Complex refractive index measurements based on Fresnel's equations and the uses of a lock-in amplifier

Der-Chin Su*[a], Ming-Horng Chiu*[b], and Ju-Yi Lee[a]

[a]Institute of Electro-Optical Engineering, National Chiao Tung University
1001, Ta-Hsueh Road, Hsin-Chu, 300, Taiwan, R.O.C.
[b]Department of Electric Engineering, Chien Hsin College of Technology & Commerce
229, Chien Hsin Road, Chung-Li, 320, Taiwan, R.O.C.

ABSTRACT

A new method for measuring complex refractive index is presented based on Fresnel's equations and the uses of a lock-in amplifier. A lock-in amplifier is introduced into a common path interferometer to measure the product of the amplitude reflection coefficients of s and p polarizations and their corresponding phase difference of a light beam reflected from a medium with complex refractive index. Then, these data are substituted into the special equations derived from Fresnel's equations, the complex refractive index can be obtained by numerical calculations. The resolution of this method is better than 0.01. It has both merits of a conventional common-path interferometer and a heterodyne interferometer. And its validity is demonstrated.

Keywords: complex refractive index, heterodyne interferometry,

1. INTRODUCTION

Complex refractive index $N(n,k)$ is an important characteristic constant of thin film materials, where n is the refractive index and k is the extinction coefficient. There are several methods for measuring the complex refractive index of a material, e.g., $R - vs - \theta$ (reflectance versus incidence angle) methods[1-6] and ellipsometry[7-8]. In those methods, the reflectances of s- and p- polarizations under several different incident angles or polarization conditions must be measured. So, the measurement processes become tedions.

In this paper, a new method for measuring complex refractive index is presented based on Fresnel's equations[9] and the uses of a lock-in amplifier. Firstly, special equations are derived based on Fresnel's equations. From these equations, it is obvious that the product of amplitude reflection coefficients of s and p polarizations and their corresponding phase difference of a light reflected from a medium with complex refractive index are the function of complex refractive index and the incident angle. Next, a common-path heterodyne interferometer is designed for measuring their relations. The light

Further author information-

* D.C.S. (correspondence): Email: t7503@cc.nctu.edu.tw; Telephone: 886-3-5731951; Fax: 886-3-5716631

* M.H.C.: Email: mhchiu@ccsun1.chs.edu.tw; Telephone: 886-3-4581196 ext. 821; Fax: 886-3-4588924

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source includes a He-Ne laser and an electro-optic modulator driven by a signal generator. And a lock-in amplifier is used to measure the product of the amplitude reflection coefficients and the corresponding phase difference simultaneously. Then, these data are substituted into the special equations, the complex refractive index can be obtained by numerical calculations. The resolution of this method is better than 0.01. It has both merits of a conventional common-path interferometer and a heterodyne interferometer. And its validity is demonstrated.

2. PRINCIPLE

2.1 Amplitude reflection coefficients for an absorption material

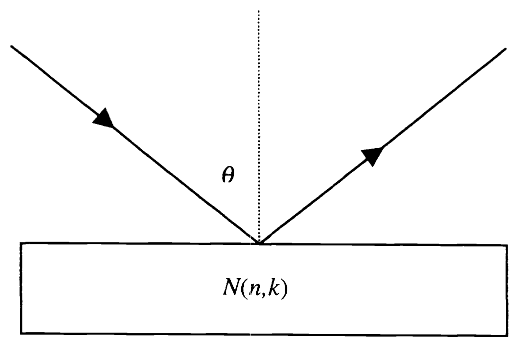


Fig.1 Reflection at the surface of an absoption material.

A ray of light in air is incident at θ on an absorption material with a complex refractive index $N(n,k)$, as shown in Fig.1. According to Fresnel's equations[9], the amplitude reflection coefficients of s and p polarizations can be expressed as

$$r_s = \frac{\cos\theta - (u+iv)}{\cos\theta + (u+iv)} = |r_s|\exp(i\delta_s), \tag{1}$$

$$r_p = \frac{N^2\cos\theta - (u+iv)}{N^2\cos\theta + (u+iv)} = |r_p|\exp(i\delta_p), \tag{2}$$

respectively, where

$$u^2 = \frac{1}{2}\left\{\left(n^2 - k^2 - \sin^2\theta\right) + \left[\left(n^2 - k^2 - \sin^2\theta\right)^2 + 4n^2k^2\right]^{1/2}\right\}, \tag{3}$$

$$v^2 = \frac{1}{2}\left\{-\left(n^2 - k^2 - \sin^2\theta\right) + \left[\left(n^2 - k^2 - \sin^2\theta\right)^2 + 4n^2k^2\right]^{\frac{1}{2}}\right\}, \tag{4}$$

δ_s and δ_p are the phase shifts of s and p polarizations, and can be expressed as

$$\delta_s = \tan^{-1}\left(\frac{2v\cos\theta}{u^2 + v^2 - \cos^2\theta}\right), \tag{5}$$

$$\delta_p = \tan^{-1}\left[\frac{2v\cos\theta\left[n^2 - k^2 - 2u^2\right]}{u^2 + v^2 - \left(n^2 + k^2\right)^2\cos^2\theta}\right], \tag{6}$$

respectively. Hence, the phase difference of s polarization relative to p polarization is

$$\phi = \delta_s - \delta_p = \tan^{-1}\left(\frac{ad - bc}{ac + bd}\right), \tag{7}$$

where

$$\left.\begin{aligned} a &= 2v\cos\theta, \\ b &= u^2 + v^2 - \cos^2\theta, \\ c &= 2v\cos\theta\left(n^2 - k^2 - 2u^2\right), \\ d &= u^2 + v^2 - \left(n^2 + k^2\right)^2\cos^2\theta. \end{aligned}\right\} \tag{8}$$

From Eqs.(1) ~ (8), it is obvious that the product $|r_s||r_p|$ and phase difference ϕ are the functions of n, k, and θ. If $|r_s||r_p|$ and ϕ can be experimentally measured for a given θ, then the complex refractive index of the medium can be estimated by solving the following set of simultaneous equations

$$V = V(n, k, \theta), \tag{9}$$

$$\phi = \phi(n, k, \theta), \tag{10}$$

where $V = |r_s||r_p|$.

2.2 Product of amplitude reflection coefficients and phase difference measurements with heterodyne interferometry

Chiu *et al.* [10] had proposed a method for measuring the refractive index of a transparent material by using the total internal reflection heterodyne interferometry (TIRHI). The schematic diagram of the optical arrangement of our method, which is based on similar considerations, is designed and shown in Fig.2(a). The linearly polarized light passing through an electro-optic modulator EO is incident on a beam splitter BS and divided into two parts: the reference beam and the test beam. The reference beam passes through an analyzer AN_r, and enters the photodetector D_r. If the amplitude of light detected by D_r is E_r, then the intensity measured by D_r is $I_r = |E_r|^2$. Here I_r is the reference signal. On the other hand, the test beam is incident at θ on a test material. Finally, the reflected light passes through an analyzer AN_t, and is detected by another photodetector D_t. If the amplitude of the test beam detected by D_t is E_t, then D_t measures the intensity $I_t = |E_t|^2$. And I_t is the test signal.

For convenience the +z axis is in the propagation direction and the y axis is in the vertical direction. Let the incident light be linearly polarized at 45° with respect to the x axis, the fast axis of EO under an applied electric field be in the horizontal direction, and both the transmission axes of AN_r and AN_t be at 45° with respect to the x axis. If a sawtooth signal of a frequency f and an amplitude $V_{\lambda/2}$, the half-wave voltage of the EO, is applied to the electro-optic modulator, then by using the Jones calculus we can get[11,12]

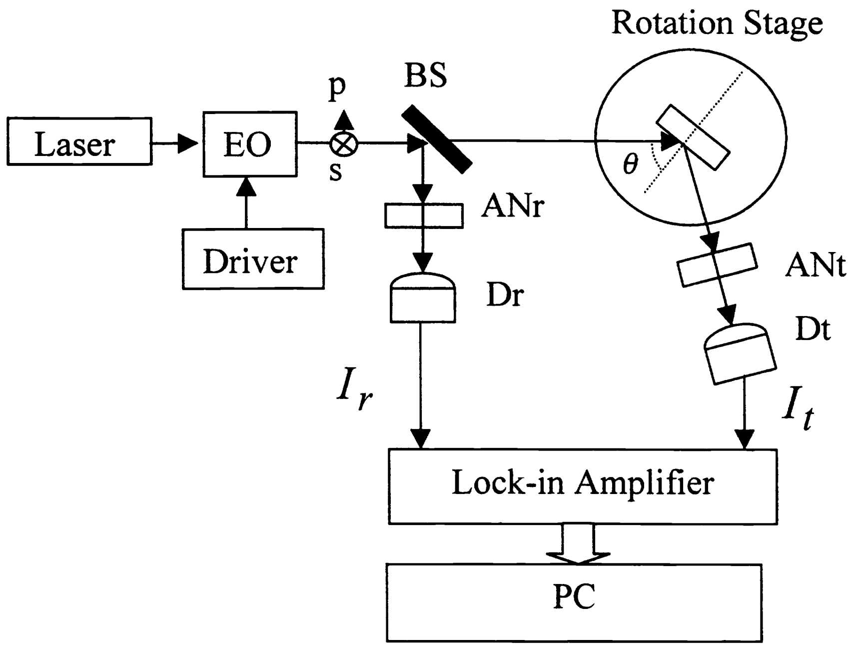
$$I_r = \frac{1}{2}\left[1 + \cos(2\pi ft - \phi_{BS})\right], \tag{11}$$

$$I_t = \frac{1}{2}\left[\frac{|r_s|^2}{2} + \frac{|r_p|^2}{2} + |r_s||r_p|\cos(2\pi f - \phi)\right], \tag{12}$$

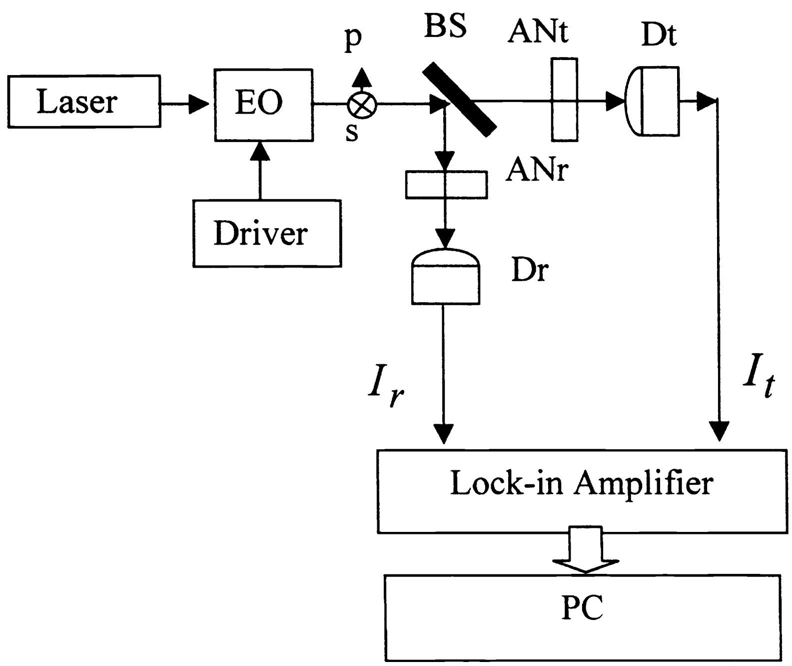
where ϕ_{BS} is the phase difference between s and p polarizations due to the reflection at BS. These two sinusoidal signals are sent to a lock-in amplifier, the product of $|r_s||r_p|$ and their phase difference

$$\phi' = \phi - \phi_{BS}, \tag{13}$$

can be obtained. Next, let the test beam enter the photodetector D_t directly without the reflection from the test material, as shown in Fig.2(b). The test signal still has the form of Eq.(12), but this time with $\phi = 0$. Therefore the phase meter in Fig.2(b) represents $-\phi_{BS}$. Substituting $-\phi_{BS}$ into Eq.(13), we can obtain the phase difference ϕ.



(a)



(b)

Fig.2 Schematic diagram for measuring the phase differences owing to the reflection at (a) an absoption material, and (b) BS.EO: electro-optic modulator; BS: beam-splitter;AN: analyzer; D: photodetector; I_r: reference signal; I_t: test signal; PC: personal computer.

3. EXPERIMENTS AND RESULTS

To show the feasibility of this technique, the complex refractive index of nickel (Ni) was measured. A He-Ne laser with 632.8nm wavelength and an electro-optic modulator (Model PC200/2, manufactured by England Electro-Optics Developments Ltd.) with a half-wave voltage 170V were used in this test. The frequency of the sawtooth signal which was applied to the electro-optic modulator was 800 Hz. A high precision rotation stage ($PS-\theta-90$) with angular resolution of 0.005° manufactured by Japan Chuo Precision Industrial Company Ltd. was used to mount and rotate the test material. And a high-resolution lock-in amplifier was used to measure the product of $|r_s||r_p|$ and the phase difference ϕ. In addition, a personal computer was used to record and analyze the data. The experimental conditions and results are summarized in Table 1. The average experimental value of the complex refractive indices of Ni is $N(Ni)=1.98+i3.76$.

4. DISCUSSIONS

From Eqs.(8) and (9), we can get

$$\Delta n \cong \frac{\left|\frac{\partial \phi}{\partial k}\right||\Delta V|+\left|\frac{\partial V}{\partial k}\right||\Delta \phi|}{\left|\frac{\partial V}{\partial n}\frac{\partial \phi}{\partial k}-\frac{\partial \phi}{\partial n}\frac{\partial V}{\partial k}\right|}, \tag{14}$$

and

$$\Delta k \cong \frac{\left|\frac{\partial \phi}{\partial n}\right||\Delta V|+\left|\frac{\partial V}{\partial n}\right||\Delta \phi|}{\left|\frac{\partial V}{\partial n}\frac{\partial \phi}{\partial k}-\frac{\partial \phi}{\partial n}\frac{\partial V}{\partial k}\right|}, \tag{15}$$

where Δn and Δk are the errors in n and k, and ΔV and $\Delta \phi$ are the errors of the product of $|r_s||r_p|$ and the phase difference ϕ, respectively. The errors of phase differences include the angular resolution[14] of a lock-in amplifier, the second harmonic error[15], and the polarization-mixing error[16,17]. In our experments, ΔV and $|\Delta\phi|$ are nearly equal to 0.001 and 0.03° , respectively. Consequently, the curves of measurement resolution versus incident angle θ for Δn and Δk can be obtained by substituting these data into Eqs.(14) and (15). The results are shown in Fig. 3. Obviously, the best resolution can be obtained as θ is in the neighborhood near 70° . And Δn and Δk corresponding to our experimental conditions are calculated and added into Tab.1.

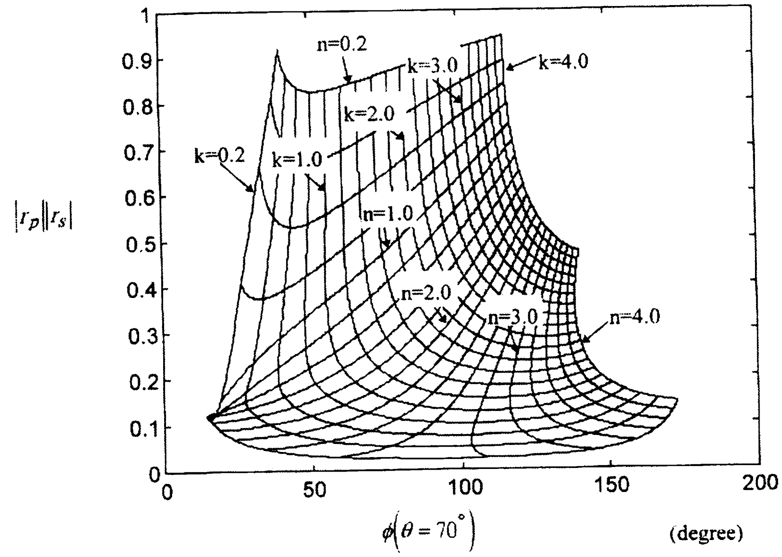


Fig. 3 Curves of measurememt resolution versus θ for Δn and Δk of nickle.

θ	60°	70°	80°
$\|r_s\|\|r_p\|$	0.609	0.571	0.585
ϕ	144.75°	121.01°	73.80°
n	1.9807	1.986	1.993
k	3.766	3.765	3.760
Δn	6.8×10^{-3}	6.4×10^{-3}	7.1×10^{-3}
Δk	7.1×10^{-3}	5.6×10^{-3}	4.0×10^{-3}

reference values[13] : $N(Ni)$=1.98+i3.74

Table 1 : Experimental conditions and measurement results.

To investigate the effects of experimental conditions on the measurements, the relation curves of $|r_s||r_p|$ versus ϕ for $\theta = 70^\circ$ are shown in Fig. 4. In the figure, the values of n and k are between 0.2 and 4.0 in 0.2 steps. According to Humphreys-Owen[4], the sensitivity to the experimental conditions is indicated by the spacing between contours — if the spacing is large, the sensitivity is good and vice versa. It is obvious that our experimental conditions are useful if n and k are small. For good sensitivity, it is better to choose optimum incidence angles with the method proposed by Logofatu *et al.* [18]. In additions, curves of cinstant n and k as functions of Δn and Δk are shown in Fig. 5. We obtained them by substituting the experimental conditions $\theta = 70^\circ$, ΔV =0.001, and $|\Delta\phi|$=0.03° into Eqs. (14) and (15). In this figure, the values of *n* and *k* are 0.5 to 4 in steps of 0.5. It can seen that both Δn and Δk are smaller than 1×10^{-2}.

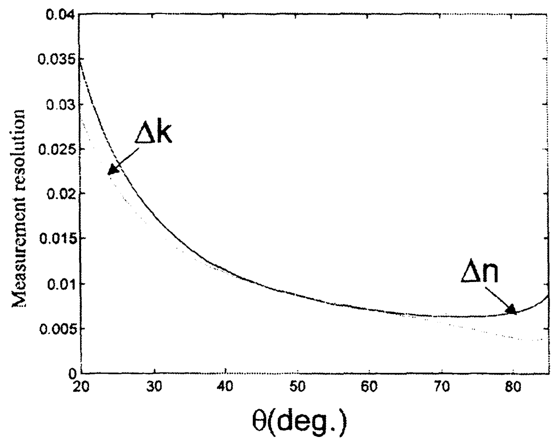


Fig. 4 Curves of constant *n* and *k* as a function of $|r_s||r_p|$ and ϕ at $\theta = 70^\circ$, where the values of *n* and *k* are between 0.2 and 4 in 0.2 steps.

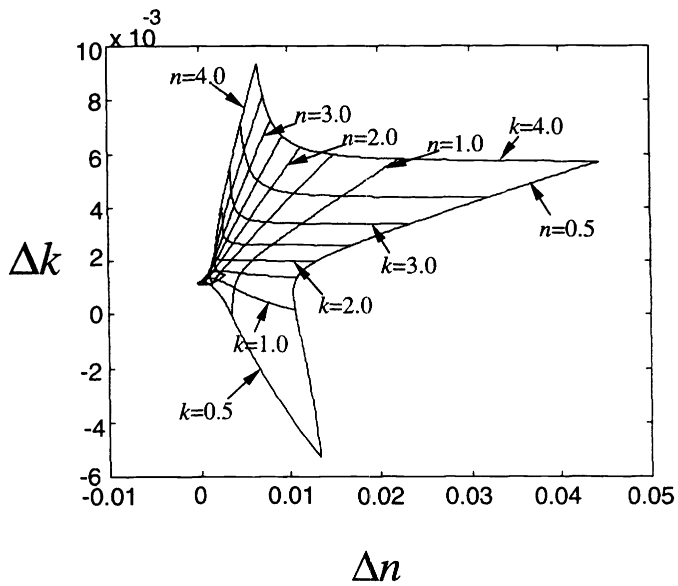


Fig.5 Curves of constant *n* and *k* as a function of Δn and Δk at $\theta_1 = 60^\circ$ and $\theta_2 = 80^\circ$, where the values of *n* and *k* are between 0.5 and 4 in 0.5 steps.

5. CONCLUSION

Based on Fresnel's equations and the uses of a lock-in amplifier, a new method for measuring the complex refractive index is proposed. The product of the amplitude reflection coefficients of s and p polarizations and their phase difference of a light beam reflected from a complex refractive index are measured with a lock-in amplifier in a common-path interferometer. Then these data are substituted into the special equations derived from Fresnel's equations, and the complex refractive index of the material can be estimated. Its resolution is better than 0.01. It has both merits of a conventional common-path interferometer and a heterodyne interferometer, such as, simple optical setup, easy operation, high stability, high measurement accuracy, and rapid measurement. Its validity has been demonstrated.

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