# On the Extremal Number of Edges in Hamiltonian Graphs*

TUNG-YANG HO+, CHENG-KUAN LIN[1], JIMMY J. M. TAN[1], D. FRANK HSU[2] AND LIH-HSING HSU[3]

+*Department of Information Management*
*Ta Hwa Institute of Technology*
*Hsinchu, 307 Taiwan*
*E-mail: hoho@thit.edu.tw*
[1]*Department of Computer Science*
*National Chiao Tung University*
*Hsinchu, 300 Taiwan*
[2]*Department of Computer and Information Science*
*Fordham University*
*New York, NY 10023, U.S.A.*
[3]*Department of Computer Science and Information Engineering*
*Providence University*
*Taichung, 433 Taiwan*

Assume that $n$ and $\delta$ are positive integers with $2 \le \delta < n$. Let $h(n, \delta)$ be the minimum number of edges required to guarantee an $n$-vertex graph with minimum degree $\delta(G) \ge \delta$ to be hamiltonian, *i.e.*, any $n$-vertex graph $G$ with $\delta(G) \ge \delta$ is hamiltonian if $|E(G)| \ge h(n, \delta)$. We prove that $h(n, \delta) = C(n - \delta, 2) + \delta^2 + 1$ if $\delta \le \left\lfloor \frac{n+1+3\times((n+1) \bmod 2)}{6} \right\rfloor$, $h(n, \delta) = C(n - \left\lfloor \frac{n-1}{2} \right\rfloor, 2) + \left\lfloor \frac{n-1}{2} \right\rfloor^2 + 1$ if $\left\lfloor \frac{n+1+3\times((n+1) \bmod 2)}{6} \right\rfloor < \delta \le \left\lfloor \frac{n-1}{2} \right\rfloor$, and $h(n, \delta) = \left\lceil \frac{n\delta}{2} \right\rceil$ if $\delta > \left\lfloor \frac{n-1}{2} \right\rfloor$.

***Keywords:*** complete graph, cycle, hamiltonian, hamiltonian cycle, edge-fault tolerant hamiltonian

## 1. INTRODUCTION

Throughout this paper, we use $C(a, b)$ to denote the number of combinations of "$a$" numbers taking "$b$" numbers at a time, where $a$, $b$ are positive integers and $a \ge b$. For the graph definitions and notations, we follow [1]. Let $G = (V, E)$ be a simple graph if $V$ is a finite set and $E$ is a subset of $\{(u, v) | (u, v)$ is an unordered pair of $V\}$. We say that $V$ is the *vertex set* and $E$ is the *edge set*. Two vertices $u$ and $v$ are *adjacent* if $(u, v) \in E$. The *complete graph* $K_n$ is the graph with $n$ vertices such that any two distinct vertices are adjacent. The *degree* of a vertex $u$ in $G$, denoted by $\deg_G(u)$, is the number of vertices adjacent to $u$. We use $\delta(G)$ to denote $\min\{\deg_G(u) | u \in V(G)\}$. We use $c(G)$ to denote the number of connected components in $G$. A *path*, $\langle v_0, v_1, \ldots, v_{m-1}\rangle$, is an ordered list of distinct vertices such that $v_i$ and $v_{i+1}$ are adjacent for $0 \le i \le m - 2$. A *cycle* is a path with at least three vertices such that the first vertex is the same as the last one. A *hamiltonian cycle* of $G$ is a

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+ Corresponding author.

cycle that traverses every vertex of $G$ exactly once. A graph is *hamiltonian* if it has a hamiltonian cycle.

In the past years, the studies on hamiltonian graphs have largely focused on their relationship to the Four Color Problem. More recently, the study of hamiltonian cycle in general graphs has been fueled by practical applications and by the issue of complexity. No easily testable characterization is known for hamiltonian graphs. Some sufficient conditions have been investigated. Two milestones of these sufficient conditions are obtained by Ore and Dirac. Ore [9] proved that any $n$-vertex graph with at least $C(n, 2) - (n - 3)$ edges is hamiltonian, and there exists an $n$-vertex non-hamiltonian graph with $C(n, 2) - (n - 2)$ edges. Dirac [4] obtains the following sufficient condition based on the minimum degree.

**Theorem 1** Let $G$ be an $n$-vertex graph with $n \geq 3$ and $\delta(G) \geq \frac{n}{2}$. Then $G$ is hamiltonian. Moreover, there exists an $n$-vertex non-hamiltonian graph $G$ with $\delta(G) < \frac{n}{2}$.

Erdős [5] presents the following sufficient condition based on the combination of the number of edges and the minimum degree.

**Theorem 2** Let $G$ be an $n$-vertex graph with $n \geq 6\delta(G)$. Then $G$ is hamiltonian if $|E(G)| > C(n - \delta(G), 2) + \delta(G)^2$.

In this paper, we consider a result in a setting more general than Theorem 2. Our result (Theorem A) will also include Theorem 1 as a special case. Since a graph $G$ with $\delta(G) = 1$ is not hamiltonian, we consider graph $G$ with $\delta(G) \geq 2$ in the following. Assume that $n$ and $\delta$ are positive integers with $2 \leq \delta < n$. Let $h(n, \delta)$ be the minimum number of edges required to guarantee an $n$-vertex graph with $\delta(G) \geq \delta$ to be hamiltonian. So any $n$-vertex graph $G$ with $\delta(G) \geq \delta$ is hamiltonian if $|E(G)| \geq h(n, \delta)$. We will prove the following theorem.

**Theorem A** Assume that $n$ and $\delta$ are positive integers with $2 \leq \delta < n$. Then

$$h(n,\delta) = \begin{cases} C(n-\delta, 2) + \delta^2 + 1 & \text{if } \delta \leq \left\lfloor \frac{n+1+3\times((n+1) \bmod 2)}{6} \right\rfloor, \\ C\left(n - \left\lfloor \frac{n-1}{2} \right\rfloor, 2\right) + \left\lfloor \frac{n-1}{2} \right\rfloor^2 + 1 & \text{if } \left\lfloor \frac{n+1+3\times((n+1) \bmod 2)}{6} \right\rfloor < \delta \leq \left\lfloor \frac{n-1}{2} \right\rfloor, \\ \left\lceil \frac{n\delta}{2} \right\rceil & \text{if } \delta > \left\lfloor \frac{n-1}{2} \right\rfloor. \end{cases}$$

Roughly speaking, $h(n, \delta)$ depends on $n$ and $\delta$ when $\delta \leq \frac{n}{6}$ or $\delta > \frac{n}{2}$, $h(n, \delta)$ depends only on $n$ when $\frac{n}{6} < \delta \leq \frac{n}{2}$. The latter is our main contribution. We use an example to illustrate Theorem A with the case that $n = 16$.

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 | 16 |
| $\delta$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| $h(n, \delta)$ | 96 | 88 | 86 | 86 | 86 | 86 | 64 | 72 | 80 | 88 | 96 | 104 | 112 | 120 |

Any 16-vertex graph $G$ with $\delta(G) \geq 2$ is hamiltonian if $|E(G)| \geq 96$, with $\delta(G) \geq 3$ is hamiltonian if $|E(G)| \geq 88$, with $\delta(G) \geq 4, \ldots, 7$ is hamiltonian if $|E(G)| \geq 86$, with $\delta(G) \geq 8$ is hamiltonian if $|E(G)| \geq 64$, with $\delta(G) \geq 9$ is hamiltonian if $|E(G)| \geq 72$, with $\delta(G) \geq 15$ is hamiltonian if $|E(G)| \geq 120$.

We compare our result Theorem A with Theorems 1 and 2, and make some remarks here. Theorem 1 considers the case $\delta \geq \frac{n}{2}$, which is the same as the case $\delta > \left\lfloor \frac{n-1}{2} \right\rfloor$. And Theorem 2 discusses the case $\delta \leq \frac{n}{6}$. We notice that there are still cases for $\frac{n}{6} < \delta \leq \left\lfloor \frac{n-1}{2} \right\rfloor$ left open. So our result fills up the gap and unifies the previous results.

We defer the proof of Theorem A to section 4. We first give an application of Theorem A, which is the original motivation of this paper. In particular, we establish the relationship between $h(n, g)$ and $g$-conditional edge-fault tolerant hamiltonicity of the complete graph $K_n$. Then we give some preliminary results in section 3. Finally, section 4 gives the proof of Theorem A.

## 2. APPLICATIONS

A hamiltonian graph $G$ is *k edge-fault tolerant hamiltonian* if $G - F$ remains hamiltonian for every $F \subset E(G)$ with $|F| \leq k$. The *edge-fault tolerant hamiltonicity*, $\mathcal{H}_e(G)$, is defined as the maximum integer $k$ such that $G$ is $k$ edge-fault tolerant hamiltonian if $G$ is hamiltonian and is undefined otherwise. Assume that $G$ is a hamiltonian graph and $x$ is a vertex such that $\deg_G(x) = \delta(G)$. We arbitrary choose $\deg_G(x) - 1$ edges from those edges incident to $x$ to form an edge faulty set $F$. Obviously, $\deg_{G-F}(x) = 1$ and hence, $G - F$ is not hamiltonian. Therefore, $\mathcal{H}_e(G) \leq \delta(G) - 2$ if $\mathcal{H}_e(G)$ is defined. It is easy to check that $\mathcal{H}_e(K_n) = n - 3$ for $n \geq 3$. In Latifi *et al.* [7], it is proved that $\mathcal{H}_e(Q_n) = n - 2$ for $n \geq 2$ where $Q_n$ is the $n$-dimensional hypercube. In Li *et al.* [8], it is proved that $\mathcal{H}_e(S_n) = n - 3$ for $n \geq 3$ where $S_n$ is the $n$-dimensional star graph.

Chan and Lee [2] began the study of the existence of hamiltonian cycle in a graph such that each vertex is incident to at least $g$ nonfaulty edges. A graph $G$ is *g-conditional k edge-fault tolerant hamiltonian* if $G - F$ is hamiltonian for every $F \subset E(G)$ with $|F| \leq k$ and minimum degree $\delta(G - F) \geq g$. The *g-conditional edge-fault tolerant hamiltonicity*, $\mathcal{H}_e^g(G)$, is defined as the maximum integer $k$ such that $G$ is $g$-conditional $k$ edge-fault tolerant hamiltonian if $G$ is hamiltonian and is undefined otherwise. Chan and Lee [2] proved that $\mathcal{H}_e^g(Q_n) \leq 2^{g-1}(n - g) - 1$ for $n > g \geq 2$ and the equality holds for $g = 2$.

Fu [6] studied the 2-conditional edge-fault tolerant hamiltonicity of the complete graph. The following result is in [6]:

> Suppose $F \subset E(K_n)$ and $\delta(K_n - F) \geq 2$, where $n \geq 4$. If $n \notin \{7, 9\}$ (respectively, $n \in \{7, 9\}$) then $K_n - F$ is hamiltonian, where $|F| \leq 2n - 8$ (respectively, $|F| \leq 2n - 9$).

In the conclusion of [6], it is claimed that the above statement is optimal. We restate this result using our terminology.

> $\mathcal{H}_e^2(K_n) = 2n - 8$ for $n \notin \{7, 9\}$ and $n \geq 4$, $\mathcal{H}_e^2(K_7) = 5$, and $\mathcal{H}_e^2(K_9) = 9$.
> Yet, it is easy to check that $\mathcal{H}_e^2(K_3)$ is 0 and $\mathcal{H}_e^2(K_4)$ is 2 (not 0).

Now, we extend the result in [6] and use our main result Theorem A to compute $\mathcal{H}_e^g(K_n)$ for any $1 \le g < n$.

**Theorem 3** $\mathcal{H}_e^g(K_n) = C(n, 2) - h(n, g)$ for any $1 \le g < n$.

***Proof:*** Let $F$ be any faulty edge set of $K_n$ with $|F| \le C(n, 2) - h(n, g)$ such that $\delta(K_n - F) \ge g$. Obviously, $|E(K_n - F)| \ge h(n, g)$. By Theorem A, $K_n - F$ is hamiltonian. Thus, $\mathcal{H}_e^g(K_n) \ge C(n, 2) - h(n, g)$.

Now, we prove that $\mathcal{H}_e^g(K_n) \le C(n, 2) - h(n, g)$. Assume that $\mathcal{H}_e^g(K_n) \ge C(n, 2) - h(n, g) + 1$. Let $G$ be any graph with $h(n, g) - 1$ edges such that $\delta(G) \ge g$. Let $F$ be $E(K_n) - E(G)$. In other words, $G = K_n - F$. Obviously, $|F| = C(n, 2) - h(n, g) + 1$. Since $\mathcal{H}_e^g(K_n) \ge C(n, 2) - h(n, g) + 1$, $G$ is hamiltonian. This contradicts to the definition of $h(n, g)$. Thus, $\mathcal{H}_e^g(K_n) \le C(n, 2) - h(n, g)$.

Therefore, $\mathcal{H}_e^g(K_n) = C(n, 2) - h(n, g)$ for any $1 \le g < n$. ❑

## 3. PRELIMINARY RESULTS

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. The *union* of $G_1$ and $G_2$, $G_1 + G_2$, has edge set $E_1 \cup E_2$ and vertex set $V_1 \cup V_2$ with $V_1 \cap V_2 = \phi$. The *join* of $G_1$ and $G_2$, $G_1 \vee G_2$, is obtained from $G_1 + G_2$ by joining each vertex of $G_1$ to each vertex of $G_2$.

For $1 \le m < n/2$, let $C_{m,n}$ be the graph $(\overline{K}_m + K_{n-2m}) \vee K_m$ and $S$ be the set of vertices in $K_m$; *i.e.*, $C_{m,n} - S = \overline{K}_m + K_{n-2m}$. We know that $c(C_{m,n} - S) = m + 1 > |S|$. Therefore, $C_{m,n}$ is not hamiltonian.

The *degree sequence* of an $n$-vertex graph is the list of vertices degree, in nondecreasing order, as $d_1 \le d_2 \le \ldots \le d_n$. A sequence of real numbers $(p_1, p_2, \ldots, p_n)$ is said to be *majorised* by another sequence $(q_1, q_2, \ldots, q_n)$ if $p_i \le q_i$ for $1 \le i \le n$. A graph $G$ is *degree-majorised* by a graph $H$ if $|V(G)| = |V(H)|$ and the nondecreasing degree sequence of $G$ is majorised by that of $H$. For instance, the 5-cycle is degree-majorised by the complete bipartite graph $K_{2,3}$ because (2, 2, 2, 2, 2) is majorised by (2, 2, 2, 3, 3).

Chvátal [3] points out that the family of degree-maximal non-hamiltonian graphs (those are not degree-majorised by others) are exactly $C_{m,n}$'s, *i.e.*, any $n$-vertex non-hamiltonian graph is degree-majorised by some $C_{m,n}$.

**Corollary 1** Let $n \ge 5$. Assume that $G$ is an $n$-vertex non-hamiltonian graph. Then $\delta(G) \le \left\lfloor \frac{n-1}{2} \right\rfloor$ and $|E(G)| \le \max\{|E(C_{\delta(G),n})|, |E(C_{\lfloor \frac{n-1}{2} \rfloor,n})|\}$.

***Proof:*** Let $G$ be any $n$-vertex non-hamiltonian graph. With Theorem 1, $\delta(G) \le \left\lfloor \frac{n-1}{2} \right\rfloor$. And we know that $G$ is degree-majorised by $C_{m,n}$ for some integer $m$. Since $\delta(C_{m,n}) = m$, $\delta(G) \le m \le \left\lfloor \frac{n-1}{2} \right\rfloor$. Therefore, $|E(G)| \le \max\{|E(C_{m,n})| \mid \delta(G) \le m \le \left\lfloor \frac{n-1}{2} \right\rfloor\}$. Since $|E(C_{m,n})| = \frac{1}{2}(m^2 + (n - 2m)(n - m - 1) + m(n - 1))$ is a quadratic function with respect to $m$ and the the maximum value of it occurs at the boundary $m = \delta(G)$ or $m = \left\lfloor \frac{n-1}{2} \right\rfloor$, $|E(G)| \le \max\{|E(C_{\delta(G),n})|, |E(C_{\lfloor \frac{n-1}{2} \rfloor,n})|\}$. ❑

By Corollary 1, we have the following corollary.

**Corollary 2** Assume that $G$ is an $n$-vertex graph with $n \geq 5$. Then $G$ is hamiltonian if $|E(G)| \geq \max\{|E(C_{\delta(G),n})|, |E(C_{\lfloor \frac{n-1}{2} \rfloor,n})|\} + 1$.

**Lemma 1** Assume that $n$ and $k$ are integers with $n \geq 5$ and $1 \leq k \leq \left\lfloor \frac{n-1}{2} \right\rfloor$. Then $|E(C_{k,n})| \geq |E(C_{\lfloor \frac{n-1}{2} \rfloor,n})|$ if and only if $1 \leq k \leq \left\lfloor \frac{n+1+3\times((n+1)\bmod 2)}{6} \right\rfloor$ or $k = \left\lfloor \frac{n-1}{2} \right\rfloor$.

***Proof:*** We first prove the case that $n$ is even. We claim that $|E(C_{k,n})| \geq |E(C_{\frac{n}{2}-1,n})|$ if and only if $1 \leq k \leq \left\lfloor \frac{n+4}{6} \right\rfloor$ or $k = \frac{n}{2} - 1$. Suppose that $|E(C_{k,n})| < |E(C_{\frac{n}{2}-1,n})|$. Then $|E(C_{k,n})| = \frac{1}{2}(k^2 + (n-2k)(n-k-1) + k(n-1)) < |E(C_{\frac{n}{2}-1,n})| = \frac{1}{2}((\frac{n}{2}-1)^2 + (n-2(\frac{n}{2}-1))(n-(\frac{n}{2}-1)-1) + (\frac{n}{2}-1)(n-1))$. This implies $3k^2 + (1-2n)k + (\frac{1}{4}n^2 + \frac{1}{2}n - 2) < 0$, which means $(k - \frac{n}{2} + 1)(3k - \frac{n}{2} - 2) < 0$. Thus, $|E(C_{k,n})| < |E(C_{\frac{n}{2}-1,n})|$ if and only if $\frac{n+4}{6} < k < \frac{n}{2} - 1$. Note that $n$ and $k$ are integers with $n$ being even, $n \geq 6$, and $1 \leq k \leq \frac{n}{2} - 1$. Thus, $|E(C_{k,n})| \geq |E(C_{\frac{n}{2}-1,n})|$ if and only if $1 \leq k \leq \left\lfloor \frac{n+4}{6} \right\rfloor$ or $k = \frac{n}{2} - 1$.

For odd integer $n$, using the same method, we can prove that $|E(C_{k,n})| < |E(C_{\frac{n}{2}-1,n})|$ if and only if $\frac{n+1}{6} < k < \frac{n-1}{2}$. Given that $n \geq 5$, and $1 \leq k < \frac{n-1}{2}$, then $|E(C_{k,n})| \geq |E(C_{\frac{n-1}{2},n})|$ if and only if $1 \leq k \leq \left\lfloor \frac{n+1}{6} \right\rfloor$ or $k = \frac{n-1}{2}$. Therefore, the result follows. ❑

## 4. PROOF OF THEOREM A

By brute force, we can check that $h(3, 2) = 3$, $h(4, 2) = 4$, and $h(4, 3) = 6$. Therefore, the theorem holds for $n = 3, 4$. Next, we consider the cases that $1 \leq \delta \leq \left\lfloor \frac{n-1}{2} \right\rfloor$ and $n \geq 5$.

Suppose that $1 \leq \delta \leq \left\lfloor \frac{n+1+3\times((n+1)\bmod 2)}{6} \right\rfloor$. By Lemma 1, $|E(C_{\delta,n})| \geq |E(C_{\lfloor \frac{n-1}{2} \rfloor,n})|$. Let $G$ be any $n$-vertex graph with $\delta(G) \geq \delta$ and $|E(G)| \geq |E(C_{\delta,n})| + 1 = C(n-\delta, 2) + \delta^2 + 1$. By Corollary 2, $G$ is hamiltonian. Therefore, $h(n, \delta) \leq C(n-\delta, 2) + \delta^2 + 1$. Since $\delta < \frac{n}{2}$, $C_{\delta,n}$ is not hamiltonian. Thus, $h(n, \delta) > |E(C_{\delta,n})| = C(n-\delta, 2) + \delta^2$. Hence, $h(n, \delta) = C(n-\delta, 2) + \delta^2 + 1$.

Suppose that $\left\lfloor \frac{n+1+3\times((n+1)\bmod 2)}{6} \right\rfloor < \delta \leq \left\lfloor \frac{n-1}{2} \right\rfloor$. By Lemma 1, $|E(C_{\delta,n})| \leq |E(C_{\lfloor \frac{n-1}{2} \rfloor,n})|$. Let $G$ be any $n$-vertex graph with $\delta(G) \geq \delta$ and $|E(G)| \geq |E(C_{\frac{n}{2}-1,n})| + 1 = C(n - \frac{n}{2} - 1, 2) + (\frac{n}{2} - 1)^2 + 1$. By Corollary 2, $G$ is hamiltonian. Therefore, $h(n, \delta) \leq C(n - \frac{n}{2} - 1, 2) + (\frac{n}{2} - 1)^2 + 1$. And we know that $C_{\frac{n}{2}-1,n}$ is not hamiltonian. Thus, $h(n, \delta) > |E(C_{\frac{n}{2}-1,n})| = C(n - \frac{n}{2} - 1, 2) + (\frac{n}{2} - 1)^2$. Hence, $h(n, \delta) = C(n - \frac{n}{2} - 1, 2) + (\frac{n}{2} - 1)^2 + 1$.

Finally, we consider the case that $\delta > \left\lfloor \frac{n-1}{2} \right\rfloor$ and $n \geq 5$. Let $G$ be any graph with $\delta(G) \geq \delta > \left\lfloor \frac{n-1}{2} \right\rfloor$. By Theorem 1, $G$ is hamiltonian. Obviously, $|E(G)| \geq \left\lceil \frac{n\delta}{2} \right\rceil$. Thus, $h(n, \delta) = \left\lceil \frac{n\delta}{2} \right\rceil$.

This completes the proof of our main result Theorem A.

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**Tung-Yang Ho (何東洋)** received the B.S. and M.S. degrees in Applied Mathematics in 1984 and 1989, respectively, and the Ph.D. degree in Computer Science in 1995, all from National Chiao Tung University, Taiwan, R.O.C. Currently, he is a Professor in the Department of Information Management, Ta Hwa Institute of Technology, Hsinchu, Taiwan, R.O.C. His research interests are applications of graph theory in network architectures and graph algorithms.

**Cheng-Kuan Lin (林政寬)** received his M.S. degree in Mathematics from National Central University, Taiwan, R.O.C. in 2002. His research interests include interconnection networks and graph theory.

**Jimmy J. M. Tan (譚建民)** received the B.S. and M.S. degrees in Mathematics from the National Taiwan University in 1970 and 1973, respectively, and the Ph.D. degree from Carleton University, Ottawa, in 1981. He has been on the faculty of the Department of Computer Science, National Chiao Tung University, since 1983. His research interests include design and analysis of algorithms, combinatorial optimization, and interconnection networks.

**D. Frank Hsu** is the Clavius Distinguished Professor of Science and a Professor of Computer and Information Science at Fordham University in New York. He has been visiting professor/scholar at Keio University (Tokyo, Japan), JAIST (Kanazawa, Japan), Taiwan University, TsingHua University (Hsinchu, Taiwan), University of Paris-Sud and CNRS, MIT and Boston University. Dr. Hsu's research interests are combinatorics, algorithms, optimization, network interconnection and communication, machine learning, data mining and information fusion, applied informatics in biomedicine, health, finance, business, neuroscience, and cybersecurity. Dr. Hsu has served on several editorial boards and currently is an associate editor of Pattern Recognition Letter and Special Issue Editor for the Journal of Interconnection Networks (JOIN). He was Conference Co-Chair for the International Conference on Cyber Security (ICCS 2009) held in New York City, January 6-8, 2009, and for International Symposium on Pervasive Systems, Algorithms and Networks (I-SPAN 2009) held in Kaohsiung, Taiwan December 14-16, 2009. Dr. Hsu is a senior member of IEEE, a Foundation Fellow of the Institute of Combinatorics and Applications (ICA), a Fellow of the New York Academy of Sciences (NYAS) and the International Society of Intelligent Biological Medicine (ISIBM).

**Lih-Hsing Hsu (徐力行)** received his B.S. degree in Mathematics from Chung Yuan Christian University, Taiwan, R.O.C., in 1975, and his Ph.D. degree in Mathematics from the State University of New York at Stony Brook in 1981. From 1981 to 1985, he was Associate Professor at Department of Applied Mathematics at National Chiao Tung University in Taiwan. In 1985, he was a Professor in National Chiao Tung University. After 1988, he joined with Department of Computer and Information Science of National Chiao Tung University. In 2004, he is retied from National Chiao Tung University by holding a title as honorary scholar of National Chiao Tung University. He is currently a Professor in the Department of Computer Science and Information Engineering, Providence University, Taichung, Taiwan, R.O.C. His research interests include interconnection networks, algorithms, graph theory, and VLSI layout.