Resonator configuration dependent nonlinear dynamics with application to Kerr-lens mode-locked lasers

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Abstract

The nonlinear dynamics of the Gaussian beam propagation in resonator was studied by constructing the iterative map of q-parameter. From the Greene's residue theorem, there are specific configurations sensitive to nonlinear effect existing in the geometrically stable region. By applying to Kerr-lens mode-locking (KLM) resonators, we found that multiple solution, period doubling, period tripling and period quadrupling can occur at the configurations with product of cavity G-parameters equal to 0, 1/2, 1/4 (or 3/4) and $(2\pm\sqrt{2})/4$, respectively. Moreover, the systems will result in classical chaos if further increasing the nonlinear effect. We will also report appearance of unexpected transverse beam profiles and shrinkage of beam waist causes to lower laser threshold in an end-pumped Nd:YVO_4 laser and chaotic behavior in the KLM laser around these critical resonator configurations.

Summary

For a Gaussian beam, the relation of q-parameter to radius of curvature, R, and spot size, w, is well known as $1/q = 1/R - i\lambda/(\pi w^2)$, where λ is the wavelength of cavity beam. Owing to the inherent clock provided by the round-trip time of the resonator, we can obtain the iterative map of beam parameter from the relationship between before and after a round-trip. The method borrowed from the nonlinear dynamics had used to discuss the dynamics of resonator beam in many studies.[1-3]

Consider a resonator that the elements of all transfer matrices are real, the map is a conservative one.[3] Analyzing the stability of this conservative map by using the Greene's residue theorem,[4] the residue at the period-1 fixed point in linear system is[2]

$$\mathrm{Re}\,s = 1 - \left(2G_1G_2 - 1\right)^2. \tag{1}$$

Here we had defined G_1=a-b/ρ_1 and G_2=d-b/ρ_2 as the G-parameter for general optical resonators, where $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is the transfer matrix of one-way pass between the two end mirrors with ρ_1 and ρ_2 being their

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radii of curvature. When 0<Res<1, the tangent space orbits rotate about the fixed point on ellipses and the system is stable. From Eq.(1), one can obtain the stable condition with $0<G_1G_2<1$ which is identical to the geometrically stable one. Whereas the system is unstable with either Res>1 or Res<0.

If the nonlinear effect is considered, complicated phenomena will be found in some specific residues. The pitchfork or saddle-node bifurcation occurs at the critical condition Res=0 and period doubling bifurcation takes place at Res=1.[5] Even within the geometrically stable region, the configurations with Res=3/4 and sometimes 1/2 become unstable. The special cases with Res=0, 1, 3/4 and 1/2 correspond to the low order resonances $\chi^p=1$ with p=1, 2, 3 and 4, respectively. Here χ is the eigenvalue of multipliers.

Verifying the previous discussion, we numerically study the nonlinear dynamics of resonator configuration dependence in Kerr-lens mode-locked (KLM) lasers. Besides, the multiple solution is found at the confocal configuration $G_1G_2=1$ with Res=0, the pitchfork bifurcation exists in an equal arm system and saddle-node bifurcation takes place in an unequal arm system results from the symmetry breaking. Period doubling bifurcation is found at $G_1G_2=1/2$ with Res=1 agrees with the previous predict. Moreover, period tripling and quadrupling can occur at the configurations $G_1G_2= 1/4$ (or 3/4) with Res=3/4 and $(2\pm\sqrt{2})/4$ with Res=1/2, respectively. By increasing the pumping power, the classical chaos is easier to happen.

Multiple solution, period doubling, period tripling and period quadrupling are found in KLM resonator at resonator configurations $G_1G_2=0$, 1/2, 1/4 (or 3/4) and $(2\pm\sqrt{2})/4$, respectively. These configuration dependent phenomena may aid researchers in the resonator design for studying the nonlinear effect of Gaussian beam propagation in general. We will also report some recent experimental results on appearance of unexpected transverse beam profiles and shrinkage of beam waist causes to lower laser threshold in an end-pumped $Nd:YVO_4$ laser and chaotic behavior in the KLM laser around these critical resonator configurations.

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