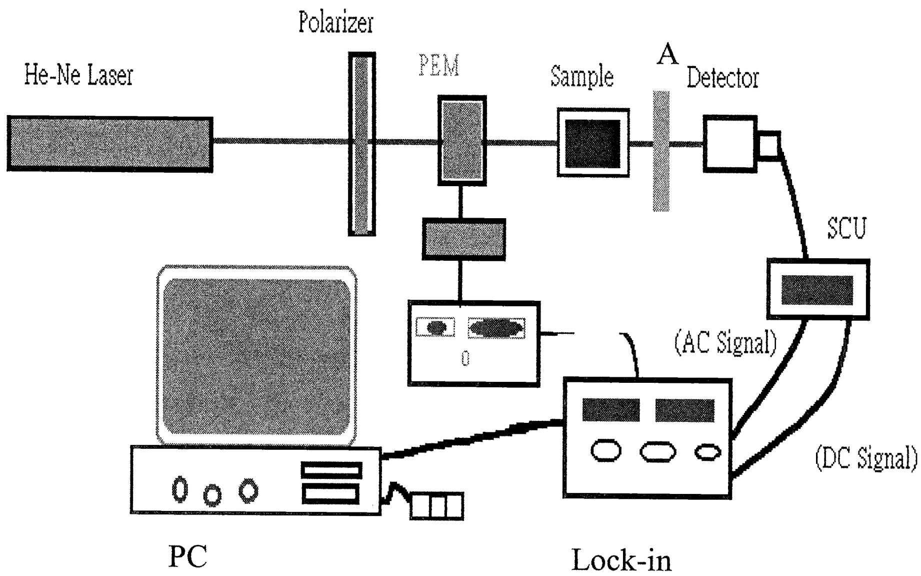
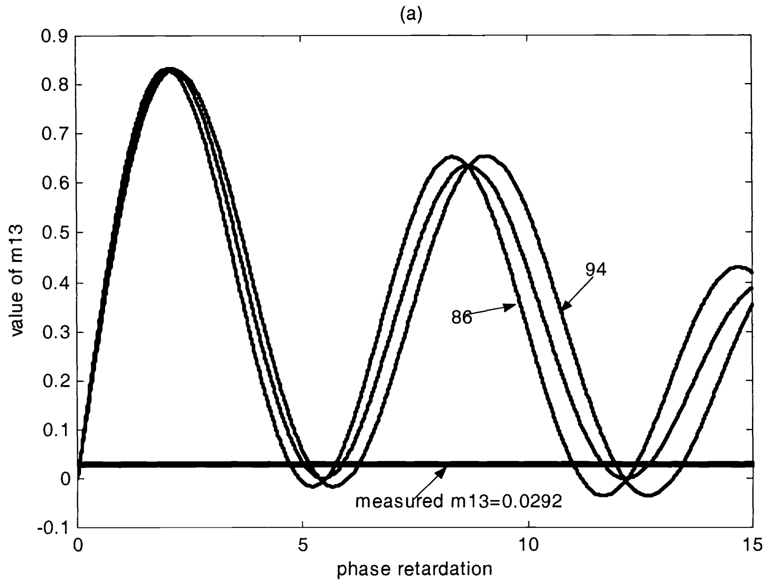


Fig. 1 Schematic setup of PEM Polarimeter



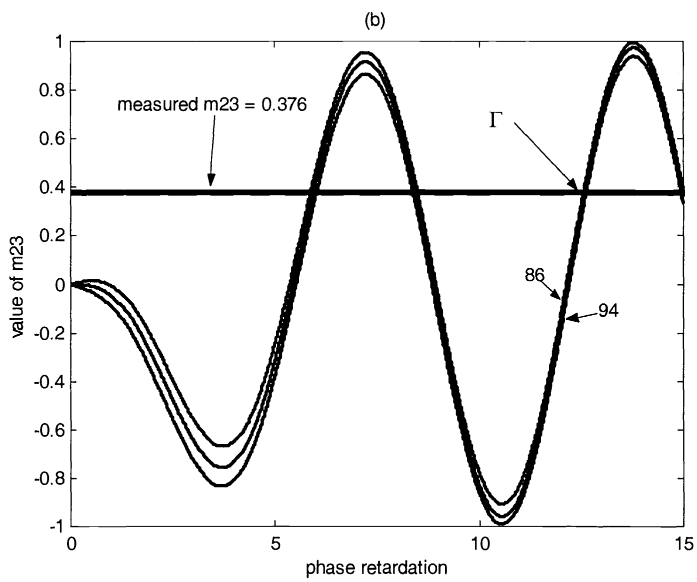


Fig. 2 The simulated value of the element of (a) m13 and (b) m23 for TN-LCD under varies phase retardation and twisted angle

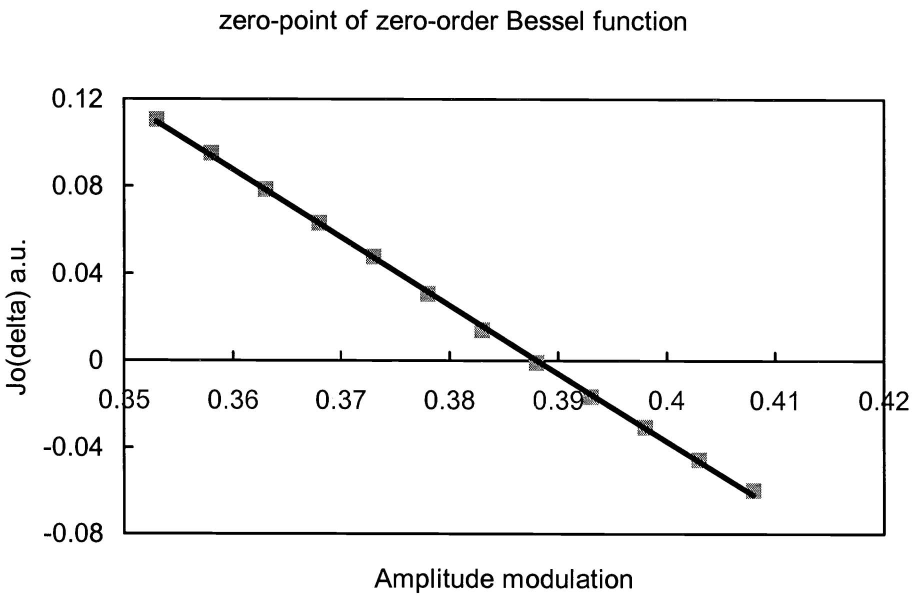


Fig. 3 Zero order Bessel function under varies amplitude of modulation

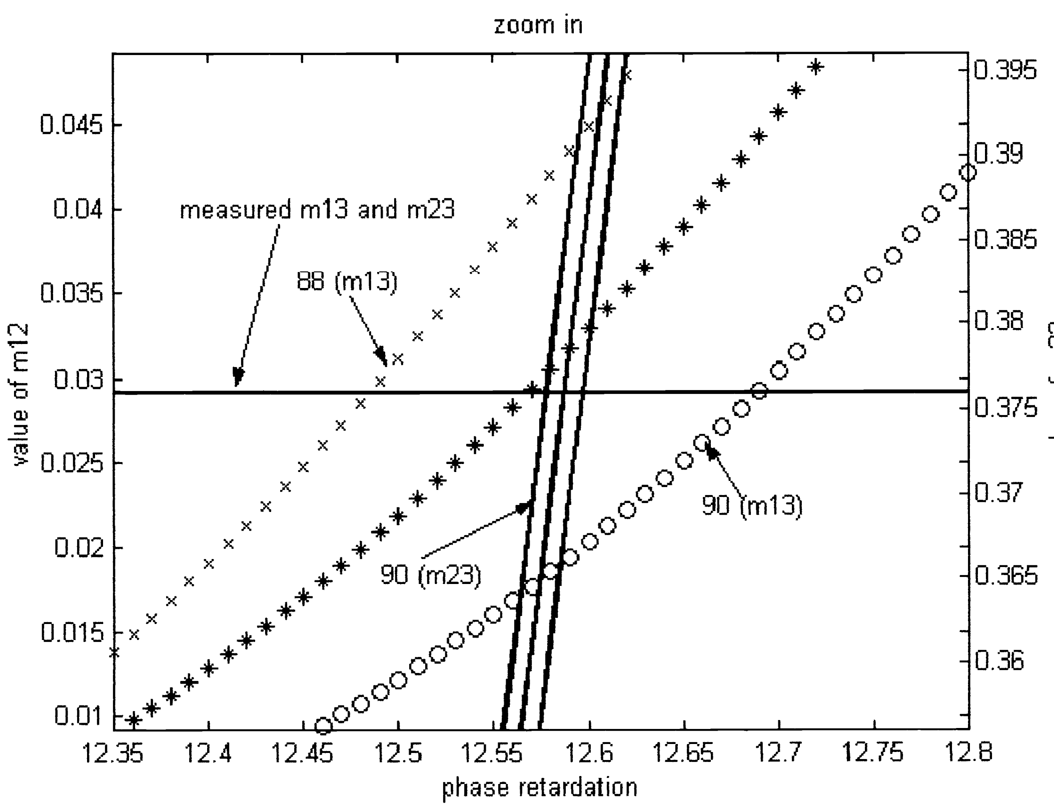


Fig. 4 Graphical relation of the simulated value of m13 and m23 for TN-LCD under varies phase retardation and twisted angle